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Load Prediction of In-Service Bridges by Using an Unbiased Grey Markov Forecasting Model

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Abstract: Reliable prediction of traffic loads is essential for bridge planning and design. In the study, an improved unbiased Grey Markov forecasting model is applied to predict traffic loads on highway bridges. The comparison between the predicted results and existing practical measurements indicates that high accuracy can be achieved by using the developed model for bridge load prediction. The developed model shows great promise in structural design and durability assessment of highway bridges.

Keywords: Bridge Load Prediction; Unbiased Gray Markov Model;Bridge Structural Durability.

1 Introduction

Structural weakness, degradation and failure due to load have become serious issues that threaten the safety, reliability, and integrity of in-service bridges. When designing a bridge, engineers usually seek underestimate of the load such that the real structure can carry more load than predicted and structural safety can be achieved. Therefore, reliable prediction of extreme load influences expected during the proposed or remaining life of the structure is highly needed for the design and evaluation of highway bridges [1].
The prediction process of bridge loads is somehow complicated and burdensome. The most commonly used approaches include exponential smoothing, regression analysis, and combined forecasting prediction [2, 3]. The traffic flow information has the apparently layer complexity of structure, the fuzzy relation of construction, and the uncertainties of data [4]. Due to the constraints of present techniques, unaffectedly environmental variation and some artificial factors, the statistics or forecast data may have some mistakes, errors or scarcity [4]. The Grey model initially proposed by Deng [5] has advantages in model establishment with few data and incomplete data to realize the prediction of the system, but grey prediction does not match to random data with large fluctuation and the forecast precision is thus relatively low. The Markov model [6] has also been utilized extensively for prediction of various problems, but it is suitable for the solution to predict steady stochastic data sequences, which are not practically obtained in traffic environment.
Various strategies of combining the Grey model and the Markov model have been therefore explored recently. Chen et al. [7] developed a hybrid Grey Markov model to predict traffic volume, in which the error produced by the Grey model in the next step could be compensated by using the Markov forecasting method for error estimation. Li et al. [8] presented an improved prediction analysis combining the GM (1,1) model and the time series Autoregressive Integrated Moving Average (ARIMA) model. A new metabolic unbiased Grey Markov model was proposed by Dong et al. [9] predict stochastic fluctuating dynamic process, where the newest data was gradually added with the old one removed from original data sequence.

In this paper, the improved unbiased Grey Markov forecasting model [9] is applied for bridge load prediction by conducting equal-dimensional information processing and updating the original data. Attention is paid on the load prediction of the bridge in Quzhou city of Zhenjiang province in the study. Prediction results are compared with practical measurement to evaluate the capability of the proposed model. It can be found that the presented method can provide important data with high accuracy for bridge durability analysis.

2. UNBIASED GREY MARKOV MODEL

2.1. Unbiased Grey Forecasting Model

We first establish unbiased GM (1,1) model and conduct trend processing for original data, more details can be found in [9]. Assume that the original raw series $X^{(0)}$ with *n* samples is expressed as:

$$X^{(0)} = \{x^{(0)}_{(1)}, x^{(0)}_{(2)}, \ldots. x^{(0)}_{(n)}\}, \text{ where } x^{(0)}_{(k)} \geq 0, k = 1,2,\ldots, n \tag{1}$$

(1) A new series $X^{(1)}$ is produced by accumulated generating operation (AGO)

$$X^{(1)} = \{x^{(1)}_{(1)}, x^{(1)}_{(2)}, \ldots. x^{(1)}_{(n)}\} \tag{2}$$

where $x^{(1)}_{(k)} = \sum_{i=1}^{k} x^{1}_{i}, k = 1,2,\ldots, n$

(2) Determine data matrices B and Yn

$$B = \begin{bmatrix} -\frac{1}{2}(x^{(0)}_{(1)} + x^{(1)}_{(2)}) & 1 \\ -\frac{1}{2}(x^{(1)}_{(2)} + x^{(1)}_{(3)}) & 1 \\ \ldots & \ldots \\ -\frac{1}{2}(x^{(1)}_{(n-1)} + x^{(1)}_{(n)}) & 1 \end{bmatrix} \text{ and } \quad Yn = \begin{bmatrix} x^{(0)}_{(2)} \\ x^{(0)}_{(3)} \\ \ldots \\ x^{(0)}_{(n)} \end{bmatrix} \tag{3}$$

(3) Calculate unknown parameters $\hat{a}$ and $\hat{u}$ in first-order differential equations using least squares methods

$$\begin{bmatrix}\hat{a} & \hat{u}\end{bmatrix} = (B^T B)^{-1} B^T Yn \tag{4}$$

(4) Calculate GM(1,1) model parameter b and A

First-order accumulation of the original exponential data series is

$$x^{(0)}_{(k)} = Ae^{b(k-1)}(k = 1,2,\ldots,n) \tag{5}$$

which yields

$$X^{(1)}_{(k)} = \sum_{i=1}^{k} x^{(1)}_{(i)} = A\frac{1-e^{bk}}{1-e^{b}}(k = 1,2,\ldots,n) \tag{6}$$

Conventional GM (1,1) method [4, 5, 10, 11] leads to

$$\begin{bmatrix}\hat{a} \\ \hat{u}\end{bmatrix} = (B^T B)^{-1} B^T Yn = \begin{bmatrix} 2(1-e^{b})/(1+e^{b}) \\ 2A/(1+e^{b}) \end{bmatrix} \tag{7}$$

which estimates the parameters $\hat{b}$ and A in the unbiased GM model in terms of $\hat{a}$ and $\hat{u}$ as

$$\hat{b} = \ln\frac{2-\hat{a}}{2+\hat{a}}, \quad A = \frac{2\hat{u}}{2+\hat{a}} \tag{8}$$

(5) Establish original data series model

$$\hat{x}^{(0)}_{(1)} = x^{(0)}_{(1)}, \hat{x}^{(0)}_{(k+1)} = Ae^{bk}(k = 0,1,\ldots, n-1) \tag{9}$$

which is the fitting value of the original data series, and $\hat{x}^{(0)}_{(k+1)}, (k \geq n)$ is the predicted value of the original data series.

The unbiased GM (1,1) model eliminates not only the intrinsic error from the conventional GM (1,1) but the failure in treating the original data with high increasing rate. Moreover, the improved model does not require accumulation regression and it simplifies the modeling procedures, which thus enhances the computational efficiency [12].

2.2 .Markov Improvement for the Grey Forecasting Model Results [13]

2.2.1. State Classification

The state $\otimes_i$ denotes original data series with respective the deviation to the prediction curve $\hat{y}_k$.

$\hat{y}_k = x^{(0)}_{(k+1)} = Ae^{bk}$ curve for the center to the division of the system curve with the $\hat{y}_k$ m a strip parallel to the regional, any division is described as below:

$$\otimes_i = [\otimes_{i1} \quad , \quad \otimes_{i2}] \tag{10}$$

where $\otimes_{i1} = \hat{y}_k + A_i$,

$\otimes_{i2} = \hat{y}_k + B_i$, A_i and B_i constant translation（m, A_i and B_i are determined by the objects and the original data）.

2.2.2. Calculation of Transition Matrix

Supposing M_{ij} is the original sample data which is from the state $\otimes_i$ to the state $\otimes_j$ through the transition of m. M_i is the sample data of state $\otimes_j$ and $P_{ij} = M_{ij} / M_i (i = 1,2,..., m)$ is called the state transition probability from the state $\otimes_i$ to the state $\otimes_j$. The transition probability matrix of construction m is

$$R(m) = \begin{bmatrix} P_{11}(m) & P_{12}(m) & \dots & P_{1n}(m) \\ P_{21}(m) & P_{22}(m) & \dots & P_{2n}(m) \\ - & \dots & \dots & \dots \\ P_{n1}(m) & P_{n2}(m) & \dots & P_{nn}(m) \end{bmatrix} \tag{11}$$

The transition probability matrix reflects the transition rules of different states in the system. The future developing trend can be defined from the state of transition probability matrix and the initial status. The next transition state of the target can also be defined according to the state transition matrix R(1).When there are two or more than two sameness or similarities in the line of R1, the future transition of the state can be referred to.

2.2.3. The Definition of Predictive Values

When the future state， this is $\otimes_i$,of transition is defined, the predictive value, this is $[\otimes_{i1} \quad , \quad \otimes_{i2}]$,of interval fluctuation is also defined. If the midpoint is chosen, the predictive results can be obtained.

2.3. The Improved Unbiased Grey Markov Forecasting Model

The target of the Markov prediction model is a dynamic system, which is based on the Markov process. The characteristics of this process are as follows: n state Markov chain is defined by the state set and the transition probability, which is only in one state at any moment. If it is in the state $\otimes_i$ in the K process, the probability Pij will be obtained in the state $\otimes_j$ in the K+1 process. All the characteristics of the Markov chain show that the prediction of the future developing trend is defined by the transition probability in different states. The transition probability reflects the impact of different random factors, which proves that Markov chain can be used to predict the problems of random undulatory property. This is a way to supplement the Grey Prediction. But the target of Markov chain requires the mean value, such as the stationary process. While the prediction of the bridge load is a random non-stationary process, the unbiased GM can be used to find the developing trend and supplement the prediction of Markov chain [13].

As time goes by, some actuation factors will constantly appear in the development of grey system and affect it. It is the same as the unbiased Grey Markov Forecasting Model. The proper data is only one or two after the punch mark. The accuracy of forecasting becomes lower when it develops. So the actuation factors must be considered in the practical application. It is suggested to trivialize the historical data, put the new information into the system at times, build up new models and improve the accuracy of long-term prediction.

Equal dimension message processing means that the unbiased GM is used to predict the data, supplement the known data with it and get rid of the oldest one. The next step is then to keep the equal dimension, rebuild the unbiased GM model, predict the next value, supplement the series of numbers with the original results and take out the oldest value. Finally, one need fill vacancies in the proper order and predict one by one until the forecast targeting achieved or certain accuracy is obtained.

3. STEPS OF UNBIASED GREY MARKOV MODEL

The procedures of use of the unbiased Grey Markov model for prediction are summarized as follows [14, 15]:

(1) Originate data series

(2) Generate models

(3) Determine data and define parameter

(4) Suppose $\hat{b} = \ln \frac{2 - \hat{a}}{2 + \hat{a}}$ and come up with data series

(5) Suppose the improved Markov predictive value to be $\hat{y}_k$ in the k stage, $\hat{y}_k = x^{(0)}_{(k+1)} = Ae^{bk}$, $\hat{y}_k$ to be centered and divided the system into m states.

(6) Calculate one step state matrix
(7) Calculate one step state-transition matrix
(8) Judge the state of the value to be predicted
(9) Calculate the predict value
(10) Update data series

(11) Go back to step (2), repeat step (2) to (10), until the calculation of all the predict values is finished.

4. EXAMPLE AND ANALYSIS

Take the Qujiang Bridge in Quzhou City Zhejiang Province for example, the predict value of the load, including trucks, buses and crowds, can be obtained by using the unbiased Grey Markov forecasting model. The accuracy of the forecast precision can be evaluated by the comparison of the predict values with the actual observations in [16].

4.1. Prediction of Load of Different Kinds of Trucks

The predicted values of the truck load are shown in Table 1. The comparison of the predicted values and the actual observed values [16] can be seen in Fig, 1. It is found that excellent consistency with consistency measurements [16] can be achieved by using the current model, which further indicates the high precision of the model.

4.2. Prediction of Load of Different Kinds of Buses

The predicted values of the bus load are demonstrated in Table 2. The comparison of the predicted values and the actual observed values [16] can be seen in Fig. 2. Again, it is seen that the prediction results by the developed model agree well with the practical measurements [16].

4.3. Prediction of Load of different kinds of Crowd

The predicted values of the crowd load are demonstrated in Table 3. The comparison of the predicted values and the actual observed values [16] can be seen in Fig. 3. Compared to the existing measurements [16], similar high precision can be observed as that in Figs. 1 and 2, which shows the versatile applicability of the current model.

Table 1 The number of trucks loading prediction

K	Analog value	Measured [16]	Residual	The relative value of residual（%）	Relative error（%）
2（2005）	94367.326	94027.841	-339.485	-0.361	0.361
3（2006）	92534.238	96321.657	3787.419	3.932	3.932
4（2007）	89589.741	89620.457	30.716	0.034	0.034
5（2008）	87758.378	87360.482	-397.896	-0.455	0.455
6（2009）	85056.472	85147.209	90.737	0.107	0.107

Table 2. The number of bus load forecast results

K	Analog value	Measured [16]	Residual	The relative value of residual（%）	Relative error（%）
2 (2005)	184367.326	184527.841	160.515	0.086987	0.086987
3(2006)	186534.238	184321.658	-2212.58	-1.20039	1.20039
4 (2007)	188589.741	189620.457	1030.716	0.543568	0.543568
5(2008)	190758.378	190260.482	-497.896	-0.26169	0.26169
6 (2009)	193169.472	193247.209	77.737	0.040227	0.040227

Table 3. Groups of the number of predicted load

K	Analog value	Measured [16]	Residual	The relative value of residual（%）	Relative error（%）
2（2005）	144367.326	144027.841	-339.485	-0.23571	0.23571
3（2006）	145534.238	150321.657	1787.419	1.189063	1.189063
4（2007）	147589.741	150620.457	-1969.28	-1.30745	1.30745
5（2008）	149758.378	157360.482	602.104	0.382627	0.382627
6（2009）	152569.472	159147.209	-422.263	-0.26533	0.26533

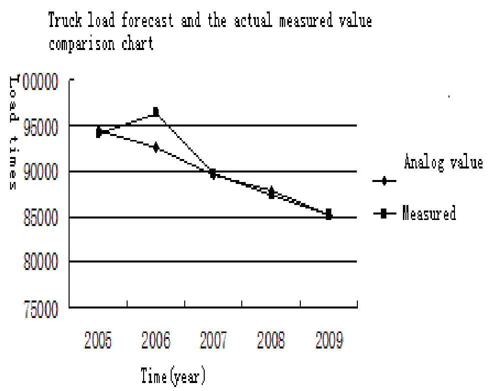

Fig. 1. The number of goods vehicle load forecasting results and the measured comparison chart.

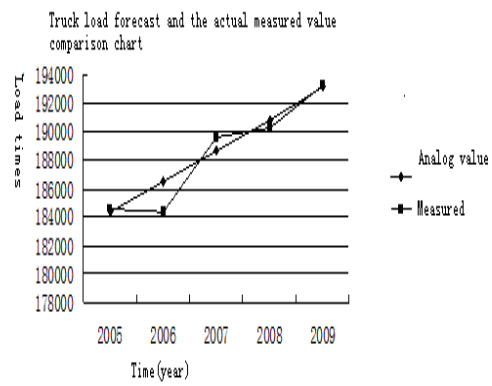

Fig. 2. The number of bus load forecasting results and the measured value comparison Chart.

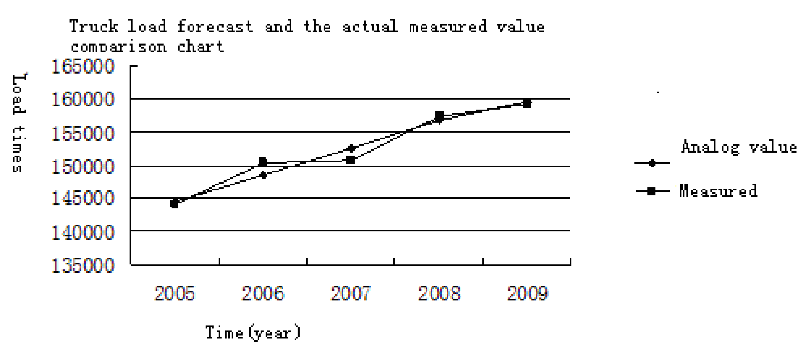

Fig. 3. Groups of the number of load forecasting results and the actual value comparison chart

5. CONCLUSIONS

(1) From the prediction results of three types of highway loads, it is shown that the more updated the prediction data is used, the more accurate the prediction results can be achieved. By comparison, it is seen that the prediction results of three load types for year 2009 are closer to the exact value, and the average error is only 3%. This prediction method is thus developed with high accuracy, and can be directly applied in practical engineering application.
(2) This prediction model could deviate a lot with less unknown data, i.e., when the deviated values cannot be corrected, more difference can be produced. Therefore, this prediction model applies for the bridges with more than three year service ages. For newly built bridges (serve age is smaller than three years), the beam load prediction need correcting before it is applied in the calculation. Since newly built bridges do not need fixture, this method is of practical significance.

(3) The prediction of bridge loads is a complicated process. This study does not address natural and policy factors, when combining Grey system theory and Markov Chain Forecasting model to conduct equal-dimension processing for original data. The developed unbiased Grey Markov Forecasting Model is applied to predict the traffic loads for Quzhou Bridge, and the reliability and accuracy of this method can be verified by very good applicability. This method can provide reliable data for the evaluation of bridge structural durability, and also serves as references for the consolidation and maintenance of in-service bridges.

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