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On Inferences in Acceptance Sampling for Inverse Rayleigh Distributed Life Time

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Abstract: The development of advanced computer technology makes easier to construct an effective acceptance sampling plan for reliability inspection satisfying both the producer's and consumer's quality and risk requirements. This paper construct the reliability inspection plans for inverse Rayleigh distributed lifetime using the ratio of index average lifetime and testing time for two values of average lifetime-acceptable and non-acceptable ones. A relationship between index and reliability function is also obtained. An example is provided to illustrate its use.

Keywords: Acceptance sampling, Quality and risk, Inverse Rayleigh distribution, Sampling Plan, Average lifetime, Testing time, Reliability.

1 Introduction

Quality plays a fundamental role in the scope of industries and business. It is a core theme in business strategies in order to become more and more competitive. That's why; the developing and competitive world now has great importance of statistical quality control techniques. As such the inspection of the various manufacture products is considered essential for ensuring the trustworthiness of an item with regards to its lifetime. In this situation, it becomes necessary to ascertain the visible operating characteristic values of the proposed plan. Acceptance sampling plan plays an important role to ensure that the lifetime of the product is according to the specified/desired level/standard of the consumer or not. If the life test follows that the average life of the product is above the specified/desired level/standard, the submitted lot is accepted otherwise the same is rejected.

Quality has become a cutting-edge factor in consumers' choices of products and services. Consequently, Statistical Process Control (SPC) has been amply used in order to achieve improved quality in products, processes and services. No matter how good is a design, how performing is the production process, how careful is handled and exploited a technical system there is no way to stop its final decay. After a certain period of time- every human made object sooner or later will fail. This item failure is due to natural causes or due to some spurious ones i.e. use of the item in inappropriate conditions (environment, lack of maintenance actions, mishandling, intensive operational tasks etc.). If the item failure occurs after a certain period of time i.e. the system was operating satisfactorily. Since we can't explain the exact time of a specified object that it will fail, we are in position to explain it in terms of probabilities and expected time as its main parameters. Now, the failure behaviour of that specific object has to be modelled and hence choose the most suitable class of life distributions describing this time-to-failure phenomenon.

The document MILSTD 781 Reliability test: exponential distribution used the ratio where $E(T)$ is the average lifetime or durability of underlying objects and is the testing time in the exponential case [1,2]. Later, Isaic Maniu- Voda [3] obtained the some inferences for Weibull distribution and explained its application/utility in case of Rayleigh distribution. But due to monotone increasing failure rate, Rayleigh distribution has an inherent defect, which makes it

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unsuitable for many applications/situations. An alternative to this distribution is an inverse Rayleigh distribution which has an increasing or decreasing failure rate depending upon the X$>1.069543/\theta$ or X$<1.069543/\theta$. First, the distribution was introduced by [4,5] and explored the distribution of lifetimes of several types of experimental units that can be approximated by the inverse Rayleigh distribution. Paper [6,7] studied some properties of the inverse Rayleigh distribution. Khan and Islam [8] obtained the strength reliability for inverse Rayleigh distributed stress. The inverse Rayleigh distribution in acceptance sampling has been studied by [9,10] and obtained single sampling and economic acceptance sampling plan. Aslam and Jun [11] studied the group acceptance sampling plan following inverse Rayleigh lifetime. The present study deals with some new results on the index average lifetime/testing time in the construction of acceptance sampling plans for reliability inspection, when time-to-failure distribution is following inverse Rayleigh distribution.

2 Background and Assumptions for Reliability Inspection

Acceptance Sampling is used to make decisions on accepting or rejecting a lot (or batch) of product. For this purpose, a sample is taken from the lot, and some quality characteristic of the units in the sample is inspected. On this inspection report, we decide whether or not the lot is likely to be acceptable, not to estimate the quality of the lot. There are several Acceptance Sampling methods for attributes and variables. The attribute sampling is a simple statistical method that utilizes representative samples to analyze traits of a large body of data and decides based on the number of defectives in a lot. Variables sampling is designed to predict the value of a given variable and to decide based on measurement values. Thus, statistically valid sampling plan tells us the probability of accepting bad lots and the probability of rejecting good lots in the manufacturing system.

As in the procedure of batch inspection, the characteristic of interest is reliability or durability of underlying items. So, we must take into account their failure behaviour which is concern with time, the important characteristic under economical condition, in order to construct suitable sampling acceptance plans. The attributive method never matters the nature of the investigated quality characteristics of batch inspection. The nature of attributive method lies in the fact that products are classified into categories: conforming and nonconforming (defective) ones for some specified criteria. In the case of reliability/durability inspection, this attributive approach ignores the very nature of failure behaviour of inspected objects and this could lead to a larger sample to be tested. If the items are quite expensive and since the specific test in this case is destructive, the procedure appears to be non-economic. For the reliability or durability case; the attributive approach not takes care of the following elements:

1. assumption about distribution for time-to-failure;
2. types of samples for inspection: complete or censored ones;
3. about with or without replacement sampling;
4. accelerated testing or normal testing conditions;
5. the relationship between testing time and the actual operating life of the items;
6. the items are repairable or non-repairable;

If they are non- restoring, then is just the mean durability and , the sample mean is computed with , the first and last failure values of the ith item on the test; it is worthless to describe about Mean Time Between Failures. In this special case, most useful methods are based on average operating time or on hazard rate associated to the failure time model specific for each attributive instance.

3 The Inverse Rayleigh distribution

The inverse Rayleigh distribution is an important lifetime distribution in survival analysis that has many applications in the area of reliability studies such as infant mortality, useful life and wear-out periods. Reliability and failure data both from life testing and in service records are often modeled by the life time distributions such as the inverse Rayleigh distributions.

Let T be the distribution function of inverse Rayleigh distribution

$$f(t) = \frac{2}{\theta t^3} \exp\left[-\left(\frac{1}{\theta t^2}\right)\right],\ \ t > 0,\ \theta > 0.$$

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and cumulative density function is $F(t) = e^{-(1/(\theta t^2))}$; $t > 0$, $\theta > 0$.

The corresponding reliability function is $R(t) = 1 - e^{-(1/(\theta t^2))}$ and mean value is $E(T) = \sqrt{\left(\frac{\pi}{\theta}\right)}$.

Hence, we have to consider it as

$\theta = \left[\frac{\sqrt{(\pi)}}{E(T)}\right]^2$ and consequently we get $R(t) = 1 - exp\left[-\left(\frac{E(T)}{t\sqrt{(\pi)}}\right)^2\right]$.

Therefore, for inverse Rayleigh distribution the ratio $E(T)/t$ depends on its reliability function. If we fix $t = T_0$ we have either to estimate $R(T_0)$ or to fix lower acceptable bound for it. Now

$R(T_0) = 1 - exp\left[-\left(\frac{E_0}{T_0\sqrt{(\pi)}}\right)^2\right]$.

4 Design of the Proposed Sampling Plan

The procedure for the construction of a sampling plan with the following assumptions:

1. The items subjected to inspection are non-reparable;
2. The failure time distribution is following inverse Rayleigh life time;
3. We use only one sample with no replacement, its size has to be determined;
4. There is fixed an acceptable average lifetime $[E(T)]_1$ corresponding to a given risk α ;
5. There is fixed a non-acceptable average lifetime $[E(T)]_2$ corresponding to a given risk β;
6. There is a fixed testing time T_0 smaller than the actual operating life of the underling items.

Therefore, the sampling plan will be the system of objects $\{(n, c|T_0)\}$ where n and c are respectively the sample size and acceptance number which has to be determined and T_0 is the previously fixed testing time. The decision on the lot is taken as follows: submit to the specific reliability/durability test a sample of size n drawn randomly from a lot of size $N(n < N)$, during a period of units of T_0; record then the number (d) of failed elements in the interval $[0, T_0]$; if $d \leq c$, then the lot is accepted - otherwise, if $d > c$, then the lot is rejected. The values of n and c are determined via the OC - function (Operative Characteristic) of the plan which has the function

$$L(p) = \sum_{d=0}^{c} \frac{1}{d!}(np)^d e^{-np}$$

where $d! = 0, 1, 2, \ldots, c$ and p is the defective fraction in the lot given by $p = e^{-(1/\theta t^2)}$; $\theta > 0, t \geq 0$. d is the number of failed elements during the testing period T_0 [12].

Let's define two values for p (say, p_1 and p_2) for which $L(p_1) = 1 - \alpha$ and $L(p_2) = 1 - \beta$. Using the ratios $[E(T)]_1/T_0$ and $[E(T)]_2/T_0$, we obtain a system which provides the values of n and c of the specified plan. Table presents some values for n and c for the inverse Rayleigh lifetime. The input data being the following quantities: $100T_0/[E(T)]_1$ for which $L(p_1) = 0.95$ and $100T_0/[E(T)]_2$ $L(p_2) = 0.10$ (the first figure is given in brackets). This approach avoided the knowledge of $R(T_0)$ since the input values are only T_0 and $[E(T)]_{1,2}$ which are fixed previously taking into account the pre-specific case at hand.

Table 1: Values of Single sampling plan $[(n, c)|T_0]$

c	n			
	Values of $100T_0/[E(T)]_1$ **for which** $L(p_2) = 0.10$			
	100	**50**	**25**	**15**
0	3 (12)	10 (6.8)	41 (3)	123 (2)
1	5 (26)	17 (11)	77 (7.3)	224 (4)
2	8 (35)	29 (17)	105 (7.7)	285 (5.5)
3	10 (39)	33 (20)	129 (10)	359 (6.3)

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5 An Illustrative example:

Assume that we have an acceptable durability $[E(T)]_1 = 4000$ hours and a non-acceptable one as $[E(T)]_2 = 800$ hours. Testing time was fixed at the value $T_0 = 400$ hours. The usual consumer risk $\alpha = 0.05$ and producer's risk $\beta = 0.10$.

Therefore, to find the plan, we evaluate

$$\frac{100T_0}{[E(T)]_2} = \frac{100\times 400}{800} = 50 \text{ and } \frac{100T_0}{[E(T)]_1} = \frac{100\times 400}{4000} = 10$$

From the table 1, the nearest value of $100T_0/[E(T)]_1$ for $100T_0/[E(T)]_2 = 50$ is 11 and hence for the couple 50 (11). We choose $n = 17$ sampling units with the acceptance number $c = 1$. Thus, the number of tests made is considerably smaller than the complete sample size. This test saving can be important when the testing of these experiments are costly. The sampling plan is then $(17,1)|400$ and as a consequence we shall test $n = 17$ items on a period of 400 hours and record d - the number of failed items. If $d = 0$ or 1, we shall accept the lot - otherwise we shall reject it.

6 Conclusions

Acceptance Sampling Plans are being widely used to protect against the irregular degradation of quality levels in the submitted lots. A good sampling will also protect the producer/customer in the sense that lots produced at permissible/pre-assign levels of quality and it will have a good chance to be accepted. To determine an economical acceptance sampling plan, it is obvious to fix the time and then estimate its reliability or to fix the lower acceptable criteria. This methodological approach helps us in fixing/deciding the sample number for inspection and for a suitable combination of the parameters as per our requirement we have to select the small sample number for the plan.

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