Mathematical Sciences Letters
An International Journal

http://dx.doi.org/10.12785/msl/040210

New Exact Solutions to the KdV and $(N+1)$-Dimensional Double Sinh-Gordon Equation by the Extended Trial Equation Method

Abdullah Sonmezoglu[1], *Mehmet Ekici*[1,*] *and Elsayed M. E. Zayed*[2]

[1] Department of Mathematics, Faculty of Science and Arts, Bozok University, 66100 Yozgat, Turkey
[2] Department of Mathematics, Faculty of Science, Zagazig University, P.O. Box 44519, Zagazig, Egypt

Received: 13 Aug. 2014, Revised: 18 Oct. 2014, Accepted: 20 Oct. 2014
Published online: 1 May 2015

Abstract: In this paper we introduce a new version of the trial equation method for solving non-integrable partial differential equations in mathematical physics. Some exact solutions including soliton solutions, rational and elliptic function solutions to the KdV equation and the $(N+1)$-dimensional double sinh-Gordon equation are obtained by this method.

Keywords: Extended trial equation method, KdV equation, $(N+1)$-dimensional double sinh-Gordon equation, soliton solution, elliptic solutions

1 Introduction

The exact solutions of nonlinear partial differential equations (NLPDEs) have been investigated by many authors who are interested in nonlinear phenomena which exist in all fields including either the scientific works or engineering fields, such as fluid mechanics, chemical physics, chemical kinematics, plasma physics, elastic media, optical fibers, solid state physics, biology, biophysics and so on. The research of traveling wave solutions of some nonlinear evolution equations derived from such fields played an important role in the analysis of some phenomena.

The theory of nonlinear evolution equations (NLEEs) has come a long way through. There are a large number of nonlinear evolution equations that are studied nowadays. These equations are especially generated as a generalized and combined versions of the existing version of the well-known equations like the Korteweg-de Vries (KdV) equations [1], sine-Gordon equations and many more.

The objective of our present work is to apply the extended trial equation method to the two generalized nonlinear equations: the KdV equation and the $(N+1)$-dimensional double sinh-Gordon equation. The KdV equation has several connections to physical problems. It describes the evolution one-dimensional waves in many physical settings, including: shallow-water waves with weakly nonlinear restoring forces, acoustic waves on a crystal lattice, and more. The $(N+1)$-dimensional double sinh-Gordon equation is an important model equation and plays an important role in physics. It has numerous applications in physics, such as nonlinear optics, Josephson array, ferromagnetic materials, charge density waves, and the study of liquid helium, and so on. For example, this equation arises in resonant nonlinear optics in the theory of self-induced transparency when the atoms of the resonant medium have degenerate energy levels.

Many powerful methods, such as the Backlund transformation, the inverse scattering method [2], bilinear transformation, the tanh-sech method [3], the extended tanh method, the pseudo-spectral method [4], the trial function and the sine-cosine method [5], Hirota method [6], tanh-coth method [7,8], the exponential function method [9], (G'/G)-expansion method [10,11], homogeneous balance method [12], the trial equation method [13,14,15,16,17,18,19,20,21,22] have been used to investigate nonlinear partial differential equations problems. The types of solutions of NLEEs, that are integrated using various mathematical techniques, are very important and appear in various areas of physics,

* Corresponding author e-mail: mehmet.ekici@bozok.edu.tr

applied mathematics and engineering. There are a lot of nonlinear evolution equations that are integrated using these and other mathematical methods.

In this paper, the KdV equation with power law nonlinearity and double sinh-Gordon equation will be studied by extended trial equation. By virtue of the solitary wave ansatz method, an exact 1-soliton solution will be obtained. The extended trial equation method will be employed to back up our analysis in obtaining exact solutions with distinct physical structures.

2 The extended trial equation method

Step 1. For a given nonlinear partial differential equation with rank inhomogeneous

$$P(u,u_t,u_x,u_{xx},\dots)=0, \tag{1}$$

take the wave transformation

$$u(x_1,\dots,x_N,t)=u(\eta), \quad \eta=\lambda\left(\sum_{j=1}^{N}x_j-ct\right), \tag{2}$$

where $\lambda \neq 0$ and $c \neq 0$. Substituting Eq. (2) into Eq. (1) yields a nonlinear ordinary differential equation,

$$N(u,u',u'',\ldots)=0. \tag{3}$$

Step 2. Take transformation and trial equation as follows:

$$u=\sum_{i=0}^{\delta}\tau_i\Gamma^i, \tag{4}$$

in which

$$(\Gamma')^2=\Lambda(\Gamma)=\frac{\Phi(\Gamma)}{\Psi(\Gamma)}=\frac{\xi_\theta\Gamma^\theta+\ldots+\xi_1\Gamma+\xi_0}{\zeta_\varepsilon\Gamma^\varepsilon+\ldots+\zeta_1\Gamma+\zeta_0}. \tag{5}$$

where $\tau_i\,(i=0,\ldots,\delta)$, $\xi_i\,(i=0,\ldots,\theta)$ and $\zeta_i\,(i=0,\ldots,\varepsilon)$ are constants. Using the relations (4) and (5), we can find

$$(u')^2=\frac{\Phi(\Gamma)}{\Psi(\Gamma)}\left(\sum_{i=0}^{\delta}i\tau_i\Gamma^{i-1}\right)^2, \tag{6}$$

$$u''=\frac{\Phi'(\Gamma)\Psi(\Gamma)-\Phi(\Gamma)\Psi'(\Gamma)}{2\Psi^2(\Gamma)}\left(\sum_{i=0}^{\delta}i\tau_i\Gamma^{i-1}\right)+\frac{\Phi(\Gamma)}{\Psi(\Gamma)}\left(\sum_{i=0}^{\delta}i(i-1)\tau_i\Gamma^{i-2}\right), \tag{7}$$

where $\Phi(\Gamma)$ and $\Psi(\Gamma)$ are polynomials. Substituting these terms into Eq. (3) yields an equation of polynomial $\Omega(\Gamma)$ of Γ :

$$\Omega(\Gamma)=\rho_s\Gamma^s+\ldots+\rho_1\Gamma+\rho_0=0. \tag{8}$$

According to the balance principle we can determine a relation of θ, ε, and δ. We can take some values of θ, ε, and δ.

Step 3. Let the coefficients of $\Omega(\Gamma)$ all be zero will yield an algebraic equations system:

$$\rho_i=0, \quad i=0,\ldots,s. \tag{9}$$

Solving this equations system (9), we will determine the values of $\xi_0,\ldots,\xi_\theta$; $\zeta_0,\ldots,\zeta_\varepsilon$ and $\tau_0,\ldots,\tau_\delta$.

Step 4. Reduce Eq. (5) to the elementary integral form,

$$\pm(\eta-\eta_0)=\int\frac{d\Gamma}{\sqrt{\Lambda(\Gamma)}}=\int\sqrt{\frac{\Psi(\Gamma)}{\Phi(\Gamma)}}d\Gamma. \tag{10}$$

Using a complete discrimination system for polynomial to classify the roots of $\Phi(\Gamma)$, we solve the infinite integral (10) and obtain the exact solutions to Eq. (3). Furthermore, we can write the exact traveling wave solutions to Eq. (1) respectively.

3 Applications of the extended trial equation method

To illustrate the necessity of our new approach concerning the trial equation method, we introduce two case studies.

3.1 The KdV equation

We start our application by considering the KdV equation with power-law nonlinearity [23,24],

$$u_t+\alpha(n+1)u^nu_x+\beta u_{xxx}=0, \tag{11}$$

where α and β are two nonzero coefficients and $n>2$. We note that when $n=1$ and $\alpha=3$, Eq. (11) is known as the KdV equation and when $n=2$ and $\alpha=2$ it is known as the mKdV equation. Now as far as the domain restriction is considered, it must be very clearly stated that $n\neq 4$ (see [25] for more details).

In order to look for solutions of Eq. (11), we make the transformation

$$u(x,t)=u(\eta), \quad \eta=x-ct, \tag{12}$$

where c is an arbitrary constant. Then, Eq. (13) becomes

$$-cu'+\alpha(n+1)u^nu'+\beta u'''=0. \tag{13}$$

Integrating Eq. (13) ones with respect to η and setting the integration constant equal to zero, we obtain:

$$-cu+\alpha u^{n+1}+\beta u''=0. \tag{14}$$

Eq. (14), with the transformation

$$u=\omega^{1/n}, \tag{15}$$

reduces to

$$n\beta\omega\omega''+\beta(1-n)(\omega')^2-cn^2\omega^2+n^2\alpha\omega^3=0. \quad (16)$$

Substituting Eqs. (6) and (7) into Eq. (16) and using balance principle yields $\theta=\varepsilon+\delta+2$.
If we take $\theta=3$, $\varepsilon=0$ and $\delta=1$, then

$$(\omega')^2=\frac{\tau_1^2(\xi_3\Gamma^3+\xi_2\Gamma^2+\xi_1\Gamma+\xi_0)}{\zeta_0}, \quad (17)$$

where $\xi_3\neq 0$, $\zeta_0\neq 0$. Respectively, solving the algebraic equation system (9) yields

$$\begin{aligned}&\zeta_0=\zeta_0,\quad \xi_2=\xi_2,\quad \tau_0=\tau_0,\quad \tau_1=\tau_1,\\ &\xi_0=\frac{\tau_0^2(4n^2\alpha\zeta_0\tau_0+\beta\xi_2(n+2))}{\beta\tau_1^2(n+2)},\\ &\xi_1=\frac{2\tau_0(3n^2\alpha\zeta_0\tau_0+\beta\xi_2(n+2))}{\beta\tau_1(n+2)},\\ &\xi_3=-\frac{2n^2\alpha\zeta_0\tau_1}{\beta(n+2)},\quad c=\frac{6n^2\alpha\zeta_0\tau_0+\beta\xi_2(n+2)}{n^2\zeta_0(n+2)}.\end{aligned} \quad (18)$$

Substituting these results into Eq. (5) and Eq. (10), we can write

$$\pm(\eta-\eta_0)=\sqrt{-\frac{\beta(n+2)}{2n^2\alpha\tau_1}}\times\int\frac{d\Gamma}{\sqrt{\Gamma^3+\ell_2\Gamma^2+\ell_1\Gamma+\ell_0}}, \quad (19)$$

where

$$\begin{aligned}&\ell_2=-\frac{\beta\xi_2(n+2)}{2n^2\alpha\zeta_0\tau_1},\\ &\ell_1=-\frac{\tau_0(3n^2\alpha\zeta_0\tau_0+\beta\xi_2(n+2))}{n^2\alpha\zeta_0\tau_1^2},\\ &\ell_0=-\frac{\tau_0^2(4n^2\alpha\zeta_0\tau_0+\beta\xi_2(n+2))}{2n^2\alpha\zeta_0\tau_1^3}.\end{aligned} \quad (20)$$

Integrating Eq. (19), we obtain the solutions to the Eq. (11) as follows:

$$\pm(\eta-\eta_0)=-2\sqrt{\frac{A}{\Gamma-\alpha_1}}, \quad (21)$$

$$\pm(\eta-\eta_0)=2\sqrt{\frac{A}{\alpha_2-\alpha_1}}\arctan\sqrt{\frac{\Gamma-\alpha_2}{\alpha_2-\alpha_1}},\quad \alpha_2>\alpha_1, \quad (22)$$

$$\pm(\eta-\eta_0)=\sqrt{\frac{A}{\alpha_1-\alpha_2}}\ln\left|\frac{\sqrt{\Gamma-\alpha_2}-\sqrt{\alpha_1-\alpha_2}}{\sqrt{\Gamma-\alpha_2}+\sqrt{\alpha_1-\alpha_2}}\right|,\quad \alpha_1>\alpha_2, \quad (23)$$

$$\pm(\eta-\eta_0)=2\sqrt{\frac{A}{\alpha_1-\alpha_3}}F(\varphi,l),\quad \alpha_1>\alpha_2>\alpha_3, \quad (24)$$

where

$$F(\varphi,l)=\int_0^{\varphi}\frac{d\psi}{\sqrt{1-l^2\sin^2\psi}},\quad \varphi=\arcsin\sqrt{\frac{\Gamma-\alpha_3}{\alpha_2-\alpha_3}}, \quad (25)$$

and

$$l^2=\frac{\alpha_2-\alpha_3}{\alpha_1-\alpha_3},\quad A=-\frac{\beta(n+2)}{2n^2\alpha\tau_1}. \quad (26)$$

Also α_1, α_2 and α_3 are the roots of the polynomial equation

$$\Gamma^3+\frac{\xi_2}{\xi_3}\Gamma^2+\frac{\xi_1}{\xi_3}\Gamma+\frac{\xi_0}{\xi_3}=0. \quad (27)$$

Substituting the solutions (21), (22), (23) into (4) and (15), denoting $\overline{\tau}=\tau_0+\tau_1\alpha_1$, and setting

$$v=\frac{6n^2\alpha\zeta_0\tau_0+\beta\xi_2(n+2)}{n^2\zeta_0(n+2)}, \quad (28)$$

we get

$$u(x,t)=\left[\overline{\tau}+\frac{4\tau_1 A}{(x-vt-\eta_0)^2}\right]^{\frac{1}{n}}, \quad (29)$$

$$u(x,t)=\left\{\overline{\tau}+\tau_1(\alpha_2-\alpha_1)\left[1-\tanh^2\left(\mp B(x-vt-\eta_0)\right)\right]\right\}^{\frac{1}{n}}, \quad (30)$$

$$u(x,t)=\left\{\overline{\tau}+\tau_1(\alpha_1-\alpha_2)\mathrm{cosech}^2\left(B(x-vt)\right)\right\}^{\frac{1}{n}}, \quad (31)$$

where $B=\frac{1}{2}\sqrt{\frac{\alpha_1-\alpha_2}{A}}$. If we take $\tau_0=-\tau_1\alpha_1$, that is $\overline{\tau}=0$, and $\eta_0=0$, then the solutions (29), (30), (31) can reduce to rational function solution

$$u(x,t)=\left(\frac{2\sqrt{\tau_1 A}}{x-vt}\right)^{\frac{2}{n}}, \quad (32)$$

1-soliton solution

$$u(x,t)=\frac{A_1}{\cosh^{\frac{2}{n}}[\mp B(x-vt)]}, \quad (33)$$

and singular soliton solution

$$u(x,t)=\frac{A_2}{\sinh^{\frac{2}{n}}[B(x-vt)]}, \quad (34)$$

where

$$A_1=[\tau_1(\alpha_2-\alpha_1)]^{\frac{1}{n}},\quad A_2=[\tau_1(\alpha_1-\alpha_2)]^{\frac{1}{n}}. \quad (35)$$

Here, A_1 and A_2 are the amplitudes of the solitons, while v is the velocity and B is the inverse width of the solitons. Thus, we can say that the solitons exist for $\tau_1>0$.

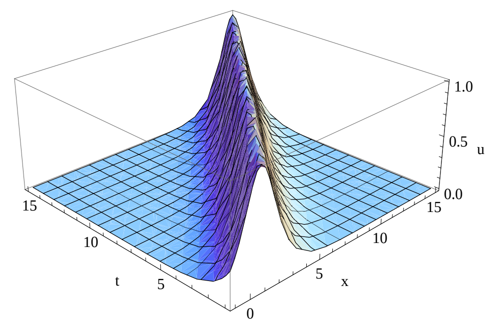

Fig. 1: Profile of a numerical solution of (33) at $n=2, A_1=1, B=1$ while $v=1$.

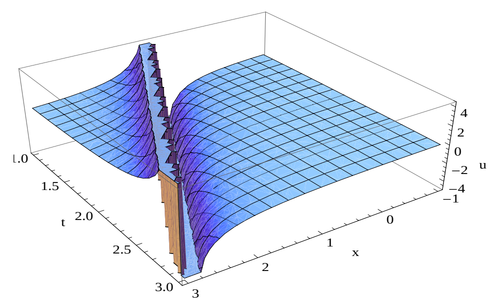

Fig. 2: Profile of a numerical solution of (34) at $n=2, A_2=1, B=1$ while $v=1$.

3.2 The $(N+1)$-dimensional double sinh-Gordon equation

Double sinh-Gordon equation is given by [26,27]

$$\sum_{j=1}^{N} u_{x_j x_j} - u_{tt} - \alpha \sinh u - \beta \sinh 2u = 0. \qquad (36)$$

We look for solutions of Eq. (36) by considering the transformation

$$u(x_1,\ldots,x_N,t) = u(\eta),\ \eta = \lambda\left(\sum_{j=1}^{N} x_j - ct\right),\ \lambda \neq 0,\ c \neq 0. \qquad (37)$$

Using the transformation (37), Eq. (36) can be rewritten in the following form:

$$\lambda^2(N-c^2)u'' - \alpha \sinh u - \beta \sinh 2u = 0, \qquad (38)$$

where the prime denotes derivative with respect to η. Next, let us consider the transformation

$$u = \ln \omega, \qquad (39)$$

then we obtain

$$u'' = \frac{\omega\omega'' - (\omega')^2}{\omega^2}, \qquad \sinh u = \frac{\omega - \omega^{-1}}{2}, \quad \sinh 2u = \frac{\omega^2 - \omega^{-2}}{2}. \qquad (40)$$

By substituting Eq. (40) in Eq. (38), we can rewrite the $(N+1)$-dimensional double sinh- Gordon Eq. (36) in the following form:

$$2\lambda^2(N-c^2)(\omega\omega'' - (\omega')^2) - \alpha(\omega^3 - \omega) - \beta(\omega^4 - 1) = 0. \qquad (41)$$

Substituting Eqs. (6) and (7) into Eq. (41) and using balance principle yields $\theta = \varepsilon + 2\delta + 2$.
If we take $\theta = 4$, $\varepsilon = 0$ and $\delta = 1$, then

$$(\omega')^2 = \frac{\tau_1^2(\xi_4\Gamma^4 + \xi_3\Gamma^3 + \xi_2\Gamma^2 + \xi_1\Gamma + \xi_0)}{\zeta_0}, \qquad (42)$$

where $\xi_4 \neq 0$, $\zeta_0 \neq 0$. Respectively, solving the algebraic equation system (9) yields

$$\xi_1 = \xi_1, \quad \xi_2 = \xi_2, \quad \zeta_0 = \zeta_0, \quad \tau_0 = \tau_0, \quad \tau_1 = \tau_1,$$
$$\xi_0 = \frac{\xi_1\tau_1(-2\alpha\tau_0 + 4\alpha\tau_0^3 - \beta + 5\beta\tau_0^4) - 2\xi_2\tau_0(\tau_0^2 - 1)(\beta + \tau_0(\alpha + \beta\tau_0))}{2\tau_1^2(\alpha(3\tau_0^2 - 1) + 4\beta\tau_0^3)},$$
$$\xi_3 = \frac{\tau_1(2\xi_2\tau_0 - \xi_1\tau_1)(\alpha + 2\beta\tau_0)}{\alpha(3\tau_0^2 - 1) + 4\beta\tau_0^3}, \quad \xi_4 = \frac{\beta\tau_1^2(2\xi_2\tau_0 - \xi_1\tau_1)}{2(\alpha(3\tau_0^2 - 1) + 4\beta\tau_0^3)},$$
$$c = \pm\frac{1}{\lambda}\sqrt{\frac{\zeta_0(\alpha - 3\alpha\tau_0^2 - 4\beta\tau_0^3) + \lambda^2(2\xi_2\tau_0 - \xi_1\tau_1)N}{2\xi_2\tau_0 - \xi_1\tau_1}}. \qquad (43)$$

Substituting these results into Eq. (5) and Eq. (10), we can write

$$\pm(\eta - \eta_0) = \sqrt{\frac{2\zeta_0(\alpha(3\tau_0^2 - 1) + 4\beta\tau_0^3)}{\beta\tau_1^2(2\xi_2\tau_0 - \xi_1\tau_1)}} \times \int \frac{d\Gamma}{\sqrt{\Gamma^4 + \ell_3\Gamma^3 + \ell_2\Gamma^2 + \ell_1\Gamma + \ell_0}}, \qquad (44)$$

where

$$\ell_3 = \frac{2(\alpha + 2\beta\tau_0)}{\beta\tau_1},$$
$$\ell_2 = \frac{2\xi_2(\alpha(3\tau_0^2 - 1) + 4\beta\tau_0^3)}{\beta\tau_1^2(2\xi_2\tau_0 - \xi_1\tau_1)},$$
$$\ell_1 = \frac{2\xi_1(\alpha(3\tau_0^2 - 1) + 4\beta\tau_0^3)}{\beta\tau_1^2(2\xi_2\tau_0 - \xi_1\tau_1)},$$
$$\ell_0 = \frac{\xi_1\tau_1(-2\alpha\tau_0 + 4\alpha\tau_0^3 - \beta + 5\beta\tau_0^4) - 2\xi_2\tau_0(\tau_0^2 - 1)(\beta + \tau_0(\alpha + \beta\tau_0))}{\beta\tau_1^4(2\xi_2\tau_0 - \xi_1\tau_1)}. \qquad (45)$$

Integrating Eq. (44), we obtain the solutions to the Eq. (36) as follows:

$$\pm(\eta - \eta_0) = -\frac{B}{\Gamma - \alpha_1}, \qquad (46)$$

$$\pm(\eta-\eta_0)=\frac{2B}{\alpha_1-\alpha_2}\sqrt{\frac{\Gamma-\alpha_2}{\Gamma-\alpha_1}},\quad \alpha_2>\alpha_1, \tag{47}$$

$$\pm(\eta-\eta_0)=\frac{B}{\alpha_1-\alpha_2}\ln\left|\frac{\Gamma-\alpha_1}{\Gamma-\alpha_2}\right|, \tag{48}$$

$$\begin{aligned}\pm(\eta-\eta_0)&=\frac{B}{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}\\ &\times\ln\left|\frac{\sqrt{(\Gamma-\alpha_2)(\alpha_1-\alpha_3)}-\sqrt{(\Gamma-\alpha_3)(\alpha_1-\alpha_2)}}{\sqrt{(\Gamma-\alpha_2)(\alpha_1-\alpha_3)}+\sqrt{(\Gamma-\alpha_3)(\alpha_1-\alpha_2)}}\right|,\\ &\alpha_1>\alpha_2>\alpha_3,\end{aligned} \tag{49}$$

$$\pm(\eta-\eta_0)=2\sqrt{\frac{B}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}F(\varphi,l),\quad \alpha_1>\alpha_2>\alpha_3>\alpha_4, \tag{50}$$

where

$$\begin{aligned}F(\varphi,l)&=\int_0^{\varphi}\frac{d\psi}{\sqrt{1-l^2\sin^2\psi}},\\ \varphi&=\arcsin\sqrt{\frac{(\Gamma-\alpha_1)(\alpha_2-\alpha_4)}{(\Gamma-\alpha_2)(\alpha_1-\alpha_4)}},\\ l^2&=\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)},\\ B&=\sqrt{\frac{2\zeta_0(\alpha(3\tau_0^2-1)+4\beta\tau_0^3)}{\beta\tau_1^2(2\xi_2\tau_0-\xi_1\tau_1)}}.\end{aligned} \tag{51}$$

Also α_1, α_2, α_3 and α_4 are the roots of the polynomial equation

$$\Gamma^4+\frac{\xi_3}{\xi_4}\Gamma^3+\frac{\xi_2}{\xi_4}\Gamma^2+\frac{\xi_1}{\xi_4}\Gamma+\frac{\xi_0}{\xi_4}=0. \tag{52}$$

Substituting the solutions (46), (47), (48), (49) into (4) and (39), and setting

$$v=\pm\frac{1}{\lambda}\sqrt{\frac{\zeta_0(\alpha-3\alpha\tau_0^2-4\beta\tau_0^3)+\lambda^2(2\xi_2\tau_0-\xi_1\tau_1)N}{2\xi_2\tau_0-\xi_1\tau_1}}, \tag{53}$$

we obtain

$$u(x_1,\ldots,x_N,t)=ln\left\{\tau_0+\tau_1\alpha_1\mp\frac{\tau_1B}{\lambda\left(\sum_{j=1}^{N}x_j-vt-\frac{\eta_0}{\lambda}\right)}\right\}, \tag{54}$$

$$u(x_1,\ldots,x_N,t)=ln\left\{\tau_0+\tau_1\alpha_1+\frac{4B^2(\alpha_2-\alpha_1)\tau_1}{4B^2-\left[\lambda(\alpha_1-\alpha_2)\left(\sum_{j=1}^{N}x_j-vt-\frac{\eta_0}{\lambda}\right)\right]^2}\right\}, \tag{55}$$

$$u(x_1,\ldots,x_N,t)=ln\left\{\tau_0+\tau_1\alpha_2+\frac{(\alpha_2-\alpha_1)\tau_1}{\exp\left(\frac{\lambda(\alpha_1-\alpha_2)}{B}\left(\sum_{j=1}^{N}x_j-vt-\frac{\eta_0}{\lambda}\right)\right)-1}\right\}, \tag{56}$$

$$u(x_1,\ldots,x_N,t)=ln\left\{\tau_0+\tau_1\alpha_1+\frac{(\alpha_1-\alpha_2)\tau_1}{\exp\left(\frac{\lambda(\alpha_1-\alpha_2)}{B}\left(\sum_{j=1}^{N}x_j-vt-\frac{\eta_0}{\lambda}\right)\right)-1}\right\}, \tag{57}$$

$$u(x_1,\ldots,x_N,t)=ln\left\{\tau_0+\tau_1\alpha_1-\frac{2(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)\tau_1}{2\alpha_1-\alpha_2-\alpha_3+(\alpha_3-\alpha_2)\cosh\left(B_1\left(\sum_{j=1}^{N}x_j-vt\right)\right)}\right\}, \tag{58}$$

where $B_1=\frac{\lambda\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}{B}$. If we take $\tau_0=-\tau_1\alpha_1$ and $\eta_0=0$, then the solutions (54), (55), (56), (57), (58) can reduce to rational function solutions

$$u(x_1,\ldots,x_N,t)=ln\left\{\mp\frac{\tau_1B}{\lambda\left(\sum_{j=1}^{N}x_j-vt\right)}\right\}, \tag{59}$$

$$u(x_1,\ldots,x_N,t)=ln\frac{4B^2(\alpha_2-\alpha_1)\tau_1}{4B^2-\left[\lambda(\alpha_1-\alpha_2)\left(\sum_{j=1}^{N}x_j-vt\right)\right]^2}, \tag{60}$$

traveling wave solutions

$$u(x_1,\ldots,x_N,t)=ln\left\{\frac{(\alpha_2-\alpha_1)\tau_1}{2}\times\left[1\mp\coth\left(\frac{\lambda(\alpha_1-\alpha_2)}{2B}\left(\sum_{j=1}^{N}x_j-vt\right)\right)\right]\right\}, \tag{61}$$

and soliton solution

$$u(x_1,\ldots,x_N,t)=ln\frac{A_3}{D+\cosh\left[B_1\left(\sum_{j=1}^{N}x_j-vt\right)\right]}, \tag{62}$$

where

$$A_3 = \frac{2(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)\tau_1}{\alpha_3-\alpha_2}, \quad D = \frac{2\alpha_1-\alpha_2-\alpha_3}{\alpha_3-\alpha_2}. \quad (63)$$

Here, A_3 is the amplitude of the soliton, while v is the velocity and B_1 is the inverse width of the soliton. Thus, we can say that the solitons exist for $\tau_1 < 0$.

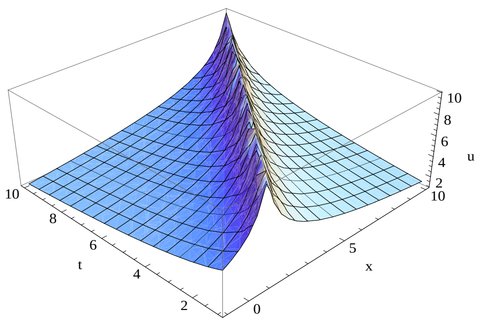

Fig. 3: Profile of a numerical solution of (62) at $N = 1, A_3 = 2, B_1 = 0.1$ and $D < 0$ while $v = 1$.

Acknowledgement

The authors are grateful to the referees for a careful checking of the details and for helpful comments that improved this paper.

4 Conclusion

In this paper we have used the extended trial equation method to derive exact solutions with distinct physical structures. This method with symbolic computation on the computer is used for constructing broad classes of periodic and soliton solutions of two nonlinear equations arising in nonlinear physics. Basic features of the 1-soliton solution and singular soliton solution were analytically and numerically discussed. We proposed a new trial equation method as an alternative approach to obtain the analytic solutions of nonlinear partial differential equations with generalized evolution in mathematical physics. We use the extended trial equation method aided with symbolic computation to construct the soliton solutions, the elliptic function and rational function solutions for the KdV equation with power law nonlinearity and $(N+1)$-dimensional double sinh-Gordon equation.

References

[1] A. Biswas, Nonlinear Dyn. **58**, 345-348 (2009).
[2] M.J. Ablowitz and P.A. Clarkson, Solitons, Nonlinear Evolution Equations and Inverse Scattering, Cambridge: Cambridge University press, 1991.
[3] A.M. Wazwaz, Appl. Math. Comput. **154**, 713-723 (2004).
[4] P. Rosenau and J.M. Hyman, Physical Review Letters **70**, 564-567 (1993).
[5] A.M. Wazwaz, Mathematics and Computers in Simulation **63**, 35-44 (2003).
[6] R. Hirota, Phys. Lett. A **27**, 1192-1194 (1971).
[7] W. Malfliet and W. Hereman, Phys. Scr. **54**, 563-568 (1996).
[8] M.A. Abdou, Appl. Math. Comput. **190**, 988-996 (2007).
[9] H.X. Wu and J.H. He, Chaos, Solitons & Fractals **30**, 700-708 (2006).
[10] M. Wang, X. Li and J. Zhang, Phys. Lett. A **372**, 417-423 (2008).
[11] G. Ebadi and A. Biswas, Commun. Nonlinear Sci. Numer. Simulat. **16**, 2377-2382 (2011).
[12] M. Wang, Phys. Lett. A **199**(3-4), 169-172 (1995).
[13] C.S. Liu, Acta. Phys. Sin. **54**, 2505-2509 (2005).
[14] C.S. Liu, Commun. Theor. Phys. **45**, 395-397 (2006).
[15] C.S. Liu, Commun. Theor. Phys. **45**, 219-223 (2006).
[16] C.S. Liu, Acta. Phys. Sin. **54**, 4506-4510 (2005).
[17] C.S. Liu, Comput. Phys. Commun. **181**, 317-324 (2010).
[18] Y. Gurefe, A. Sonmezoglu and E. Misirli, Pramana-J. Phys. **77**(6), 1023-1029 (2011).
[19] Y. Gurefe, A. Sonmezoglu and E. Misirli, J. Adv. Math. Stud. **5**(1), 41-47 (2012).
[20] Y. Pandir, Y. Gurefe, U. Kadak and E. Misirli, Abstr. Appl. Anal. **2012**, Art. ID 478531, 16 pp (2012).
[21] Y. Gurefe, E. Misirli, A. Sonmezoglu and M. Ekici, Appl. Math. Comput. **219**, 5253-5260 (2013).
[22] Y. Gurefe, E. Misirli, Y. Pandir, A. Sonmezoglu and M. Ekici, Bull. Malays. Math. Sci. Soc. (2015), DOI 10.1007/s40840-014-0075-z.
[23] A.M. Wazwaz, Numer. Methods Partial Differ. Eqn. **23**, 247-255 (2006).
[24] M. Hayek, Appl. Math. Comput. **217**, 212-221 (2010).
[25] M. Antonova and A. Biswas, Commun. Nonlinear Sci. Numer. Simulat. **14**, 734-748 (2009).
[26] D. S. Wang, Z. Yan and H. Li, Comput. Math. Appl. **56**, 1569-1579 (2008).
[27] J. Lee and R. Sakthivel, Pramana-J. Phys. **75** 565-578 (2010).

Abdullah Sonmezoglu is an assistant professor in Department of Mathematics at Bozok University, Yozgat, Turkey. He received his B.Sc. degree from Ondokuz Mayıs University, M.Sc. degree from Gaziosmanpaşa University, and Ph.D. degree from Erciyes University. He is interested in summability of Fourier series, elliptic functions and integrals, probability and statistics, nonlinear partial differential equations in Mathematical Physics, nonlinear sciences.

Mehmet Ekici is a Ph.D. student in Department of Mathematics at Erciyes University. He received his M.Sc. degree from Adnan Menderes University and B.Sc. degree from Celal Bayar University. He is a Lecturer at Department of Mathematics, Bozok University, Yozgat, Turkey. He is interested in numerical analysis, analytical and numerical solutions of the linear or nonlinear partial differential equations, nonlinear sciences, mathematical physics, mathematical biology, control theory and its applications.

Elsayed M. E. Zayed, Professor of Mathematics (from 1989 till now) at Zagazig University. His interests are the nonlinear PDEs in Mathematical Physics, Inverse problems in differential equations, Nonlinear difference equations. He Published about 225 articles in famous International Journals around the world. He was the head of Mathematics Department at Zagazig University, Egypt from 2001 till 2006. He is an Editor of WSEAS Transaction on Mathematics. He has got some Mathematical prizes by the Egyptian Academy of scientific research and Technology. He got the Medal of Science and Arts of the first class from the president of Egypt. He got the Medal of Excellent of the first class from the president of Egypt. He is one of the Editorial Board of the Punjab University Journal of Mathematics. He have reviewed many articles for many international Journals . He has got his BSC of Mathematics from Tanta University, Egypt 1973 . He has got two MSC degrees in Mathematics. The first one from Al-Azher University, Egypt 1977, while the second one from Dundee University, Scotland, UK 1978. He has got his PHD in Mathematics from Dundee University, Scotland, UK 1981. He has got the Man of the Year award, (1993), by the American Biographical, Institute, U.S.A. He has got the Research Fellow Medal, (1993), by the American Biographical Institute, U.S.A. The Curriculum Vitae of Professor E.M.E. Zayed has been published by Men of achievement of the International Biographical center, Cambridge, vol. 16, (1995), p.534 The Curriculum Vitae of Professor E.M.E. Zayed has been published by Dictionary of International Biography of the International Biographical center, Cambridge, vol. 23, (1995), p.714. He was a Professor of Mathematics at Taif University, Saudi Arabia (2006-2010).