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Self-tuning Information Fusion Kalman Filter for Multisensor Multi-channel ARMA Signals with Colored Measurement Noises and its Convergence

Guili Tao[1,2] *and Zili Deng*[1]

[1] Department of Automation, Heilongjiang University, Harbin, 150080, People's Republic of China
[2] Heilongjiang Institute of Science and Technology, Harbin 150027, People's Republic of China

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Abstract: For the multisensor multi-channel autoregressive moving average (ARMA) signals with white measurement noises and an ARMA colored measurement noise as a common disturbance noise, a multi-stage information fusion identification method is presented when model parameters and noise variances are partially unknown. The local estimators of model parameters and noise variances are obtained by the multi-dimensional recursive instrumental variable (MRIV) algorithm, correlation method, and the Gevers-Wouters algorithm, and the fused estimators are obtained by taking the average of the local estimators. They have the consistency. Substituting them into the optimal fusion Kalman filter weighted by scalars, a self-tuning fusion Kalman filter for multi-channel ARMA signals is presented. It requires a less computational burden, and is suitable for real time applications. Applying the dynamic error system analysis (DESA) method, it is proved that the proposed self-tuning fusion Kalman filter converges to the optimal fusion Kalman filter in a realization, so that it has asymptotic optimality. A simulation example shows its effectiveness.

Keywords: Information fusion Kalman filter,identification,convergence analysis, self-tuning Kalman filter.

1. Introduction

Multi-sensor data fusion is a technique, which seeks to combine data from multiple sensors to achieve improved accuracies that could not be achieved by using a single sensor. Although there have been a significant amount of research efforts reported during the past twenty years, fusion of multi-sensor data is still a challenging problem [1–5]. As we all known, two basic data fusion methods are centralized and distributed (or decentralized) fusion methods [6,7]. The centralized filter method can provide the globally optimal state estimation by directly combining local measurement data. However, the centralized filter can cause a large computational burden in the fusion center due to the high-dimension computation and large data memory [8]. In distributed fusion, the information from local estimators can yield the global optimal or suboptimal state estimation according to certain information fusion criteria. The Bayesian algorithm [9] and the genetic algorithm [10] were also studied for distributed fusion of multisensor data. Compared to centralized data fusion, distributed data fusion effectively utilizes information from a lot of different sensors. It has many advantages such as lighter processing load, lower communication load, easy fault detection and isolation, and high reliability [11]. And, the existing information fusion Kalman filtering is mainly focused on the information fusion Kalman filtering with known model parameter and noise statistics. However, in many applications, the model parameters and/or noise variances are usually unknown. The filtering for the systems with unknown model parameters and/or noise variances is called self-tuning filtering [12]. Several self-tuning weighted fusion estimators [13–16] were presented. Their drawbacks are that only the noise variances are assumed to be unknown, while the model parameters are assumed to be known. The self-tuning information fusion Kalman or Wiener filter was presented for the multisensor single-channel ARMA signals with unknown model parameters and unknown noise variances in [17,18]. However, only a few results were re-

* Corresponding author: e-mail: dzl@hlju.edu.cn

ported for the multisensor multi-channel ARMA signals with unknown model parameters and unknown noise variances [19,20]. And in [19], for the multisensor multi-channel ARMA signals, using the modern time series analysis method, based on the online identification of the autoregressive moving average (ARMA) model, the self-tuning weighted measurement fusion Wiener filter was proposed. Its limitation is that the measurement noises of sensors are assumed to be white measurement noises. The convergence analysis method of the self-tuning Kalman or Wiener fuser has been proved in [13,14], which is called dynamic error system analysis (DESA) method, and a new concept of convergence in a realization was presented in [13,14], which is weaker than the convergence with probability one. The convergence analysis method of the self-tuning Riccati equation and Lyapunov equation was proved in [16], which is called dynamic variance error system analysis (DVESA) method.

In this paper, using the classical Kalman filtering method, the self-tuning fusion Kalman filter weighted by scalars is presented for the multisensor systems with colored measurement noises, and with partially unknown model parameters and noise variances. By the DESA method, the convergence of self-tuning information fusion Kalman filter was proved by the DESA method, i.e., the self-tuning fusion Kalman filter converges to the optimal fusion Kalman filter in a realization, so it has asymptotic optimality. Compared with [19,20], the paper presents the information fusion Kalman filter for multisensor multi-channel ARMA signals with an ARMA colored common disturbance measurement noise. The results proposed in [21] are extended to multi-channel case with an ARMA colored measurement noise.

The rest of this paper is organized as follows: In Section 2 we give the optimal fusion Kalman signal filter. Multi-stage information fusion estimators of model parameters and noise variances is presented in Section 3. Self-tuning fusion Kalman filter is presented in Section 4. Section 5 presents convergence analysis. Section 6 gives one simulation example. The conclusion is presented in Section 7.

2. The optimal fusion Kalman signal filter

Consider the multisensor multi-channel ARMA signals with a colored common disturbance measurement noise

$$A(q^{-1})s(t) = C(q^{-1})w(t) \tag{1}$$

$$y_i(t) = s(t) + \eta(t) + v_i(t), i = 1, \cdots, L \tag{2}$$

$$B(q^{-1})\eta(t) = R(q^{-1})\xi(t) \tag{3}$$

where t is the discrete time, $s(t) \in R^m$ is the signal to be estimated, $y_i(t) \in R^m$ is the measurement of the ith sensor, $\xi(t)$ is a white noise, $w(t) \in R^r$, $v_i(t) \in R^m$ and $\eta(t) \in R^m$ are a white process noises, white measurement noises and an ARMA colored common disturbance measurement noise, respectively. $A(q^{-1})$, $C(q^{-1})$, $B(q^{-1})$ and $R(q^{-1})$ are polynomials of backward shift operator q^{-1} with the following form:

$$X(q^{-1}) = X_0 + X_1 q^{-1} + \cdots X_{n_x} q^{-n_x} \tag{4}$$

where $X_0 = I_m, I_m$ is the $m \times m$ identity matrix, n_a, n_c, n_b, n_r are the orders of polynomial matrices $A(q^{-1})$, $C(q^{-1})$, $B(q^{-1})$, $R(q^{-1})$, respectively.

Assumption 1. $w(t)$, $\xi(t)$ and $v_i(t)$ $(i = 1, \cdots, L)$ are independent white noises with zero mean and variances Q_w, Q_ξ and Q_{vi}, respectively.

Assumption 2. $A(q^{-1})$, $C(q^{-1})$, $B(q^{-1})$ and $R(q^{-1})$ are stable polynomials. $(A(q^{-1}), C(q^{-1}))$ and $(B(q^{-1}), R(q^{-1}))$ are left coprime.

Assumption 3. $A(q^{-1}), C(q^{-1})$ and Q_w are unknown, but $B(q^{-1}), R(q^{-1}), Q_\xi$ and Q_{vi} are known.

Assumption 4. A realization of measurement stochastic process $y_i(t) (i = 1, 2, \cdots, L)$ is bounded for t .

The problem is to find self-tuning information fusion Kalman signal filter weighted by scalars when the ARMA model parameters and noise variances are partially unknown.

2.1. The conversion of ARMA signal model into state space model

Setting $w(t) = \underline{w}(t-1)$, yields that $\underline{w}(t)$ has the variance Q_w. The ARMA signal (1) can be rewritten as

$$A(q^{-1})s(t) = \underline{C}(q^{-1})\underline{w}(t) \tag{5}$$

where $\underline{C}(q^{-1}) = \underline{C}_0 + \underline{C}_1 q^{-1} \cdots \underline{C}_{n_{\underline{c}}} q^{-n_{\underline{c}}}, \underline{C}_0 = 0, \underline{C}_i = C_{i-1}, i = 1, 2, \cdots, n_{\underline{c}}, n_{\underline{c}} = n_c + 1$.

The signal system (5) has the state space model with the companion form

$$\alpha(t+1) = \underline{A}\alpha(t) + \underline{C}\underline{w}(t) \tag{6}$$

$$s(t) = H_\alpha \alpha(t) \tag{7}$$

$$\underline{A} = \begin{bmatrix} -A_1 & & \\ -A_2 & I_{m(n_a-1)} & \\ \vdots & & \\ -A_{n_{ac}} & 0 \cdots & 0 \end{bmatrix}, \underline{C} = \begin{bmatrix} I_m \\ C_1 \\ \vdots \\ C_{n_{ac}} \end{bmatrix},$$

$$H_\alpha = \begin{bmatrix} I_m & 0 \cdots 0 \end{bmatrix} \tag{8}$$

with definition $n_{ac} = \max(n_a, n_c + 1), A_j = 0 (j > n_{ac}), C_j = 0 (j > n_{ac})$.

Similarly, making $\xi(t) = \underline{\xi}(t-1)$, yields that $\underline{\xi}(t)$ has also the variance Q_ξ. And the colored noise model (3) can be transformed to the equivalent state space model

$$\beta(t+1) = \underline{P}\beta(t) + \underline{R}\underline{\xi}(t) \tag{9}$$

$$\eta(t) = H_\beta \beta(t) \tag{10}$$

where

$$\underline{P} = \begin{bmatrix} -B_1 & & \\ -B_2 & I_{m(n_b-1)} & \\ \vdots & & \\ -B_{n_{br}} & 0 \cdots & 0 \end{bmatrix}, \underline{R} = \begin{bmatrix} I_m \\ R_1 \\ \vdots \\ R_{n_{br}} \end{bmatrix}, \tag{11}$$

$$H_\beta = \begin{bmatrix} I_m & 0 \cdots 0 \end{bmatrix}$$

with $n_{br} = \max(n_b, n_r + 1), B_j = 0(j > n_{br}), R_j = 0(j > n_{br})$.

Introducing the augmented state $x(t)$, augmented input noise $\bar{w}(t)$,

$$x(t) = \begin{bmatrix} \alpha(t) \\ \beta(t) \end{bmatrix}, \bar{w}(t) = \begin{bmatrix} \underline{w}(t) \\ \underline{\xi}(t) \end{bmatrix} \quad (12)$$

Then augmented system is given as

$$x(t+1) = \Phi x(t) + \Gamma \bar{w}(t) \quad (13)$$
$$y_i(t) = Hx(t) + v_i(t) \quad (14)$$
$$s(t) = H_s x(t) \quad (15)$$

where

$$\Phi = \begin{bmatrix} \underline{A} & 0 \\ 0 & \underline{P} \end{bmatrix}, \Gamma = \begin{bmatrix} \underline{C} & 0 \\ 0 & \underline{R} \end{bmatrix},$$
$$H = \begin{bmatrix} H_\alpha & H_\beta \end{bmatrix}, H_s = \begin{bmatrix} H_\alpha & 0 \end{bmatrix}. \quad (16)$$

Noting $\bar{w}(t)$ and $v_i(t)$ are uncorrelated white noises with zero mean and variance matrices $Q_{\bar{w}}$ and Q_{vi}, respectively, and

$$Q_{\bar{w}} = \begin{bmatrix} Q_w & 0 \\ 0 & Q_\xi \end{bmatrix}, Q_{vij} = 0, S_i = \mathrm{E}[\bar{w}(t)v_i^{\mathrm{T}}(t)] = 0 \quad (17)$$

2.2. *The optimal fusion Kalman signal filter weighted by scalars*

Lemma 1 [22]. For the multi-sensor systems (13)-(15) with known model parameters and noise variances, the ith sensor subsystem has the local optimal Kalman filter $\hat{x}_i(t|t)$ of $x(t)$ as

$$\hat{x}_i(t|t) = \Psi_{fi}(t)\hat{x}_i(t-1|t-1) + K_{fi}(t)y_i(t) \quad (18)$$
$$\Psi_{fi}(t) = [I_{m(n_a+n_b)} - K_{fi}(t)H]\Phi \quad (19)$$
$$K_{fi}(t) = \Sigma_i(t|t-1)H^{\mathrm{T}}(H\Sigma_i(t|t-1)H^{\mathrm{T}} + Q_{vi})^{-1} \quad (20)$$
$$P_i(t|t) = [I_{m(n_a+n_b)} - K_{fi}(t)H]\Sigma_i(t|t-1) \quad (21)$$
$$\hat{s}_i(t|t) = H_s\hat{x}_i(t|t) \quad (22)$$
$$\hat{x}_i(0|0) = \mu, P_i(0|0) = P_0 \quad (23)$$

where the prediction error variance matrices $\Sigma_i(t|t-1)$ satisfy the time-varying optimal Riccati equation

$$\Sigma_i(t+1|t) = \Phi[\Sigma_i(t|t-1) - \Sigma_i(t|t-1)H^{\mathrm{T}}(H \times\Sigma_i(t|t-1)H^{\mathrm{T}} + Q_{vi})^{-1}H\Sigma_i(t|t-1)] \times\Phi^{\mathrm{T}} + \Gamma Q_{\bar{w}}\Gamma^{\mathrm{T}} \quad (24)$$

The local filter error cross-covariances $P_{ij}(t|t) = \mathrm{E}[\tilde{x}_i(t|t)\tilde{x}_j^{\mathrm{T}}(t|t)]$, $i,j = 1,2,\cdots,L$, with $\tilde{x}_i(t|t) = x(t) - \hat{x}_i(t|t)$, satisfy the time-varying Lyapunov equation

$$P_{ij}(t|t) = \Psi_{fi}(t)P_{ij}(t-1|t-1)\Psi_{fj}^{\mathrm{T}}(t) + [I_{m(n_a+n_b)} - K_{fi}(t)H]\Gamma Q_{\bar{w}}\Gamma^{\mathrm{T}} \times[I_{m(n_a+n_b)} - K_{fj}(t)H]^{\mathrm{T}}, i \neq j \quad (25)$$

with the definition $P_{ii}(t|t) = P_i(t|t)$, where the cross-covariances among the filter errors $\tilde{s}_i(t|t) = s(t) - \hat{s}_i(t|t)$, $P_{sij} = \mathrm{E}[\tilde{s}_i(t|t)\tilde{s}_j^{\mathrm{T}}(t|t)]$, are

$$P_{sij}(t|t) = H_s P_{ij}(t|t) H_s^{\mathrm{T}} \quad (26)$$

Lemma 2[22] For the multi-sensor systems (13)-(15) with assumptions 1 and 2, we have the optimal information fusion Kalman signal filter weighted by scalars

$$\hat{s}_0(t|t) = \sum_{j=1}^{L} \omega_j(t|t)\hat{s}_j(t|t) \quad (27)$$

where the optimal scalars weighting coefficient vectors $\omega_j(t|t)$ are given by

$$\begin{bmatrix} \omega_1(t|t), \cdots, \omega_L(t|t) \end{bmatrix} = [e^{\mathrm{T}}(P_s^{-1}(t|t))e]^{-1}e^{\mathrm{T}}P_s^{-1}(t|t) \quad (28)$$

where $e^{\mathrm{T}} = [1,\cdots,1]$, and $L\times L$ matrix $P_s(t|t)$ is defined as

$$P_s(t|t) = (\mathrm{tr}P_{sij}(t|t)), i,j = 1,\cdots,L \quad (29)$$

whose $(i,j)\mathrm{th}$ element $P_s(t|t)$ are $\mathrm{tr}P_{sij}(t|t)$.

3. Multi-stage information fusion estimators of model parameters and noise variances

For the multisensor systems (1) -(3) with assumptions 1-4, we can apply the multi-stage information fusion identification method [23] to obtain estimators of the model parameters and noise variances. The information fusion estimators can be obtained by the following three stages.

3.1. *Information fusion autoregressive (AR) parameter estimator*

Substituting (1) and (3) into (2) yields

$$y_i(t) = A^{-1}(q^{-1})C(q^{-1})w(t) + B^{-1}(q^{-1})R(q^{-1})\xi(t) + v_i(t) \quad (30)$$

(30) can be rewritten as

$$\det B(q^{-1})A(q^{-1})y_i(t) = \det B(q^{-1})C(q^{-1})w(t) + A(q^{-1})\mathrm{adj}B(q^{-1}) \times R(q^{-1})\xi(t) + \det B(q^{-1}) \times A(q^{-1})v_i(t) \quad (31)$$

Setting $\det B(q^{-1})y_i(t) = z_i(t)$, so we have

$$A(q^{-1})z_i(t) = \det B(q^{-1})C(q^{-1})w(t) + A(q^{-1})\mathrm{adj}B(q^{-1})R(q^{-1})\xi(t) + \det B(q^{-1})A(q^{-1})v_i(t) \quad (32)$$

Defining

$$\begin{aligned}\bar{C}(q^{-1}) &= \det B(q^{-1})C(q^{-1})\\ &= I_m + \bar{C}_1 q^{-1} + \cdots + \bar{C}_{n_{\bar{c}}} q^{-n_{\bar{c}}},\\ \bar{R}(q^{-1}) &= A(q^{-1})\mathrm{adj}B(q^{-1})R(q^{-1})\\ &= I_m + \bar{R}_1 q^{-1} + \cdots + \bar{R}_{n_{\bar{r}}} q^{-n_{\bar{r}}},\\ \bar{A}(q^{-1}) &= \det B(q^{-1})A(q^{-1})\\ &= I_m + \bar{A}_1 q^{-1} + \cdots + \bar{A}_{n_{\bar{a}}} q^{-n_{\bar{a}}} \end{aligned} \quad (33)$$

with $n_{\bar{r}} = n_a + n_b + n_r, n_{\bar{c}} = mn_b + n_c, n_{\bar{a}} = mn_b + n_a$. (32) can be written as

$$A(q^{-1})z_i(t) = \bar{C}(q^{-1})w(t) + \bar{R}(q^{-1})\xi(t) + \bar{A}(q^{-1})v_i(t) \quad (34)$$

Hence for the ith sensor, we have the least squares (LS) structure as

$$z_i(t) = \Theta\varphi_i(t) + r_i(t), i = 1, 2, \cdots, L \quad (35)$$

with the definition

$$\Theta = \left[\, A_1, \cdots, A_{n_a} \,\right] \quad (36)$$

$$\varphi_i(t) = \left[-z_i(t-1); \cdots; -z_i(t-n_a) \right] \quad (37)$$

$$r_i(t) = \bar{C}(q^{-1})w(t) + \bar{R}(q^{-1})\xi(t) + \bar{A}(q^{-1})v_i(t) \quad (38)$$

Lemma 3 [23] For the ith sensor subsystem with multichannel stationary ARMA model (1)-(3), the multidimensional recursive instrumental variable (MRIV) local estimator $\hat{\Theta}_i(t)$ of Θ are

$$\hat{\Theta}_i(t) = \hat{\Theta}_i(t-1) + \frac{[z_i(t) - \hat{\Theta}_i(t-1)\varphi_i(t)]\hat{\varphi}_i^{\mathrm{T}}(t)}{1 + \hat{\varphi}_i^{\mathrm{T}}(t)P_i(t-1)\varphi_i(t)} \times P_i(t-1) \quad (39)$$

$$P_i(t) = P_i(t-1) - \frac{P_i(t-1)\varphi_i(t)\hat{\varphi}_i^{\mathrm{T}}(t)P_i(t-1)}{1 + \hat{\varphi}_i^{\mathrm{T}}(t)P_i(t-1)\varphi_i(t)} \quad (40)$$

$$\hat{\varphi}_i(t) = \varphi_i(t - n_0) \quad (41)$$

with initial value $\hat{\Theta}_i(t) = \hat{\Theta}_{i0}$, $P_i(t_0) = \alpha I_{mn_a}, z_i(k) = 0(k \le 0)$, $n_0 = \max(n_{\bar{a}}, n_{\bar{c}}, n_{\bar{r}})$. Applying the multidimensional recursive instrumental variable (MRIV) algorithm [23], the multidimensional local RIV estimators $\hat{\Theta}_i(t)$ of Θ converge to the true value with probability one, i.e.

$$\hat{\Theta}_i(t) \to \Theta, as \quad t \to \infty, \mathrm{w.p.1}, i = 1, \cdots, L \quad (42)$$

where “w.p.1” denotes “with probability one”. The information fusion estimator $\hat{\Theta}_f(t)$ of Θ is defined as

$$\hat{\Theta}_f(t) = \frac{1}{L}\sum_{i=1}^{L} \hat{\Theta}_i(t) = \left[\, \hat{A}_1(t), \cdots, \hat{A}_{n_a}(t) \,\right] \quad (43)$$

so that from (43) it is also strongly consistent, i.e.

$$\hat{\Theta}_f(t) \to \Theta, as \quad t \to \infty, \mathrm{w.p.1} \quad (44)$$

From (43), $\hat{A}_l(t)$ is the lth element of $\hat{\Theta}_f(t)$. Hence, we have

$$\hat{A}_l(t) \to A_l, l = 1, \cdots, n_a, as \quad t \to \infty, \mathrm{w.p.1} \quad (45)$$

Hence, $\hat{A}(q^{-1})$ is obtained. Then substituting $\hat{A}(q^{-1})$ into (33), we can yield $\hat{\bar{A}}(q^{-1})$ and $\hat{\bar{R}}(q^{-1})$, i.e. $\hat{\bar{A}}_l(t)(l = 1, \cdots, n_{\bar{a}})$ and $\hat{\bar{R}}_l(t)(l = 1, \cdots, n_{\bar{r}})$ are obtained.

3.2. Information fusion estimators of moving average (MA) parameters and variance

From (31), (33) and (38), we have

$$r_i(t) = \bar{A}(q^{-1})y_i(t) \quad (46)$$

It is clearly that $\bar{A}(q^{-1})$ is a stable polynomials of q^{-1}, $y_i(t)$ is a stationary stochastic process, so it yields that $r_i(t)$ is also a stationary stochastic process with cross-correlation function as

$$R_{rij}(k) = \mathrm{E}[r_i(t)r_j^{\mathrm{T}}(t-k)], \quad k = 0, \cdots, n_{\bar{a}} \quad (47)$$

with a cut-off as lag $n_{\bar{a}}$. At time t, the estimate of the measurement process $r_i(t)$ is defined as

$$\hat{r}_i(t) = \hat{\bar{A}}(q^{-1})y_i(t) \quad (48)$$

The on-line sampled correlation function $\hat{R}^t_{rij}(k)$ of $R_{rij}(k)$ is defined as

$$\hat{R}^t_{rij}(k) = \frac{1}{t}\sum_{u=1}^{t} \hat{r}_i(u)\hat{r}_j^{\mathrm{T}}(u-k) \quad (49)$$

which has the recursive form

$$\hat{R}^t_{rij}(k) = \hat{R}^{t-1}_{rij}(k) + \frac{1}{t}[\hat{r}_i(t)\hat{r}_j^{\mathrm{T}}(t-k) - \hat{R}^{t-1}_{rij}(k)] \quad (50)$$

with the initial value $\hat{R}^1_{rij}(k) = \frac{1}{t}\sum\limits_{j=1}^{t} \hat{r}_i(1)\hat{r}_j^{\mathrm{T}}(1-k)$.

Defining the MA process as

$$m(t) = \bar{C}(q^{-1})w(t) \quad (51)$$

from (38), we can obtain that

$$r_i(t) = m(t) + \bar{R}(q^{-1})\xi(t) + \bar{A}(q^{-1})v_i(t) \quad (52)$$

and $m(t)$ is a stationary stochastic process, whose correlation function $R_m(k)$ is defined as $R_m(k) = \mathrm{E}[m(t)m^{\mathrm{T}}(t-k)]$. From (51), we have that

$$R_m(k) = \sum_{u=k}^{n_{\bar{c}}} \bar{C}_u Q_w \bar{C}^{\mathrm{T}}_{u-k}, k = 0, \cdots, n_{\bar{c}} \quad (53)$$

In order to yield $\bar{C}_u$ and Q_w, we need to find $R_m(k)$. Computing the correlation function of the two sides of (52) yields that

$$R_m(k) = R_{rij}(k) - \sum_{u=k}^{n_{\bar{r}}} \bar{R}_u Q_\xi \bar{R}_{u-k}^{\mathrm{T}} - \sum_{u=k}^{n_{\bar{a}}} \bar{A}_u Q_{vi} \bar{A}_{u-k}^{\mathrm{T}} \delta_{ij}, k = 0, \cdots, n_{\bar{c}} \tag{54}$$

Substituting the sample estimates $\hat{R}_{rij}^t(k)$, and fused estimates $\hat{\bar{A}}_i(t)$ and $\hat{\bar{R}}_i(t)$ into (54) yields the local estimates of $R_m(k)$ as

$$\hat{R}_{mij}^t(k) = \hat{R}_{rij}^t(k) - \sum_{u=k}^{n_{\bar{r}}} \hat{\bar{R}}_u(t) Q_\xi \hat{\bar{R}}_{u-k}^{\mathrm{T}}(t) - \sum_{u=k}^{n_{\bar{a}}} \hat{\bar{A}}_u(t) Q_{vi} \hat{\bar{A}}_{u-k}^{\mathrm{T}}(t) \delta_{ij}, k = 0, \cdots, n_{\bar{c}} \tag{55}$$

Bases on the estimates $\hat{R}_{mij}^t(k)$, using the Gevers-Wouters algorithm [24] with a dead band, we can obtain the local estimates $\hat{\bar{C}}_{uij}(t)(u = 1, \cdots, n_{\bar{c}}, i, j = 1, \cdots, L)$ and $\hat{Q}_{wij}(t)$ as

$$\hat{Q}_{wij}(t) = \lim_{l\to\infty} R_{mwij}(l, l) \tag{56}$$

$$\hat{\bar{C}}_{uij}(t) = \lim_{l\to\infty} R_{mwij}(l, l-u) R_{mwij}^{-1}(l, l), \quad u = 1, \cdots, n_{\bar{c}} \tag{57}$$

$$R_{mwij}(l, l-u) = \hat{R}_{mij}^t(u) - \sum_{r=u+1}^{n_{\bar{c}}} R_{mwij}(l, l-r) \times R_{mwij}^{-1}(l-r, l-r) \times R_{mwij}(l-u, l-r) \tag{58}$$

with the definitions

$$R_{mwij}(0,0) = \hat{R}_{mij}^t(0), R_{mwij}(l, l-r) = 0 (l < r)$$
$$R_{mwij}^{-1}(l-r, l-r) = 0 (l < r) \tag{59}$$

Then the information fusion estimates $\hat{\bar{C}}_u(t), \hat{Q}_w(t)$ based on all sensors are defined as

$$\hat{Q}_w(t) = \frac{1}{L^2} \sum_{i=1}^{L} \sum_{j=1}^{L} \hat{Q}_{wij}(t), \tag{60}$$

$$\hat{\bar{C}}_u(t) = \frac{1}{L^2} \sum_{i=1}^{L} \sum_{j=1}^{L} \hat{\bar{C}}_{uij}(t), u = 1, \cdots, n_{\bar{c}} \tag{61}$$

Defining

$$\det B(q^{-1}) = 1 + g_1 q^{-1} + \cdots g_{mn_b} q^{-mn_b} \tag{62}$$

$$M = \left[C_1^{\mathrm{T}}, \cdots, C_{n_c}^{\mathrm{T}} \right]^{\mathrm{T}} \tag{63}$$

From (33), we have

$$\varOmega M = \varUpsilon \tag{64}$$

where

$$\varOmega = \begin{bmatrix} I_m & & 0 \\ g_1 I_m & \ddots & \\ \vdots & \ddots & I_m \\ g_{mn_b} I_m & & g_1 I_m \\ & \ddots & \\ 0 & & g_{mn_b} I_m \end{bmatrix}, \varUpsilon = \begin{bmatrix} \bar{C}_1 - g_1 I_m \\ \vdots \\ \bar{C}_{mn_b} - g_{mn_b} I_m \\ \bar{C}_{mn_b+1} \\ \vdots \\ \bar{C}_{n_{\bar{c}}} \end{bmatrix} \tag{65}$$

which yields

$$M = (\varOmega^{\mathrm{T}} \varOmega)^{-1} \varOmega^{\mathrm{T}} \varUpsilon \tag{66}$$

Substituting the estimators $\hat{\bar{C}}_u(t)(u = 1, \cdots, n_{\bar{c}})$ into the first formula of (33) yields

$$\hat{\bar{C}}(q^{-1}) = \det B(q^{-1}) \hat{C}(q^{-1}) = I_m + \hat{\bar{C}}_1(t) q^{-1} + \cdots \hat{\bar{C}}_{n_{\bar{c}}}(t) q^{-n_{\bar{c}}} \tag{67}$$

where

$$\hat{C}(q^{-1}) = I_m + \hat{C}_1(t) q^{-1} + \cdots + \hat{C}_{n_c}(t) q^{-n_c} \tag{68}$$

Defining

$$\hat{M}(t) = \left[\hat{C}_1^{\mathrm{T}}(t), \cdots, \hat{C}_{n_c}^{\mathrm{T}}(t) \right]^{\mathrm{T}} \tag{69}$$

Substituting $\hat{C}_u(t)$ into (64) yields

$$\varOmega \hat{M}(t) = \hat{\varUpsilon}(t) \tag{70}$$

where

$$\hat{\varUpsilon}(t) = \begin{bmatrix} \hat{\bar{C}}_1(t) - g_1 I_m \\ \vdots \\ \hat{\bar{C}}_{mn_b}(t) - g_{mn_b} I_m \\ \hat{\bar{C}}_{mn_b+1}(t) \\ \vdots \\ \hat{\bar{C}}_{n_{\bar{c}}}(t) \end{bmatrix} \tag{71}$$

Solving (70) by Pseudo inverse yields that

$$\hat{M}(t) = (\varOmega^{\mathrm{T}} \varOmega)^{-1} \varOmega^{\mathrm{T}} \hat{\varUpsilon}(t) \tag{72}$$

Theorem 1. For the multisensor systems (1)-(3) with assumptions 1-4, the fused estimators of model parameters and noise variances are consistent, i.e.

$$\hat{\bar{A}}_i(t) \to \bar{A}_i, i = 1, \cdots, n_{\bar{a}}, \text{as} \quad t \to \infty, \text{i.a.r} \tag{73}$$

$$\hat{\bar{R}}_i(t) \to \bar{R}_i, i = 1, \cdots, n_{\bar{r}}, \text{as} \quad t \to \infty, \text{i.a.r} \tag{74}$$

$$\hat{\bar{C}}_u(t) \to \bar{C}_u, u = 1, \cdots, n_{\bar{c}}, \text{as} \quad t \to \infty, \text{i.a.r} \tag{75}$$

$$\hat{Q}_w(t) \to Q_w, \text{as} \quad t \to \infty, \text{i.a.r} \tag{76}$$

$$\hat{C}_u(t) \to C_u, i = 1, \cdots, n_c, \text{as} \quad t \to \infty, \text{i.a.r} \tag{77}$$

where the notation “i.a.r” denotes the convergence “in a realization”[13].

Proof. From (33) and (45), we have that (73) and (74) hold. From (53) and the existence theorem of implicit function [25], $\bar{C}_u$ and Q_w are the continuous functions of the elements of $R_m(k)(k=0,\cdots,n_{\bar{c}})$ in a sufficiently small neighborhood, i.e.

$$\bar{C}_u = f_{\bar{c}}(R_m(0),\cdots,R_m(n_{\bar{c}})) \quad (78)$$
$$Q_w = f_w(R_m(0),\cdots,R_m(n_{\bar{c}})) \quad (79)$$

where $f_{\bar{c}}$ and f_w are the continuous functions, when t is sufficiently large, we have relations

$$\hat{\bar{C}}_{uij}(t) = f_{\bar{c}}(\hat{R}^t_{mij}(0),\cdots,\hat{R}^t_{mij}(n_{\bar{c}})) \quad (80)$$
$$\hat{Q}_{wij}(t) = f_w(\hat{R}^t_{mij}(0),\cdots,\hat{R}^t_{mij}(n_{\bar{c}})) \quad (81)$$

According to the ergodicity [26], we have

$$\hat{R}^t_{rij}(k) \to R_{rij}(k), \text{as} \quad t\to\infty, \text{i.a.r} \quad (82)$$

and from (54),(55),(73), (74) and (82), we have

$$\hat{R}^t_{mij}(k) \to R_m(k), \text{as} \quad t\to\infty, \text{i.a.r} \quad (83)$$

From (78)-(81), (83), and the continuity of $f_{\bar{c}}$ and f_w, we have

$$\hat{\bar{C}}_{uij}(t) \to \bar{C}_u, \hat{Q}_{wij}(t) \to Q_w, \text{as} \quad t\to\infty, \text{i.a.r} \quad (84)$$

Therefore, from (60), (61) and (84), it follows that (75) and (76) hold. From (66) and (72), the each element of M and $\hat{M}(t)$ is a continuous function of elements of $\bar{C}_u$ and $\hat{\bar{C}}_u(t)(u=1,\cdots,n_{\bar{c}})$, which yields $\hat{M}(t)\to M$, as $t\to\infty$, i.a.r, i.e., (77) holds.

4. Self-tuning fusion Kalman filter

When model parameters and noise variances are partially unknown, substituting their estimators into the optimal fusion Kalman filter yields the self-tuning fusion Kalman filter. It consists of the following steps:

Step 1.a) Applying the multidimensional recursive instrumental variable (MRIV) algorithm [23], yields the information fusion parameter estimators $\hat{A}_l(t)(l=1,\cdots,n_a)$ at time t. And based on (33), $\hat{\bar{A}}_l(t), \hat{\bar{R}}_l(t)$ can be obtained.

b) Based on the estimates $\hat{\bar{A}}_l(t), \hat{\bar{R}}_l(t)$ and the sampled correlation function estimates $\hat{R}^t_{rij}(k)$, and applying the Gevers-Wouters algorithm with a dead band to (55), the information fusion estimates $\hat{\bar{C}}_u(t)$ and $\hat{Q}_w(t)$ can be obtained. From (72), the fused estimates $\hat{C}_u(t)(u=1,\cdots,n_c)$ can be obtained .

Step 2. Substituting all the estimates obtained by step 1 into (16) and (17) yields the estimates $\hat{\Phi}(t)$, $\hat{\Gamma}(t)$ and $\hat{Q}_{\bar{w}}(t)$ of Φ, Γ and $Q_{\bar{w}}$, and we have that

$$[\hat{\Phi}(t)-\Phi]\to 0, [\hat{\Gamma}(t)-\Gamma]\to 0, [\hat{Q}_{\bar{w}}(t)-Q_{\bar{w}}]\to 0, \quad \text{as} \quad t\to\infty, \text{i.a.r} \quad (85)$$

Step 3. In Lemma 1, Φ, Γ and $Q_{\bar{w}}$ are replaced by $\hat{\Phi}(t)$, $\hat{\Gamma}(t)$ and $\hat{Q}_{\bar{w}}(t)$ respectively. Substituting the estimates into (18)-(26) , the self-tuning local Kalman signal filter can be given as

$$\hat{x}^s_i(t|t) = \hat{\Psi}_{fi}(t)\hat{x}^s_i(t-1|t-1) + \hat{K}_{fi}(t)y_i(t) \quad (86)$$
$$\hat{\Psi}_{fi}(t) = [I_{m(n_a+n_b)} - \hat{K}_{fi}(t)H]\hat{\Phi}(t) \quad (87)$$
$$\hat{K}_{fi}(t) = \hat{\Sigma}_i(t|t-1)H^{\mathrm{T}}(H\hat{\Sigma}_i(t-1|t)H^{\mathrm{T}} +Q_{vi})^{-1} \quad (88)$$
$$\hat{P}_i(t|t) = [I_{m(n_a+n_b)} - \hat{K}_{fi}(t)H]\hat{\Sigma}_i(t|t-1) \quad (89)$$
$$\hat{s}^s_i(t|t) = H_s\hat{x}^s_i(t|t) \quad (90)$$

where the prediction error variance matrice $\hat{\Sigma}_i(t|t-1)$ satisfy the self-tuning Riccati equation

$$\hat{\Sigma}_i(t+1|t) = \hat{\Phi}(t)[\hat{\Sigma}_i(t|t-1) - \hat{\Sigma}_i(t|t-1)H^{\mathrm{T}} \times(H\hat{\Sigma}_i(t|t-1)H^{\mathrm{T}}+Q_{vi})^{-1}H \times\hat{\Sigma}_i(t|t-1)]\hat{\Phi}^{\mathrm{T}}(t) + \hat{\Gamma}(t)\hat{Q}_{\bar{w}}(t)\hat{\Gamma}^{\mathrm{T}}(t) \quad (91)$$

where the cross-covariances among the signal filter errors $\tilde{s}^s_i(t|t) = s(t) - \hat{s}^s_i(t|t), \hat{P}_{sij}(t) = \mathrm{E}[\tilde{s}^s_i(t|t)\tilde{s}^{\mathrm{T}}_j(t|t)]$, are

$$\hat{P}_{sij}(t|t) = H_s\hat{P}_{ij}(t|t)H^{\mathrm{T}}_s \quad (92)$$

The local filter error cross-covariances satisfy the self-tuning Lyapunov equation

$$\hat{P}_{ij}(t|t) = \hat{\Psi}_{fi}(t)\hat{P}_{ij}(t-1|t-1)\hat{\Psi}^{\mathrm{T}}_{fj}(t) +[I_{m(n_a+n_b)} - \hat{K}_{fi}(t)H]\hat{\Gamma}(t)\hat{Q}_{\bar{w}}(t)\hat{\Gamma}^{\mathrm{T}}(t) \times[I_{m(n_a+n_b)} - \hat{K}_{fj}(t)H]^{\mathrm{T}}, i\neq j \quad (93)$$

with the definition $\hat{P}_{ii}(t|t) = \hat{P}_i(t|t)$.

Step 4. By (28) and (29), the estimates $\hat{P}_s(t|t)$ and $\hat{\omega}_j(t|t)$ can be obtained and the self-tuning fused Kalman signal filter is given as

$$\hat{s}^s_0(t|t) = \sum_{j=1}^{L}\hat{\omega}_j(t|t)\hat{s}^s_j(t|t) \quad (94)$$

The above four steps are repeated at each time t.

5. Convergence analysis

Lemma 4[16] Consider the discrete dynamic error system

$$\delta(t) = F(t)\delta(t-1) + u(t) \quad (95)$$

where $t\geq 0$, $\delta(t)\in R^n$ is the output, $u(t)\in R^n$ is the input, and the matrix $F(t)\in R^{n\times n}$ is uniformly asymptotically stable. If $u(t)$ is bounded, then $\delta(t)$ is bounded. If $u(t)\to 0$, as $t\to\infty$, then $\delta(t)\to 0$, as $t\to\infty$.

Theorem 2. For the multisensor systems (1)-(3) with the assumptions 1-4, the self-tuning local Kalman signal filters $\hat{s}_i^s(t|t)$ converge to the local optimal Kalman signal filters $\hat{s}_i(t|t)$ in a realization, i.e.

$$[\hat{s}_i^s(t|t) - \hat{s}_i(t|t)] \to 0, as \quad t \to \infty, \text{i.a.r} \tag{96}$$

Proof. According to [27], it can be proved similarly that

$$(\hat{\Sigma}_i(t+1|t) - \Sigma_i(t+1|t)) \to 0, \text{as} \quad t \to \infty, \text{i.a.r} \tag{97}$$

According to the stability theory of the Kalman filtering [28], from (20), the steady-state Kalman filter gains K_{fi} are the continuous functions of Σ_i, i.e., $K_{fi} = f_k(\Sigma_i)$. Hence, from (20) and (88), applying the continuity of f_k, it can be obtained that

$$\hat{K}_{fi}(t) \to K_{fi}(t), \text{as} \quad t \to \infty, \text{i.a.r} \tag{98}$$

From (19),the steady-state Kalman filter transition matrices Ψ_{fi} are the continuous functions of K_{fi} and Φ,i.e., $\Psi_{fi} = g_f(K_{fi}, \Phi)$. Hence, from (85) and (98), applying the continuity of g_f, it can be obtained that

$$\hat{\Psi}_{fi}(t) \to \Psi_{fi}(t), \text{as} \quad t \to \infty, \text{i.a.r} \tag{99}$$

Setting $\hat{K}_{fi}(t) = K_{fi}(t) + \Delta\hat{K}_{fi}(t)$, and $\hat{\Psi}_{fi}(t) = \Psi_{fi}(t) + \Delta\hat{\Psi}_{fi}(t)$, from (98) and (99), we have that

$$\Delta\hat{K}_{fi}(t) \to 0, \Delta\hat{\Psi}_{fi}(t) \to 0, as \quad t \to \infty, \text{i.a.r} \tag{100}$$

Notice that $\hat{K}_{fi}(t)$ and $\hat{K}_{fi}(t)y_i(t)$ are bounded, and $\hat{\Psi}_{fi}(t)$ is uniformly asymptotically stable [26]. Hence, applying the Lemma 4 to (86), we have that $\hat{x}_i^s(t|t)$ are bounded. Subtracting (18) from (86), and setting $\delta_i(t) = \hat{x}_i^s(t|t) - \hat{x}_i(t|t)$, yields the dynamic error equation

$$\delta_i(t) = \Psi_{fi}(t)\delta_i(t-1) + u_i(t) \tag{101}$$

where $u_i(t) = \Delta\hat{\Psi}_{fi}(t)\hat{x}_i^s(t-1|t-1) + \Delta\hat{K}_{fi}(t)y_i(t)$. From the boundedness of $\hat{x}_i^s(t|t)$ and $y_i(t)$, and from (100), we obtain $u_i(t) \to 0$. Hence, applying the Lemma 4 to (101), we can yield $\delta_i(t) \to 0$, and from (22) and (90), we have that (96) holds. This completes to the proof.

Theorem 3 For the multisensor systems (1)-(3) with the assumptions 1-4, and with the estimators $\hat{\Phi}(t)$, $\hat{\Gamma}(t)$ and $\hat{Q}_{\bar{w}}(t)$, the self-tuning fusion Kalman signal filter $\hat{s}_0^s(t|t)$ given in (94) converges to the optimal fusion Kalman signal filter $\hat{s}_0(t|t)$ given in (27) in a realization, i.e.

$$[\hat{s}_0^s(t|t) - \hat{s}_0(t|t)] \to 0, as \quad t \to \infty, \text{i.a.r} \tag{102}$$

Proof. Applying dynamic variance error system analysis (DVESA) method [29], it has been proved that

$$\hat{P}_{ij}(t|t) \to P_{ij}(t|t), \text{as} \quad t \to \infty, \text{i.a.r} \tag{103}$$

From (26), (92) and (103), we also have

$$\hat{P}_{sij}(t|t) \to P_{sij}(t|t), \text{as} \quad t \to \infty, \text{i.a.r} \tag{104}$$

Setting $\hat{\omega}(t|t) = \omega(t|t) + \Delta\hat{\omega}(t|t), i = 1, 2, \cdots, L$. From (28) and (104), we have yield

$$\hat{\omega}(t|t) \to \omega(t|t), \text{as} \quad t \to \infty, \text{i.a.r} \tag{105}$$

From (105), we have that $\Delta\hat{\omega}(t|t) \to 0$, as $t \to \infty$, i.a.r. Subtracting (27) from (94) yields

$$\hat{s}_0^s(t|t) - \hat{s}_0(t|t) = \sum_{i=1}^{L} \omega(t|t)[\hat{s}_i^s(t|t) - \hat{s}_i(t|t)] + \sum_{i=1}^{L} \Delta\hat{\omega}(t|t)\hat{s}_i^s(t|t) \tag{106}$$

From (90) and the boundedness of $\hat{x}_i^s(t|t)$, we have that $\hat{s}_i^s(t|t)$ is bounded. Applying the boundedness of $P_{sij}(t|t)$, we have that $\omega(t|t)$ is bounded. Applying (96), $\Delta\hat{\omega}(t|t) \to 0$ and the boundedness of $\omega(t|t)$ and $\hat{s}_i^s(t|t)$, yields that (102) holds.

6. Simulation example

Consider the multisensor multi-channel autoregressive moving average (ARMA) signal with white measurement noises and a colored noise

$$(I_2 + A_1q^{-1} + A_2q^{-2})s(t) = (I_2 + C_1q^{-1})w(t) \tag{107}$$

$$y_i(t) = s(t) + \eta(t) + v_i(t), i = 1, \cdots, L \tag{108}$$

$$(I_2 + B_1q^{-1})\eta(t) = (I_2 + R_1q^{-1})\xi(t) \tag{109}$$

where the signal $s(t) = \begin{bmatrix} s_1(t) \ s_2(t) \end{bmatrix}^{\mathrm{T}}$, $y_i(t) \in R^2$ is the measurement of the ith sensor, $w(t)$, $\xi(t)$ and $v_i(t)$ are independent white noises with zero mean and unknown variances Q_w, Q_ξ and Q_{vi}, respectively. Assume A_1, A_2, C_1 and Q_w are unknown. (107)-(109) have the equivalent state model (13)-(15). Hence the problem of finding self-tuning fusion Kalman filter $\hat{s}_0^s(t|t)$ of signal $s(t)$ can be converted into the problem of finding the self-tuning fusion Kalman filter weighted by scalars of state $x(t)$'s first component.

In simulation we take that

$$A_1 = \begin{bmatrix} 1.2, & -0.5 \\ -0.2, & 0.7 \end{bmatrix}, A_2 = \begin{bmatrix} 0.5, & -0.7 \\ 0.3, & 0.9 \end{bmatrix},$$

$$Q_w = \begin{bmatrix} 6, & 0 \\ 0, & 8 \end{bmatrix}, C_1 = \begin{bmatrix} -0.1, & -0.3 \\ 0.1, & 0.5 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.5, & -0.3 \\ 0.1, & -0.4 \end{bmatrix}, R_1 = \begin{bmatrix} -0.3, & -0.1 \\ 0.2, & -0.5 \end{bmatrix},$$

$$Q_\xi = \begin{bmatrix} 0.1, & 0 \\ 0, & 0.1 \end{bmatrix}, Q_{v1} = \begin{bmatrix} 0.1, & 0 \\ 0, & 0.4 \end{bmatrix},$$

$$Q_{v2} = \begin{bmatrix} 0.2, & 0 \\ 0, & 0.6 \end{bmatrix}, Q_{v3} = \begin{bmatrix} 0.15, & 0 \\ 0, & 0.35 \end{bmatrix}$$

In Figure 1-Figure 10, the straight lines denote the true values, and $M(k,r)$ denotes the(k,r)th element of the matrix M. Applying Lemma 3, we can obtain the fused estimates $\hat{A}_l(t) (l = 1, 2)$.The curves of the fused estimates

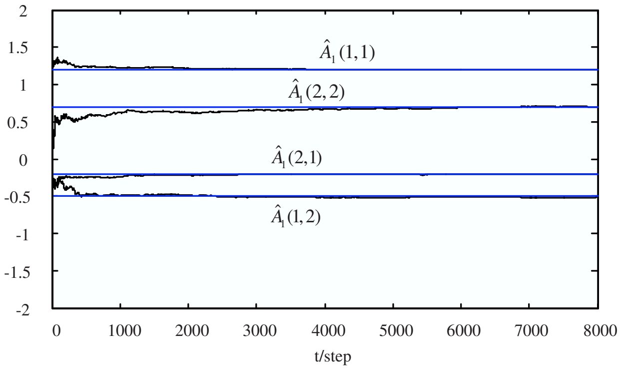

Figure 1 The curves of the fused parameter estimate A_1.

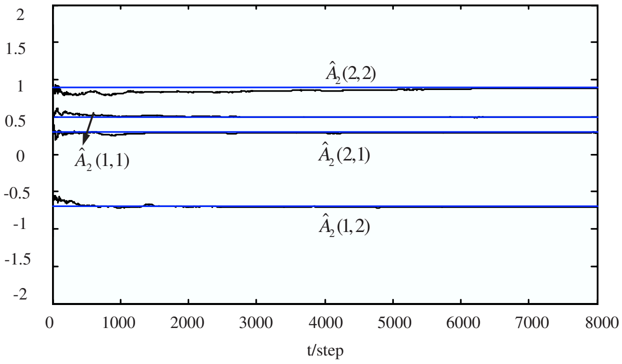

Figure 2 The curves of the fused parameter estimate A_2.

$\hat{A}_1(t)$ and $\hat{A}_2(t)$ are shown in Figure 1 and Figure 2. Applying the correlation method and Gevers-Wouters algorithm with the dead band $T_d = 200$, we can obtain $\hat{C}_1(t)$ and $\hat{Q}_w(t)$. The curves of the fused estimate $\hat{Q}_w(t)$ are shown in Figure 3. The curves of the fused estimate $\hat{C}_1(t)$ are shown in Figure 4. The curves of the signal $s(t)$ and measurement $y_i(t) = \left[y_{i1}(t)\ y_{i2}(t)\right]^{\mathrm{T}}$ $(i = 1, 2, 3)$ are given in Figure 5 and Figure 7. From Figure 6 and Figure 8, we can see that the optimal fused filter $\hat{s}_0(t|t) = \left[\hat{s}_{01}(t|t)\ \hat{s}_{02}(t|t)\right]^{\mathrm{T}}$ and the self-tuning filter $\hat{s}_0^s(t|t) = \left[\hat{s}_{01}^s(t|t)\ \hat{s}_{02}^s(t|t)\right]^{\mathrm{T}}$ approximate to the true signal $s(t)$. The error curves between the self-tuning and optimal fused Kalman signal filters are presented in Figure 9 and Figure 10. We see the errors approximate to zero, which verify the self-tuning fusion Kalman signal filter converges to the optimal fusion Kalman signal filter .

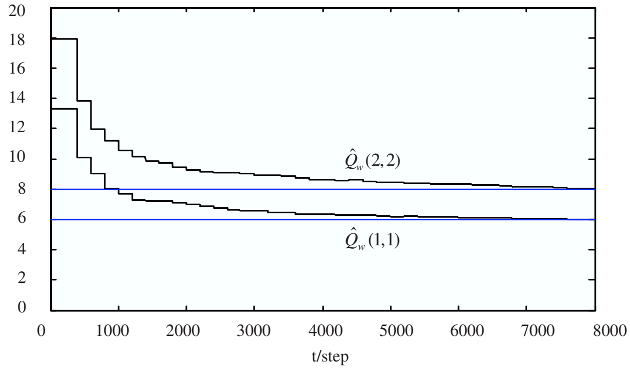

Figure 3 The curves of the fused noise variance estimate Q_w.

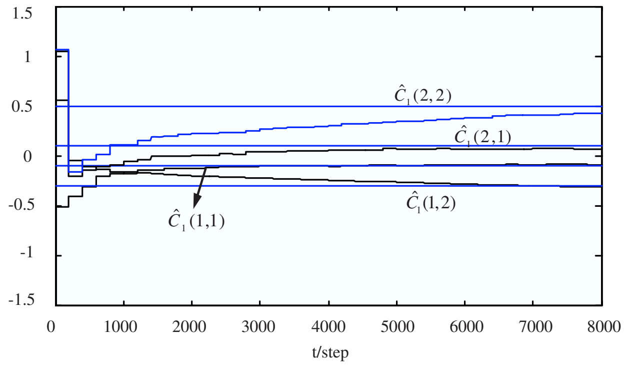

Figure 4 The curves of the fused parameter estimate C_1.

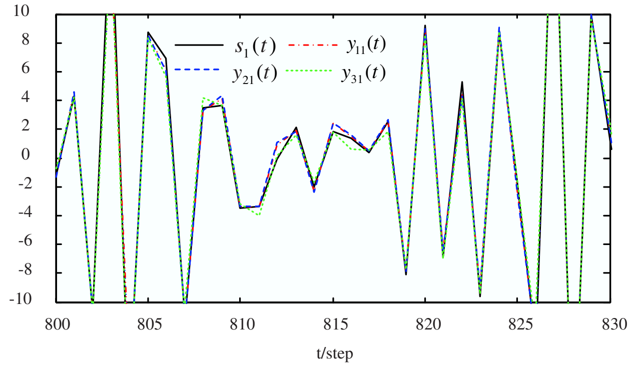

Figure 5 The curves of the signal $s_1(t)$, the measurement $y_{11}(t)$, the measurement $y_{21}(t)$ and the measurement $y_{31}(t)$.

7. Conclusion

For the multisensor multi-channel ARMA signal with white measurement noises and an ARMA colored measurement noise, when the model parameters and noise variances are partially unknown, a self-tuning information fusion Kalman filter weighted by scalars has been presented by the classical Kalman filter method. In this paper, four main contributions are as follows: (i) The multi-stage information fusion identification method has been presented for the multisensor multi-channel ARMA signal with an ARMA colored

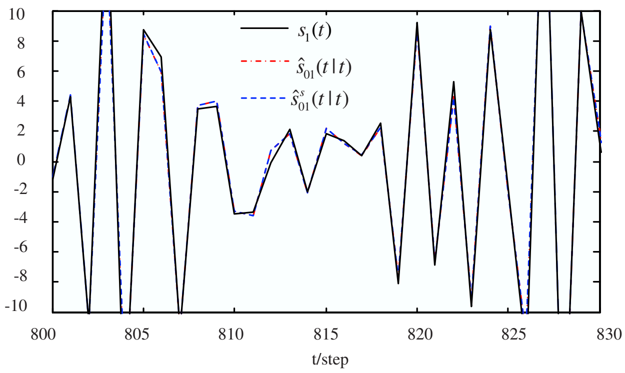

Figure 6 The signal $s_1(t)$, the optimal fused filter $\hat{s}_{01}(t|t)$, and the self-tuning fused filter $\hat{s}^s_{01}(t|t)$.

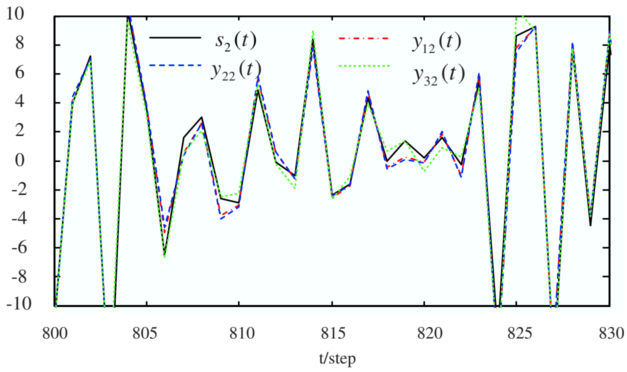

Figure 7 The curves of the signal $s_2(t)$, the measurement $y_{12}(t)$, the measurement $y_{22}(t)$ and the measurement $y_{32}(t)$.

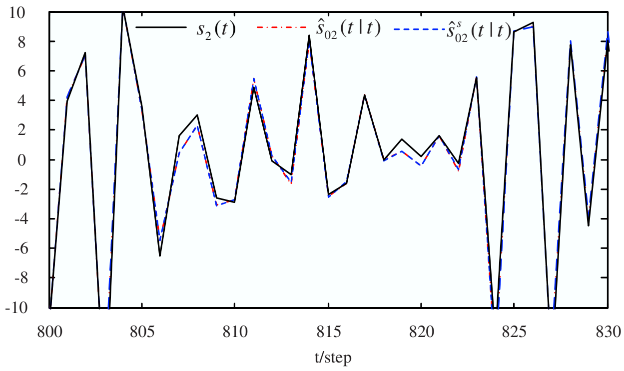

Figure 8 The signal $s_2(t)$, the optimal fused filter $\hat{s}_{02}(t|t)$, and the self-tuning fused filter $\hat{s}^s_{02}(t|t)$.

noise, which consists of the multidimensional RIV algorithm, the correlation method, and the Gevers-Wouters algorithm with a dead band. It solves the online identification problem of the multi-channel ARMA signal system

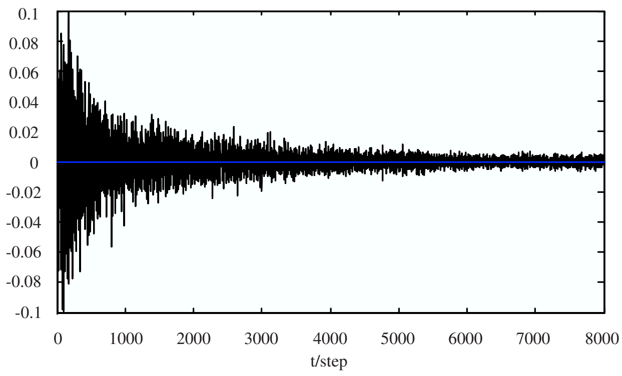

Figure 9 The error curve $e_1(t) = \hat{s}^s_{01}(t|t) - \hat{s}_{01}(t|t)$.

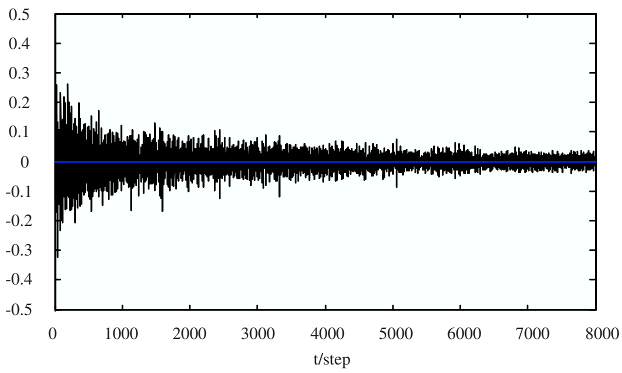

Figure 10 The error curve $e_2(t) = \hat{s}^s_{02}(t|t) - \hat{s}_{02}(t|t)$.

with a colored measurement noise, which has not been solved in the existing papers [19–21,23]. The classical system identification method [26] has been developed to handle the multi-sensor system with a colored noise. (ii) By converting the ARMA signal model to the state space model, the signal can be regarded as a component of the state. Then using the classical Kalman filtering method, the information fusion Kalman signal filter is obtained. (iii) The proposed self-tuning information fusion Kalman signal filter overcomes the limitation that the existing results are only suitable for multisensor system with single-channel white measurement noises. (iv) By the DESA method, it has been proved strictly that the self-tuning fused Kalman filter converges to the optimal fused Kalman filter in a realization, so that it has the asymptotic optimality.

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Guili Tao was born in Heilongjiang,China,in 1980. She received the B.Sc.and M.Sc. degrees from the Department of Automation,Heilongjiang University,Heilongjiang, respectively. Currently,she is working towards the Ph.D.degree at the Department of Automation, Heilongjiang University. She has been a Lecturer in Heilongjiang Institute of Science and Technology.Her research interests include multisensor information fusion, signal processing, and self-tuning filtering.

Zili Deng is a Professor at the Department of Automation, Heilongjiang University. He has published more than 400 papers and 8 books in the fields of optimal filtering, self-tuning filtering, time series analysis, and multisensor information fusion. He is the author the books ”Kalman Filtering and Wiener Filtering-Modern Time Series Analysis Method”(2001), ”Self-tuning Filtering Theory with Applications-Modern Time Series Analysis Method”(2003), ”Optimal Estimation Theory with Applications-Modeling, Filtering, Information Fusion Estimation”(2005), and ”Information Fusion Filtering Theory and Applications”(2007). His research interests include optimal and self-tuning filtering, deconvolution, state estimation, signal processing, estimation theory, identification, time-series analysis, and multisensor information fusion.