Parameter Estimation of Power Lindley Distribution under Hybrid Censoring

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Abstract: The present study deals with the classical and Bayesian estimation of the hybrid censored lifetime data under the assumption that the lifetimes follow the power Lindley distribution. By assuming Jeffrey's invariant and gamma priors of the unknown parameters, Bayes estimates along with posterior standard error and highest posterior density credible intervals of the parameters are obtained. A Markov Chain Monte Carlo technique such as Metropolis-Hastings algorithm has been utilized to generate draws from the posterior density of the parameters. A real data set has been analyzed for illustration purpose.

Key words: Power Lindley distribution, hybrid censoring, Bayes estimate, Gibbs sampler, Metropolis-Hastings algorithm, highest posterior density credible interval.

1. Introduction

In reliability literature, Type-I and Type-II censoring schemes are the most regularly used censoring schemes. The mixture of Type-I and Type-II censoring scheme is known as hybrid censoring scheme. In this censoring scheme, n items are put on test and the test is terminated when the pre-chosen number R out of n items are failed or when a pre-decided time T on the test has been reached. In other words, we can say that the termination point of the test is $T^{\dagger} = \min\{X_{R:n}, T\}$. Epstein [1] was the first to introduce this censoring scheme and it is quite applicable in reliability acceptance test [2]. Since then, hybrid censoring scheme is used by many authors like Chen and Bhattacharya [3], Childs et al. [4], and Draper and Guttmen [5] and the reference cited therein.

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Although, this censoring scheme is very useful in reliability/survival analysis, the limited attention has been paid in analyzing hybrid censored lifetime data. Some recent studies on hybrid censoring are Kundu [6], Banerjee and Kundu [7], Kundu and Pradhan [8], Ganguly et al. [9], Dube et al. [10] and Gupta and Singh [18]. For a comprehensive review of hybrid censoring, see Balakrishnan and Kundu [11].

The one parameter Lindley distribution was introduced by Lindley [12] in the context of Bayesian statistics, as a counter example of fudicial statistics. Recently, many authors make use of one parameter Lindley distribution as a lifetime model in various fields of reliability/survival analysis like stress-strength model [14], load-share model [13], competing risk model [15], discrete model [16], censoring scheme [17,18, 26] etc. However, we may encounter data which is incompatible with one parameter Lindley distribution. To overcome this situation, very recently, Ghitany et al. [19] proposed the Power Lindley (PL) distribution indexed by both shapes and scale parameter. The Lindley distribution family justifies the real phenomenon as it does not permit the constant hazard rate because there are hardly any real life systems that have time independent failure rate. PL distribution shows the decreasing and increasing hazard rates with decreasing and the uni-model distribution functions respectively these situations are very common in life-testing experiments.

In lieu of above considerations, the paper is organized as follows. In section 2, we describe the model under the assumption of hybrid censored data from power Lindley distribution. In section 3, we obtain the maximum likelihood estimators (MLE) of the unknown parameters. Further, by assuming Jeffrey's invariant and gamma priors of the unknown parameters, Bayes estimates along with their posterior standard error (PSE) and highest posterior density credible (HPD) interval of the parameters are obtained in section 4. In Section 5, a real data set has been analyzed for illustration purpose.

2. Model Description

Suppose n identical units are put to test under the same environmental conditions and test is terminated when a pre-chosen number R, out of n items have failed or a pre determined time T, on test has been reached. It is assumed that the failed item not replaced and at least one failure is observed during the experiment. Therefore, under this censoring scheme we have one of the following types of observations:

Case I: $\left\{ x_{1:n} < \ldots\ldots < x_{R:n} \right\}$ if $x_{R:n} < T$

Case II: $\left\{ x_{1:n} < \ldots\ldots < x_{d:n} \right\}$ if $1 \le d < R$ and $x_{d:n} < T < x_{d+1:n}$

Here, $x_{1:n} < x_{2:n} < \ldots$ denote the observed failure times of the experimental units. For schematic representation of the hybrid censoring scheme refer to Kundu and Pradhan [8]. It may be mentioned that although we do not observe $x_{d+1:n}$, but $x_{d:n} < T < x_{d+1:n}$ means that the dth failure took place before T and no failure took place between $x_{d:n}$ and T. Let the life time random variable X has a power Lindley distribution with parameter β and α i.e. the probability density function (PDF) of x is given by;

$$f_X(x \mid \beta, \alpha) = \frac{\alpha\beta^2}{(1+\beta)}\left(1 + x^\alpha\right)x^{\alpha-1}e^{-\beta x^\alpha} \quad ; \qquad x, \beta, \alpha > 0$$

Based on the observed data, the likelihood function is given by
Case I:

$$L\left(\underset{\sim}{x} \mid \alpha, \beta\right) = \frac{\alpha^R \beta^{2R}}{(1+\beta)^R} \prod_{i=1}^{R}\left(1+x_{i:n}^\alpha\right)\prod_{i=1}^{R} x_{i:n}^{\alpha-1} e^{-\beta\left\{\sum\limits_{i=1}^{R} x_{i:n}^\alpha + (n-R)x_{R:n}^\alpha\right\}}\left(1+\frac{\beta}{1+\beta}x_{R:n}^\alpha\right)^{n-R} \qquad \ldots (1)$$

Case II:

$$L\left(\underset{\sim}{x} \mid \alpha, \beta\right) = \frac{\alpha^d \beta^{2d}}{(1+\beta)^d} \prod_{i=1}^{d}\left(1+x_{i:n}^\alpha\right)\prod_{i=1}^{d} x_{i:n}^{\alpha-1} e^{-\beta\left\{\sum\limits_{i=1}^{d} x_{i:n}^\alpha + (n-d)T^\alpha\right\}}\left(1+\frac{\beta}{1+\beta}T^\alpha\right)^{n-d} \qquad \ldots (2)$$

Where $\underset{\sim}{x} = \left(x_{1:n}, x_{2:n}, \ldots\ldots\right)$

The combined likelihood for Case I and case II can be written as

$$L = L\left(\underset{\sim}{x} \mid \alpha, \beta\right) = \frac{\alpha^r \beta^{2r}}{(1+\beta)^r} \prod_{i=1}^{r}\left(1+x_{i:n}^\alpha\right)\prod_{i=1}^{r} x_{i:n}^{\alpha-1} e^{-\beta\left\{\sum\limits_{i=1}^{r} x_{i:n}^\alpha + (n-r)c^\alpha\right\}}\left(1+\frac{\beta}{1+\beta}c^\alpha\right)^{n-r} \qquad \ldots (3)$$

Where,

$$r = \begin{cases} R & \text{for case I} \\ d & \text{for case II} \end{cases} \qquad\qquad c = \begin{cases} x_{R:n} & \text{for case I} \\ T & \text{for case II} \end{cases}$$

3. Maximum Likelihood Estimators

The log-likelihood function for equation (3) can be written as

$$\log L = r\log(\alpha) + 2r\log(\beta) - n\log(1+\beta) + (n-r)\log\left[1+\beta\left(1+c^\alpha\right)\right] + \sum_{i=1}^{r}\log\left(1+x_{i:n}^\alpha\right)$$
$$+ (\alpha-1)\sum_{i=1}^{r} x_{i:n} - \beta\left[\sum_{i=1}^{r} x_{i:n}^\alpha + (n-r)c^\alpha\right] \qquad \ldots (4)$$

To obtain the MLEs of (β, α) say $(\hat{\beta}, \hat{\alpha})$, one can solve the following equations

$$\frac{\partial \log L}{\partial \beta} = \frac{2r}{\theta} - \frac{n}{(1+\beta)} - \sum_{i=1}^{n} x_i^{\alpha} + (n-r)\frac{(1+c^{\alpha})}{[1+\beta(1+c^{\alpha})]} \quad \dots (5)$$

$$\frac{\partial \log L}{\partial \alpha} = \frac{r}{\alpha} + \sum_{i=1}^{n} \frac{x_{i:n}^{\alpha} \log(x_{i:n})}{(1+x_{i:n}^{\alpha})} \sum_{i=1}^{r} \log x_{i:n} - \beta \sum_{i=1}^{n} x_{i:n}^{\alpha} \log(x_{i:n}) - (n-r)c^{\alpha} \log(c) + \frac{(n-r)\beta c^{\alpha} \log(c)}{[1+\beta(1+c^{\alpha})]}$$
$$\dots (6)$$

Equation (5) and (6) can be solved for $\hat{\beta}$ and $\hat{\alpha}$ using any numerical iterative procedure. Since, the MLEs of β and α are not in the closed forms, therefore, it is not possible to derive their exact distributions. Thus, using large sample theory of MLE, the asymptotic sampling distribution of $\begin{pmatrix} \hat{\beta} - \beta \\ \hat{\alpha} - \alpha \end{pmatrix}$ is $N_2(0, \Delta^{-1})$ where, Δ is the observed Fisher information matrix. The elements of Δ are given by

$$\Delta_{11} = \left| -\frac{\partial^2 \log L}{\partial \beta^2} \right|_{\beta=\hat{\beta}}, \quad \Delta_{12} = \Delta_{21} = \left| -\frac{\partial^2 \log L}{\partial \beta \partial \alpha} \right|_{\beta=\hat{\beta}, \alpha=\hat{\alpha}} \quad \text{and} \quad \Delta_{22} = \left| -\frac{\partial^2 \log L}{\partial \alpha^2} \right|_{\alpha=\hat{\alpha}}$$

Here, $\dfrac{\partial^2 \log L}{\partial \beta^2} = -\dfrac{2r}{\beta^2} + \dfrac{n}{(1+\beta)^2} - (n-r)\dfrac{(1+c^{\alpha})^2}{[1+\beta(1+c^{\alpha})]^2}$

$$\frac{\partial^2 \log L}{\partial \beta \partial \alpha} = \frac{\partial^2 \log L}{\partial \alpha \partial \beta} = -\sum_{i=1}^{r} x_{i:n}^{\alpha} \log(x_{i:n}) - (n-r)c^{\alpha} \log(c) + (n-r)\frac{c^{\alpha} \log(c)}{[1+\beta(1+c^{\alpha})]^2}$$

$$\frac{\partial^2 \log L}{\partial \alpha^2} = -\frac{r}{\alpha^2} + \sum_{i=1}^{r} \frac{x_{i:n}^{\alpha} (\log x_{i:n})^2}{(1+x_{i:n}^{\alpha})^2} - (n-r)c^{\alpha} (\log c)^2 + \frac{(n-r)\beta c^{\alpha} (\log(c))^2 (1+\beta)}{[1+\beta(1+c^{\alpha})]^2}$$

The respective asymptotic $100(1-\gamma)\%$ confidence intervals (C.I.) for β and α are $\hat{\beta} \pm z_{\gamma/2}\sqrt{V(\hat{\beta})}$ and $\hat{\alpha} \pm z_{\gamma/2}\sqrt{V(\hat{\alpha})}$ where $V(\hat{\beta})$ and $V(\hat{\alpha})$ are the variances of $\hat{\beta}$ and $\hat{\alpha}$, which can be obtained using fisher information matrix. Here $z_{\gamma/2}$ is the upper $100*(\gamma/2)^{th}$ percentile of a standard normal distribution.

4. Bayesian Estimation

In real life situations, generally, it is noted that the manners of the parameters representing the lifetime model characteristics cannot be treated as constant throughout the life-testing period. Therefore, it would be reasonable to assume the parameters involved in the lifetime model as random variables. In view of this, we propose Bayesian estimation procedure by assuming the following independent gamma priors for β and α;

$$g_1(\beta) = \frac{v_1^{\delta_1}}{\Gamma\delta_1}\beta^{\delta_1-1}\exp(-v_1\beta); \ (v_1, \delta_1, \beta > 0) \qquad \dots (7)$$

and

$$g_2(\alpha) = \frac{v_2^{\delta_2}}{\Gamma\delta_2}\alpha^{\delta_2-1}\exp(-v_2\alpha); \ (v_2, \delta_2, \alpha > 0) \qquad \dots (8)$$

Here the hyper parameters are assumed to be known real numbers. Using likelihood function in (5) and prior distributions in (7) and (8), the joint posterior distribution of β, α and the data is given by

$$\phi(\beta, \alpha|\underset{\sim}{x}) = L(\underset{\sim}{x}|\beta, \alpha)g_1(\beta)g_2(\alpha) \qquad \dots (9)$$

It can be seen that the above expression cannot be obtained in nice closed form and one needs numerical approximation. Here, we use Gibbs sampler, a MCMC method, proposed by Geman and Geman [20]. It allows us to generate observations from the conditional distribution of each of the parameters using the current values of the given parameters. MCMC is a class of methods in which one can simulate draws that are slightly dependent and approximately from the posterior distribution. By means of this procedure, our aim is to get the ergodic chains of the parameters which are irreducible, aperiodic and positive recurrent. For implementing Gibbs sampling procedure, the full conditional posterior distributions of β and α are

$$\pi_1(\beta|\underset{\sim}{x}, \alpha) \propto \frac{\beta^{2r+\delta_1-1}}{(1+\beta)^r}\exp\left[-\beta\left(\sum_{i=1}^{r}x_{i:n}^{\alpha} + (n-r)c^{\alpha} + v_1\right)\right]\left(1+\frac{\beta}{1+\beta}c^{\alpha}\right)^{n-r} \quad \dots (10)$$

$$\pi_2(\alpha|\beta,\underset{\sim}{x}) \propto \alpha^{r+\delta_2-1}e\prod_{i=1}^{r}\left(1+x_{i:n}^{\alpha}\right)\prod_{i=1}^{r}x_{i:n}^{\alpha-1}\exp\left[-\beta\left(\sum_{i=1}^{r}x_{i:n}^{\alpha} + (n-r)c^{\alpha}\right) - v_2\alpha\right]\left(1+\frac{\beta}{1+\beta}c^{\alpha}\right)^{n-r}$$
$$\dots (11)$$

On putting $v_1 = \delta_1 = v_2 = \delta_2 = 0$ in (18) and (19), one gets the respective conditional posterior distributions of the parameters under the assumptions of Jeffrey's prior.

Gibbs algorithm:

1. Generate β from $\pi_1\left(\beta|\alpha,\underset{\sim}{x}\right)$.

2. Generate α from $\pi_2\left(\alpha|\beta,\underset{\sim}{x}\right)$ for β generated in step 1.

3. Repeat step 1-2 B-times.

4. Bayes estimate of $\Omega=(\beta,\alpha)$ say $\Omega^*=\left(\beta^*,\alpha^*\right)$ under squared error loss function is

$$\Omega^*=\frac{1}{B}\sum_{i=1}^{B}\Omega_i$$

5. The posterior variance of Ω is $V\left(\Omega^*\right)=\frac{1}{B}\sum_{i=1}^{B}\left(\Omega_i-\Omega^*\right)^2$

6. Let $\Omega^*_{(1)}\leq\Omega^*_{(2)}\leq....\leq\Omega^*_{(B)}$ respectively denote the ordered values of $\Omega^*_1,\Omega^*_2,....,\Omega^*_B$. Then, following Chen and Shao [21], the $(1-\gamma)\times100\%$ HPD intervals for Ω is $\left(\Omega^*_{\left(j^*\right)},\Omega^*_{\left(j^*+[(1-\gamma)B]\right)}\right)$, Where, j^* is chosen so that

$$\Omega^*_{j^*+(1-\gamma)B}-\Omega^*_{\left(j^*\right)}=\min_{1\leq j\leq\gamma B}\left(\Omega^{\wedge}_{(j+\gamma B)}-\Omega^*_{(j)}\right)$$

Since, the posterior densities given in (10) and (11) are not easy to simulate. Therefore, we utilize the Metropolis-Hastings algorithm [22, 23] by choosing the suitable proposal densities for both β and α.

5. Data Analysis

Here, we conduct a real data analysis for illustrative purpose. It is strength data originally reported by Badar and Priest [24]. However, we use the transformed data considered by Raqab and Kundu [25]. This data set consists of the strength measurements in GPA, for single carbon fibers and impregnated 1000-carbon fiber tows. Single fibers were tested under at gauge lengthof 1, 10, 20, 50 mm. impregnated tows of 1000 fibers were tested at gauge length of 20, 50, 150, and 300 mm. The data is given below:

Data Set:

0.312, 0.314, 0.479, 0.552, 0.700, 0.803, 0.861, 0.865, 0.944, 0.958, 0.966, 0.997, 1.006, 1.021, 1.027, 1.055, 1.063, 1.098, 1.140, 1.179, 1.224, 1.240, 1.253, 1.270, 1.272, 1.274, 1.301, 1.301, 1.359, 1.382, 1.382, 1.426, 1.434, 1.435, 1.478, 1.490, 1.511, 1.514, 1.535, 1.554, 1.566, 1.570, 1.586, 1.629, 1.633, 1.642, 1.648, 1.684, 1.697, 1.726, 1.770, 1.773, 1.800, 1.809, 1.818, 1.821, 1.848, 1.880, 1.954, 2.012, 2.067, 2.084, 2.090, 2.096, 2.128, 2.233, 2.433, 2.585, 2.585.

Before analyzing further, first we check whether the data fits the PL distribution or not, and for this we have used the complete data. In this case, The MLE and Bayes estimates with informative and non-informative priors of the unknown parameters (β, α) are (0.4863, 2.6959), (0.4882, 2.6955) and (0.4878, 2.6961) respectively. The corresponding Kolmogorov-Smirnov (KS) distances are 0.0405, 0.0388 and 0.0391 respectively. For all the method of estimation we adopted in this paper (ML, Jeffrey Bayes and Gamma Bayes), the empirical and fitted distribution function has been plotted in Fig. 1. It is observed that all the three methods of estimation fitted the data very well and hence we may try the above data set for hybrid censoring scheme also.

For analyzing this data set with hybrid censoring, we have created three artificially hybrid censored data sets from the above complete (uncensored) data under the following censoring schemes:

Scheme 1: R = 50, T=2.5

Scheme 2: R = 35, T=2.0

Scheme 3: R = 20, T=1.0

In all the cases, we have estimated the unknown parameter using the ML and Bayes methods of estimation. For obtaining MLEs and 95% asymptotic confidence intervals, we have used nlm() function of R package. Bayes estimates of the parameters and HPD intervals are obtained using gamma and Jeffrey priors. The summary for the above three schemes is given in Table 1. From the Table 1, we observed that both the methods of estimation used in this paper are precisely estimating the parameter (in terms of standard error and length of the confidence/HPD interval). Bayes estimation with gamma prior provides more precise estimates as compared to the Jeffrey prior and MLEs. Also the performance of MLEs and Jeffrey prior are quite similar. The length of the HPD credible intervals based on Gamma prior are smaller than the corresponding length of the HPD credible intervals based on Jeffrey's prior.

Acknowledgement

Table 1: **Summary for the three schemes from hybrid censored Lindley distribution:**

	β		
Estimates\Scheme	Scheme 1: R=5,T=2.5	Scheme 2 : R=35, T=2.0	Scheme 3: R=20, T=1.5
ML [SE] CI {Width}	$\hat{\beta}$ = 0.4797[0.0735] $\beta \in$ (0.3356 0.6239),{0.2822}	$\hat{\beta}$ = 0.4860 [0.0746] $\theta \in$ (0.3396, 0.6324),{0.2927}	$\hat{\beta}$ = 0.4916 [0.0749] $\theta \in$ (0.3397, 0.6333),{0.2935}
Jeffrey Bayes [PSE] HPD{Width}	β^{*} = 0.4834 [0.0717] $\beta \in$ (0.3454, 0.6158),{0.2704}	β^{*} = 0.4896 [0.0745] $\beta \in$ (0.3459, 0.6305),{0.2846}	β^{*} = 0.4896 [0.0744] $\beta \in$ (0.3483, 0.6356),{0.2831}
Gamma Bayes [PSE] HPD{Width}	$\beta^{\#}$ = 0.4846 [0.0698] $\beta \in$ (0.3499, 0.6163),{0.2633}	$\beta^{\#}$ = 0.4869 [0.0710] $\beta \in$ (0.3452, 0.6204),{0.2752}	$\beta^{\#}$ = 0.4884 [0.0715] $\beta \in$ (0.3520, 0.6295),{0.2774}
	α		
ML [SE] CI {Width}	$\hat{\alpha}$ = 2.7608 [0.3168] $\alpha \in$ (2.1399, 3.3818),{1.2419}	$\hat{\alpha}$ = 2.6175 [0.3899] $\alpha \in$ (1.8532, 3.3817),{1.5284}	$\hat{\alpha}$ = 2.6025 [0.5066] $\alpha \in$ (1.6995, 3.6855),{1.9859}
Jeffrey Bayes [PSE] HPD{Width}	α^{*} = 2.7514 [0.3113] $\alpha \in$ (2.1115, 3.3203),{1.2088}	α^{*} = 2.6046 [0.3933] $\alpha \in$ (1.8334, 3.3540),{1.5205}	α^{*} = 2.6699 [0.4957] $\alpha \in$ (1.7843, 3.6745),{1.9301}
Gamma Bayes [PSE] HPD{Width}	$\alpha^{\#}$ =2.7667 [0.2993] $\alpha \in$ (2.1399, 3.2953),{1.1593}	$\alpha^{\#}$ =2.6243 [0.3709] $\alpha \in$ (1.9206, 3.3551),{1.4344}	$\alpha^{\#}$ =2.6854 [0.4574] $\alpha \in$ (1.8167, 3.6099),{1.7931)

Fig.1:- Empirical and fitted survival function for the complete data set
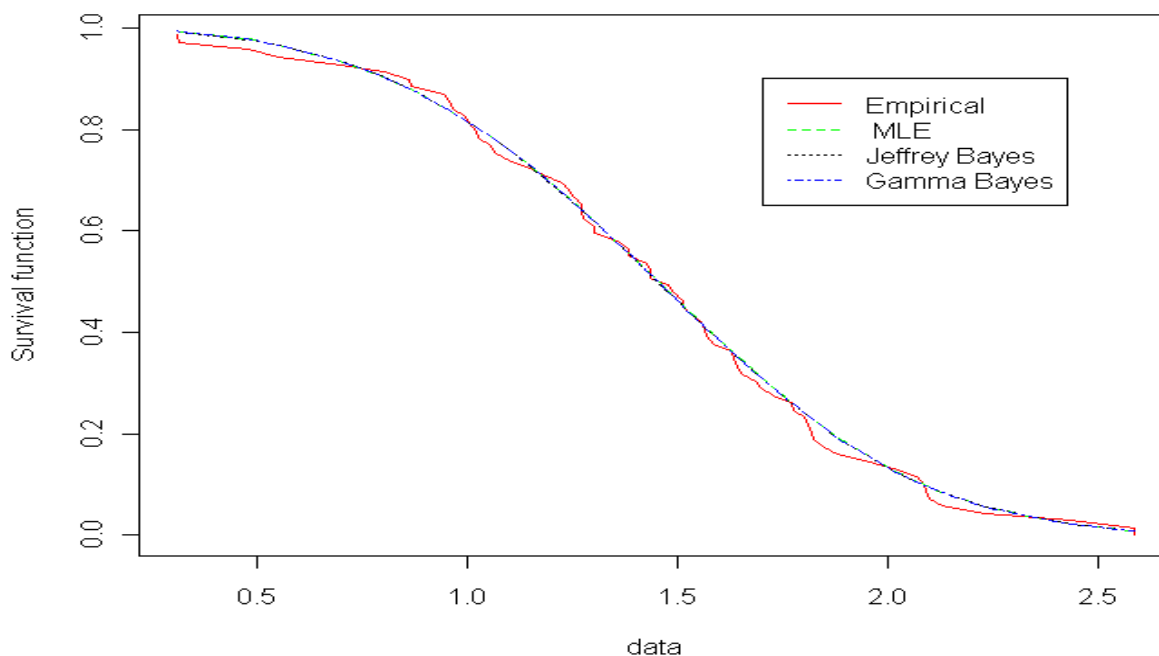
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