Sohag Journal of Mathematics
An International Journal

http://dx.doi.org/10.12785/sjm/010101

Coupled Fixed Point Theorems for Nonlinear Contractions in G-Metric Spaces

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Received: 09 Feb. 2014, Revised: 20 Mar. 2014, Accepted: 23 Mar. 2014
Published online: 1 Sep. 2014

Abstract: In this manuscript, we establish some coupled fixed point results for a nonlinear contraction using mixed monotone property in G-metric spaces. We also give some examples in support of our results.

Keywords: G-metric space, coupled fixed point, mixed monotone property

1 Introduction

Fixed point theory has a wide application in almost all fields of quantitative sciences such as biology, chemistry, comport science, economics, physics and many introduced branches of engineering. Banach contraction principle is one of the care subject that has been studied. It has so many different generalizations with different approaches.

φ-contraction, was given by Boyd and Wong [3] in 1969. Mustafa and Sims [1] introduced the notion of generalized metric spaces or G-metric spaces as a generalization of metric spaces in 2006. Based on the concept of G-metric spaces, Mustafa et al [4,5,6] proved several fixed point theorems for mappings satisfying different contractive conditions. In 2006, Gnana-Bhaskar and Lakshmikantham [2] introduced the notion of coupled fixed point and proved some fixed point theorems under certain conditions. Many authors focused on coupled fixed point theory and proved remarkable results (see e.g. [7,8,9,10,11]).

The aim of this paper is to show that most of coupled fixed point theorems in metric spaces can be easily obtained in G-metric spaces from well known fixed point theorems in the literature.

2 Preliminaries

Here we recall some basic definitions. Throughout this paper, N^* is the set of non-negative integers.

Definition 1 (see [1]). Let X be a nonempty set, and let $G : X \times X \times X \to R^+$, be a function satisfying:

(G1)$G(x,y,z) = 0$ if $x = y = z$
(G2)$0 < G(x,x,y)$, for all $x,y \in X$; with $x \neq y$,
(G3)$G(x,x,y) \leq G(x,y,z)$, for all $x,y,z \in X$ with $z \neq y$,
(G4)$G(x,y,z) = G(x,z,y) = G(y,z,x) = \cdots$, (symmetry in all three variables), and
(G5)$G(x,y,z) \leq G(x,a,a) + G(a,y,z)$, for all $x,y,z,a \in X$, (rectangle inequality),

then the function G is called a generalized metric, or, more specifically a G-metric on X, and the pair (X,G) is a G-metric space.

Clearly these properties are satisfied when $G(x,y,z)$ is the perimeter of the triangle with vertices at x, y and z in R^2, further taking a in the interior of the triangle shows that (G5) is best possible.

Definition 2 (see [1]). Let (X,G) be a G-metric space. The sequence $\{x_n\} \in X$ is G-convergent to x if it converges to x in the G-metric topology, $\tau(G)$.

Definition 3 (see [2]). Let $(X,\leq)$ be a partially ordered set and $F : X \times X \to X$ be a mapping. F is said to have the mixed monotone property if $F(x,y)$ is monotone non-decreasing in x and is monotone non-increasing in y, that is, for any $x,y \in X$,

$$x_1 \leq x_2 \Rightarrow F(x_1,y) \leq F(x_2,y), \quad \text{for } x_1,x_2 \in X \text{ and}$$
$$y_1 \leq y_2 \Rightarrow F(x,y_2) \leq F(x,y_1), \quad \text{for } y_1,y_2 \in X.$$

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Definition 4 (see [2]). An element $(x,y) \in X \times X$ is said to be a coupled fixed point of the mapping $F : X \times X \to X$ if

$$F(x,y) = x \quad \text{and} \quad F(y,x) = y.$$

Throughout this paper, $(X,\le)$ will denote a partially ordered set and G will be a G-metric on X such that (X,G) is a complete metric space. Further, the product space $X \times X$ satisfies the following:

$$(u,v) \le (x,y) \Leftrightarrow u \le x, \quad y \le v; \qquad \text{for all } (x,y),(u,v) \in X \times X. \tag{1}$$

Theorem 5. Let $(X,\le)$ be a partially ordered set and (X,G) be a complete G-metric space.

Let $F : X \times X \to X$ be a mapping having the mixed monotone property on X

Assume that for all $x \ge u$, $y \le v$

$$\begin{aligned}
&\psi(G(F(x,y),F(u,v),F(u,v))\\
&\le \frac{1}{2}\psi(G(x,u,u)+G(y,v,v))\\
&\quad -\varphi(G(x,u,u)+G(y,v,v))
\end{aligned} \tag{2}$$

where $\varphi \in \Phi$ and $\psi \in \Psi$. Suppose that there exist $x_0, y_0 \in X$ such that

$$x_0 \le F(x_0,y_0), \quad y_0 \le F(y_0,x_0).$$

Suppose that either

(a)F is continuous, or
(b)X has the following property:
(i)if non-decreasing sequence x_n tends to x, then $x_n \le x$ for all n,
(ii)if non-increasing sequence y_n tends to y, then $y_n \ge y$ for all n,
then there exist $x, y \in X$ such that

$$F(x,y) = x, \quad F(y,x) = y$$

Proof. Let $x_0, y_0 \in X$ be such that

$$x_0 \le F(x_0,y_0), \quad y_0 \ge F(y_0,x_0)$$

We construct the sequences $\{x_n\}$ and $\{y_n\}$ as follows:

$$x_n = F(x_{n-1},y_{n-1}), \; y_n = F(y_{n-1},x_{n-1}), \qquad \text{for } n = 1,2,3,\dots \tag{3}$$

By mixed monotone property of F, we can easily show that

$$\begin{aligned}
&x_0 \le x_1 \le \cdots \le x_{n-1} \le x_n \le \cdots\\
&y_0 \ge y_1 \ge \cdots \ge y_{n-1} \ge y_n \ge \cdots
\end{aligned} \tag{4}$$

From (2), (3) and (4), we have

$$\begin{aligned}
&\psi(G(x_{n+1},x_{n+2},x_{n+2}))\\
&= \psi(G(F(x_n,y_n),F(x_{n+1},y_{n+1}),F(x_{n+1},y_{n+1}))\\
&\le \frac{1}{2}(G(x_n,x_{n+1},x_{n+1})+G(y_n,y_{n+1},y_{n+1}))\\
&\quad -\varphi(G(x_n,x_{n+1},x_{n+1})+G(y_n,y_{n+1},y_{n+1}))
\end{aligned} \tag{5a}$$

$$\begin{aligned}
&\psi(G(y_{n+1},y_{n+2},y_{n+2}))\\
&= \psi(G(F(y_n,x_n),F(y_{n+1},x_{n+1}),F(y_{n+1},x_{n+1}))\\
&\le \frac{1}{2}(G(y_n,y_{n+1},y_{n+1})+G(x_n,x_{n+1},x_{n+1}))\\
&\quad -\varphi(G(y_n,y_{n+1},y_{n+1})+G(x_n,x_{n+1},x_{n+1}))
\end{aligned} \tag{5b}$$

Now equations (5a) and (5b), gives

$$\begin{aligned}
&\psi(G(x_{n+1},x_{n+2},x_{n+2}))+\psi(G(y_{n+1},y_{n+2},y_{n+2}))\\
&\quad \le \frac{1}{2}\psi(G(x_n,x_{n+1},x_{n+1})+G(y_n,y_{n+1},y_{n+1}))\\
&\quad\quad -2\varphi(G(x_n,x_{n+1},x_{n+1})+G(y_n,y_{n+1},y_{n+1}))
\end{aligned} \tag{6}$$

Due to the property of ψ, we have

$$\begin{aligned}
&\psi(G(x_{n+1},x_{n+2},x_{n+2})+G(y_{n+1},y_{n+2},y_{n+2}))\\
&\quad \le \psi(G(x_{n+1},x_{n+2},x_{n+2}))+\psi(G(y_{n+1},y_{n+2},y_{n+2}))
\end{aligned} \tag{7}$$

Combining (6) and (7), we get

$$\begin{aligned}
&\psi(G(x_{n+1},x_{n+2},x_{n+2})+G(y_{n+1},y_{n+2},y_{n+2}))\\
&\le \psi(G(x_n,x_{n+1},x_{n+1})+G(y_n,y_{n+1},y_{n+1}))\\
&\quad -2\varphi(G(x_n,x_{n+1},x_{n+1})+G(y_n,y_{n+1},y_{n+1}))
\end{aligned}$$

Suppose $\delta_n = [G(x_n,x_{n-1},x_{n-1})+G(y_n,y_{n-1},y_{n-1})]$. Then we get

$$\psi(\delta_{n+2}) \le \psi(\delta_{n+1}) - 2\varphi(\delta_{n+1}) \quad \text{for all } n, \tag{8}$$

which becomes

$$\psi(\delta_{n+2}) \le \psi(\delta_{n+1}) \quad \text{for all } n.$$

Since ψ is non-decreasing, we get that $\delta_{n+2} \ge \delta_{n+1}$ for all n. Hence $\{\delta_n\}$ is a non-increasing sequence. Since it is bounded below, there is some $\delta \ge 0$ such that

$$\lim_{n\to\infty} \delta_n = \delta.$$

We shall prove that $\delta = 0$. Suppose, on the contrary, that $\delta > 0$.

Letting $n \to \infty$ in (8) and having in mind that we suppose $\lim_{t\to r}\varphi(t) > 0$ for all $r > 0$ and $\lim_{t\to 0^+}\varphi(t) = 0$, we have

$$\delta \le \delta - 2\varphi(\delta) < \delta$$

which is a contradiction. Thus, $\delta = 0$, that is,

$$\lim_{n\to\infty} \delta_n = \lim_{n\to\infty}[G(x_n,x_{n-1},x_{n-1})+G(y_n,y_{n-1},y_{n-1})] = 0 \tag{9}$$

Now, we shall show that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences. Suppose on the contrary that at least one of $\{x_n\}$ and $\{y_n\}$ is not Cauchy. So, there exists $\varepsilon > 0$ for which we can find subsequences $\{x_{n(k)}\}$, $\{x_{m(k)}\}$ of $\{x_n\}$ and $\{y_{n(k)}\}$, $\{y_{m(k)}\}$ of $\{y_n\}$ with $n(k) > m(k) = k$ such that

$$G(x_{n(k)}, x_{m(k)}, x_{m(k)}) + G(y_{n(k)}, y_{m(k)}, y_{m(k)})) = \varepsilon. \quad (10)$$

Additionally, corresponding to $m(k)$, we may select $n(k)$ such that it is the smallest integer satisfying (10) and $n(k) > m(k) = k$. So, we get

$$G(x_{n(k)-1}, x_{m(k)}, x_{m(k)}) + G(y_{n(k)-1}, y_{m(k)}, y_{m(k)})) \\ < \varepsilon. \quad (11)$$

By using the triangle inequality and having (10) and (11) in mind we obtain

$$\begin{aligned}
\varepsilon \le t_k \\
&= G(x_{n(k)}, x_{m(k)}, x_{m(k)}) + G(y_{n(k)}, y_{m(k)}, x_{m(k)})) \\
&\le G(x_{n(k)}, x_{n(k)-1}, x_{n(k)-1}) \\
&\quad + G(x_{n(k)-1}, x_{m(k)}, x_{m(k)}) \\
&\quad + G(y_{n(k)}, y_{n(k)-1}, y_{n(k)-1}) \\
&\quad + G(y_{n(k)-1}, y_{m(k)}, y_{m(k)})) \\
&< G(x_{n(k)}, x_{n(k)-1}, x_{n(k)-1}) \\
&\quad + G(y_{n(k)}, y_{n(k)-1}, y_{n(k)-1}) + \varepsilon. \quad (12)
\end{aligned}$$

Supposing $k \to \infty$ in (12) and using (9) we get

$$\lim_{n\to\infty} = \lim_{n\to\infty} [G(x_n, x_{n-1}, x_{n-1}) + G(y_n, y_{n-1}, y_{n-1})] = \varepsilon.$$

Again by the triangle inequality

$$\begin{aligned}
t_k &= G(x_{n(k)}, x_{m(k)}, x_{m(k)}) + G(y_{n(k)}, y_{m(k)}, y_{m(k)}) \\
&\le G(x_{n(k)}, x_{n(k)+1}, x_{n(k)+1}) \\
&\quad + G(x_{n(k)+1}, x_{m(k)+1}, x_{m(k)+1}) \\
&\quad + G(x_{m(k)+1}, x_{m(k)}, x_{m(k)}) \\
&\quad + G(y_{n(k)}, y_{n(k)+1}, y_{n(k)+1}) \\
&\quad + G(y_{n(k)+1}, y_{m(k)+1}, y_{m(k)+1}) \\
&\quad + G(y_{m(k)+1}, y_{m(k)}, y_{m(k)}) \\
&\le \delta_{n(k)+1} + \delta_{m(k)+1} \\
&\quad + G(x_{n(k)+1}, x_{m(k)+1}, x_{m(k)+1}) \\
&\quad + G(y_{n(k)+1}, y_{m(k)+1}, y_{m(k)+1}). \quad (13)
\end{aligned}$$

Since $n(k) > m(k)$, then

$$x_{n(k)} \ge x_{m(k)} \quad \text{and} \quad y_{n(k)} \le y_{m(k)}. \quad (14)$$

Hence by (2), (3) and (14), we get,

$$\begin{aligned}
&\psi(G(x_{n(k)+1}, x_{m(k)+1}, x_{m(k)+1})) \\
&= \psi(G(F(x_{n(k)}, y_{n(k)}), F(x_{m(k)}, y_{m(k)}, F(x_{m(k)}, y_{m(k)}) \\
&\le \frac{1}{2}\psi(G(x_{n(k)}, x_{m(k)}, x_{m(k)}) + G(y_{n(k)}, y_{m(k)}, y_{m(k)})) \\
&\quad - \varphi(G(x_{n(k)}, x_{m(k)}, x_{m(k)}) + G(y_{n(k)}, y_{m(k)}, y_{m(k)})) \quad (15a)
\end{aligned}$$

$$\begin{aligned}
&\psi(G(y_{n(k)+1}, y_{m(k)+1}, y_{m(k)+1})) \\
&= \psi G(F(y_{n(k)}, x_{n(k)}), F(y_{m(k)}, x_{m(k)}), F(y_{m(k)}, x_{m(k)}) \\
&\le \frac{1}{2}\psi(G(y_{n(k)}, y_{m(k)}, y_{m(k)}) + G(x_{n(k)}, x_{m(k)}, x_{m(k)})) \\
&\quad - \varphi(G(y_{n(k)}, y_{m(k)}, y_{m(k)}) + G(x_{n(k)}, x_{m(k)}, x_{m(k)})) \quad (15b)
\end{aligned}$$

Combining (13) with (15a) and (15b), we obtain

$$\begin{aligned}
\psi(t_k) &\le \psi(\delta_{n(k)+1} + \delta_{m(k)+1} + G(x_{n(k)+1}, x_{m(k)+1}, x_{m(k)+1}) \\
&\quad + G(y_{n(k)+1}, y_{m(k)+1}, y_{m(k)+1})) \\
&\le \frac{1}{2}\psi(\delta_{n(k)+1} + \delta_{m(k)+1}) + \psi(t_k) - 2\varphi(t_k).
\end{aligned}$$

Letting $k \to \infty$, we get a contradiction. This shows that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences.

Since X is a complete metric space, there exist $x, y \in X$ such that

$$\lim_{n\to\infty} x_n = n \quad \text{and} \quad \lim_{n\to\infty} y_n = y \quad (16)$$

suppose that the condition (a) holds. Then from (3) and (16), we have

$$\begin{aligned}
x &= \lim_{n\to\infty} x_n \\
&= \lim_{n\to\infty} F(x_{n-1}, y_{n-1}) \\
&= F(\lim_{n\to\infty} x_{n-1}, \lim_{n\to\infty} y_{n-1}) \\
&= F(x, y).
\end{aligned}$$

Analogously, we also observe that

$$y = \lim_{n\to\infty} y_n = \lim_{n\to\infty} F(y_{n-1}, x_{n-1}) = F(x, y).$$

Thus, we have

$$F(x, y) = x, \quad F(y, x) = y.$$

Let us assume that the assumption (b) holds. Since $\{x_n\}$ is non-decreasing and x_n tends to x and also $\{y_n\}$ is non-increasing and y_n tends to y, we have $x_n \le x$, $y_n \ge y$, for all n, by the condition (b).

Consider

$$\begin{aligned}
&G(x, F(x, y), F(x, y)) \\
&\le G(x, x_{n+1}, x_{n+1}) + G(x_{n+1}, F(x, y), F(x, y)) \\
&= G(x, x_{n+1}, x_{n+1}) + G(F(x_n, y_n), F(x, y), F(x, y)) \\
&\le G(x, x_{n+1}, x_{n+1}) + \frac{1}{2}\psi(G(x_n, x, x) + G(y_n, y, y)) \\
&\quad - \varphi(G(x_n, x, x) + G(y_n, y, y)). \quad (17)
\end{aligned}$$

As n goes to ∞ in (17) and using (16), we get that $G(x,F(x,y),F(x,y))=0$.

Thus, $x=F(x,y)$. Analogously, we get that

$$F(y,x)=y.$$

Thus, we proved that F has a coupled fixed point.

Corollary 6. Let $(X,\leq)$ be a partially ordered set and (X,G) be a complete G-metric space. Let $F:X\times X\to X$ be a mapping having the mixed monotone property on X. Suppose that there exists $k\in[0,1)$ such that

$$G(F(x,y),F(u,v),F(u,v))\leq\frac{k}{2}[G(x,u,u)+G(y,v,v)]$$

for all $x\geq u$, and $y\leq v$. Suppose that there exist $x_0,y_0\in X$ such that

$$x_0\leq F(x_0,y_0),\quad y_0\geq F(y_0,x_0).$$

Suppose that either

(a)F is continuous, or
(b)X has the following property:
(i)if non-decreasing sequence $x_n\to x$, then $x_n\leq x$ for all n,
(ii)if non-increasing sequence $y_n\to y$, then $y_n\geq y$ for all n,
then there exist $x,y\in X$ such that

$$F(x,y)=x,\quad F(y,x)=y.$$

Proof. It is sufficient to take $\psi(t)=t$ and $\varphi(t)=\dfrac{k-t}{2}$ in the above theorem.

3 Uniqueness of coupled fixed point

In this section we shall prove the uniqueness of the coupled fixed point. For a product $X\times X$ of a partial ordered set $(X,\leq)$ we define a partial ordering in the following way:

For all $(x,y),(u,v)\in X\times X$

$$(x,y)\leq(u,v)\quad\Leftrightarrow\quad x\leq u,\ y\geq v.$$

We say that (x,y) is equal to (u,v) if and only if $x=u$ and $y=v$.

Theorem 7. In addition to the hypothesis of Theorem 5, suppose that for all $(x,y),(u,v)\in X\times X$, there exists $(a,b)\in X\times X$ that is comparable to (x,y) and (u,v), then F has a unique coupled fixed point.

Proof. The set of coupled fixed points of F is not empty due to Theorem 5. Assume that (x,y) and (u,v) are coupled fixed points of F, that is,

$$F(x,y)=x,\quad F(u,v)=u,$$
$$F(y,x)=y,\quad F(v,u)=v.$$

We shall show that (x,y) and (u,v) are equal. By the assumption of the theorem, there exists $(a,b)\in X\times X$ that is comparable to (x,y) and (u,v). Define sequences $\{a_n\}$ and $\{b_n\}$ such that

$$a=a_0,\quad b=b_0$$

and

$$a_n=F(a_{n-1},b_{n-1}),$$
$$b_n=F(b_{n-1},a_{n-1})$$

for all n. Since (x,y) is comparable with (a,b), we may assume that $(x,y)=(a,b)=(a_0,b_0)$.

Recursively, we get that

$$(x,y)=(a_n,b_n)\quad\text{for all } n. \tag{18}$$

By (2) and (18), we have

$$\begin{aligned}&\psi(G(x,a_{n+1},a_{n+1}))\\&=\psi(G(F(x,y),F(a_n,b_n),F(a_n,b_n))\\&\leq\frac{1}{2}\psi(G(x,a_n,a_n)+G(y,b_n,b_n))\\&\quad-\varphi(G(x,a_n,a_n)+G(y,b_n,b_n)),\end{aligned}\tag{19a}$$

$$\begin{aligned}&\psi(G(y,b_{n+1},b_{n+1}))\\&=\psi(G(F(y,x),F(y,x),F(b_n,a_n))\\&\leq\frac{1}{2}\psi(G(y,b_n,b_n)+G(x,a_n,a_n))\\&\quad-\varphi(G(y,b_n,b_n)+G(x,a_n,a_n))\end{aligned}\tag{19b}$$

Set $\gamma_n=G(x,a_n,a_n)+G(y,b_n,b_n)$. Then, from (19a) and (19b), we have

$$\psi(\gamma_{n+1})=\psi(\gamma_n)-2\varphi(\gamma_n)\quad\text{for all } n,$$

which implies

$$\gamma_{n+1}\leq\gamma_n.$$

Hence, the sequence $\{\gamma_n\}$ is decreasing and bounded below. Thus, there exists $\gamma\geq0$ such that

$$\lim_{n\to\infty}\gamma_n=\gamma.$$

Now, we shall show that $\gamma=0$. Suppose to the contrary that $\gamma>0$.

Letting $n\to\infty$ in

$$\psi(\gamma_{n+1})\leq\psi(\gamma_n)-2\varphi(\gamma_n),$$

we get that

$$\psi(\gamma)\leq\psi(\gamma)-\lim\varphi(\gamma_n)<\psi(\gamma)$$

which is a contradiction. Therefore, $\gamma=0$. That is,

$$\lim_{n\to\infty}\gamma_n=0.$$

Consequently, we have

$$\lim_{n\to\infty} G(x,a_n,a_n)=0 \text{ and } \lim_{n\to\infty} G(y,b_nb_n)=0 \tag{20}$$

Similarly, we show that

$$\lim_{n\to\infty} G(u,a_n,a_n)=0 \text{ and } \lim_{n\to\infty} G(v,b_nb_n)=0 \tag{21}$$

Combining (20) and (21) gives that (x,y) and (u,v) are equal.

4 Examples

We give some examples to show that our results are effective.

Example 8. Let $X=[0,\infty)$ with the metric $G(x,y,z)=|x-y-z|$, for all $x,y,z\in X$ and the following order relation:

$$x,y\in X,\quad x\preccurlyeq y \Leftrightarrow x=y \text{ or } (x,y\in Z \text{ and } x=y),$$

where Z is the set of integers and $\leq$ is the usual ordering.

Let $F: X\times X\to X$ be given by

$$F(x,y)=\begin{cases}\frac{1}{2}, & \text{if } xy\neq 0\\ 0, & \text{if } xy=0\end{cases}$$

Let $\psi,\varphi:[0,\infty)\to[0,\infty)$ be given by

$$\psi(t)=2t,\ \text{ and }\ \varphi(t)=\frac{t}{5}\ \text{ for all } t\in[0,\infty).$$

It is easy to check that all the conditions of Theorem 5 are satisfied.

By applying Theorem 5 we conclude that F has a coupled fixed point.

In fact, F has two coupled fixed points. They are $(0,0)$ and $(\frac{1}{2},\frac{1}{2})$. Therefore, the conditions of Theorem 5 are not sufficient for the uniqueness of a coupled fixed point.

Acknowledgements

The Authors are very thankful to the referees for their useful suggestions in this paper.

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