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Soliton Solutions of the Time Fractional Generalized Hirota-satsuma Coupled KdV System

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Abstract: In this present study, the exact traveling wave solutions to the time fractional generalized Hirota-Satsuma coupled KdV system are studied by using the direct algebraic method. The exact and complex solutions obtained during the present investigation are new, whereas literature survey has revealed generalizations of solutions. The solutions obtained during the present work demonstrate the fact that solutions to the time fractional Generalized Hirota-Satsuma coupled KdV system can exhibit a variety of behaviors. It is also exhibited that the proposed method is more effective and general in nature.

Keywords: Hirota-Satsuma coupled KdV system, direct algebraic method, exact wave solutions.

1 Introduction

As is well known, solitons are universal phenomenon, appearing in a great array of contexts such as, for example, nonlinear optics, plasma physics, fluid dynamics, semiconductors and many other systems [1,2, 3].

Studying of nonlinear evolution equations (NLEEs) modeling various physical phenomena has played a significant role in many scientific applications such as water waves, nonlinear optics, plasma physics and solid state physics [4,5,6,7].

Many powerful methods for finding exact solutions of NLEEs have been proposed, such as ansatz method and topological solitons [8,9,10,11], tanh method [12,13], multiple exp-function method [14], simplest equation method [15,16,17,18], Hirotas direct method [19,20], transformed rational function method [21] and so on.

This work aims to find out the solitary and complex wave solutions to the time fractional Generalized Hirota–Satsuma coupled KdV system [22] with following form:

$$\begin{aligned} D_t^\alpha u &= \tfrac{1}{4}u_{xxx} + 3u_x u + 3(-v^2+w)_x, \\ D_t^\alpha v &= \tfrac{1}{4}v_{xxx} - 3u_x, \quad 0 < \alpha \le 1, \\ D_t^\alpha w &= -\tfrac{1}{2}w_{xxx} - 3uw_x. \end{aligned} \tag{1}$$

The Generalized Hirota-Satsuma coupled KdV system is one of the essential nonlinear equations in mathematics and physics. Therefore, it is important to find solutions for this equation. This equation arises as a special case of the Toda lattice equation, a well- known soliton equation in one space and one time dimension, which can be used to model the interaction of neighboring particles of equal mass in a lattice formation with a crystal. The Generalized Hirota-Satsuma coupled KdV system has many applications in many branches of nonlinear science. One application of the Generalized Hirota-Satsuma coupled KdV system is that it can be used to describe generic properties of string dynamics for strings and multi-strings in constant curvature space. Another application of the sinh-Gordon equation is in the field of thermodynamics, where it can be applied to exactly calculate partition and correlation functions.

The paper is arranged as follows. In Section 2, we describe briefly the Modified Riemann–Liouville derivative with properties and simplest equation method. In Section 3, we apply this method to the time fractional Generalized Hirota-Satsuma coupled KdV system.

2 Modified Riemann-Liouville derivative and direct algebraic method

In this section, we first give some definitions and properties of the modified Riemann–Liouville derivative which are used further in this paper. [23,24,25]

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Assume that $f:\mathbb{R}\to\mathbb{R}, x\to f(x)$ denote a continuous (but not necessarily differentiable) function. The Jumarie modified Riemann–Liouville derivative of order α is defined by the expression

$$D_x^\alpha f(x)=\begin{cases}\frac{1}{\Gamma(-\alpha)}\int_0^x (x-\xi)^{-\alpha-1}[f(\xi)-f(0)]\,d\xi\,, & \alpha\prec 0\\ \frac{d}{dx}\frac{1}{\Gamma(1-\alpha)}\int_0^x (x-\xi)^{-\alpha}[f(\xi)-f(0)]\,d\xi\,, & 0\prec\alpha\prec 1\\ (f^{(n)}(x))^{(\alpha-n)} & n\le\alpha\le n+1, n\ge 1\end{cases} \tag{2}$$

Some properties of the fractional modified Riemann-Liouville derivative were summarized and three useful formulas of them are

$$D_x^\alpha x^y=\frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma-\alpha)}x^{\gamma-\alpha},\gamma\succ 0 \tag{3}$$

$$D_x^\alpha(u(x)v(x))=v(x)D_x^\alpha u(x)+u(x)D_x^\alpha v(x), \tag{4}$$

$$D_x^\alpha[f(u(x))]=f'(u)D_x^\alpha u(x)=D_x^\alpha f(u)(u_x')^\alpha, \tag{5}$$

Which are direct consequences of the equality $d^\alpha x(t)=\Gamma(1+\alpha)dx(t)$.

Next, let us consider the time fractional differential equation with independent variables $x=(x_1,x_2,...,x_m,t)$ and a dependent variable u,

$$F(u\ , D_t^\alpha u, u_{x_1}, u_{x_2}, u_{x_3}... \\ , D_t^{2\alpha}u, u_{x_1x_1}, u_{x_2x_2}, u_{x_3x_3},...)=0. \tag{6}$$

Using the variable transformation

$$u(x_1,x_2,...,x_m,t)=U(\xi),$$
$$\xi=x_1+l_1x_2+...+l_{m-1}x_m+\frac{\lambda t^\alpha}{\Gamma(1+\alpha)}\beta, \tag{7}$$

where k,l_i and λ are constants to be determined later; the fractional differential equation (6) is reduced to a nonlinear ordinary differential equation

$$H=(U(\xi),U'(\xi),U''(\xi),...), \tag{8}$$

Where $"'"=\frac{d}{d\xi}$.

We assume that Eq. (8) has a solution in the form

$$u(\xi)=\sum_{i=0}^{n}a_iF^i(\xi), \tag{9}$$

Where $a_i(i=1,2,..,n)$ are real constants to be determined later. $F(\xi)$ expresses the solution of the auxiliary ordinary differential equation

$$F'(\xi)=b+F^2(\xi), \tag{10}$$

Eq. (10) admits the following solutions:

$$F(\xi)=\begin{cases}-\sqrt{-b}\tanh(\sqrt{-b}\xi), & b\prec 0\\ -\sqrt{-b}\coth(\sqrt{-b}\xi), & b\prec 0\end{cases}$$
$$F(\xi)=\begin{cases}\sqrt{b}\tan(\sqrt{b}\xi), & b\succ 0\\ -\sqrt{b}\cot(\sqrt{b}\xi), & b\succ 0\end{cases} \tag{11}$$
$$F(\xi)=-\tfrac{1}{\xi}, \quad b=0$$

Integer n in (9) can be determined by considering homogeneous balance [8] between the nonlinear terms and the highest derivatives of $u(\xi)$in Eq. (8).

Substituting (9) into (8) with (10), then the left hand side of Eq. (8) is converted into a polynomial in $F(\xi)$, equating each coefficient of the polynomial to zero yields a set of algebraic equations for a_i,k,c. Solving the algebraic equations obtained and substituting the results into (9), then we obtain the exact traveling wave solutions for Eq. (1).

3 Application to the time fractional Generalized Hirota–Satsuma coupled KdV system:

Next, we study Eq. (1). Considering the following complex transformation:

$$\begin{aligned}u(x,t)&=\tfrac{1}{\lambda}U^2(\xi),\\ v(x,t)&=-\lambda+U(\xi),\\ w(x,t)&=2\lambda^2-2\lambda U(\xi),\end{aligned} \tag{12}$$

and $\xi=x-\frac{\lambda t^\alpha}{\Gamma(1+\alpha)}$.

Substituting Eq. (12) into Eq. (1), we can know that Eq. (12) is reduced into ordinary differential equations:

$$\lambda(U_\xi)^2+\lambda UU_{\xi\xi}+3U^4-4\lambda^2U^2+6\lambda^4+2\lambda^2R=0, \tag{13}$$

and

$$\lambda U_{\xi\xi}+2U^3-2\lambda^2U=0, \tag{14}$$

where $R$is an integration constant to be determined later.

Case 1: Balancing $UU_{\xi\xi}$ with U^4 in Eq. (13) gives $n=1$. Therefore, we may choose

$$U=a_1F+a_0, \tag{15}$$

Substituting Eq. (15) along with Eq. (9) in Eq. (13) and setting all the coefficients of powers $F$to be zero, then we obtain a system of nonlinear algebraic equations and by

solving it, we obtain

$$\begin{aligned} a_1 &= \pm\tfrac{\sqrt{2}}{4}\sqrt{4b+3}, \\ a_0 &= \pm\tfrac{\sqrt{2}}{24}\sqrt{9-16b^2}, \\ \lambda &= -\tfrac{4b+3}{8}, \\ R &= -\tfrac{1}{4b+3}\left(\tfrac{3}{8}(4b+3)^2+\tfrac{(4b+3)^2}{1152}(9-16b^2)-\right. \\ &\left.\tfrac{1}{6912}(9-16b^2)^2+\tfrac{4b+3}{16}(4b+3)b^2\right) \end{aligned} \tag{16}$$

From (11),(15) and (16), we obtain the solitary wave solutions of (13) as follows

$$\begin{aligned} U_1 &= \pm\frac{\sqrt{2}}{4}\sqrt{4b+3}[\sqrt{-b}\tanh\sqrt{-b}(x+\frac{4b+3t^\alpha}{8\Gamma(1+\alpha)})] \\ &\pm\frac{\sqrt{2}}{24}\sqrt{9-16b^2}, \end{aligned} \tag{17}$$

Where $b \prec 0$and k is an arbitrary real constant. From (12) we have

$$\begin{aligned} u_1(x,t) &= -\frac{8}{4b+3}(-\frac{b}{8}(4b+3) \\ &\times \tanh^2\sqrt{-b}(x+\frac{(4b+3)t^\alpha}{8\Gamma(1+\alpha)} \\ &\pm\frac{1}{24}\sqrt{(4b+3)(9-16b^2)} \\ &\times [\sqrt{-b}\tanh\sqrt{-b}(x+\frac{(4b+3)t^\alpha}{8\Gamma(1+\alpha)})] \\ &+\frac{1}{288}(9-16b^2)) \end{aligned}$$

$$\begin{aligned} v_1(x,t) &= \frac{8}{4b+3}\pm\frac{\sqrt{2}}{4}\sqrt{4b+3}[\sqrt{-b} \\ &\times \tanh\sqrt{-b}(x+\frac{(4b+3)t^\alpha}{8\Gamma(1+\alpha)})] \\ &\pm\frac{\sqrt{2}}{24}\sqrt{9-16b^2}, \end{aligned}$$

$$\begin{aligned} w_1(x,t) &= 2(\frac{8}{4b+3})^2-2(\frac{8}{4b+3})[\pm\frac{\sqrt{2}}{4}\sqrt{4b+3}[\sqrt{-b} \\ &\times \tanh\sqrt{-b}(x+\frac{(4b+3)t^\alpha}{8\Gamma(1+\alpha)})] \\ &\pm\frac{\sqrt{2}}{24}\sqrt{9-16b^2} \end{aligned}$$

And

$$\begin{aligned} U_2 &= \pm\frac{\sqrt{2}}{4}\sqrt{4b+3}[\sqrt{-b}\coth\sqrt{-b}(x+\frac{(4b+3)t^\alpha}{8\Gamma(1+\alpha)})] \\ &\pm\frac{\sqrt{2}}{24}\sqrt{9-16b^2}, \end{aligned}$$

So

$$\begin{aligned} u_2(x,t) &= -\frac{8}{4b+3}(-\frac{b}{8}(4b+3) \\ &\times \coth^2\sqrt{-b}(x+\frac{(4b+3)t^\alpha}{8\Gamma(1+\alpha)}) \\ &\pm\frac{1}{24}\sqrt{(4b+3)(9-16b^2)} \\ &\times [\sqrt{-b}\coth\sqrt{-b}(x+\frac{(4b+3)t^\alpha}{8\Gamma(1+\alpha)})] \\ &+\frac{1}{288}(9-16b^2)). \end{aligned}$$

$$\begin{aligned} v_2(x,t) &= \frac{8}{4b+3}\pm\frac{\sqrt{2}}{4}\sqrt{4b+3}[\sqrt{-b} \\ &\times \coth\sqrt{-b}(x+\frac{(4b+3)t^\alpha}{8\Gamma(1+\alpha)})] \\ &\pm\frac{\sqrt{2}}{24}\sqrt{9-16b^2}, \end{aligned}$$

$$\begin{aligned} w_2(x,t) &= 2(\frac{8}{4b+3})^2-2(\frac{8}{4b+3}) \\ &\times [\pm\frac{\sqrt{2}}{4}\sqrt{4b+3}[\sqrt{-b} \\ &\times \coth\sqrt{-b}(x+\frac{(4b+3)t^\alpha}{8\Gamma(1+\alpha)})] \\ &\pm\frac{\sqrt{2}}{24}\sqrt{9-16b^2}], \end{aligned}$$

Where $b \prec 0$and k is an arbitrary real constant.

$$\begin{aligned} U_3 &= \pm\frac{\sqrt{2}}{4}\sqrt{4b+3}[\sqrt{b}\tan\sqrt{b}(x+\frac{(4b+3)t^\alpha}{8\Gamma(1+\alpha)})] \\ &\pm\frac{\sqrt{2}}{24}\sqrt{9-16b^2}, \end{aligned} \tag{18}$$

Therefore

$$\begin{aligned} u_3(x,t) &= -\tfrac{8}{4b+3}\left(\tfrac{b}{8}(4b+3)\tan^2\sqrt{b}\left(x+\tfrac{(4b+3)t^\alpha}{8\Gamma(1+\alpha)}\right)\right. \\ &\pm\tfrac{1}{24}\sqrt{(4b+3)(9-16b^2)}\times\left[\sqrt{b}\tan\sqrt{b}(\,x+\tfrac{(4b+3)t^\alpha}{8\Gamma(1+\alpha)})\right] \\ &\left.+\tfrac{1}{288}\left(9-16b^2\right)\right), \end{aligned}$$

$$\begin{aligned} v_3(x,t) &= \frac{8}{4b+3}\pm\frac{\sqrt{2}}{4}\sqrt{4b+3}[\sqrt{b} \\ &\times \tan\sqrt{b}(x+\frac{(4b+3)t^\alpha}{8\Gamma(1+\alpha)})] \\ &\pm\frac{\sqrt{2}}{24}\sqrt{9-16b^2}, \end{aligned}$$

$$w_3(x,t) = 2(\frac{8}{4b+3})^2 - 2(\frac{8}{4b+3})[\pm\frac{\sqrt{2}}{4}\sqrt{4b+3}[\sqrt{b} \times \tan\sqrt{b}(x+\frac{(4b+3)t^\alpha}{8\Gamma(1+\alpha)})] \pm \frac{\sqrt{2}}{24}\sqrt{9-16b^2}$$

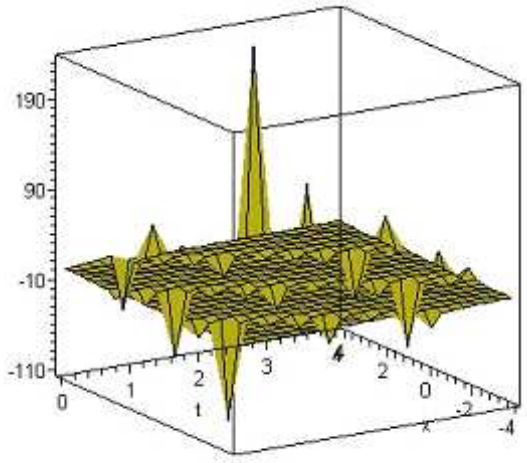

Figure 1: Complex solitary waves described by solution; case1, $v_3(x,t)$ for $b=1,\alpha=1$.

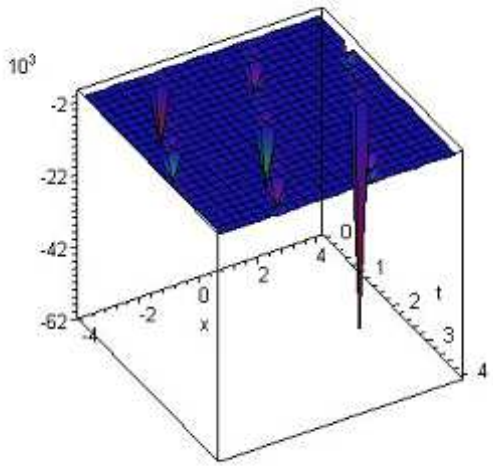

Figure 2: solitary waves described by solution; case1, $u_3(x,t)$ for $b=1,\alpha=1$.

Where $b \succ 0$and k is an arbitrary real constant. And

$$U_4 = \pm\frac{\sqrt{2}}{4}\sqrt{4b+3}[-\sqrt{b}\tan\sqrt{b}(x+\frac{(4b+3)t^\alpha}{8\Gamma(1+\alpha)})] \pm \frac{\sqrt{2}}{24}\sqrt{9-16b^2}, \quad (19)$$

Now,

$$u_4(x,t) = -\frac{8}{4b+3}[\pm\frac{\sqrt{2}}{4}\sqrt{4b+3}[-\sqrt{b} \times \cot\sqrt{b}(x+\frac{(4b+3)t^\alpha}{8\Gamma(1+\alpha)})] \pm \frac{\sqrt{2}}{24}\sqrt{9-16b^2}]^2,$$

$$v_4(x,t) = \frac{8}{4b+3} \pm \frac{\sqrt{2}}{4}\sqrt{4b+3}[-\sqrt{b} \times \cot\sqrt{b}(x+\frac{(4b+3)t^\alpha}{8\Gamma(1+\alpha)})] \pm \frac{\sqrt{2}}{24}\sqrt{9-16b^2},$$

$$w_4(x,t) = 2(\frac{8}{4b+3})^2 - 2(\frac{8}{4b+3}) \times [\pm\frac{\sqrt{2}}{4}\sqrt{4b+3}[-\sqrt{b} \times \cot\sqrt{b}(x+\frac{(4b+3)t^\alpha}{8\Gamma(1+\alpha)})] \pm \frac{\sqrt{2}}{24}\sqrt{9-16b^2}],$$

Where $b \succ 0$and k is an arbitrary real constant. For $b=0$

$$U_5 = \mp\frac{2\sqrt{6}\Gamma(1+\alpha)}{8x\Gamma(1+\alpha)+3t^\alpha} \pm \frac{\sqrt{2}}{8}$$

Substituting in (12) we have

$$u_5(x,t) = -\frac{8}{4b+3}\left[\mp\frac{2\sqrt{6}\Gamma(1+\alpha)}{8x\Gamma(1+\alpha)+3t^\alpha} \pm \frac{\sqrt{2}}{8}\right]^2 = -\frac{192}{4b+3}(\frac{\Gamma(1+\alpha)}{8x\Gamma(1+\alpha)+3t^\alpha})^2 \mp \frac{\sqrt{3}\Gamma(1+\alpha)}{8x\Gamma(1+\alpha)+3t^\alpha} + \frac{1}{32},$$

$$v_5(x,t) = -\lambda \mp \frac{2\sqrt{6}\Gamma(1+\alpha)}{8x\Gamma(1+\alpha)+3t^\alpha} \pm \frac{\sqrt{2}}{8},$$

$$w_5(x,t) = \frac{(4b+3)^2}{32} + \frac{4b+3}{4}\left(\mp\frac{2\sqrt{6}\Gamma(1+\alpha)}{8x\Gamma(1+\alpha)+3t^\alpha} \pm \frac{\sqrt{2}}{8}\right),$$

Case 2: Balancing $U_{\xi\xi}$ with U^3 in Eq. (14) gives N= 1. Therefore, we may choose

$$U = a_1F + a_0, \quad (20)$$

Substituting Eq. (20) along with Eq. (10) in Eq. (14) and setting all the coefficients of powers $F$to be zero, then we obtain a system of nonlinear algebraic equations and by solving it, we obtain

$$\begin{aligned} a_1 &= \pm\sqrt{-\tfrac{b}{5}}, \\ a_0 &= \pm\tfrac{2\sqrt{3}bi}{15}, \\ \lambda &= \tfrac{b}{5} \end{aligned} \quad (21)$$

From (11),(20) and (21), we obtain the complex travelling wave solutions of (13) as follows

$$U_1 = \pm\sqrt{-\frac{b}{5}}\left[\sqrt{-b}\tanh\sqrt{-b}(\ x-\frac{bt^{\alpha}}{5\Gamma(1+\alpha)})\right]\pm\frac{2\sqrt{3}bi}{15},$$

Substituting in (12) we have

$$u_1(x,t) = (-\frac{b}{5})[\sqrt{-b}\tanh\sqrt{-b}(x-\frac{bt^{\alpha}}{5\Gamma(1+\alpha)})]^2 \pm \frac{4bi}{15}\sqrt{-\frac{3b}{5}}[\sqrt{-b}\tanh\sqrt{-b}(x-\frac{bt^{\alpha}}{5\Gamma(1+\alpha)})] - \frac{4}{75}$$

$$v_1(x,t) = -\frac{b}{5}+[\pm\sqrt{-\frac{b}{5}}[\sqrt{-b}\tanh\sqrt{-b}(x-\frac{bt^{\alpha}}{5\Gamma(1+\alpha)})] \pm \frac{2\sqrt{3}bi}{15}],$$

$$w_1(x,t) = \frac{2b}{25}-[\pm\frac{2b}{5}\sqrt{-\frac{b}{5}}[\sqrt{-b}\tanh\sqrt{-b}(x-\frac{bt^{\alpha}}{5\Gamma(1+\alpha)})] \pm \frac{4\sqrt{3}b^2i}{75}],$$

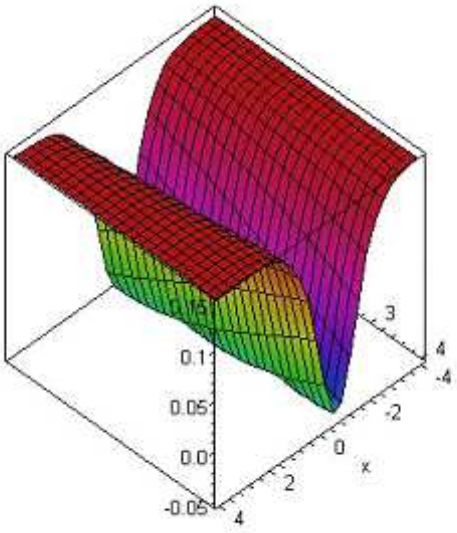

Figure 3: Complex solitary waves described by solution; case2, $u_1(x,t)$ for $b=-1,\alpha=1$.

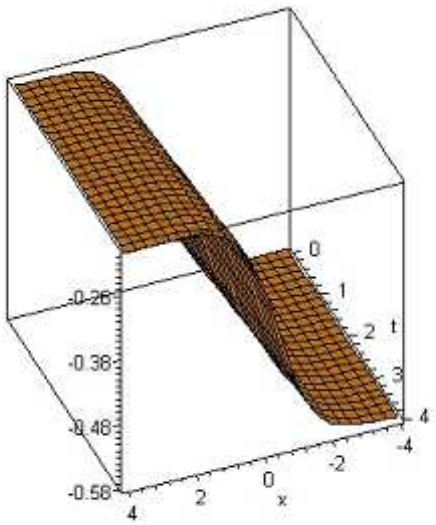

Figure 4: Complex solitary waves described by solution; case2, $w_1(x,t)$ for $b=-1,\alpha=1,$.

Where $b\prec 0$ and k is an arbitrary real constant. And

$$U_2 = \pm\sqrt{-\frac{b}{5}}[\sqrt{-b}\coth\sqrt{-b}(x-\frac{bt^{\alpha}}{5\Gamma(1+\alpha)})] \pm \frac{2\sqrt{3}bi}{15},$$

So

$$u_2(x,t) = (-\tfrac{b}{5})\left[\sqrt{-b}\coth\sqrt{-b}(\ x-\tfrac{bt^{\alpha}}{5\Gamma(1+\alpha)})\right]^2 \pm \tfrac{4bi}{15}\sqrt{-\tfrac{3b}{5}}\left[\sqrt{-b}\coth\sqrt{-b}(\ x-\tfrac{bt^{\alpha}}{5\Gamma(1+\alpha)})\right]-\tfrac{4}{75},$$

$$v_2(x,t) = -\frac{b}{5}+[\pm\sqrt{-\frac{b}{5}}[\sqrt{-b}\coth\sqrt{-b}(x-\frac{bt^{\alpha}}{5\Gamma(1+\alpha)})] \pm \frac{2\sqrt{3}bi}{15}],$$

$$w_2(x,t) = \frac{2b}{25}-[\pm\frac{2b}{5}\sqrt{-\frac{b}{5}}[\sqrt{-b}\coth\sqrt{-b}(x-\frac{bt^{\alpha}}{5\Gamma(1+\alpha)})] \pm \frac{4\sqrt{3}b^2i}{75}],$$

Where $b\prec 0$and k is an arbitrary real constant.

$$U_3 = \pm\sqrt{-\frac{b}{5}}\left[\sqrt{b}\tan\sqrt{b}(\ x-\frac{bt^{\alpha}}{5\Gamma(1+\alpha)})\right]\pm\frac{2\sqrt{3}bi}{15},$$

Now

$$u_3(x,t) = (-\tfrac{b}{5})\left[\sqrt{b}\tan\sqrt{b}(\ x-\tfrac{bt^{\alpha}}{5\Gamma(1+\alpha)})\right]^2 \pm \tfrac{4bi}{15}\sqrt{-\tfrac{3b}{5}}\left[\sqrt{b}\tan\sqrt{b}(\ x-\tfrac{bt^{\alpha}}{5\Gamma(1+\alpha)})\right]-\tfrac{4}{75},$$

$$v_3(x,t) = -\frac{b}{5} + [\pm\sqrt{-\frac{b}{5}}[\sqrt{b}\tan\sqrt{b}(x - \frac{bt^\alpha}{5\Gamma(1+\alpha)})] \pm \frac{2\sqrt{3}bi}{15}],$$

$$w_3(x,t) = \frac{2b}{25} - [\pm\frac{2b}{5}\sqrt{-\frac{b}{5}}[\sqrt{b}\tan\sqrt{b}(x - \frac{bt^\alpha}{5\Gamma(1+\alpha)})] \pm \frac{4\sqrt{3}b^2i}{75}],$$

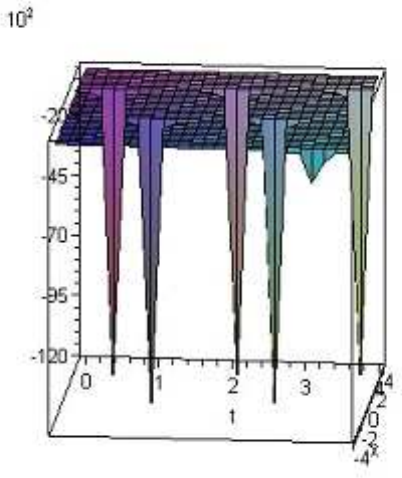

Figure 5: solitary waves described by solution; case2 $u_3(x,t)$ for $b = 1, \alpha = 1$.
Where $b \succ 0$and k is an arbitrary real constant.

$$U_4 = \pm\sqrt{-\frac{b}{5}}\left[-\sqrt{b}\cot\sqrt{b}(\ x - \frac{bt^\alpha}{5\Gamma(1+\alpha)})\right] \pm \frac{2\sqrt{3}bi}{15},$$

and

$$u_4(x,t) = (-\frac{b}{5})[-\sqrt{b}\cot\sqrt{b}(x - \frac{bt^\alpha}{5\Gamma(1+\alpha)})]^2 \pm \frac{4bi}{15}\sqrt{-\frac{3b}{5}}[-\sqrt{b}\cot\sqrt{b}(x - \frac{bt^\alpha}{5\Gamma(1+\alpha)})] - \frac{4}{75},$$

$$v_4(x,t) = -\frac{b}{5} + \pm\sqrt{-\frac{b}{5}}[-\sqrt{b}\cot\sqrt{b}(x - \frac{bt^\alpha}{5\Gamma(1+\alpha)})] \pm \frac{2\sqrt{3}bi}{15},$$

$$w_4(x,t) = \frac{2b}{25} - \pm\frac{2b}{5}\sqrt{-\frac{b}{5}}[-\sqrt{b}\cot\sqrt{b}(x - \frac{bt^\alpha}{5\Gamma(1+\alpha)})] \pm \frac{4\sqrt{3}b^2i}{75},$$

Where $b \succ 0$ and k is an arbitrary real constant. For $b = 0$ don't have any solution.

4 Conclusion

In this present work we deduce that the referred method can be extended to solve many systems of nonlinear partial differential equations which are arising in the theory of solitons and other areas such as physics, biology, and chemistry. With the help of symbolic computation (Maple), a rich variety of exact solutions are obtained by applying direct algebraic method, and the method can be applied to other nonlinear evolution equations.

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