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Design of the Distribution Network for a "Collect-on-Delivery" Company in a Metropolitan Context using Simulated Annealing with Path Relinking

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Abstract: Parcel delivery service by collect-on-delivery (COD, a.k.a. cash-on-delivery) companies is a new distribution mode in China. The design of the distribution network of a COD company in urban areas is of great importance and a relatively new research direction. In this paper, we aim to model this distribution network as a two-echelon location routing problem (2E-LRP) and propose a novel efficient optimization algorithm by combining simulated annealing (SA) and path relinking (PR) to solve this NP combinatorial optimization problem. The experimental results are reported for 84 test problems from Nguyen's, Prodhon's, and Sterle's benchmark. They indicate that the proposed SA-PR algorithm is competitive with other existing state-of-art algorithms for 2E-LRP from the perspectives of both quality of solution and computational time. Therefore, SA-PR is an effective metaheuristic to solve 2E-LRP problems, i.e., design distribution network of a COD company in a metropolitan context.

Keywords: Simulated annealing, COD company, location routing, two echelon

1 Introduction

The CEP sector (courier, express, and parcel service), especially the "last-mile" stage of delivery in urban cities, such as parcel delivery, has changed drastically in the past few years in China. On the one hand, municipal governments are devoted to urban freight consolidation to reduce environmental pollution and increase city attractiveness. On the other hand, the private sectors of E-commerce and CEP companies fight for market share by price competition. Thus COD (collect-on-delivery or cash-on-delivery) companies, such as City 100 Freight Consolidation Co. in Beijing, have emerged to restructure the supply chain and to cope with these challenges. COD companies have been suggested as a possible solution for parcel delivery within urban areas if the objective is to reduce the costs and contain the environmental impact.

In the literature, the design of a distribution network of a COD company has attracted attention because it combines two decision levels, strategic and tactical, for a CEP system aimed at guaranteeing parcel delivery in cities. The strategic decisions assign the locations and number of facilities, and the tactical decisions focus on the vehicle routing and scheduling.

However, most scholars pay attention to designing the parcel delivery network [1,2] by employing the location routing problem model (a.k.a. one echelon LRP, 1E-LRP) in which parcels are delivered to and from the city by facilities called Distribution Centers (DC, the terms Urban Consolidation Centers [3], and Urban Logistics Platforms [4] are also used), as shown in Fig. 1. Papers on 1E-LRP have appeared from the 1980s. Nagy and Salhi [5] and Prodhon and Prins [6] have provided excellent surveys of models and applications of 1E-LRP. However, the 1E-LRP system does not appear interesting for large urban cities. Since metropolises are very constrained areas, characterized by limited access times for trucks and vans, one-way streets, traffic congestion in peak hours, and inaccessible curb space, among other things, large vehicles find it difficult to move within urban areas. On the contrary, the human-powered or electrically aided freight tricycle is a more efficient way to deliver parcels.

Therefore a more general two-echelon system, combining major Central Distribution Centers (CDCs) and Regional Distribution Centers (RDCs), appears to be

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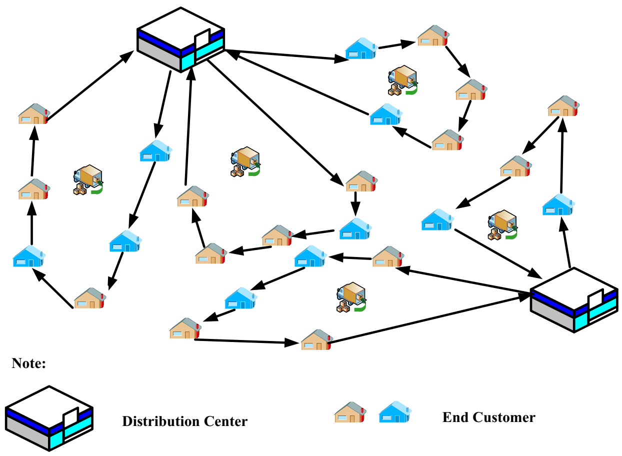

Fig. 1: Distribution network of “COD” company in a metropolis.

promising network structures for “COD” companies in a metropolis [7], as presented in Fig. 2. CDCs are located on the outskirts of the metropolis and serve as platforms to consolidate input and output urban parcel flow. RDCs are located in the metropolis and serve as transit centers in which the parcels coming from the CDCs by vans are transferred and consolidated into smaller vehicles, such as electrically aided tricycles, to End Customers (ECs). This network is considered as a 2E-LRP model, which is an NP-hard problem, since it combines FLP (facility location problem) and VRP (vehicle routing problem), and both are NP-hard problems.

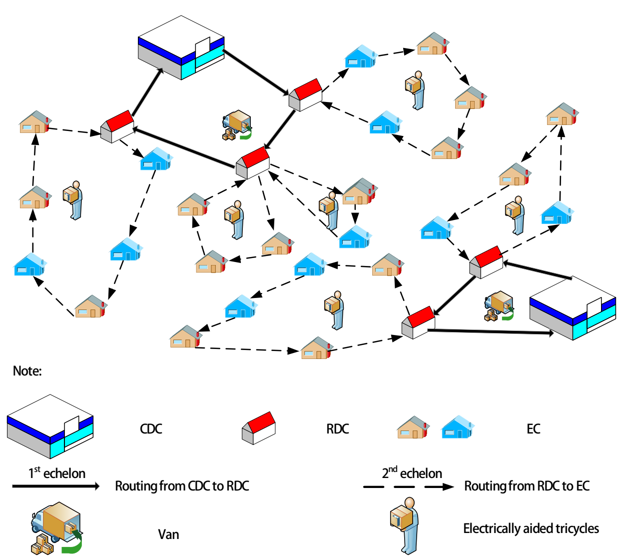

Fig. 2: Distribution network of parcel delivery based on 2E-LRP model.

Simulated annealing (SA), because of its simplicity in coding and its global and local exploration abilities [8], may be usefully applied in combinatorial optimization problems. However, it requires a very slow cooling-down procedure to guarantee the solution quality. Moreover, its computational time increases quadratically with an increasing number of customers. To solve the 2E-LRP problem on a large-scale network, more efficient algorithms are needed.

The purpose of the present study is to develop an efficient metaheuristic to design the distribution network of “COD” companies in a metropolitan context, which is also a 2E-LRP model, by a simulated annealing with path relinking (SA-PR). The SA-PR algorithm combines the recently proposed path relinking (PR) method with the SA algorithm. This paper is organized as follows. Section 2 introduces a basic assumption of the distribution network of “COD” companies in urban areas and the related literature. Section 3 gives a detailed description of how the proposed SA-PR algorithm is implemented on the 2E-LRP problem. Section 4 provides comparisons to the 2E-LRP benchmark. Lastly are conclusions and future research opportunities presented in Section 5.

2 Problem definition and literature review

The distribution network of a “COD” company in a metropolis (2E-LRP model) is formed as follows. CDCs form the first level of the distribution network, which are located far from the center of a city. Parcels transported by heavy trucks from different E-commercial and CEP companies are consolidated prior to delivery to geographically scattered customers in the city. RDCs form the second level of the distribution network, which receive parcels coming from CDCs by vans and use smaller vehicles for local distribution in dense city zones. Fig. 3 provides a representative structure of a COD company’s distribution network in a metropolis. In this distribution network, three types of decisions are to be made to minimize the sum of the costs associated with locating depots and distribution to the customers. They are (1) location decisions: the number and locations of CDCs and RDCs; (2) allocation decisions: assign open RDCs to open CDCs, and assign ECs to open RDCs; (3) routing decisions: the design of routes originating at the CDCs to serve the RDCs and the design of routes emanating from RDCs to serve ECs.

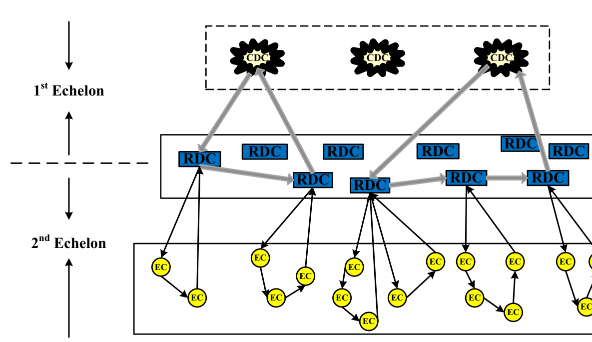

Fig. 3: Distribution network structure of a COD company.

In this distribution network, the following assumptions are summarized:

- All parcels (freight) start from CDCs and finish at ECs.
- CDCs and RDCs have different capacity limits.
- Demands come from the ECs. A single demand cannot be split among different vehicles, but more demands can be loaded on the same vehicle.
- Direct shipments from CDCs to ECs are not allowed. The freight must be consolidated by a CDC to an RDC and then from the RDC to a specific EC.
- The same type of van is used in the first echelon (from CDC to RDC), and the same type of electrically aided tricycle is used in the second echelon (from RDC to EC). Therefore the capacity of first-echelon vehicles is apparently much higher than the capacity of second-echelon vehicles. And the number of vehicles on each echelon is not known in advance.
- The first-echelon routes start at a CDC, visit one or more RDCs, and return to the same CDC. The second-echelon routes start at an RDC, visit one or more ECs, and return to the same RDC.

Before presenting the formulations, the parameter setting of the problem is given.

Sets:

$P = 1, 2, \dots p$ Set of the possible CDC locations

$S = 1, 2, \dots s$ Set of the possible RDC locations

$C = 1, 2, \dots, c$ Set of the ECs

$V = 1, 2, \dots, v$ Set of the first-echelon vehicles, vans

$E = 1, 2, \dots, e$ Set of the second-echelon, electrically aided tricycle

Parameters:

K_p Capacity of CDC p, $p \in P$

FC_p Fixing cost for opening a CDC p, $p \in P$

CTA_{ij} Transportation cost on the first echelon from node i to node j, $i, j \in P \cup S$

F_s Capacity of RDC s, $s \in S$

FC_s Fixing cost for opening an RDC s, $s \in S$

CTB_{ij} Transportation cost on the second echelon from node i to node j, $i, j \in S \cup C$

FCV Fixing cost for using a van v, $v \in V$

CAV Capacity of vans v, $v \in V$

FCE Fixing cost for using electrically aided tricycle e, $e \in E$

CAE Capacity of electrically aided tricycle e, $e \in E$

D_c Demand of each EC, $c \in C$

Variables:

$x_{ij}^v = \{0,1\}$ $x_{ij}^v = 1$, if i precedes j in the routing of the first echelon, performed by van v, otherwise $x_{ij}^v = 0$.

$y_{ij}^e = \{0,1\}$ $y_{ij}^e = 1$, if i precedes j in the routing of the second echelon, performed by an electrically aided tricycle, otherwise $y_{ij}^v = 0$.

$z_{sc} = \{0,1\}$ $z_{sc} = 1$, if the customer c ($c \in C$) is assigned to satellite $s (s \in S)$; otherwise $z_{sc} = 0$.

$m_p = \{0,1\}$ $m_p = 1$, if a CDC is opened at node p, $p \in P$; otherwise $m_p = 0$.

$m_s = \{0,1\}$ $m_s = 1$, if an RDC is opened at node s, $s \in S$; otherwise $n_s = 0$.

$n^v = \{0,1\}$ $n^v = 1$, if a van v is used, $v \in V$; otherwise $n^v = 0$.

$n^e = \{0,1\}$ $n^e = 1$, if an electrically aided tricycle e is used, $e \in E$, otherwise $n^e = 0$.

$q_{ps}^v \geq 0$ is the quantity of goods transported by the CDC p ($p \in P$) to the RDC s ($s \in S$) with van v ($v \in V$).

Thus the distribution network of a COD company in a metropolis can be formulated as follows [9]:

$$\begin{aligned} & Min \sum_{p\in P} FC_p \times m_p + \sum_{s\in S} FC_s \times m_s + \sum_{v\in V} FCV \times n^v \\ & + \sum_{e\in E} FCE \times n^e + \sum_{v\in V} \sum_{i\in P\cup S} \sum_{j\in P\cup S} CTA_{ij} \times x_{ij}^v \\ & + \sum_{e\in E} \sum_{i\in S\cup C} \sum_{j\in S\cup C} CTB_{ij} \times y_{ij}^e \end{aligned} \tag{1}$$

The objective function (1) is the sum of six cost components: location cost for CDC, location cost for RDC, fixed cost for usage of vans, fixed cost for usage of electrically aided tricycles, and transportation costs on the second and on the first echelons.

Routing constrains for the first echelon:

Each open RDC ($s \in S$) is served by exactly one van $v \in V$

$$\sum_{v\in V} \sum_{j\in P\cup S} x_{lj}^v = m_l \quad \forall l \in S \tag{2}$$

Each van ($v \in V$) enters into a node and also must leave the same node, which ensures that the number of arcs entering a node is equal to the number of arcs leaving it.

$$\sum_{l\in P\cup S} x_{lh}^v = \sum_{l\in P\cup S} x_{hl}^v \quad \forall h \in P \cup S, \forall v \in V \tag{3}$$

Subtour elimination constraints, which guarantee open RDC, are served by one van.

$$\begin{aligned} & \sum_{l\in\Omega} \sum_{h\in\overline{\Omega}} \sum_{v\in V} x_{lh}^v \geq m_j \quad \forall j \in S, \forall \Omega \subset P \cup S, \\ & and P \subseteq \Omega, \overline{\Omega} \cap \{j\} \neq \emptyset \end{aligned} \tag{4}$$

Each van can perform only one route.

$$\sum_{l\in P\cup S} \sum_{j\in P} x_{lj}^v \leq 1 \quad \forall v \in V \tag{5}$$

Routing constraints for the second echelon:

Each customer ($c \in C$) is served by exactly one electrically aided tricycle $e \in E$

$$\sum_{e\in E} \sum_{j\in S\cup C} y_{cj}^e = 1 \quad \forall c \in C \tag{6}$$

Each electrically aided tricycle ($e \in E$) enters a node and also must leave the same node, which ensures that the number of arcs entering a node is equal to the number of arcs leaving it.

$$\sum_{l\in S\cup C} x_{lj}^e = \sum_{l\in S\cup C} x_{jl}^e \quad \forall j \in S \cup C, \forall e \in E \tag{7}$$

Subtour elimination constraints, which guarantee that the EC is served by one electrically aided tricycle.

$$\sum_{l\in\Omega}\sum_{h\in\overline{\Omega}}\sum_{e\in E} x^{e}_{lh} \geq 1 \quad \forall\Omega \subset S\cup C, and P \subseteq \Omega \tag{8}$$

Each electrically aided tricycle can perform only one route.

$$\sum_{l\in S\cup C}\sum_{j\in S} x^{e}_{lj} \leq 1 \quad \forall e \in E \tag{9}$$

Capacity constraints for vehicles:

The amount of flow transferred by a van $v\,(v \in V)$ from CDC, $p(p \in P)$, to RDC $s\,(s \in S)$, must be less than its own capacity if the vehicle is used.

$$\sum_{p\in P}\sum_{s\in S} q^{v}_{ps} \leq CAVn^{v} \quad \forall v \in V \tag{10}$$

The demand assigned to an electrically aided tricycle e $(e \in E)$ must be less than its own capacity if the vehicle is used.

$$\sum_{c\in C} D_c \sum_{j\in S\cup C} y^{e}_{cj} \leq CAEn^{e} \quad \forall e \in E \tag{11}$$

Flow conservation constraints:

The amount of flow leaving the CDCs is to be equal to the total demand of the ECs.

$$\sum_{p\in P}\sum_{v\in V} q^{v}_{ps} = \sum_{c\in C} D_c z_{sc} \quad \forall s \in S \tag{12}$$

Capacity constraints for facilities:

If a CDC is open, the flow leaving it, $p(p \in P)$, must be less than its own capacity.

$$\sum_{s\in S}\sum_{v\in V} q^{v}_{ps} \leq K_p m_p \quad \forall p \in P \tag{13}$$

If an RDC is open, the flow leaving it, $s\,(s \in S)$, must be less than its own capacity.

$$\sum_{p\in P}\sum_{v\in V} q^{v}_{ps} \leq F_s m_s \quad \forall s \in S \tag{14}$$

The binary variables constraints:

$$x^{v}_{ij} = \{0,1\} \quad \forall i,j \in P\cup S, v \in V \tag{15}$$
$$y^{e}_{ij} = \{0,1\} \quad \forall i,j \in S\cup C, e \in E \tag{16}$$
$$z_{sc} = \{0,1\} \quad \forall s \in S, c \in C \tag{17}$$
$$m_p = \{0,1\} \quad \forall p \in P \tag{18}$$
$$m_s = \{0,1\} \quad \forall s \in S \tag{19}$$
$$n_v = \{0,1\} \quad \forall v \in V \tag{20}$$
$$n_e = \{0,1\} \quad \forall e \in E \tag{21}$$
$$q^{v}_{ps} \geq 0 \quad \forall p \in P, s \in S, v \in V \tag{22}$$
$$r^{e}_{sc} = \{0,1\} \quad \forall s \in S, c \in C, e \in E \tag{23}$$

Obviously, the 2E-LRP belongs to the class of NP-hard problems, since it combines two difficult subproblems: the facility location problem (FLP) and the vehicle routing problem (VRP), both of which are NP-hard [10,11]. Because of the 2E-LPR problem's complexity, technical literature on it is somewhat limited. To the best of our knowledge, a branch-and-cut algorithm by Contardo, Hemmelmayr, and Crainic [12] is the only exact algorithm implemented to address this problem.

Because of the computational challenge associated with this applied problem, researchers are interested in developing heuristics to solve this very difficult optimization problem. Lin and Lei [13] formulated a 2E-LRP system, including a set of plants, big clients, and small customers. A genetic algorithm, followed by a cluster-based routing heuristic and local search, is proposed. Boccia et al. [14] handled the 2E-LRP problem as two capacitated 1E-LRP problems and solved each by a tabu search. The same problem was then studied by Boccia et al. [15]. A three-index, two-index, and one-set partitioning formulations for the 2E-LRP problem were built, and small instances were solved by Xpress-MP. Nguyen, Prins, and Prodhon [16] addressed a 2E-LRP problem with a single fixed CDC and a set of potential RDCs with limited capacities and opening cost. A hybrid metaheuristic, which was a greedy randomized adaptive search procedure (GRASP), complemented by a learning process (AP) and path relinking (PR) was presented. Nguyen, Prins, and Prodhon [17], the same authors, then proposed a multi-start iterated local search, MS-ILS+PR (multi-start iterated local search + path-relinking), which consists of three GRASPs for restarts, two local search procedures, a tabu list for short-term diversification, a PR, and two search spaces. Contardo, Hemmelmayr, and Crainic [12] proposed an adaptive large-neighborhood search (ALNS) metaheuristic to solve the 2E-LRP problem.

3 Simulated Annealing with Path Relinking

The problem-solving methodology based on the SA with LR is presented in this section. SA implements a Metropolis sampling strategy with probability mutability, which randomizes the local search procedure and obtains the global optimum with a slow cooling schedule. The most important feature of SA compared with other local search algorithms is that SA always accepts a better or unchanged solution as a new current solution, and it accepts a worse solution with a certain probability. This avoids the procedure being trapped prematurely in a local minimum, and it thus becomes a global optimum algorithm in theory. PR is then used to speed up SA to solve the 2E-LRP problem.

The pseudo-code shown in Fig. 4 describes steps in the SA-PR algorithm as applied to the design of a distribution network of a "COD" company in a metropolitan context. Step 1 is for initialization of a feasible solution, and the temperature is in line 3 of the SA-PR algorithm. Step 2 is the local search in the neighborhood, which is used to improve the solutions. Step 3 is the PR post-optimization to explore the feasible space among these obtained local optima to achieve better results. At each iteration, new optima, subject to further consideration, using PR by relinking to the most similar ones (from line 7 to line 8 of SA-PR) in an elite set, maintain several top candidate solutions. Moreover, we also generate paths by relinking members in the elite set. The final solution is the one with the best objective functional value along any of the relinked paths connecting pairs of local optima (line 12 of SA-PR).

```
1.  P^elite := ∅
2.  WHILE (stopping criterion is not satisfied) DO
3.      s := s_0, T_0 := γ × cost(s_0)
4.      s := LocalSearch(s)
5.      IF ( |P^elite| < N^elite )
6.              P^elite := P^elite ∪ {s}   ELSE
7.              s* := Select ( P^elite )
8.              s* := PathRelinking ( s, s* )
9.              Update ( P^elite , s* )
10.     END IF
11. END WHILE
12. P*_ij := PathRelinking ( P^elite(i), P^elite(j) ), i < j , i, j ∈ N
13. s_best := SelectBest ( P* )
```

Fig. 4: Pseudo-code of SA-PR for distribution network.

The text below describes steps in the simulated annealing algorithm applied to design a distribution network of a "COD" company. The problem is decomposed into two main components, i.e., two 1E-LRP problems. In each, capacitated FLP (CFLP) and multi-depot VRP (MDVRP) are solved separately, and sub-problem solutions are combined.

3.1 Phase I: Initialization

The pseudo-code in Fig. 5 describes, steps in the SA algorithm applied to solve the 2E-LRP problem. As pointed out by Dowsland [18], the "temperature" is used to imitate the cooling process in physical annealing. It is merely a control parameter that controls the probability of accepting an increase in the total cost in the 2E-LRP problem. A high temperature translates into a high probability of accepting a solution s'_k as the new solution. The SA algorithm has two loops. In the inner loop, a local search is executed for a certain temperature in the neighborhood of the current solution to generate a new feasible solution; the decision of whether to accept a new solution is based on lines 8–11 in Fig. 5, and the search continues until the maximum number of iterations is reached. In the outer loop, the temperature is lowered gradually until the stopping criterion is met.

3.1.1 Generate initial feasible solution

The initial feasible solution of the 2E-CFLP problem is constructed. In this problem, each node is assigned to a

```
1.  s := s_0, T_0 := γ × cost(s_0)
2.  k := 0, ω := 0
3.  WHILE ( ω < ω_max ) DO
4.  l := 0
5.      WHILE ( l < L ) DO
6.              BEGIN
7.                  s'_k = perturbation function(s_k) s'_k ∈ N(s_k)
8.                  IF  cost(s'_k) < cost(s_k) , THEN
9.                      s_k := s'_k , l = 0 , updateOptimalSolution = true , ELSE
10.                     s_k := s'_k with probability  e^((cost(s_k)−cost(s'_k))/T_k) , l := l + 1
11.                 END IF
12.     END WHILE
13.     k := k + 1,  T_{k+1} := T(k)
14.     IF  ! updateOptimalSolution
15.         ω := ω + 1
16. END WHILE
17. Return  s_k
```

Fig. 5: Pseudo-code of sequential SA for 2E-LRP.

vehicle and consequently to a route. Although this method does not find a good initial solution, it satisfies the constraints of the problem and serves as an initial guess.

The purpose of this heuristic is to find the minimum number of facilities opened in each echelon. Because the demands of ECs are determined, it is convenient to first optimize the number of open RDCs in the second echelon. The capacity of open RDCs is set so that the demand of all ECs are satisfied, as shown in Equation (24). The capacity of an RDC is noted as FC_i^{RDC} , and the demand of an EC is noted as D_j . The parameter $\alpha(\alpha \in [90\% - 95\%])$ is a percentage parameter, which assures a higher probability of determining a feasible assignment.

$$\alpha \sum_{i=1,2...m} FC_i^{RDC} \geq \sum_{j \in EC} D_j \tag{24}$$

In this procedure, after assignment of EC demands to each open RDC, the method is repeated to find the minimum number of open CDCs in the first echelon. The structure of a feasible initial solution is demonstrated in Fig. 6.

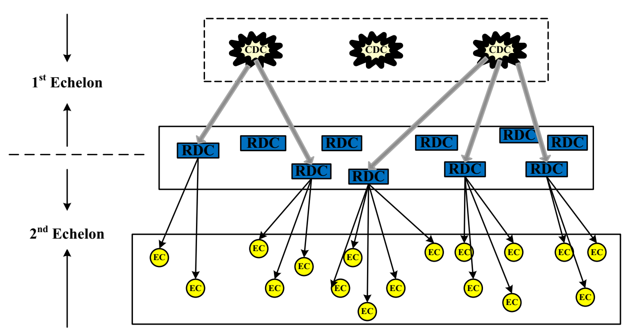

Fig. 6: A feasible initial solution for distribution network.

3.1.2 Initial temperature

The setting of an initial temperature determines whether the initial stage of the annealing process can accept poorer solutions with high probability. The cost of the distribution network (2E-LRP solution) consists of location cost and routing cost. The cooling temperature of the 2E-LRP solution is computed as follows:

$$T_k = \gamma \times \mathrm{cost}(s_k) \tag{25}$$

where $\gamma < 1$ is a constant.

Van Laarhoven and Aarts [19] proposed a method of decreasing the temperature, which appears to be the most widely used method reported in the literature:

$$T_{k+1} = \beta \times T_k \tag{26}$$

where β is the coefficient controlling the cooling schedule and a constant less than 1.

When $k = 0$, the initial temperature is:

$$T_0 = \gamma \times \mathrm{cost}(s_0) \tag{27}$$

3.2 Phase II: Local Search

A standard SA procedure with a random neighborhood structure is used to solve the 2E-LRP problem. This method does a local search in the random neighborhood of the current solution S_k to generate a new feasible solution S'_k. The neighborhood $N(s_k)$ is critical. A small neighborhood does not allow a good exploration of the solution space, but a large neighborhood could be ineffective. In this step, several types of moves will be employed to find a better solution. These moves are commonly embedded in SA heuristics and other meta-heuristics.

3.2.1 Location moves

The location moves determine the number and location of facilities. Two basic types of moves are featured: swap move and insertion move. At each iteration, the next solution S'_k is selected from $N(s_k)$ by a swap move and an insertion move iteratively.

The swap move is performed by randomly selecting the ith facility and the jth facility in the same echelon, and then exchanging the position of the two, i.e., an open facility is closed, and a closed facility is opened. This move does not change the number of open facilities.

The insertion move is performed by randomly selecting the ith facility and inserting it into the position immediately before another randomly selected jth facility, i.e., we increase the number of open facilities.

3.2.2 Routing moves

In this step, several moves will be applied to improve existing routes. From the first feasible solution, the definition and the optimization of the routes is based on three phases.

Phase 1: definition of multi-stop routes and improvements of a single route assigned to a single facility:

- 2-opt move [20] and Or-opt [21] move for intra-route improvement;
- swap/shift [22] and 2-opt* [23] move for inter-route improvement.

Phase 2: optimization of multiple routes assigned to a single facility:

- swap move for a single facility;
- insertion move for a single facility.

Phase 3: optimization of multiple routes assigned to multiple facilities:

- swap moves for multiple facilities;
- insertion moves for multiple facilities.

After applying the location moves and routing moves on each echelon, four sub-problems are locally optimized. At this point, two other steps are performed to obtain a global solution. First, the sub-problems of each echelon are combined; then the sub-problems of the two echelons are combined.

3.3 Phase III: Path Relinking (PR)

Path relinking was originally introduced by Glover and Laguna [24] as an intensification strategy with tabu search as a way of exploring trajectories between elite solutions. This method is based on the idea that good solutions to a problem should share some common characteristics. By generating paths (i.e., sequences of intermediate solutions) between elite solutions, a person could reasonably hope to find better ones. Glover [25,26] and Glover, Laguna and Marti. [27] have provided excellent surverys of PR. In the PR procedure, an initial solution and a guiding solution are chosen in a reference set of elite solutions to represent the starting and ending points of the path. And the attributes contained in the guiding solutions are then incorporated into the intermediate solutions initially originated in the initiating solution; these solutions ultimately contain fewer characteristics from the initial solution and more from guiding solutions as one moves along the path.

In Path II, each solution obtained is feasible and unique. Top candidates among these solutions with better objective function values are kept in the elite set P^{elite}. In the proposed SA-PR algorithm (Fig. 4), the PR is used in two places. From line 7 to line 9, a new local optimum and its most similar solution in P^{elite} are relinked. In line 13, all pairs of elite solutions in P^{elite} are linked.

Table 1: Main features of benchmark data sets

Set	Nguyen	Prodhon	Sterle
File format	P-S-NM or n-m-N	P-S-β a or P-S-β b	Test set P-S-C
No. of instances	24	30	93 (Usually use 30)
No of CDCs	1	1	[2,5]
No. of RDCs	{5, 10}	{5, 10}	[3-20]
No. of customers	{25, 50, 100, 200}	{20, 50, 100, 200}	[8,200]
1st level vehicle capacity	{750, 850} Suffix "b" means capacity is 850.	1.5 $\times$ max {RDCs}	500 (up to 10 customers) 1500 (up to 100 customers) 3000 (up to 200 customers)
2nd level vehicle capacity	{100, 150}	{70, 150}	100 (up to 10 customers) 200 (up to 100 customers) 500 (up to 200 customers)
Customer location	Either a normal distribution (N in the file name) or a multinormal distribution (NM).	Cluster $\beta = \{1, 2, 3\}$ $\beta = 1$ is a uniform distribution. Suffix "bis" means strongly separated clusters.	Randomly distributed
Customer demand	A normal distribution with mean $\mu = 15$ and variance $\sigma^2 = 25$	Uniform distribution in [10,20]	Randomly generated in the range [1,100].

4 Evaluation and discussion

In this section, standard benchmark data sets for 2E-LRP were used to test the effectiveness of our proposed SA-PR meta-heuristic. These include Nguyen instances, Prodhon instances, and Sterle instances. The main features of these three data sets are summarized in Table 1.

The SA-PR algorithm was developed in a JDK 7 environment, and all experiments were executed on a desktop Intel Xeon CPU E5-2650 (2 processors) 2.0 GHZ with 16G of RAM.

4.1 Configuration of parameters

Prior to conducting the experiments, several trial runs were undertaken to tune the algorithm parameters, thus ensuring convergence and speed, and to obtain high-quality solutions within a reasonable computation time. These parameters are α, γ, β, Δ, L, $\omega_{\max}$, and N^{elite}. Parameter α assures a higher probability of determining a feasible assignment. Parameter γ relates the cost and temperature, and as suggested by Czech and Czarnas [28], γ is set to 1. The parameter β is the cooling ratio. The temperature should be decreased in such a way to avoid excessively long Markov chains, since we obtain one Markov chain node for each temperature value. The parameter Δ represents the probability of accepting worse solutions. The random() returns a random value in the range $[0,1]$. If the acceptance probability function $P(T_k,T,D) >$ random(), accept the solution; otherwise do not accept it. Parameter L is the maximum number of iterations for temperature T. The parameter $\omega_{\max}$ is the maximum number of iterations for which a non-improved solution is obtained, which should be sufficient to achieve a good solution. But this must be balanced against the fact that a larger $\omega_{\max}$ value will increase the computation time, and the number of moves actually accepted is rather small at low temperatures. N^{elite} denotes the size of the elite set.

Our experiments verify the expected behavior. Higher cooling-ratio values correspond to slower cooling schedules, and therefore more reduction steps are required for the algorithm to stop. The above parameter values were obtained through fine tuning and are listed in Table 2. In general, the performance of the p-SA algorithms is highly sensitive to the cooling schedule.

Table 2: Parameter-setting summary

Parameter	α	γ	β	L	$\omega_{\max}$	Δ	N^{elite}
SA	0.93	1	0.99	40	200	0.4	–
SA-PR	0.95	1	0.92	40	200	0.4	5

4.2 Computational analysis

This subsection presents the computational results of the proposed SA-PR for the 2E-LRP problem. Two criteria for comparison were adopted: (i) the solution difference between the proposed algorithm and the best-known solution (BKS) in the literature; (ii) the CPU time that obtains the best solution by the SA-PR and the existing approaches.

4.2.1 Results for 2E-LRP instances with one main depot

This subsection reports our results on the instances with a single main depot (Nguyen and Prodhon), and the next subsection concerns Sterle's instances with multiple depots.

After Nguyen, Prins and Prodhon [29] generated the Nguyen data set (24 test instances) and the Prodhon data set (30 test instances), using GRASP-LP for solving the 2E-LRP problem, other metaheuristics have been developed, such as MS-ILS [17], MS-ILS-PR [17],

Table 3: Results comparison from literature for the Nguyen data set [29]

Instance	BKS	GRASP-LP			MS-ILS			MS-ILS-PR			GRASP-LP-PR			ALNS			VNS (20 runs)			SA			SA-PR		
		Cost	Gap%	T_1	Cost	Gap%	T_2	Cost	Gap%	T_3	Cost	Gap%	T_4	Cost	Gap%	T_5	Cost	Gap%	T_6	Cost	Gap%	T_*	Cost	Gap%	T_{**}
25-5N	80370	81152	0.97	0.9	**80370**	0.00	1.6	**80370**	0.00	1.6	**80370**	0.00	3.1	**80370**	0.00	2.2	**80370**	0.00	38.0	**80370**	0.00	1.4	**80370**	0.00	1.21
25-5Nb	64562	64572	0.02	0.8	**64562**	0.00	1.1	**64562**	0.00	1.1	**64562**	0.00	2.6	**64562**	0.00	1.8	**64562**	0.00	36.4	**64562**	0.00	1.1	**64562**	0.00	0.85
25-5MN	78947	80412	1.86	0.9	79674	0.92	1.6	79674	0.92	1.6	**78947**	0.00	3.2	79674	0.92	1.6	**78947**	0.00	50.6	**78947**	0.00	2.4	**78947**	0.00	2.09
25-5MNb	64438	64438	0.00	0.8	**64438**	0.00	1.5	**64438**	0.00	1.5	**64438**	0.00	4.1	**64438**	0.00	6.8	**64438**	0.00	30.6	**64438**	0.00	3.0	**64438**	0.00	2.44
50-5N	137815	145942	5.90	2.4	138126	0.23	10.4	138126	0.23	10.4	138126	0.23	13.7	143328	4.00	3.9	**137815**	0.00	74.2	**137815**	0.00	10.4	**137815**	0.00	9.13
50-5Nb	110094	113234	2.85	2.3	111290	1.09	6.3	111290	1.09	6.3	111062	0.88	11.7	112764	2.43	3.6	**110094**	0.00	113.8	**110094**	0.00	9.3	**110094**	0.00	7.17
50-5MN	123484	126313	2.29	2.2	**123484**	0.00	5.2	**123484**	0.00	5.2	**123484**	0.00	9.1	123920	0.35	3.1	**123484**	0.00	105.2	**123484**	0.00	10.9	**123484**	0.00	9.06
50-5MNb	105401	106033	0.60	2.3	**105401**	0.00	7.7	**105401**	0.00	7.7	**105401**	0.00	13.6	105846	0.42	3.9	**105401**	0.00	77.5	**105401**	0.00	12.1	**105401**	0.00	10.15
50-10N	115725	116709	0.85	4.5	116132	0.35	36.8	116132	0.35	36.8	116132	0.35	46.6	116132	0.35	7.1	**115725**	0.00	90.3	115732	0.01	27.8	**115725**	0.00	24.35
50-10Nb	87315	90559	3.72	6.4	**87315**	0.00	15.9	**87315**	0.00	15.9	**87315**	0.00	22.4	**87315**	0.00	10.0	**87315**	0.00	102.0	87354	0.04	17.9	**87315**	0.00	14.09
50-10MN	135519	137321	1.33	5.4	136123	0.45	21.8	136123	0.45	21.8	135748	0.17	37.5	136337	0.60	9.5	**135519**	0.00	76.3	135519	0.00	28.1	**135519**	0.00	24.26
50-10MNb	110613	110703	0.08	6.7	**110613**	0.00	19.4	**110613**	0.00	19.4	**110613**	0.00	42.4	**110613**	0.00	10.5	**110613**	0.00	49.2	110684	0.06	39.4	**110613**	0.00	30.88
100-5N	193228	200974	4.01	5.9	196910	1.91	10.7	196910	1.91	10.7	196910	1.91	13.1	196999	1.95	8.0	**193228**	0.00	224.5	194087	0.44	17.5	194087	0.44	13.58
100-5Nb	158927	160488	0.98	5.3	159989	0.67	21.0	159989	0.67	21.0	159086	0.10	33.1	159714	0.50	8.5	**158927**	0.00	234.9	159837	0.57	18.7	159837	0.57	16.32
100-5MN	204682	210381	2.78	5.3	208177	1.71	17.0	208177	1.71	17.0	207119	1.19	25.5	207141	1.20	8.4	**204682**	0.00	271.2	205218	0.26	23.8	205218	0.26	18.69
100-5MNb	165744	170513	2.88	6.1	166640	0.54	28.3	166640	0.54	28.3	166115	0.22	41.3	167466	1.04	9.1	**165744**	0.00	220.9	166141	0.24	38.1	166141	0.24	32.21
100-10N	212729	229246	7.76	20.7	218040	2.50	126.0	218040	2.50	126.0	215792	1.44	132.5	215792	1.44	32.4	212847	0.06	150.8	213684	0.45	107.3	213684	0.45	84.24
100-10Nb	155489	162308	4.39	20.3	157267	1.14	57.8	157267	1.14	57.8	156401	0.59	76.9	160322	3.11	29.5	**155489**	0.00	177.2	156451	0.62	48.1	156451	0.62	38.68
100-10MN	201275	210496	4.58	17.9	206450	2.57	91.6	206450	2.57	91.6	205964	2.33	156.1	209478	4.08	26.3	**201275**	0.00	155.4	202084	0.40	108.4	202084	0.40	84.24
100-10MNb	170625	172276	0.97	21.0	170706	0.05	111.9	170706	0.05	111.9	170706	0.05	192.4	171872	0.73	38.5	**170625**	0.00	178.7	170894	0.16	148.7	170894	0.16	128.18
200-10N	346181	361971	4.56	29.8	355185	2.60	299.4	355185	2.60	299.4	353685	2.17	240.8	357286	3.21	35.9	347395	0.35	420.5	351841	1.63	227.3	351841	1.63	183.29
200-10Nb	256171	267733	4.51	52.3	263157	2.73	281.1	263157	2.73	281.1	262072	2.30	358.8	264241	3.15	77.6	**256171**	0.00	492.4	260771	1.80	214.5	260242	1.59	176.10
200-10MN	325747	348866	7.10	34.4	336097	3.18	483.5	336097	3.18	483.5	332345	2.03	523.1	337748	3.68	48.5	326454	0.22	547.3	331775	1.85	387.4	331056	1.63	331.18
200-10MNb	289239	302500	4.58	59.6	292523	1.14	549.2	292523	1.14	549.2	292654	1.18	690.0	298556	3.22	85.5	289742	0.17	689.5	294681	1.88	690.0	293627	1.52	577.43
Ave gap (%)			2.90			0.99			0.71			1.52			0.03			0.15			0.43			0.40	
Std dev (%)			2.23			1.06			0.87			1.44			0.09			0.67			0.65			0.58	
BKS found			1			7			8			5			20			22			9			12	
Avg CPU (s)				13.1			92.0			112.2			19.7			192.0			274.7			91.4			75.8

Note: Values written in bold represent BKS.

T_1, T_2, T_3, and T_4: CPU times in seconds executed on a 3.4 GHz pentium 4 PC with 1 GB of RAM.

T5: CPU times in seconds executed on a Xeon E5462 3.0 GHz.

T6: CPU times in seconds executed on a CPU times on 2.53 GHZ Xeon E5540.

GRASP-LP-PR [16], ALNS [12], and VNS [30]. The solutions provided by these algorithms were compared, and the algorithm that produced the best solutions was found to vary from instance to instance. Table 3 and Table 4 compare the results achieved by these algorithms, which includes these performance metrics: cost, Gap%, and CPU time. (the Gap% shown in this table is given by (BKS − the algorithm)/BKS × 100%).

We first present the results in Table 3 for the Nguyen data set. SA outperforms on average the GRASP and MS-ILS algorithms. It achieves a smaller solution gap (0.43%), better than MS-ILS-PR (0.71%), MS-ILS (0.99%), GRAPS-LP-PR (1.52%), and GRASP-LP (2.90%), and worse than ALNS (0.03%) and VNS (0.15%). The SA results were improved further by adding path relinking. The average gap% was reduced from 0.43% to 0.40%, the standard deviation was reduced from 0.65% to 0.58%, the number of BKS was increased from 9 to 12, and the average CPU times was down from 91.4s to 75.8s. From the perspective of CPU time, the SA-PR algorithm outperformed the other 7 algorithms. However, these reported times were performed on different computer configurations.

The result for the 30 2E-LRP instances derived from Prodhon's CLRP benchmarks are presented in Table 4. The average solution gap and the number of BKS led to the same hierarchy as before: VNS (0.04%) < ALNS (0.27%) < SA (0.36%) < SA-PR (0.43%) < MS-ILS-PR (0.93%) < MS-ILS (1.16%) < GRASP-LP-PR (1.79%) < GRASP-LP (2.57%), again at the expense of augmented running times. On average, these instances with a uniform distribution of customers or a partition of clusters look a bit easier than Nguyen's instances, which are based on normal and multinormal distributions. All heuristics require less time and produce slightly reduced gaps. It should again be noted that the reported CPU times are for different computer configurations.

In addition to the best solutions, SA (0.56%) and SA-PR (0.58%) are also more robust than the GRASP (1.72%), MS-ILS (1.27%), MS-ILS-PR (1.08%), and GRASP-LP-PR (1.32%), when we consider the standard deviation of gaps to BKS over the set of instances (Std dev in the tables).

For the two sets of instances, we performed a Friedman test to compare statistically the four algorithms in terms of solution costs. On Nguyen's instance, the test gives a chi-square value of 114.68 and p-value smaller than 0.001. On Prodhon's instances, we get chi-square = 155.47 and again a p-value less than 0.001. Therefore the null hypothesis (the eight heuristics have equivalent performances) can be rejected for significance level 0.05 and 0.01.

Table 4: Results comparison from literature for the Prodhon data set [29]

Instance	BKS	GRASP-LP[1]			MS-ILS[2]			MS-ILS-PR[3]			GRASP-LP-PR[4]			ALNS[5]			VNS (20 runs)[6]			SA			SA-PR		
		Cost	Gap%	T_1	Cost	Gap%	T_2	Cost	Gap%	T_3	Cost	Gap%	T_4	Cost	Gap%	T_5	Cost	Gap%	T_6	Cost	Gap%	T_*	Cost	Gap%	T_{**}
20-5-1	89075	92633	3.99	0.6	**89075**	0.00	0.9	**89075**	0.00	2.4	91062	2.23	1.2	**89075**	0.00	26.3	**89075**	0.00	63.46	**89075**	0.00	1.6	**89075**	0.00	1.22
20-5-1b	61863	63094	1.99	0.6	**61863**	0.00	0.5	**61863**	0.00	2.6	64090	3.60	1.7	**61863**	0.00	26.1	**61863**	0.00	82.84	**61863**	0.00	1.7	**61863**	0.00	1.29
20-5-2	84478	86212	2.05	0.4	85290	0.96	0.7	85290	0.96	1.6	86118	1.94	0.6	**84478**	0.00	28.9	**84478**	0.00	62.22	**84478**	0.00	1.0	**84478**	0.00	0.72
20-5-2b	60838	**60838**	0.00	0.4	**60838**	0.00	0.4	**60838**	0.00	1.4	**60838**	0.00	1.1	**60838**	0.00	29.1	**60838**	0.00	125.43	**60838**	0.00	1.2	**60838**	0.00	0.89
50-5-1	130843	136137	4.05	2.2	134855	3.07	5.8	134855	3.07	9.2	135549	3.60	3.5	**130843**	0.00	96.9	**130843**	0.00	80.1	**130843**	0.00	7.3	**130843**	0.00	5.43
50-5-1b	101530	109173	7.53	1.9	101687	0.15	6.9	**101530**	0.00	15.8	102286	0.74	3.8	**101530**	0.00	94.2	**101530**	0.00	127.87	**101530**	0.00	12.7	**101530**	0.00	9.53
50-5-2	131825	136785	3.76	2.1	132159	0.25	5.6	132159	0.25	12.9	132364	0.41	3.0	**131825**	0.00	130.6	**131825**	0.00	96.71	**131825**	0.00	10.1	**131825**	0.00	7.49
50-5-2b	110332	111440	1.00	2.1	110658	0.30	8.2	110547	0.19	18.6	112810	2.25	3.3	**110332**	0.00	160.7	**110332**	0.00	198.21	**110332**	0.00	15.7	**110332**	0.00	12.06
50-5-2BIS	122599	124440	1.50	3.0	122654	0.04	19.4	122654	0.04	27.6	124375	1.45	5.1	**122599**	0.00	121.2	**122599**	0.00	111.58	**122599**	0.00	22.5	**122599**	0.00	17.89
50-5-2bBIS	105696	108718	2.86	2.6	106197	0.47	19.7	105776	0.08	30.0	105776	0.08	6.6	**105696**	0.00	131.6	**105696**	0.00	197.73	**105696**	0.00	24.8	**105696**	0.00	19.38
50-5-3	128379	129225	0.66	3.1	**128379**	0.00	11.8	**128379**	0.00	16.2	128903	0.41	4.5	**128379**	0.00	88.4	**128379**	0.00	79.75	**128379**	0.00	13.7	**128379**	0.00	10.80
50-5-3b	104006	104057	0.05	2.4	**104006**	0.00	9.2	**104006**	0.00	17.0	**104006**	0.00	3.8	**104006**	0.00	114.9	**104006**	0.00	131.25	**104006**	0.00	14.1	**104006**	0.00	10.34
100-5-1	318225	324823	2.07	5.9	321102	0.90	27.5	320130	0.60	37.3	321870	1.15	7.5	319137	0.29	646.9	**318225**	0.00	225.73	319138	0.29	28.0	**318225**	0.00	20.16
100-5-1b	256991	261529	1.77	5.4	259156	0.84	17.2	258256	0.49	32.0	259791	1.09	8.7	257349	0.14	1179.9	**256991**	0.00	301.11	258317	0.52	24.5	**256991**	0.00	17.86
100-5-2	231305	236937	2.43	4.6	234574	1.41	16.9	234574	1.41	29.6	235277	1.72	6.5	**231305**	0.00	316.0	**231305**	0.00	203.7	231331	0.01	23.4	**231305**	0.00	17.93
100-5-2b	194729	197035	1.18	4.3	196250	0.78	11.2	196149	0.73	24.9	196228	0.77	6.4	**194729**	0.00	1641.0	**194763**	0.02	240.39	196149	0.73	19.1	**194763**	0.02	14.02
100-5-3	244194	250398	2.54	4.4	246637	1.00	40.0	244661	0.19	68.0	248893	1.92	6.2	**244194**	0.00	375.5	**244470**	0.11	174.1	244661	0.19	48.2	**244470**	0.11	35.87
100-5-3b	194110	197279	1.63	4.8	**195254**	0.59	13.8	**195254**	0.59	54.3	**195254**	0.59	8.8	**194110**	0.00	377.0	195381	0.65	180.41	195254	0.59	42.3	195254	0.59	32.85
100-10-1	352694	370603	5.08	10.4	362957	2.91	181.4	362957	2.91	336.4	364988	3.49	17.1	358058	1.52	158.9	**352694**	0.00	233.37	352899	0.06	185.7	352899	0.06	143.87
100-10-1b	297167	314277	5.76	10.2	311779	4.92	181.7	303400	2.10	333.5	313072	5.35	18.1	**297167**	0.00	155.2	**298186**	0.34	299.03	299426	0.76	178.4	299426	0.76	128.73
100-10-2	304507	310929	2.11	9.3	307302	0.92	177.0	306303	0.59	362.0	309621	1.68	13.7	305402	0.29	270.0	**304507**	0.00	247.66	306303	0.59	198.0	306303	0.59	141.43
100-10-2b	264092	264984	0.34	12.9	264543	0.17	107.2	264389	0.11	294.1	266393	0.87	17.2	265138	0.40	348.2	**264092**	0.00	307.05	264389	0.11	134.1	264389	0.11	102.96
100-10-3	311447	319806	2.68	9.2	315805	1.40	181.7	313249	0.58	370.8	318209	2.17	16.1	313517	0.66	215.7	**311447**	0.00	226.64	313293	0.59	217.2	**311447**	0.00	153.11
100-10-3b	260516	268413	3.03	11.0	266383	2.25	129.1	266657	2.36	340.4	267643	2.74	16.0	264096	1.37	256.9	**260516**	0.00	302.85	261383	0.33	192.8	261371	0.33	149.34
200-10-1	548730	570210	3.91	26.5	559428	1.95	553.0	557099	1.53	700.9	561103	2.25	39.7	552816	0.74	1039.2	**548730**	0.00	1009.49	557145	1.53	384.9	557145	1.53	295.70
200-10-1b	445791	454181	1.88	20.6	452731	1.56	673.0	452286	1.46	723.7	453286	1.68	47.1	448236	0.55	1811.9	**445791**	0.00	634.59	452392	1.48	397.0	452392	1.48	304.77
200-10-2	497451	508450	2.21	22.4	505400	1.60	211.1	505333	1.58	220.2	506345	1.79	41.9	498199	0.15	576.4	**497451**	0.00	1158.23	505489	1.62	87.4	505489	1.62	61.31
200-10-2b	422668	429075	1.52	21.9	425311	0.63	248.1	425311	0.63	267.3	427147	1.06	37.4	423048	0.09	1723.8	**422668**	0.00	730.12	425358	0.64	98.2	425215	0.60	73.83
200-10-3	527162	541754	2.77	26.0	533732	1.25	632.9	533993	1.30	676.3	538821	2.21	43.2	534569	1.41	741.3	**527162**	0.00	970.42	533772	1.25	278.9	533772	1.25	204.24
200-10-3b	402117	421585	4.84	18.3	419790	4.39	222.9	419047	4.21	323.6	419984	4.44	35.2	404284	0.54	1091.9	**402117**	0.00	591.9	409081	1.73	117.4	409081	1.73	88.90
Ave gap (%)			2.57			1.16			0.93			1.79			0.27			0.04			0.43			0.36	
Std dev (%)			1.72			1.27			1.08			1.32			0.45			0.13			0.56			0.52	
BKS found			1			5			6			2			17			26			12			16	
Avg CPU (s)				8.3			123.8			178.4			14.2			465.8			313.1			92.7			69.5

Note: Values written in bold represent BKS.

T1, T2, T3, and T4: CPU times in seconds executed on a 3.4 GHz pentium 4 PC with 1 GB of RAM.

T5: CPU times in seconds executed on a Xeon E5462 3.0 GHz.

T6: CPU times in seconds executed on an CPU times on 2.53 GHZ Xeon E5540.

4.2.2 Results for 2E-LRP instances with several depots

After Sterle [9] generated 93 test instances and used GRASP-PR for solving the 2E-LRP problem, other metaheuristics have been developed, such as Decomposition App (DA) [14], Tabu Search [14], MS-ILS [17], ALNS [12], and VNS [30]. The solutions provided by these algorithms were compared; it was found that the ALNS [12] or VNS [30] produced the best solution. Table 5 compares the results achieved by these algorithms, which includes the same statistical performance measures as the ones provided for our instances: cost, Gap%, and CPU time. (The Gap% shown in this table is given by (BKS − the algorithm)/BKS × 100%.) The BKS for each instance is noted in Table 5.

On average, our SA-PR outperforms the other methods in terms of solution quality with the hierarchy: SA-PR (0.21%) < SA (0.52%) < VNS (0.62%) < ALNS (0.83%) < MS-ILS (3.20%) < TS2 (6.65%) < DA (13.14%). Even if the respective computer speeds are considered, SA-PR only underperformed VNS. Another advantage of SA-PR is its smallest standard deviation, which means that the average stabilities of our SA-PR algorithm are the best.

5 Summary and conclusion

In this paper, we considered the design of the distribution network for a "collect-on-delivery" company in a metropolitan context, which is the two-echelon location routing problem (2E-LRP) with limited capacities for distribution centers and vehicles alike. The complexity of real-life medium-and-large scale instances prohibits exact solutions to the problem, making it necessary to implement heuristic or meta-heuristic approaches to provide approximate solutions within a reasonable computational time. This paper has proposed a simulated annealing (SA) meta-heuristic coupled with path relinking (SA-PR) to solve this problem. To the best of our knowledge, this is the first implementation of an SA-PR approach to solve this problem.

To demonstrate the effectiveness of the SA-PR, we have carried out an experimental study on the 2E-LRP

Table 5: Results comparison from literature for the Sterle data set [9]

Instance	BKS	DA[1]			TS2[2]			MS-ILS (5 runs)[3]			ALNS[4]			VNS (20 runs)[5]			SA			SA-PR		
		Cost	Gap%	T_1	Cost	Gap%	T_2	Cost	Gap%	T_3	Cost	Gap%	T_4	Cost	Gap%	T_5	Cost	Gap%	T_*	Cost	Gap%	T_{**}
I1-5850	1132.63	1226.24	8.26	4421.3	1210.27	6.85	521.7	1171.69	3.45	103.4	**1132.63**	0.00	341.8	1162.44	2.63	168.54	**1132.63**	0.00	106.76	**1132.63**	0.00	73.36
I1-51050	1132.63	1783.60	57.47	6134.9	1256.59	10.94	853.6	1207.31	6.59	168.1	1162.44	2.63	321.5	**1132.63**	0.00	189.48	**1132.63**	0.00	248.18	**1132.63**	0.00	167.34
I1-51075	1540.23	1591.60	3.34	7512.6	1591.60	3.34	1026.1	1561.50	1.38	225.9	**1540.23**	0.00	507.2	**1540.23**	0.00	237.73	1558.62	1.19	297.59	**1540.23**	0.00	203.22
I1-51575	1686.21	1783.60	5.78	6134.9	1708.79	1.34	2614.1	1700.32	0.84	645.7	**1686.21**	0.00	527.3	**1686.21**	0.00	265.8	1697.13	0.65	907.69	**1686.21**	0.00	635.33
I1-510100	2124.09	2247.32	5.80	8033.8	2257.35	6.27	1906.2	2192.14	3.20	658.9	2124.90	0.04	659.9	**2124.09**	0.00	353.37	2147.51	1.10	758.81	**2124.09**	0.00	536.88
I1-520100	1973.08	2055.88	4.20	10218.1	2071.76	5.00	3780.6	1989.48	0.83	1969.7	**1973.08**	0.00	705.2	**1973.08**	0.00	491.68	1983.64	0.54	2536.21	1983.64	0.54	1855.12
I1-510150	1883.44	2177.77	15.63	8407.1	2097.81	11.38	3740.4	1953.55	3.72	330.9	**1883.44**	0.00	1187.8	**1883.44**	0.00	1241.05	1880.31	−0.17	347.51	1880.31	−0.17	249.54
I1-520150	1869.53	1933.82	3.44	7786.6	1919.35	2.66	3271.9	1905.81	1.94	2891.3	**1869.53**	0.00	1203.1	**1869.53**	0.00	1358.06	1854.9	−0.78	3286.56	**1869.53**	0.00	2312.52
I1-510200	2443.80	2625.11	7.42	10119.5	2601.33	6.45	2239.1	2646.27	8.29	947.2	**2443.80**	0.00	1431.0	**2443.8**	0.00	2116.14	2468.21	1.00	1348.96	2468.21	1.00	943.37
I1-520200	2219.54	3140.17	41.48	12750.3	2407.33	8.46	6037.4	2307.58	3.97	1229.3	**2219.54**	0.00	1511.5	**2219.54**	0.00	1932.42	2238.61	0.86	1451.10	2238.61	0.86	991.62
Average			15.28	8151.9		6.27	2599.1		3.42	917.0		0.27	839.6		0.26			0.44			0.22	
I2-5850	1121.13	1185.75	5.76	2023.3	1185.75	5.76	665.2	1123.42	0.20	86.9	1256.44	12.07	410.8	**1121.13**	0.00	180.43	1121.13	0.00	129.89	**1121.13**	0.00	94.57
I2-51050	1121.13	1325.61	18.24	5039.5	1335.81	19.15	390.6	1265.73	12.90	89.3	**1121.13**	0.00	386.6	1256.44	12.07	189.23	1134.44	1.19	119.92	**1121.13**	0.00	85.00
I2-51075	1691.15	1768.88	4.60	7061.0	1756.13	3.84	1252.9	1718.25	1.60	156.9	**1691.15**	0.00	638.7	**1691.15**	0.00	250.38	1703.72	0.74	185.88	**1691.15**	0.00	134.23
I2-51575	1644.79	1644.79	0.00	9499.4	1644.79	0.00	944.4	1751.14	6.47	668.9	1742.25	5.93	670.1	**1644.79**	0.00	291.24	1621.62	−1.41	738.10	1621.62	−1.41	487.10
I2-510100	2231.21	2391.17	7.17	10379.6	2290.64	2.66	769.2	2324.10	4.16	212.8	**2231.21**	0.00	870.9	**2231.21**	0.00	356.15	2242.93	0.53	263.87	**2231.21**	0.00	196.86
I2-520100	1996.34	2051.39	2.76	12405.6	2041.13	2.24	2608.4	2039.25	2.15	1386.3	**1996.34**	0.00	872.0	**1996.34**	0.00	488.67	1996.34	0.00	2040.79	**1996.34**	0.00	1491.71
I2-510150	1728.05	2111.97	22.22	14060.9	1907.71	10.40	4852.9	1768.79	2.36	787.3	**1728.05**	0.00	1004.6	**1728.05**	0.00	1027.37	1734.88	0.40	836.16	1734.88	0.40	572.66
I2-520150	1630.29	1800.89	10.46	10134.5	1707.73	4.75	4540.7	1664.20	2.08	1627.2	**1630.29**	0.00	1070.3	**1630.29**	0.00	1199.85	1646.72	1.01	1771.30	**1630.29**	0.00	1319.48
I2-510200	2147.51	2430.93	13.20	8871.8	2407.88	12.12	1078.9	2292.47	6.75	1172.2	**2147.51**	0.00	1477.2	**2147.51**	0.00	1556.07	2165.62	0.84	1516.27	2165.62	0.84	1030.57
I2-520200	2049.01	2274.29	10.99	15602.1	2223.72	8.53	7850.5	2097.74	2.38	3091.9	**2049.01**	0.00	1736.0	**2049.01**	0.00	1912.19	2073.21	1.18	4022.28	2073.21	1.18	2965.68
Average			9.54	9507.8		6.95	2495.4		4.10	928.0		1.80	913.7		1.21			0.45			0.10	
I3-5850	1162.44	1298.89	11.74	7741.9	1240.80	6.74	474.9	1171.35	0.77	75.1	1207.31	3.86	341.2	**1162.44**	0.00	164.38	1162.44	0.00	78.19	**1162.44**	0.00	57.44
I3-51050	1162.44	1256.68	8.11	4929.6	1243.87	7.01	919.5	1208.43	3.96	149.8	**1162.44**	0.00	353.2	1207.31	3.86	191.93	1193.32	2.66	182.55	1193.32	2.66	132.28
I3-51075	1721.47	1879.56	9.18	13720.0	1839.38	6.85	806.9	1732.33	0.63	95.5	**1721.47**	0.00	691.0	**1721.47**	0.00	248.58	1731.71	0.59	118.19	**1721.47**	0.00	80.12
I3-51575	1483.14	1704.65	14.94	12903.9	1590.00	7.20	1910.9	1491.31	0.55	306.1	**1483.14**	0.00	634.7	**1483.14**	0.00	260.46	1487.56	0.30	418.08	**1483.14**	0.00	291.68
I3-510100	2178.35	2601.44	19.42	20599.6	2294.44	5.33	546.6	2238.70	2.77	157.3	2187.35	0.41	922.4	**2178.35**	0.00	352.98	2186.92	0.39	225.53	**2178.35**	0.00	167.27
I3-520100	2035.37	2261.36	11.10	15724.5	2170.45	6.64	696.2	2053.12	0.87	1592.6	**2035.37**	0.00	755.1	**2035.37**	0.00	481.84	2017.38	−0.88	2071.58	2017.38	−0.88	1459.62
I3-510150	1274.44	1470.77	15.41	243.9	1342.18	5.32	2635.1	1307.19	2.57	1160.1	**1274.44**	0.00	1040.0	**1274.44**	0.00	953.29	1282.33	0.62	1227.40	**1274.44**	0.00	885.42
I3-520150	1235.86	1508.07	22.03	21240.5	1343.72	8.73	3379.3	1266.83	2.51	2441.8	**1235.86**	0.00	1062.9	**1235.86**	0.00	1003.96	1246.20	0.84	3434.91	**1235.86**	0.00	2326.81
I3-510200	1766.46	2193.32	24.16	41145.1	1893.68	7.20	2633.5	1822.50	3.17	2127.1	**1766.46**	0.00	1690.0	**1766.46**	0.00	1432.12	1789.12	1.28	3163.27	1789.12	1.28	2354.06
I3-520200	2531.21	2784.47	10.01	23319.4	2692.31	6.36	2765.4	2604.56	2.90	1245.3	**2531.21**	0.00	1608.0	**2531.21**	0.00	1895.71	2557.61	1.04	1591.67	**2531.21**	0.00	1038.16
Average			14.61	16156.8		6.74	1676.8		2.07	935.1		0.43	909.9		0.39	698.5		0.68			0.31	
Ave gap (%)			13.14			6.65			3.20			0.83			0.62			0.52			0.21	
Std dev (%)			11.86			3.74			2.72			2.51			2.32			0.77			0.71	
BKS found			1			1			0			24			27			5			19	
Avg CPU (s)				11272.2			2257.1			926.7			887.7			759.7			1180.8			838.0

Note: Values written in bold represent BKS.

T1 and T2: CPU times in seconds executed on a 2.4GHz Pentium 4 PC with 4GB of RAM.

T3: CPU times in seconds executed on a 3.4 GHz pentium 4 PC with 1 GB of RAM.

T4. CPU times in seconds executed on a Xeon E5462 3.0 GHz.

T5. CPU times in seconds executed on an CPU times on 2.53 GHZ.

(three sets with 84 test problems from Nguyen, Prodhon, and Sterle's benchmark), with 1–5 main depots, 9–20 satellites, and 20–200 customers. Our methods are competitive, since they outperform GRAPS-LP, MS-ILS, MS-ILS-PR, and GRASP-LP-PR algorithm and only underperform ALNS and VNS in the Nguyen's and Prodhon's data sets; they outperform the other five algorithms in the Sterle data set.

The proposed model points to several other avenues for future work. First, additional research is needed to explore other local search techniques to improve the proposed SA-PR. Another research extension would be to apply the SA-PR to other problems, such as consideration of the specific service time of customers, which is a two-echelon location routing problem with time windows (2E-LRPTW).

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