Quant. Phys. Lett. Vol. 1 No. 2 (2012) 59-68

NSP

Quantum Physics Letters
An International Journal

Femtosecond response of quantum discrete breathers in SRR based metamaterials and the role of dielectric permittivity

B. Mandal[1,4], S. Adhikari[2], K. Choudhary[2,3], A. Dey[2], A. Biswas[2,4], A. K. Bandyopadhyay[2], A .K. Bhattacharjee[4,*] and D. Mandal[4]

[1]*Faculty of Engg. & Technology, NSHM Knowledge Campus, Durgapur-713212, India.*
[2]*Dumkal Institute of Engineering & Technology, W. B. University of Technology, Dumkal, India.*
[3]*Department of Materials Science and Engineering, University of Florida, Gainesville, FL 32611, USA.*
[4]*Department of Electronics and Communication Engineering, National Institute of Technology, India.*

Abstract: The quantum breathers (QB) state in terms of two-phonon bound state (TPBS) via detailed quantum calculations was already shown in ferroelectrics in a periodic boundary condition. In this work on metamaterials for application in antenna arrays as split-ring resonators (SRR), only the temporal variation of the number of quanta in a non-periodic boundary condition is detailed for a Klein-Gordon lattice. A generalized Hamiltonian containing dielectric permittivity is considered which was then quantized by bosonic operators to study the temporal evolution of quanta. Moreover, the time of redistribution of quanta that is proportional to QB's lifetime in femtosecond shows significant variation with different quanta at different values of dielectric permittivity, which has implications in the applications in relevant devices.

Keywords: Dielectric permittivity, Metamaterials; Quantum breathers; Femtosecond response.

I. Introduction

In the field of applied physics, a fertile ground is created by a tremendous amount of both theoretical and experimental research work on nonlinear optics in the realm of nanotechnology, particularly on discrete breathers that depend on both nonlinearity and discreteness. The majority of these investigations have been carried out with the help of nonlinear Schrodinger equation (NLSE) on optical materials, such as ferroelectrics, metamaterials, and others. In a recent work [1], it has been shown that nonlinear Klein-Gordon (NLKG) equation can also be used in describing the modal behaviour in metamaterials. A careful analysis of both NLSE and NLKG indicates that apart from showing both dark and bright discrete breathers, the presence of breather pulse can also be observed through NLKG, but not by NLSE. In further upgradation of this work from classical breathers to quantum breathers, two-phonon bound states (TPBS) are explored in terms of periodic boundary condition. Moreover, our main focus remains on the temporal evolution of quantum breathers in such engineered materials in a non-periodic boundary condition approach with an eye on THz applications, as these materials show large magnetic response at THz frequency. Before going to the discrete breathers in the quantum regime, for the sake of generality, let us briefly describe the metamaterials.

Substances with both negative dielectric constant (ε) and magnetic permeability (μ) are predicted to possess a negative index of refraction, and consequently they exhibit a variety of optical properties that are not found in positive indexed materials. This was first formulated in 1968 by Veselago [2]. With negative refraction, such materials have quite an interesting history in terms of fabrication and measurement techniques [3-8]. However, these negative indexed materials called metamaterials do not occur in nature, and only recently it has been possible to artificially fabricate them. For example, to say that metamaterials possess a negative index is a special case nowadays, if cloaking is considered [9]. Also many anisotropic materials, such as calcite, exhibit negative refraction, as discussed by Boardman et al [10].

A natural question comes to our mind: in what sort of materials can these properties be observed? This question was answered by Smith et al [11] in terms of experimental realization of such materials based on some theoretical work of Pendry et al [12], where a type of metamaterial was made as an artificial structure. From the theoretical standpoint, if one makes a good approximation, the split-ring-resonators (SRR) assembly can be considered as equivalent to a nonlinear resistor-inductor-capacitor (RLC) circuit [13,14] that features a self-inductance L from the ring, Ohmic resistance R to take care of dissipation, and capacitance C from the split in the ring. Metamaterials are then formed as a periodic array of SRRs which are coupled by mutual inductance and arrayed in a material of dielectric constant ε. SRRs being the building blocks, a large number of loosely coupled SRRs may be taken for a study of the dynamical behavior of a metamaterials system.

As RLC circuit model is considered, the system behaves as capacitively loaded loops. It is known that these loops support wave-propagation. As the coupling is due to induced voltages, the waves are referred to as magneto-inductive (MI) waves [5]. MI waves represent a vast area of active research. Due to RLC configuration of the SRRs, there exists a resonant frequency. It is observed that these MI waves propagate within a band near the resonant frequency of the SRRs. Also, the magnetic permeability does not depend on the intensity of the electromagnetic field in the linear regime of wave propagation. Accordingly, the nonlinearity is incorporated in the system by embedding the SRRs in a Kerr-type medium [15,16].

Kourakis et al [17] studied the self-modulation of the waves by nonlinear Schrodinger equation that led to spontaneous energy localization via the generation of localized envelope structures (i.e. so called envelope solitons) and the dynamics of the nonlinear RLC circuit gave rise to a governing equation for SRRs in both space and time dimensions. There are important observations made on the appearance of multisolitons by controlling various parameters by a number of workers with numerical solutions [18,19]. Lazarides and co-workers [13,14] also studied classical discrete breathers in metamaterial systems. After describing metamaterials, let us next discuss about discrete breathers.

Discrete breathers (DB) [20], also known as intrinsic localized modes, are nonlinear excitations that are produced by the nonlinearity and discreteness of the lattice. These excitations are characterized by their long time oscillations and are highly localized pulses in space that are found in the discrete nonlinear model formulation. Unlike the plane wave like modes, DBs have no counterparts in the linear system, but exist only because of the system nonlinearity in a periodic lattice [21,22]. They are formed as a self-consistent interaction or coupling between the mode and the system nonlinearity. Thus, DB modifies the local properties of the system that provides the environment for the DB to exist. In relevance to metamaterials, Smith and Pendry [23] showed that inclusions smaller than electromagnetic wavelength of interest could be considered only through homogenization of field in any periodic structure via field-averaging method for several basic metamaterial structures.

As the continuum limit formulation cannot be applied to their study, the present formulation on discrete Klein-Gordon lattice is appropriate to highly localized pulses having widths that are not large compared to that of domain of interest [24]. So, the question is about the appropriate length scale, which drives us to the nano-range in metamaterials. Thus, localization also assumes more significance. As the nonlinearity arises in such materials due to the nonlinear Kerr medium, it could also give rise to the localization, possibly with the coupling and dielectric permittivity within the SRR system that are embodied in the discrete Hamiltonian [14]. DBs are discrete solutions, periodic in time and localized in space and whose frequencies extend outside the phonon spectrum [20,21]. This may also be described by our discrete Hamiltonian [24,25] by adding magnetic components in the system. After discussing about classical breathers, next let us look for quantum breathers.

For the characterization of DBs or classical breathers [26], the bulk system was the right tool, but for systems that are very small, the laws of classical mechanics are not valid, which brings us to the quantum breathers (QBs) [27,28]. Once generated, QBs modify system properties such as lattice thermodynamics and introduce the possibility of non-dispersive energy transport [29]. Special significance is attached to the study of QB due to its possible application in the field of quantum computation. To name a few, Quach et al [30] studied reconfigurable MMs and Rakhmanov et al [31] studied quantum Archimedean screw for superconducting electronics using MMs, and many others studied them for medicine to aerospace applications. There are so many other applications, as given in Ref [32] viz. ladder array of Josephson junction for superconductors, BEC in optical lattices, optical waveguides, micro-mechanical arrays, DNA, SRR based metamaterials in antenna arrays, two-magnon bound states in antiferromagnets, and two-phonon bound states (TPBS), i.e. quantum breathers in ferroelectrics. As pointed out by Zheludev [8], the quantum-effect enabled systems via metamaterials route will bring a range of exciting applications in the future.

For QBs, it is important to consider detailed information on phonons and their bound state concept, which is sensitive to the degree of nonlinearity. The branching out of the QB state from the single-phonon continuum is quite noteworthy in nonlinear systems with charge defects [32]. Let us consider that the phonons in one sublattice may hop from one domain to another adjacent domain. This hopping might have some consequences with the change of nonlinearity, thereby a relation could be worked out for the 'hopping strength'. It is determined by finding the phonon energy gap in the eigenspectrum by analyzing the QB state [20,33]. While the role of nonlinearity is still a debatable issue, at around room temperature, there are very few phonons and hence we investigated only TPBS. It can be realized experimentally by a Raman scattering experiment similar to that done by Santori et al [34]. The evidence of such TPBS was shown in an interesting work by Cohen and Ruvalds [35]. Also, Nelson et al worked on anharmonic vibrations in ferroelectrics by impulsive stimulated Raman scattering (ISRS) [36].

As pointed out earlier that dielectric permittivity exists within the Hamiltonian. Thus, the goal of this paper is to explore whether SRR system shows any sensitivity on 'dielectric permittivity' by quantum calculations hitherto not done on metamaterials. Moreover, the effect of change of 'coupling' between different SRR elements through engineering of their geometry is also highlighted. This study is quite realistic to understand the 'quantum localization' due to nonlinearity, which is definitely very much essential for many nano devices. Quantum localization behavior in Klein-Gordon (K-G) lattice has been studied by many researchers in terms of four atom lattice with periodic Bloch function by Proville [37], dimer case for targeted energy transfer by Aubry et al [38], delocalization and spreading behavior of wave-packets by Flach and coworkers [33,39]. QBs are characterized by various methods, as presented by Schulman [40] to describe quantum tunneling and the stability of discrete breathers. Here, our main focus will be on the temporal evolution of quanta, as it is also convenient to characterize QBs by this method. It has to be noted that in K-G lattice, the levels of 'anharmonic potential energy' are non-equidistant that could have important implications in applications. In various novel methods of preparation of metamaterials, the resonant frequency in the THz range has been proposed to be an interesting field of study [6,7] thereby making the present work more relevant.

A generalized method is presented for any number of sites and quanta without periodic boundary condition to show the QB states in metamaterials. The main issue here is that: could we explain the phenomenon in its quantum-counterpart? or, could we observe some noticeable effects in the quantum-regime or not? - which are not explained in the classical regime. In such a case, we were inclined to see it by the dependence on dielectric permittivity, while plotting QB's lifetime against number of quanta (see later in Section 3). Here, we also calculate critical times of redistribution of quanta that is proportional to QB's lifetime in fs under various physical conditions with an eye on THz applications. It has to be noted that in an anharmonic K-G model, the levels in the potential energy are non-equidistant that could have implications from the application viewpoint [20,33].

Finally, it needs to be mentioned that despite the existence of an extensive literature in the area of metamaterials (see Ref [1-17] for references), this is concentrated only in the domain of classical breathers [14,17], including those on THZ applications [6,7]. Very few papers have been published from its quantum point of view [20]. Hence, we are trying to study these materials from its quantum perspective. The reason is obvious in the sense that it will lead to an information on quantum localization and hence on the application in the nano-structured devices. For example, one could make the fastest switch using metamaterials. by studying the temporal-evolution of the number of quanta. For this purpose, we have to theoretically study by varying the concerned parameters to optimize the results and then it could eventually be used in the device applications, such as antenna arrays, modulation instability based gadgets, quantum metamaterials based superconductors, etc. Hence, from the application point of view, the present study assumes special importance.

The paper is organized as follows: In Section II, the theoretical model of the one-dimensional chain of inductively coupled SRRs is described to obtain the Klein-Gordon equation for the system and then the quantization with bosonic operators is done for non-periodic boundary condition. In Section III, the results and discussion are presented for the time of redistribution of quanta against number of quanta at different values of dielectric permittivity. Section IV contains the conclusions.

II. Theoretical Model

For our model formulation, let us consider the following case of three adjacent split-ring resonators, as shown in Fig. 1. A one-dimensional discrete, periodic array of identical non-linear SRRs, that consists of the simplest realization of a metamaterial in one dimension, is considered here. This array is shown here to emphasize that the concerned SRR domains are periodic array, and as pointed out by Segev and coworkers [22] that periodicity is important in creating discrete breathers. In one dimension, the SRRs form a linear array with their centers separated by distance *d*. Each of the SRRs has self-inductance *L*, and mutual inductance *M*. However, a device-oriented optimized model can be considered for the antenna-array application having a varying *L* and *M* values. It should be mentioned that details about the above system and also about how the nearest neighbour interactions are considered for weaker coupling are given in ref. [13,14] for classical breathers. In ref. [1], wherein the effects of further than first neighbour couplings in the SRR system are also discussed. It is an anisotropic situation, since nonlinear medium does not tend to be isotropic.

As done by Lazarides et al [13,14], the Hamiltonian of such a system involving the non-dimensional charge (q) and time (t) is given by:

$$\mathrm{H} = \sum_n [(\frac{1}{2}\dot{q}_n{}^2 - \lambda \dot{q}_n \dot{q}_{n+1} + V(q_n)] \tag{1}$$

where $\dfrac{dq_n}{dt} = \dot{q}_n = i_n$ (i is the current in SRR), $\lambda = M/L$ (coupling parameter); and the term $V_n = \int_0^{q_n} f(q_n')dq_n'$ is nonlinear on-site potential and after truncation, this is expressed as: $f(q_n) \approx q_n - (\dfrac{\alpha}{3\varepsilon_l})q_n^3$

(2)

The above eq. (1) can also be tackled by our type of discrete Hamiltonian for deriving Klein-Gordon equation in case of different ferroelectrics [24,25]. With a magnetic field H and magnetization σ of each SRR, the Hamiltonian is modified as:

$$\mathrm{H} = \sum_n [(\frac{1}{2}\dot{q}_n{}^2 - \lambda \dot{q}_n \dot{q}_{n+1} + V(q_n) - \sigma H \tag{3}$$

Now, after putting the value of magnetization (σ), as done by Kivshar et al [15], we develop the Lagrangian and by using the variational principle (Euler-Lagrange equation), the governing equation in the x-direction is derived to show the Klein-Gordon dynamical equation as [1]:

$$\frac{\partial^2 q}{\partial \tau^2} - ab\frac{\partial^2 q}{\partial x^2} + b\left[q - \frac{\alpha}{3\varepsilon_l}q^3\right] - b\Lambda\Omega\sin(\Omega\tau) + \gamma\frac{\partial q}{\partial \tau} = 0 \tag{4}$$

$a = \lambda'/(1+2\lambda')$ and $b = 1/(1+2\lambda')$; here λ that is equal to ab being an interaction constant or coupling within the SRR system in a Klein-Gordon lattice, ε_l is the linear part of the dielectric permittivity that appears to be an important parameter for femtosecond response of QBs, i.e. for THz application (see later), α = +1 (−1) corresponds to a self-focusing (self-defocusing) in a nonlinear Kerr medium, Ω is a non-dimensional frequency factor and τ is the time, with non-zero external field involving a term Λ and finally a damping term (γ) **[1]**. The equ. (4) is useful for stability analysis, as normally done in a nonlinear system.

It should be pointed out that we have simply derived Klein-Gordon equation by variational principle from Lazarides Hamiltonian. It should be made clear that even our discrete Hamiltonian as given in refs. **[24,25]** with additional magnetic components could be used for this purpose. It is pertinent to mention that in our recent work **[1]**, both K-G equation and nonlinear Schrodinger equation (NLSE) show dark and bright solitons, and also dark and bright breathers; however, K-G equation in addition shows breather pulses, whereas NLSE does not show such pulses. This work could also be relevant for important nonlinear optical materials. Next, let us describe the quantum breather state.

The generalized Hamiltonian for the Klein-Gordon system for order parameter (y_n) at nth site is written as:

$$H = \sum_n \frac{p_n^2}{2m} + \frac{A}{2} y_n^2 + \frac{B}{4} y_n^4 + k\left(y_n - y_{n-1}\right)^2 \tag{5}$$

The first term is momentum at nth site (p_n), the second and third terms are nonlinear potential formulation and the last one contains a term (k) involving coupling. Here A and B are two constants having implication for breather formation. From the above equ. (5), after deducing the classical equation of motion and rescaling of time, we get the revised Hamiltonian as:

$$\tilde{H} = \sum_n \frac{1}{2} p_n^2 + \frac{1}{2} y_n^2 + \eta y_n^4 + \lambda\left(y_n - y_{n-1}\right)^2 \tag{6}$$

Now, in the corresponding equation of motion, we rescale time as follows: $t = (1/\alpha)\tau$ and we take $\alpha^2 = A/m$, where m is the electronic mass. In such a case, the scale of time is 3.048 fs for a coupling value of 0.01, where $\eta = B/4A$ and $\lambda = k/2A$. The term η involves linear dielectric permittivity (ε_l) and focusing (defocusing) nonlinearity (α) with a value of $+1$ (-1), which are also used in our numerical simulation **[41]**. Now, in terms of Klein-Gordon lattice, our generalized Hamiltonian needs to be quantized by using creation and annihilation Bosonic operators at the nth site as follows:

$$a_n^+ = \frac{\left(y_n - ip_n\right)}{\sqrt{2}}, \; a_n = \frac{\left(y_n + ip_n\right)}{\sqrt{2}} \tag{7a}$$

$$H = \sum_n \frac{1}{2} + a_n^+ a_n + \frac{\eta}{4}\left(a_n^{+4} + 4a_n^{+3} a_n + 4a_n^+ a_n^3 + 6a_n^{+2} a_n^2 + a_n^4\right) + \frac{\lambda}{2}\left(a_n^{+2} + a_n^2 + 2a_n^+ a_n + a_{n-1}^{+2} + a_{n-1}^2 + 2a_{n-1}^+ a_{n-1} + 2\right)$$
$$- \lambda\left(a_n^+ a_{n-1}^+ + a_n a_{n-1} + a_n^+ a_{n-1} + a_n a_{n-1}^+\right) \tag{7b}$$

After having done the second quantization as described above, to account for the above mentioned terms in a proper way, a general 'basis' with non-number conserving of particles needs to be formed. In an important work done by Proville **[37]** (see references therein) the periodic boundary condition approach for four sites and an arbitrary number of particles are shown**.** However, the method presented above gives a generalized way to solve the system for arbitrary number of particles on arbitrary number of sites. So, our method is clearly distinguished from the other investigations. For the characterization of quantum discrete breathers, we need to make the Hamiltonian time-dependent. Let us resort to temporal evolution of number of quanta $\langle n_i \rangle(t) = \langle \psi_t | \hat{n}_i | \psi_t \rangle$ at each site of the system. We take i-th eigenstate of the Hamiltonian, and then we make it time dependent as follows:

$$\left|\Psi_i(t)\right\rangle = \sum_i b_i \exp(-iE_i t/\hbar)\left|\psi_i\right\rangle \tag{8}$$

where ψ_i and E_i are the i-th eigenvector and eigenvalue respectively, t is time, the Planck's constant (h) taken as unity and $b_i = \langle \psi_i | \Psi(0) \rangle$ for each site i and for a given range of t, where $\Psi(0)$ stands for initial state. In our computation by 'mathematica', we need to specify the initial states at $t = 0$ wherein there is localization in the initial state and then as time is varied, there is a continuous evolution of the number of quanta **[41].** This gives rise to QB's lifetime in the femtosecond range that brings us to our main focus of relating this response to dielectric permittivity.

It should be mentioned that in contrast with the Discrete Non-Linear Schrodinger equation, where complete energy transfer takes place **[29]**, in case of nonlinear K-G lattice, complete energy transfer does not take place between the anharmonic oscillators and there is a critical time of redistribution for the quanta **[20]**. This is an important point to be noted. In our computation, this critical time is measured when the number of quanta meets or almost meets on the time axis. In the above approach, we have not used any periodic function, such as Bloch function. With this methodology, we can now proceed to deal with the application of quantum breathers in metamaterials.

III. Results and Discussion

As a preamble for the existence of quantum breather state, here we first show an eigenspectrum of two-phonon bound state (TPBS) that is a signature of the quantum breathers, by using the formalism as described in ref. **[32]** for a periodic boundary condition with a Bloch function. For SRR based metamaterials, for a very low value of permittivity of 0.002 with focusing nonlinearity α=+1 and a small value of interaction constant of 0.001, it is noted from Fig. 2 that a continuum of states with single phonon and QB state can also be observed. As revealed by our numerical simulation, the SRRs are weakly coupled and when the coupling value increases beyond 0.05, the formation of QB becomes relatively more difficult. The reason of taking a smaller value of permittivity is to merely show the presence of QB in metamaterials, even if these smaller values may be considered practically unrealistic. Having established the presence of quantum breather states in SRR based metamaterials, we may go to our main focus area of temporal evolution of the number of quanta in relation to the dielectric permittivity.

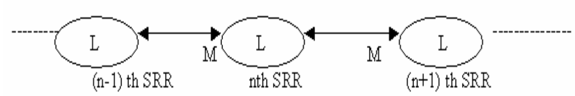

Fig. 1: Three adjacent split-ring-resonators having self-inductance *L* and mutual inductance *M*.

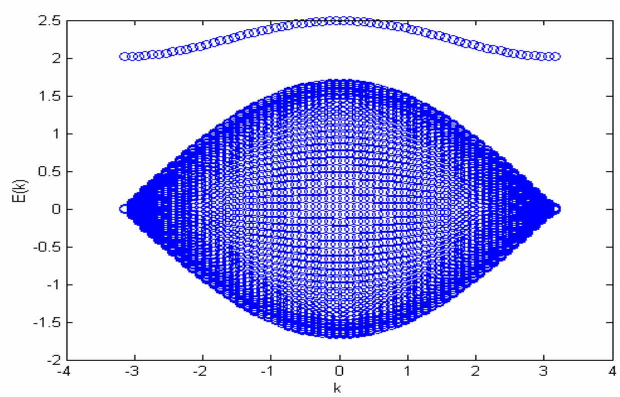

Fig. 2: Typical eigenspectrum for λ=0.001, ε_l = 0.002 and α=+1. The spectra represent single phonon continua and the quantum breather band or two-phonon bound state in the upper branch of the spectrum.

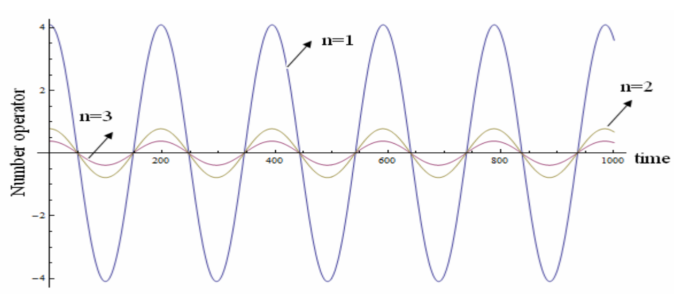

Fig. 3: Temporal-evolution spectra for metamaterials with SRR assembly for 6 particles on 3 sites with $|\psi(0)\rangle = |5,1,0\rangle$, ε_l = 2, α = +1, and λ=0.01 (t_{re} = 155.1 fs).

For a non-periodic boundary condition, the temporal evolution spectra were generated from 4 to 12 quanta on 3 sites. The time of redistribution of quanta that is proportional to QB's lifetime in fs was calculated from each spectrum. All these data are shown in Table-I for four different values of linear dielectric permittivity (ε_l) by changing 3 orders of magnitude from 0.002 to 2.0 and taking only one value of coupling of λ=0.01.

Typical spectra are shown in Fig. 3: for a relatively large value of linear dielectric permittivity, as taken by Lazarides et al **[14]** ε_l = 2, for a focusing nonlinearity α = +1, and a smaller value of coupling between the SRR elements λ=0.01. This simulation was carried out for 6 particles on 3 sites with $|\psi(0)\rangle = |5,1,0\rangle$. Here, the initial localization is mainly at the first site and then there is a fast redistribution of quanta between the other two sites until they become equal or almost equal, and the critical time for redistribution (t_{re}) is 50.89. This is about 155.1 fs for this value of coupling. This value of lifetime seems to be on the higher side as compared to that in ferroelectrics with 6 quanta (t_{re}=13.45-14.27 for a coupling between 0.1 and 0.9) despite having the same number of quanta, but with lower value of coupling within the SRR assembly. It is known that by engineering the geometry of the SRR assembly, the interaction between SRR elements can be varied. Hence, the study of QB in non-periodic boundary condition in terms of temporal evolution of the number of quanta seems to reveal a different picture for different types of materials, which both are nonlinear optical materials.

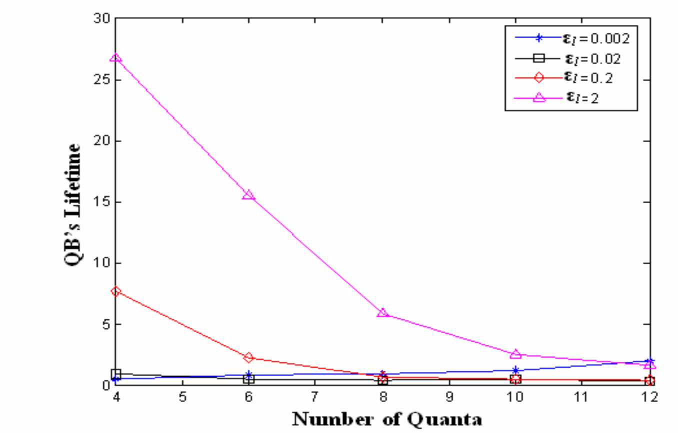

Fig. 4: The time of redistribution data (t_{re}) against number of quanta at four different values of linear permittivity showing a slight increase up to a value of ε_l = 0.2, whereas there is a significant increase afterwards towards higher value; the timescale has to be multiplied by 3.048 fs for a coupling value of 0.01.

Finally, the time of redistribution of quanta or QB's lifetime (t_{re}) is plotted against number of quanta at four different values of linear dielectric permittivity in Fig. 4. It is seen that t_{re} is low for lower permittivity values of 0.002 and 0.02, but it increases slightly up to a value of ε_l = 0.2, whereas there is a significant increase afterwards towards higher value. This change is quite noticeable at lower number of quanta in that QB's lifetime at 4 quanta changes from 76.8 fs to 266.8 fs between 0.2 and 2. TPBS parameters also show significant variation with coupling after a value of 0.05 making the quantum breather formation relatively more difficult. This shows that QB's lifetime is quite sensitive to a change of coupling as well as linear dielectric permittivity of the metamaterials.

It is pertinent to mention from the physics viewpoint that smaller values of dielectric permittivity (0.002 and 0.02) were merely taken to show the variation of number operators in our simulation. These smaller values of permittivity may be considered unrealistic from the practical point of view, but Fig. 4 shows that these values have "insignificant" role in the overall behaviour of QB's lifetime. This could be considered as an important piece of information for application in devices. For a highly absorbing sort of material, the linear part of the permittivity could be low enough, but it is difficult to ascertain its lower limit. For an increase of an order of magnitude in dielectric permittivity (i.e. 02 to 2.0), however, the variation of QB's lifetime is still very much significant. A study of QB's band gap against permittivity could possibly throw some light on the subject. It is pertinent to mention that for a macromolecule, Tretiak et al [42] observed that the lifetime of discrete breathers increases as the crystals become more and more defective. As the defect or disorder could create localization, it is congenial for the formation of QB giving us insight on quantum localization [20,27,28]. This might be an important consideration for nano device applications.

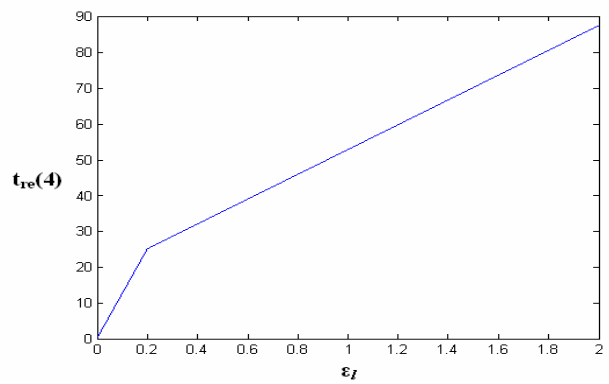

Fig. 5: Quantum breathers' lifetime at 4 quanta vs. dielectric permittivity showing the effect of permittivity at higher value; the timescale has to be multiplied by 3.048 fs for a coupling value of 0.01.

In order to highlight the role of dielectric permittivity on the femtosecond response of quantum breathers, at lower quanta, the time of redistribution at 4 quanta is plotted against dielectric permittivity in Fig. 5, wherein it is clearly seen that there is a significant increase of QB's lifetime between 0.2 and 2, as already revealed by Fig. 4 for lower quanta. From a design viewpoint of SRR assembly in an antenna array, the behavior shown in Fig. 4 and Fig. 5 indicates that for increasing QB's lifetime, i.e. the femtosecond response, the dielectric permittivity which is embodied in our Hamiltonian has an important role to play in terms of changing the temporal evolution of the number of quanta. The quantization of the Klein-Gordon lattice was not merely done to show quantum metamaterials, but to show an important variation of dielectric permittivity on the femtosecond response of quantum breathers in the realm of quantum localization. This piece of information could be useful for future directions of study in quantum coding in various relevant devices.

For the benefit of the experimentalists, one has to theoretically study temporal evolution of the quanta and vary the relevant parameters eventually to optimize the results, and only then one could make the device applications, e.g. in making antenna arrays, modulation instability based gadgets, in making quantum metamaterials based superconductors and so on. It is also emphasized in Ref. **[30]** and **[31]** that these engineered materials can be used in the quantum computation. The temporal evolution of the quanta at the correct value of coupling and dielectric permittivity could enable the required quantum coding for applications. Moreover, the proposed quantum metamaterials should allow the additional ways of controlling the 'propagation' of electromagnetic waves that are not possible by normal classical structures. For the experimentalists, this should pave the way to design better metamaterials for the required applications. Indeed, as emphasized by Rakhmanov et al **[31]**, the coherent quantum dynamics of 'qubits' determines the THZ optical properties in the system. Therefore, from the point of view of THz and other applications **[6,7]**, the present study assumes particular importance, as this is also a new area of research.

Table – I

QB's lifetime in fs at 4 different values of permittivity for 5 different quanta (coupling, λ=0.01)

No. of Quanta	ε_l =0.002	ε_l =0.02	ε_l =0.2	ε_l =2.0
4	1.5	8.8	76.8	266.8
6	2.6	4.8	22.8	155.1
8	2.9	4.4	6.3	58.0
10	3.7	4.6	4.4	24.8
12	6.4	3.2	3.6	16.2

Note: The difference of QB's lifetime between two values of permittivity is to be noted.

IV. Conclusion

In a periodic boundary condition, the branching out of two-phonon bound state from a single phonon continuum is a signature for quantum breathers in SRR based metamaterials, which is sensitive to coupling, as revealed by the eigenspectra. By quantizing a generalized Hamiltonian in Klein-Gordon lattice, for a non-periodic boundary condition, the temporal variation of the number of quanta has been studied with a main focus on its relation with dielectric permittivity. The temporal evolution spectra show a decrease of time of redistribution with increasing number of quanta for each value of dielectric permittivity except at lower value of 0.002, which does not have much significance. It is also observed that as the permittivity increases, the lifetime of QBs increases substantially from 76.8 to 266.8 fs after a value of 0.20 to 2.0 for 4 quanta and such incremental effect is reduced on increasing the number of quanta that could have implication in the design of relevant devices. This piece of information is considered useful for a future study in this new field of investigation of QBs in metamaterials, e.g. in THz applications of QBs in nonlinear optical materials. The significant role of dielectric permittivity that is embodied in our generalized Hamiltonian on the femtosecond response of quantum breathers in metamaterials might pave the way for newer quantum metamaterials. Here, for antenna array applications, this attempt in the realm of ‘quantum localization’ is expected to trigger newer investigations on this evolving subject of discrete breathers.

Acknowledgements

The authors would like to thank Dr S. Flach of Max Planck Institute (MPI) at Dresden (Germany) for his valuable guidance during the computational work.

References

[1] P. Giri, K. Choudhary, A. Sengupta, A. K. Bandyopadhyay, and A. R. McGurn, Phys. Rev. B 84, 155429 (2011).
[2] V. G. Veselago, Sov. Phys. Usp. 10, 509 (1968).
[3] Q. Cheng, W. X. Jiang and T. J. Cui, J. Phys. D: Applied Physics 43, 335406 (2010).
[4] Z. L. Mei, J. Bai, T. J. Cui, J. Phys. D: Applied Physics 43, 055404 (2010).
[5] R. R. A. Syms, E. Shamonina, and L. Solymar, Eur. Phys. J. - B 46, 301 (2005).
[6] M. Li, Z. Wen, J. Fu, X. Fang, Y. Dai, R. Liu, X Han and X. Qiu, J. Phys. D: Applied Physics 42, 15420 (2009).
[7] R. Singh, E. Plum, W. Zhang, N. I. Zheludev, Opt. Exp. 18, 13425 (2010).
[8] N. I. Zheludev, Opt. and Photon. News, March 31 (2011).
[9] A. Alu and N. Enghetta Opt. Exp. 15, 3318 (2007).
[10] A. Boardman, J. Opt. 13, 020401 (2011).
[11] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. 84, 4184 (2000).
[12] J. B. Pendry, A. J. Holden, D. J. Roberts, and W. J. Stewart, J. Phys: Condens. Matter 10, 4785 (1998).
[13] N. Lazarides,M. Eleftheriou and G. P. Tsironis, Phys. Rev. Lett. 97, 036608 (2006).
[14] M. Eleftheriou, N. Lazarides, and G. P. Tsironis, Phys. Rev. E 77, 036608 (2008).
[15] A. A. Zharov, I. V. Shadrivov, and Y. S. Kivshar, Phys Rev. Lett. 91, 037401 (2003)
[16] S. O’Brien, D. McPeake, S.A. Ramakrishna and J. B. Pendry, Phys. Rev. B 69, 241101 (2004).
[17] I. Kourakis, N. Lazarides, G. P. Tsironis, Phys. Rev. E 75, 067601 (2007).
[18] M. J. Ablowitz and G. Biondini, Opt. Lett. 23, 1668 (1998).
[19] I. Gabitov, R. Indik, L. Mollenauer, M. Shkarayev, M. Stepanov and P.M.Lushnikov, Opt. Lett. 32, 605 (2007).
[20] S. Flach and A. V. Gorbach, Phys. Rep. 467, 1 (2008).
[21] M. Sato, B. E. Hubbard and A. J. Sievers, Rev. Mod. Phys. 78, 137 (2006).
[22] J. W. Fleischer, M. Segev, N. K. Elfremidis and D. N. Christodoulides, Nature 422, 147 (2003).
[23] D. R. Smith and J. B. Pendry, J. Opt. Soc. Am.-B 23, 07403224 (2006).
[24] A. K. Bandyopadhyay, P.C. Ray, L. Vu-Quoc and A. R. McGurn, Phys. Rev. B 81, 064104 (2010).
[25] A. K. Bandyopadhyay, P. C. Ray, and V. Gopalan, J. Phys: Condens. Matter 18, 4093 (2006)
[26] P. Giri, K. Choudhary, A. Sengupta, A. K. Bandyopadhyay and P. C. Ray, J. Appl. Phys. 109, 054105 (2011).
[27] A. C. Scott, J.C. Eilbeck and H. Gilhøj, Physica D 78, 194 (1994).
[28] R. A. Pinto, M. Haque and S. Flach, Phys. Rev. A 79, 052118 (2009).

[29] G. P. Tsironis, Chaos 13, 657 (2003).
[30] J. Q. Quach, C. H. Su, A. M. Martin, A. D. Greentree, and L. C. L. Hollenberg, Opt. Exp. 19, 11018 (2011).
[31] A. L. Rakhmanov. A. M. Zagoskin, S. Savelev and F. Nori, Phys. Rev. B 77, 144507 (2008).
[32] A. Biswas, K. Choudhary, A. K. Bandyopadhyay, A .K. Bhattacharjee and D. Mandal, J. Appl. Phys. 110, 024104 (2011).
[33] J. P. Nguenang, R. A. Pinto and S. Flach , Phys. Rev. B 75, 214303 (2007).
[34] C. Santori and R. G. Beausoleil, Nature Photonics 6, 10 (2012).
[35] M. H. Cohen and J. Ruvalds, Phys. Rev. Lett. 23, 1378 (1969).
[36] C. J. Brennan and K. A. Nelson, J. Chem. Phys. 107, 9691 (1997).
[37] L. Proville, Phys. Rev. B 71, 104306 (2005).
[38] P. Maniadis, G. Kopidakis and S. Aubry, Physica D 188, 153 (2004).
[39] Ch. Skokos, D. O. Krimer, S. Komineas and S. Flach, Phys. Rev. E 79, 056211 (2009).
[40] L. S. Schulman, D Tolkunov and E. Mihokova, Chem. Phys. 322, 55 (2006).
[41] S. J. Mandal, K. Choudhary, A. Biswas, A. K. Bandyopadhyay, A. K. Bhattacharjee and D. Mandal, J. Appl. Phys. 110, 124106 (2011).
[42] S. Tretiak, A. Saxena, R. L. Martin and A. R. Bishop, Phys. Rev. Lett. 89, 7402 (2002).