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A Soft Computing Approach for Ranking Firms based on Experts' Valuation of Corporate Social Responsibility

Vicente Liern[1], *Paz Méndez-Rodríguez*[2] *and Blanca Pérez-Gladish*[2,*]

[1] Departamento de Matemáticas para la Economía y la Empresa. Universitat de València, Spain.
[2] Departamento de Economía Cuantitativa, Universidad de Oviedo, Spain.

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Abstract: In this work we focus on the obtaining of an integrative measure of Corporate Social Responsibility which does not require a unique precise definition of this concept. The proposed method will allow the ranking of firms based on this integrative measure which will incorporate all the available information from different sources. Moreover, a Soft Computing method will be applied based on interval-valued fuzzy sets reflecting the uncertain, imprecise and fuzzy nature of social performance criteria. In order to illustrate the proposed method a real case study is presented.

Keywords: Corporate Social Responsibility; Firms' ranking; Interval-valued fuzzy sets

1 Introduction

The issues surrounding Corporate Social Responsibility (CSR) have led, in recent years, to a growing interest on the part of various parties: researchers, consumers, civil servants, NGO's, governments, the media, social networks, etc. The succession of various world summits (i.e. Rio in 1992, Kyoto in 1997, Johannesburg in 2002, Copenhagen in 2009 and Rio in 2012) demonstrates the central role of social and environmental issues in the world of business. For several decades, many investors whether individuals or institutions, have tried to select those enterprises who are socially responsible or whose activities conform to their values. Recent environmental crises (i.e. Exxon in 1989; BP in 2010), financial crises (i.e. Enron in 2001; WorldCom in 2002) and social crises (i.e. Nike in 1997; Wal-Mart in 2005) have reinforced this trend of Socially Responsible Investment (SRI). In fact, the volume of SRI at the end of 2011 was 11.23% of the 33.3 trillion of total assets under management in the U.S. [24]. The recent engagement of investors, especially those from institutions, in adopting responsible practices has, amongst other consequences, allowed SRI to become a credible means of investment, and has made the social performance of a company an indispensable component of its extra-financial evaluation. Companies have, now more than ever, to integrate social and environmental concerns into their activities and into their relationships with their stakeholders. In order to respond to the needs of investors and to contribute to the growth of SRI, several tools have been developed: codes of conduct, certifications and social and environmental ratings. The United Nations, with its Principles for Responsible Investment (PRI), has also reinforced the growth of this movement (see http://www.unpri.org).

Nevertheless, despite the remarkable growth and the abundance of research around the concept of Corporate Social Responsibility, this is still an evolving concept with imprecise formulations [1,9,17]. There is currently no universal definition or consensus on the extent of the concept of the firm's Corporate Social Responsibility. The issues surrounding this concept have become increasingly large and this has resulted in a proliferation of different measures of social performance. These measures vary both, conceptually and operationally. From the proposed measures in the literature we can find: pollution indices [2,3], indices of reputation [10,20], the amount of charitable donations [4,23,28], environmental scores [15, 27] and measures from specialized rating agencies [14, 16,21,25,26]. The diversity of these measures poses a problem in terms of comparability and generalization of the results. Certain measures used in earlier studies would not be appropriate to assess the current social practices,

* Corresponding author e-mail: blancaperezgladish@gmail.com

others would not be justified theoretically [20] and others would measure only partially the social performance [8].

Nowadays, several independent agencies try to supply transparent and credible information about the social, labour and environmental performance of companies throughout the world. Some examples are the MSCI ESG STATS (known under the name of KLD Research & Analytics Inc.) database (http://www.msci.com), Ethibel (http://forumethibel.org), Vigeo (http://www.vigeo.com), Oekom Research, SAM (Sustainable Asset Management) or EIRIS (http://www.eiris.org). However, and as will be shown in section 5, each rating agency uses its own social performance measures for a different set of social criteria. In this context a company could have different social ratings depending on source database.

The aim of this work is to obtain an integrative measure of CSR which does not require a unique precise definition of this concept. The proposed method will allow the ranking of firms based on their social performance incorporating all the available information from different sources. Moreover, a Soft Computing method will be applied based on interval-valued fuzzy sets reflecting the uncertain, imprecise and fuzzy nature of social performance criteria. The ranking will rely on the definition of an ideal firm and on the definition of the similarity degree of each firm with this ideal firm.

The rest of the paper is organized as follows: in section 2 interval-valued fuzzy sets are obtained for the social performance criteria including all the available information from the rating agencies. Next section describes the construction of the interval-valued fuzzy sets for the ideal firm. In section 4, for a given exigency level, the similarity degree between the firms and the ideal firm is obtained. Section 5 includes a real example illustrating the application of the proposed method and finally, the main conclusions are presented in section 6.

2 Obtaining the interval-valued fuzzy sets for the social criteria

Let us consider r firms, $\{P_i\}_{i=1}^{r}$, each of them evaluated on n social performance criteria, $X=\{c_1,c_2,\ldots,c_n\}$, by p experts (independent rating agencies). Our objective is the attainment of a ranking of the firms based on their consensual evaluation (Figure 1).

The independent rating agencies usually provide a precise score for the performance of each firm on each social performance criterion. Nevertheless, social criteria are by nature characterized by uncertainty, vagueness and/or imprecision. Therefore, in this work valuations of social criteria will be handled by means of interval-valued fuzzy sets constructed with the help of an external to the rating agencies expert in CSR (see [6,7] for an application to the Human Resources Management problem).

Interval-valued fuzzy sets are a generalization of Fuzzy Sets introduced by Sambuc in 1975 [22,5].

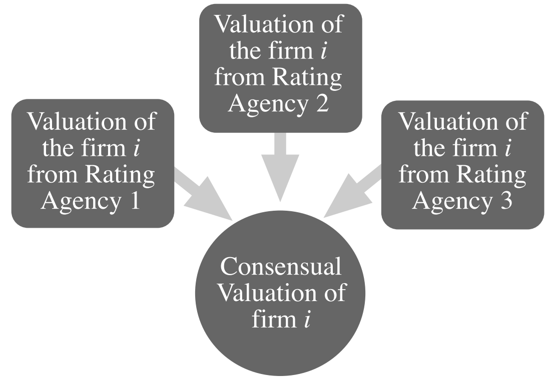

Fig. 1: Consensual valuation of a firm.

Definition 2.1. Let X be a reference set, an interval-valued fuzzy set in X is an expression given by

$$\tilde{A}^{\Phi}=\left\{\left(x,\,\mu_{\tilde{A}}^{\Phi}(x)\right),\ x\in X\right\} \tag{1}$$

where the function $\mu_{\tilde{A}}^{\Phi}: X\to D[0,1]$, given by $\mu_{\tilde{A}}(x)=\left[a_x^L,\, a_x^U\right]\in D[0,1]$, defines the degree of membership of an element x to A. The expression $D[0,1]$ denotes the set of all the closed subintervals on the interval $[0,1]$. In general, when the reference set is finite, $X=\{c_1,c_2,\ldots,c_n\}$, the interval-valued fuzzy set has the expression

$$\tilde{A}=\left\{\left(c_j,\,\mu_{\tilde{A}}(c_j)\right),\ 1\le j\le n\right\}. \tag{2}$$

In order to obtain interval-valued fuzzy sets for each social criterion we will follow the process described in Figure 2.

First step consists of the attainment from the external expert of the interval number valuation of each social criterion based on the precise score provided by the rating agencies. Intervals will be expressed as follows:

$$\left[c_{ij}^k-\varepsilon_{ij}^k, c_{ij}^k+\varepsilon_{ij}^k\right],\ 1\le i\le r, 1\le j\le n, 1\le k\le p, \tag{3}$$

where c_{ij}^k is the center and ε_{ij}^k is the radius. Table 1 displays an example of the valuation of two firms in two criteria based on the ratings of two different agencies.

Table 1: Example of the valuation of firms by two agencies.

EXPERTS VALUATION BASED ON RATING AGENCY 1		
Firm 1	$[c_{11}^1-\varepsilon_{11}^1, c_{11}^1+\varepsilon_{11}^1]$	$[c_{12}^1-\varepsilon_{12}^1, c_{12}^1+\varepsilon_{12}^1]$
Firm 2	$[c_{21}^1-\varepsilon_{21}^1, c_{21}^1+\varepsilon_{21}^1]$	$[c_{22}^1-\varepsilon_{22}^1, c_{22}^1+\varepsilon_{22}^1]$
EXPERTS VALUATION BASED ON RATING AGENCY 2		
Firm 1	$[c_{11}^2-\varepsilon_{11}^2, c_{11}^2+\varepsilon_{11}^2]$	$[c_{12}^2-\varepsilon_{12}^2, c_{12}^2+\varepsilon_{12}^2]$
Firm 2	$[c_{21}^2-\varepsilon_{21}^2, c_{21}^2+\varepsilon_{21}^2]$	$[c_{22}^2-\varepsilon_{22}^2, c_{22}^2+\varepsilon_{22}^2]$

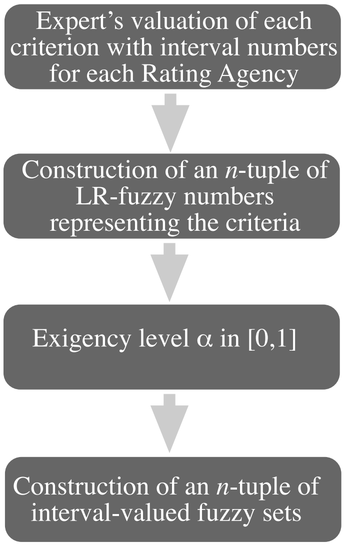

Fig. 2: Scheme for the construction of the interval-valued fuzzy sets.

Second step consists of the construction of LR-fuzzy numbers from previous intervals. The LR-fuzzy numbers will contain all the available information from the rating agencies. Dubois and Prade define a LR-fuzzy number as follows [12].

Definition 2.2. A fuzzy number $\tilde{M}$ is said to be a LR-fuzzy number

$$\tilde{M} = \left(m^L, m^R, \delta^L, \delta^R\right)_{LR}, \qquad (4)$$

if its membership function has the following form:

$$\mu_{\tilde{M}}(x) = \begin{cases} L\left(\frac{m^L - x}{\delta^L}\right) & \text{if } x \le m^L, \\ 1 & \text{if } m^L < x < m^R, \\ R\left(\frac{x - m^R}{\delta^R}\right) & \text{if } x \ge m^R, \end{cases} \qquad (5)$$

where $L, R : [0, +\infty[\longrightarrow [0,1]$ are strictly decreasing in $\mathrm{supp}(\tilde{M}) = \{x \in X \ : \ \mu_{\tilde{M}}(x) > 0\}$ and upper semi-continuous functions such that $L(0) = R(0) = 1$.

Remark 2.1. If the support of $\tilde{M}$ is a bounded set, being $m^L - \delta^L$ the infimum and $m^R + \delta^R$ the supremum in that set, then functions L and R are defined on $[0,1]$ and they satisfy that $L(1) = R(1) = 0$. When $L(z) = R(z) = \max\{0, 1 - z\}$, $\tilde{M}$ is said to be a fuzzy trapezoidal number with support $[m^L, m^R]$ and core $[M^L, M^R]$.

Proposition 2.1. We consider a family of h intervals

$$\left\{\left[c^k - \varepsilon^k, c^k + \varepsilon^k\right], \quad 1 \le k \le h\right\}$$

and two functions $L, R : [0, +\infty[\longrightarrow [0,1]$. The LR-fuzzy number $\tilde{M} = (m^L, m^R, \delta^L, \delta^R)_{LR}$ is obtained in the following way (Figure 3):

$$m^L = \min_k c^k,\ m^R = \max_k c^k,\ M^L = \min_k(c^k - \varepsilon^k),$$
$$M^R = \max_k(c^k + \varepsilon^k),\ \delta^L = m^L - M^L,\ \delta^R = M^R - m^R.$$

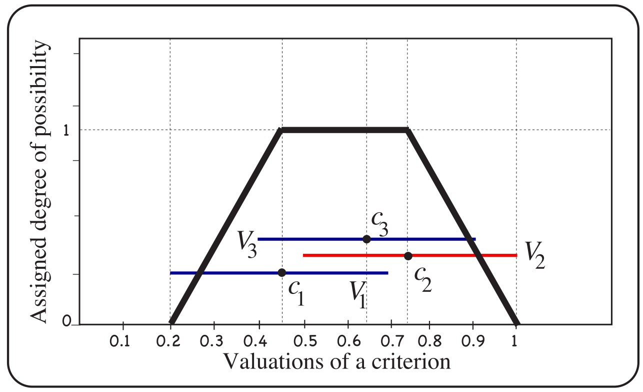

Fig. 3: Membership function of the fuzzy trapezoidal number $\tilde{M}$ constructed from three intervals.

Definition 2.3. If the inverse functions of L and R exist, the α-cuts of a LR-fuzzy number $\tilde{M}$ are defined as:

$$M(\alpha) = \left[M^L(\alpha), M^R(\alpha)\right] = \\ = \left[m^L - \delta^L L^{-1}(\alpha),\ m^R + \delta^R R^{-1}(\alpha)\right], \quad \alpha \in [0,1].$$

Remark 2.2. In particular, when L and R are linear functions, we have:

$$M(\alpha) = \left[M^L(\alpha), M^R(\alpha)\right] = \\ = \left[m^L - \delta^L \alpha, m^R + \delta^R \alpha\right], \quad \alpha \in [0,1].$$

Therefore, following the above described process we can obtain the following trapezoidal fuzzy number for criterion j and firm i (Figure 4):

$$\tilde{c}_{ij} = (m^L_{ij}, m^R_{ij}, \delta^L_{ij}, \delta^R_{ij})_{LR}, \qquad (6)$$

where

$$m^L_{ij} = \min_k c^k_{ij},\ m^R_{ij} = \max_k c^k_{ij},\ M^L_{ij} = \min_k(c^k_{ij} - \varepsilon^k_{ij})$$
$$M^R_{ij} = \max_k(c^k_{ij} + \varepsilon^k_{ij}),\ \delta^L_{ij} = m^L_{ij} - M^L_{ij},\ \delta^R_{ij} = M^R_{ij} - m^R_{ij}. \qquad (7)$$

If we repeat the process for each social criterion and for each firm, we will obtain an n-tuple of LR-fuzzy numbers $\{\tilde{c}_{i1}, \tilde{c}_{i2}, \cdots, \tilde{c}_{in}\}$ which will include all the available information about the firms provided by the rating agencies.

Once the criteria are defined by fuzzy numbers, we can state the desired exigency level and, for each firm i and for each value $\alpha \in [0,1]$, we build the interval-valued fuzzy set (Figure 5)

$$P_i^\phi = \left\{\left(\tilde{c}_{ij}, \left[\tilde{c}^L_{ij}(\alpha), \tilde{c}^R_{ij}(\alpha)\right]\right), 1 \le j \le n\right\}. \qquad (8)$$

If the referential functions of the fuzzy numbers $\{\tilde{c}_{i1}, \tilde{c}_{i2}, \cdots, \tilde{c}_{in}\}$ are linear, the criteria are given by the

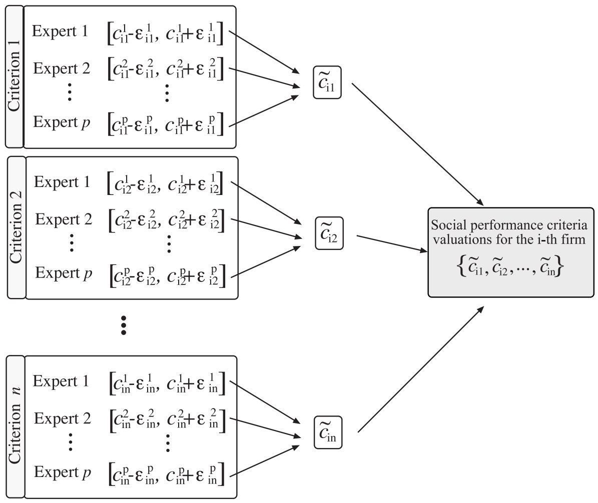

Fig. 4: Scheme for the construction of the social performance LR-fuzzy numbers for the *i-th* firm.

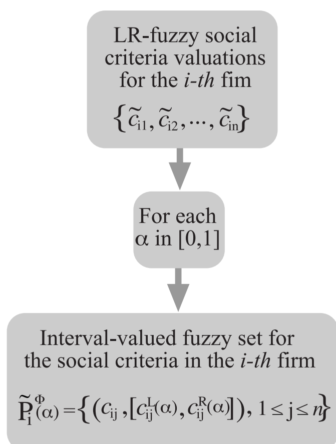

Fig. 5: Scheme describing the construction of interval-valued fuzzy sets from the LR-fuzzy numbers.

following interval-valued fuzzy sets:

$$P_i^{\phi} = \left\{ \left(\tilde{c}_{ij}, \left[M^L_{ij} + (m^L_{ij} - M^L_{ij})\alpha, M^R_{ij} + (m^R_{ij} - M^R_{ij})\alpha\right]\right), \right.$$
$$\left. 1 \le j \le n, \quad \alpha \in [0,1] \right\}. \tag{9}$$

3 Obtaining the interval-valued fuzzy sets for the ideal firm

The process described in section 2 will be followed for the construction of the interval-valued fuzzy sets for the ideal firm. For each rating agency k, intervals are obtained

Table 2: Example of the construction of the ideal intervals for each criterion.

	Rating agency k			
	Criterion 1	Criterion 2	$\cdots$	Criterion n
Firm 1	c^k_{11}	c^k_{12}	$\cdots$	c^k_{1n}
Firm 2	c^k_{21}	c^k_{22}	$\cdots$	c^k_{2n}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
Firm r	c^k_{r1}	c^k_{r2}	$\cdots$	c^k_{rn}
Max.	$c^k_{max_1}$	$c^k_{max_2}$	$\cdots$	$c^k_{max_k}$
Min.	$c^k_{min_1}$	$c^k_{min_2}$	$\cdots$	$c^k_{min_k}$
Ideal Interval	$[c^k_{min_1}, c^k_{max_1}]$	$[c^k_{min_2}, c^k_{max_2}]$	$\cdots$	$[c^k_{min_n}, c^k_{max_n}]$

Table 3: Ideal intervals for each rating agency.

	Criterion 1	Criterion 2	$\cdots$	Criterion n
Agency 1	$[c^1_{min_1}, c^1_{max_1}]$	$[c^1_{min_2}, c^1_{max_2}]$	$\cdots$	$[c^1_{min_n}, c^1_{max_n}]$
Agency 2	$[c^2_{min_1}, c^2_{max_1}]$	$[c^2_{min_2}, c^2_{max_2}]$	$\cdots$	$[c^2_{min_n}, c^2_{max_n}]$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
Agency p	$[c^p_{min_1}, c^p_{max_1}]$	$[c^p_{min_2}, c^p_{max_2}]$	$\cdots$	$[c^p_{min_n}, c^p_{max_n}]$

representing the valuations of the ideal social criteria as described in Table 2 and Table 3.

For each ideal interval number we obtain its center I^k_j and radius ε^k_j and we express the intervals as follows: $\{[I^k_j - \varepsilon^k_j, I^k_j + \varepsilon^k_j], \quad 1 \le k \le q\}$.

The following propositions will allow us to properly compare the interval-valued fuzzy set for the i-th firm with the interval-valued fuzzy set for the ideal firm.

Proposition 3.1. We consider a family of h intervals

$$\{[I^k_j - \varepsilon^k_j, I^k_j + \varepsilon^k_j], \quad 1 \le k \le h\}$$

and two functions $L, R : [0, +\infty[\longrightarrow [0,1]$. Given the values:

$$w^L = \max_k c^k,\ w^R = \sup\left\{\max_k c^k, \tfrac{1}{h}\textstyle\sum_{k=1}^h (c^k + \varepsilon^k)\right\},$$
$$W^L = \tfrac{1}{h}\textstyle\sum_{k=1}^h (c^k - \varepsilon^k), \quad W^R = \max_k (c^k + \varepsilon^k)$$
$$\delta^L = w^L - W^L, \quad \delta^R = W^R - w^R.$$

$\tilde{W} = (w^L, w^R, \delta^L, \delta^R)_{LR}$ is the LR-fuzzy number and the intervals $[w^L, w^R]$ and $[W^L, W^R]$ are the core and support, respectively, of $\tilde{W}$.

With Proposition 2.1 a fuzzy number with higher membership degree in the central part of the intervals is constructed (in our case, the fuzzy numbers for the valuation of social criteria for the $i-th$ firm). With Proposition 3.1 a fuzzy number is obtained with the higher membership degree for the right-hand side part of the intervals (our ideal fuzzy numbers). In fact, taking into account that, by construction,

$$w^L \geq m^L, \quad w^R \geq m^R, \quad W^L \geq M^L, \quad W^R \geq M^R, \quad (10)$$

and applying Definition 2.3, it is easy to prove the following result (Figure 6):

Proposition 3.2. If the numbers $\tilde{M}$, $\tilde{W}$ of the previous propositions have the same functions L and R, then:

a)For each $\alpha \in [0,1]$, the α-cuts verify $M(\alpha) \leq W(\alpha)$.

b)For each $\alpha \in [0,1]$, there are $k, k' \in \{1,2,\ldots,h\}$ such that $c^k - \varepsilon^k \leq M(\alpha)$ and $c^{k'} + \varepsilon^{k'} \geq W(\alpha)$.

With these values and applying Proposition 3.1, we now construct the trapezoidal fuzzy number $\tilde{I}_j = (w^L, w^R, \delta^L, \delta^R)_{LR}$. If we proceed in this way with each of the criteria we obtain a n-tuple of fuzzy numbers $\{\tilde{I}_1, \tilde{I}_2, \cdots, \tilde{I}_n\}$, and from them, for each value $\alpha \in [0,1]$, we have the interval-valued fuzzy sets:

$$I^\phi(\alpha) = \left\{\left(\tilde{I}_j, [I_j^L(\alpha), I_j^R(\alpha)]\right), 1 \leq j \leq n\right\}. \quad (11)$$

With the aim of simplifying, in this work linear functions L and R will be considered although any other type of function is also possible. Therefore, the fuzzy numbers $\{\tilde{I}_1, \tilde{I}_2, \cdots, \tilde{I}_n\}$ are represented by the following interval-valued fuzzy sets:

$$I^\phi(\alpha) = \left\{ \left(\tilde{I}_j, \left[W_j^L + (w_j^L - W_j^L)\alpha, W_j^R + (w_j^R - W_j^R)\alpha\right]\right), \right. \\ \left. 1 \leq j \leq n, \quad \alpha \in [0,1] \right\}. \quad (12)$$

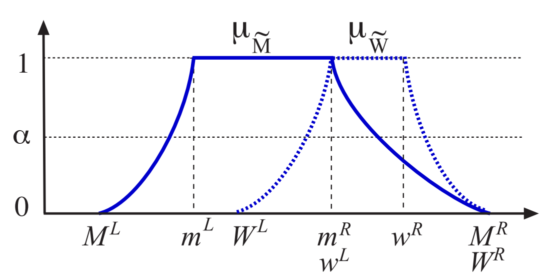

Fig. 6: Membership functions of the fuzzy numbers $\tilde{M}$ and $\tilde{W}$.

4 Measuring firms' similarity to the ideal firm

Once interval-valued fuzzy sets have been obtained for the firms and the ideal firm, we will study the similarity degree between each firm and the ideal firm using Hamming's distance [11,13]. Although other distances can be used that distance verifies suitable properties for this problem:

Definition 4.1. Given a reference set $X = \{c_1, c_2, \ldots, c_n\}$ and two interval-valued fuzzy sets $\tilde{A}^\phi$ and $\tilde{B}^\phi$, whose membership functions are respectively

$$\mu_{\tilde{A}}^\phi(c_j) = \left[a_{c_j}^L, a_{c_j}^R\right], \quad \mu_{\tilde{B}}^\phi(c_j) = \left[b_{c_j}^L, b_{c_j}^R\right], \quad 1 \leq j \leq n.$$

Hamming's normalized distance is defined as:

$$d(\tilde{A}^\phi, \tilde{B}^\phi) = \frac{1}{n}\sum_{j=1}^{n}\left|\mu_{\tilde{A}}^\phi(c_j) - \mu_{\tilde{B}}^\phi(c_j)\right| = \\ = \frac{1}{2n}\sum_{j=1}^{n}\left(\left|a_{c_j}^L - b_{c_j}^L\right| + \left|a_{c_j}^R - b_{c_j}^R\right|\right). \quad (13)$$

As we have seen previously, for each $\alpha \in [0,1]$ we have r interval-valued fuzzy sets, $P_i^\phi(\alpha)$, $1 \leq i \leq r$, which represent each of the firms, and another one $I^\phi(\alpha)$, which represents the ideal firm. The goal is to measure the distance or the similarity of each of the firms with the ideal firm

$$d_i(\alpha) = d\left(P_i^\phi(\alpha), I^\phi(\alpha)\right), \quad 1 \leq i \leq r, \quad (14)$$

where d represents Hamming's distance. Figure 7 displays the followed process.

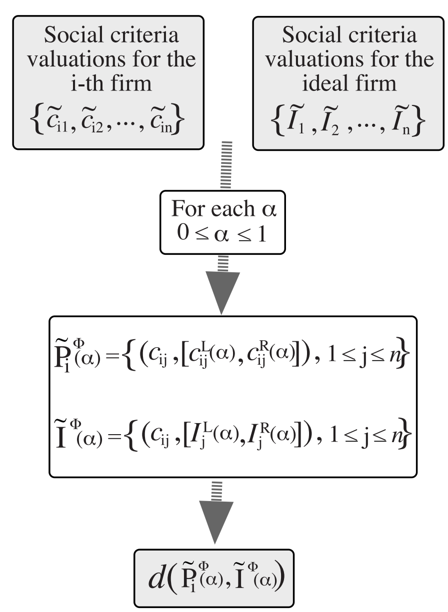

Fig. 7: Scheme for the comparison of the firms with the ideal firm.

Several works have studied similarity between interval-valued fuzzy sets [18,19]. In this paper, we present the simplest similarity degree obtained directly from Hamming's distance (see, for instance, [11]).

Definition 4.2. We call similarity degree between $\tilde{A}^{\phi}$ and $\tilde{B}^{\phi}$ to the following measure:

$$Sim(\tilde{A}^{\phi}, \tilde{B}^{\phi}) = 1 - d(\tilde{A}^{\phi}, \tilde{B}^{\phi}). \qquad (15)$$

Definition 4.3. A firm r is said to be preferred or equivalent, $\succeq$, to a firm s for a given exigency level α, and we denote it as $P_r(\alpha) \succeq P_s(\alpha)$, if firm r has a higher similarity degree with the ideal firm (i.e. if distance of the firm r with respect to the ideal is smaller than the distance of the firm s to the ideal firm, $d_r(\alpha) \leq d_s(\alpha)$).

When the set of real numbers $\{d_i(\alpha)\}_{i=1}^{r}$ is ordered from lower distance to the ideal to higher, the firms are ordered for the exigency level α. If this process is repeated for different $\alpha \in [0,1]$, an order of the firms will be obtained for each considered exigency level

$$P_{i_1}(\alpha) \succeq P_{i_2}(\alpha) \succeq \cdots \succeq P_{i_n}(\alpha), \qquad (16)$$

where the set $\{i_1, i_2, \ldots, i_r\}$ is the result of re-ordering the set $\{1, 2, \ldots, r\}$.

In general, it is possible to obtain different rankings of firms for different levels of exigency $\alpha \in [0,1]$. In this work we will consider: very low, low, medium, high and very high levels of exigency (Table 4).

Table 4: Rankings of the firms for different exigency levels.

Exigency	α-value	Ranking
Very Low	0	$P_{i_1}(0) \succeq P_{i_2}(0) \succeq \cdots \succeq P_{i_n}(0)$
Low	0.25	$P_{i_1}(0.25) \succeq P_{i_2}(0.25) \succeq \cdots \succeq P_{i_n}(0.25)$
Medium	0.5	$P_{i_1}(0.5) \succeq P_{i_2}(0.5) \succeq \cdots \succeq P_{i_n}(0.5)$
High	0.75	$P_{i_1}(0.75) \succeq P_{i_2}(0.75) \succeq \cdots \succeq P_{i_n}(0.75)$
Very High	1	$P_{i_1}(1) \succeq P_{i_2}(1) \succeq \cdots \succeq P_{i_n}(1)$

In case two firms obtain the same valuation the analysts can decide which one is the most socially responsible using additional criteria (more detailed information and/or other social indicators) and relaying on their level of expertise.

The proposed approach is able to consider as many corporate social dimensions as desired by the decision maker as it works with n-dimensional vectors where each component is the valuation of a corporate social dimension. The proposed ranking is based on the distance of the vector describing the fuzzy performance of each firm with respect to each corporate social dimension to the vector representing the ideal firm. So, with the proposed model there is no aggregation among the dimensions. Moreover, they are handled independently during all the steps of the model. This overcomes an important discussion among practitioners and academics: the convenience or not of the aggregation of different CSR dimensions within a unique measure.

5 An illustrative example

The previously presented algorithm has been applied to the ranking of 10 firms (Table 5). For this illustrative example, two different sources providing expert valuations on CSR have been considered: the MSCI ESG STATS (known under the name of Socrates KLD Research & Analytics Inc.) database and the EQUITICS® database from Vigeo. In both cases, the rigor in the valuation of the firms' social performance is considered as equivalent for all the database users. The 10 considered firms have been valued by both rating agencies.

The KLD system allows American companies to be rated according to 7 social performance dimensions that are related to key stakeholders and are evaluated on the basis of two criteria, namely strengths and concerns (Table 6).

Table 5: Example of the construction of the ideal intervals for each criterion.

Firm	Title	Sector
1	Accenture Ltd.	Business Support Services
2	Colgate-Palmolive Co.	Luxury Goods & Cosmetics
3	Delta Airlines	Travel & Tourism
4	Johnson & Johnson	Pharmaceutical & Biotech.
5	JPMorgan Chase & Co.	Banks
6	Kellogg Co.	Food
7	Nike Inc. Cl B	Specialised Retail
8	PepsiCo Inc.	Beverage
9	Whirlpool Corp.	Technology-Hardware
10	Yahoo! Inc.	Software & IT Services

The EQUITICS® database developed by Vigeo considers 38 criteria grouped in 6 clusters. They measure the companies' levels of commitment with different stakeholders at three levels: leadership, implementations and results. In this example we will use the aggregate scores for the 6 clusters provided by EQUITICS® database (Table 6). In order to obtain a common set of social criteria we will group KLD's Employees and Diversity criteria under the label Human Resources and we will consider EQUITICS® dimensions. We thus have $n = 6$ criteria, $k = 10$ firms and $p = 2$ experts (rating agencies) who value the social performance of the firms with respect to each social criterion and q=1 external expert who based on the precise scores from the rating agencies provide the intervals valuations.

For each strength and concern applied to a company, KLD attributes it with a score of 1 if the criterion applies, and a score of 0 in the opposite case. They do not aggregate strengths and concerns. Nevertheless, the majority of the scientific works, based primarily on the KLD database, use as an approximate measure of the firms social performance an aggregate index of KLD strengths and concerns. Some authors subtract the sum of

Table 6: Rating agencies social performance criteria.

KLD (Socrates)	Vigeo (Equitics)		Common Criteria
Community	Community Invest.	n=1	Community Invest.
Environment	Environment	n=2	Environment
Governance	Corp. Governance	n=3	Corp. Governance
Products	Clients & Suppliers	n=4	Clients & Suppliers
Human Rights	Human Rights	n=5	Human Rights
Employees	Human Resources	n=6	Human Resources
Diversity			

concerns score from the sum of strengths score for each dimension obtaining in this way the total score associated with each KLD dimension. Other transform concerns into strengths taking complementary binary values (e.g. if the firm is assigned the value 1 for a concern then the value zero is assigned for the corresponding strength in that criterion) and finally, some authors consider separately strengths and concerns. In this work we have chosen to transform concerns into strengths.

On the other hand, Vigeo's database EQUITICS® provides aggregated scores rated from 0-100, for each social criterion. In order to be able to compare the ratings, the scores from both rating agencies have been normalized dividing them by the total number of indicators used in the rating process (see Table 7).

Table 7: Rating Agencies valuations of the firms social performance (c^k_{ij})

		n=1	n=2	n=3	n=4	n=5	n=6
Firm 1	Vigeo	0.69	0.34	0.53	0.49	0.39	0.30
	KLD	0.27	0.54	0.46	0.38	0.57	0.64
Firm 2	Vigeo	0.29	0.46	0.45	0.53	0.43	0.36
	KLD	0.27	0.54	0.46	0.63	0.57	0.55
Firm 3	Vigeo	0.23	0.39	0.45	0.40	0.33	0.80
	KLD	0.36	0.54	0.54	0.50	0.57	0.36
Firm 4	Vigeo	0.39	0.56	0.47	0.33	0.38	0.22
	KLD	0.45	0.69	0.46	0.38	0.57	0.59
Firm 5	Vigeo	0.36	0.42	0.39	0.48	0.33	0.15
	KLD	0.55	0.54	0.38	0.13	0.43	0.55
Firm 6	Vigeo	0.79	0.30	0.46	0.38	0.39	0.29
	KLD	0.36	0.62	0.46	0.63	0.57	0.41
Firm 7	Vigeo	0.29	0.33	0.48	0.41	0.50	0.19
	KLD	0.64	0.77	0.54	0.38	0.57	0.41
Firm 8	Vigeo	0.70	0.37	0.55	0.50	0.55	0.33
	KLD	0.27	0.54	0.54	0.25	0.57	0.64
Firm 9	Vigeo	0.12	0.22	0.42	0.37	0.34	0.70
	KLD	0.55	0.62	0.46	0.63	0.57	0.41
Firm 10	Vigeo	0.74	0.37	0.44	0.26	0.42	0.13
	KLD	0.36	0.54	0.46	0.38	0.43	0.59

Based on the precise scores from the rating agencies an external expert on CSR from the academic field provides intervals valuations for each social criterion, each firm and each agency in the following way:

$$[(1-\rho_1)c_{ij},(1+\rho_2)c_{ij}], \quad \rho_1,\rho_2 \in [0,1],$$

where ρ_1,ρ_2 are tolerance levels. In this example, the expert fixes equal 15% upper and lower deviations, $\rho_1=\rho_2=0.15$. Then, for each of these intervals, the center c^k_{ij} and radius ε^k_{ij} are obtained. An example of the obtained intervals for firm $i=1$ is displayed in Table 8.

Table 8: Example of the construction of the ideal intervals for each criterion.

Social Criterion	Expert's valuation (based on Vigeo)		Expert's valuation (based on KLD)	
	$c^1_{1j}-\varepsilon^1_{1j}$	$c^1_{1j}+\varepsilon^1_{1j}$	$c^2_{1j}-\varepsilon^2_{1j}$	$c^2_{1j}-\varepsilon^2_{1j}$
n=1	0.587	0.794	0.230	0.311
n=2	0.289	0.391	0.459	0.621
n=3	0.451	0.610	0.391	0.529
n=4	0.417	0.564	0.323	0.437
n=5	0.332	0.449	0.485	0.656
n=6	0.255	0.345	0.544	0.736

The valuations of social performance criteria for the ideal firm are displayed in Table 9.

Table 9: Social performance interval valuations for the ideal firm.

Social Criterion	Ideal valuations (based on Vigeo)		Ideal valuations (based on KLD)	
	$c^1_{1j}-\varepsilon^1_{1j}$	$c^1_{1j}+\varepsilon^1_{1j}$	$c^2_{1j}-\varepsilon^2_{1j}$	$c^2_{1j}-\varepsilon^2_{1j}$
n=1	0.12	0.79	0.27	0.64
n=2	0.22	0.56	0.54	0.77
n=3	0.39	0.55	0.38	0.54
n=4	0.26	0.53	0.13	0.63
n=5	0.33	0.55	0.43	0.57
n=6	0.07	0.36	0.36	0.64

The fuzzy numbers $\widetilde{c}_{ij}=(m^L_{ij},m^R_{ij},\delta^L_{ij},\delta^R_{ij})_{LR}$ are obtained from the intervals valuations. Table 10 contains an example of the trapezoidal fuzzy numbers obtained for firm $i=1$.

The fuzzy numbers $\widetilde{I}_j=(m^L_j,m^R_j,\delta^L_j,\delta^R_j)_{LR}$ are obtained from the intervals valuations displayed in Table 9 and are shown in Table 11.

Interval-valued fuzzy sets are obtained for different exigency values. Table 12 displays the interval-valued fuzzy sets obtained for one of the firms (i=1) and for the ideal firm for a fixed exigency level $\alpha=0.25$.

Figure 8 shows as an example, the interval-valued fuzzy set $\widetilde{c}_{11}$ for the social criterion $n=1$ and for the firm $i=1$.

Table 10: Trapezoidal fuzzy numbers for firm $i = 1$.

Social Criterion	m_{ij}^L	m_{ij}^R	δ_{ij}^L	δ_{ij}^R	M_{ij}^L	M_{ij}^R
n=1	0.27	0.69	0.0405	0.1035	0.2295	0.7935
n=2	0.34	0.54	0.0510	0.0810	0.2890	0.6210
n=3	0.46	0.53	0.0690	0.0795	0.3910	0.6095
n=4	0.38	0.49	0.0570	0.0735	0.3230	0.5635
n=5	0.39	0.57	0.0585	0.0855	0.3315	0.6555
n=6	0.30	0.64	0.0450	0.0960	0.2550	0.7360

Table 11: Trapezoidal fuzzy numbers for for the ideal firm.

Social Criterion	w_j^L	w_j^R	δ_j^L	δ_j^R	W_j^L	W_j^R
n=1	0.46	0.71	0.26	0.08	0.20	0.79
n=2	0.65	0.66	0.27	0.10	0.38	0.77
n=3	0.47	0.54	0.08	0.01	0.39	0.55
n=4	0.40	0.58	0.20	0.05	0.19	0.63
n=5	0.50	0.56	0.12	0.01	0.38	0.57
n=6	0.50	0.50	0.28	0.14	0.22	0.64

Table 12: Interval-valued fuzzy sets for an exigency level $\alpha = 0.25$.

Social	Firm i=1		Ideal Firm	
Criterion	$c_{1j}^L(\alpha)$	$c_{1j}^R(\alpha)$	$I_j^L(\alpha)$	$I_j^R(\alpha)$
n=1	0.2396	0.7676	0.2610	0.7708
n=2	0.3018	0.6008	0.4479	0.7431
n=3	0.4083	0.5896	0.4080	0.5486
n=4	0.3373	0.5451	0.2431	0.6131
n=5	0.3461	0.6341	0.4095	0.5688
n=6	0.2663	0.7120	0.2876	0.6023

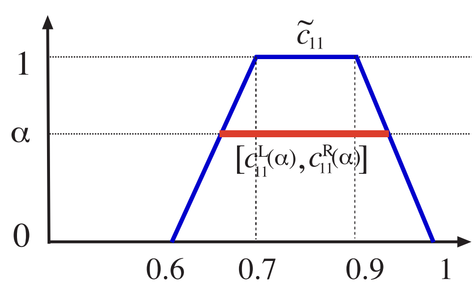

Fig. 8: Interval-valued fuzzy set.

Next step consists of calculating the distance between each firm and the ideal firm. As an example, the distance and similarity of each firm P_i with respect to the ideal firm, $d_i(\alpha) = d(\tilde{P}_i^\phi(\alpha), \tilde{I}^\phi(\alpha))$, for an exigency level $\alpha = 0.25$ is displayed in Table 13.

Table 14 displays rankings for different exigency levels. The rankings are based on the previously obtained

Table 13: Distances and similarity degrees between firm i and the ideal firm for $\alpha = 0.25$.

Firms	$d_i(\alpha)$	$Sim_i(\alpha)$	Firms	$d_i(\alpha)$	$Sim_i(\alpha)$
1	0.0647	0.9353	6	0.0771	0.9229
2	0.0956	0.9044	7	0.0848	0.9152
3	0.1169	0.8831	8	0.0636	0.9364
4	0.0816	0.9184	9	0.1053	0.8947
5	0.0984	0.9016	10	0.0805	0.9195

distances. As it can be observed the obtained ranking varies depending on the exigency level. However, the results are quite robust: firm P_1 is for all exigency levels the first or second in the ranking switching positions with firm P_8 for exigency values medium, low and very low. Firm P_6 appears first one for a high and very high exigency levels appearing for lower levels always within the first four positions in the rankings. Firms P_3, P_9 and P_5 occupy last positions for all exigency levels. These ranks incorporate the available information from the two considered rating agencies.

Table 14: Rankings of firms.

α-value	Ranking
0	$P_3 \preceq P_2 \preceq P_9 \preceq P_7 \preceq P_4 \preceq P_5 \preceq P_6 \preceq P_{10} \preceq P_8 \preceq P_1$
0.25	$P_3 \preceq P_9 \preceq P_5 \preceq P_2 \preceq P_7 \preceq P_4 \preceq P_{10} \preceq P_6 \preceq P_1 \preceq P_8$
0.5	$P_9 \preceq P_{10} \preceq P_3 \preceq P_5 \preceq P_2 \preceq P_7 \preceq P_4 \preceq P_6 \preceq P_8 \preceq P_1$
0.75	$P_5 \preceq P_3 \preceq P_9 \preceq P_{10} \preceq P_2 \preceq P_7 \preceq P_8 \preceq P_4 \preceq P_1 \preceq P_6$
1	$P_5 \preceq P_3 \preceq P_{10} \preceq P_9 \leq P_2 \preceq P_4 \preceq P_8 \preceq P_7 \preceq P_1 \preceq P_6$

In order to compare our rankings with the rating agencies' based rankings we have calculated an aggregate measure of their precise social performance scores and calculating the distance of each firm from the ideal firm which in this case is considered to reach the maximum value of 1 in all social criteria. Table 15 displays the rankings obtained for each of the rating agencies:

Table 15: Rating agencies ranking of firms.

Agency	Ranking
Vigeo	$P_9 \preceq P_3 \preceq P_5 \preceq P_7 \preceq P_4 \preceq P_{10} \preceq P_2 \preceq P_6 \preceq P_1 \preceq P_8$
KLD	$P_5 \preceq P_{10} \preceq P_8 \preceq P_1 \preceq P_3 \preceq P_2 \preceq P_6 \preceq P_4 \preceq P_9 \preceq P_7$

The rankings derived from the rating agencies are completely different between them. However, Vigeo's ranking is quite similar to our ranking. Firms P_8, P_1 and P_6 are placed in the first positions and firms P_5, P_3 and P_9 occupy the last positions. Thus, even considering the same social criteria rankings from the rating agencies can be different as the used social performance measures differ as well as the available information from the firms.

6 Conclusions

The debate around the concept of CSR continues to grow, both in practical and academic terms. Literature on CSR shows the absence of a consensus definition which has resulted in a lack of uniformity in measures of social performance.

Given the exploratory nature of this work, the results can be considered as a starting point for reflection and research. The essential point to be raised is the need for a reliable measurement of firms' social performance based on an integrative measure which integrates all the available information from the rating agencies. Social performance criteria are by their own nature imprecise and/or uncertain and this feature needs to be taken into account in the measurement of social performance.

In this work a Soft Computing method has been used for the obtaining of a ranking of a set of firms based on their social performance and from the information provided by two well known rating agencies. The proposed ranking depends on different exigency levels and overcomes the problem of a precise definition of the concept of CSR allowing firms' ranking from different measures of social performance.

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Vicente Liern holds a degree in Mathematics and a PhD in Theoretical Physics from the University of Valencia. He is a Senior Professor of Financial Economics at the same university. He began his research career in the field of mathematical physics, with the study of the motion of elementary particles. Subsequently, his research field has been mathematical optimization under uncertainty conditions with applications to portfolio management, location and management of human resources. He has published his research in prestigious international journals such as Fuzzy Sets and Systems, the European Journal of Operational Research and OMEGA.

Paz Méndez-Rodríguez is an Assistant Professor of Quantitative Economics at the Business and Management Faculty at the University of Oviedo in Spain. She has authored many articles that analyze consumer and investor behavior and that have appeared in various international scientific journals as the European Journal of Operational Research and the Journal of Multicriteria Decision Analysis.

Blanca Pérez-Gladish is a Full Time Professor of Quantitative Economics at the Business and Management Faculty at the University of Oviedo in Spain. She has authored numerous articles on multicriteria decision making and portfolio selection in publications such as the European Journal of Operational Research, Applied Mathematics and Computation, Journal of the Operational Research Society and OMEGA.