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Parameter Estimation in Marshall-Olkin Exponential Distribution Under Type-I Hybrid Censoring Scheme

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Abstract: The two most popular censoring schemes used in life testing experiments are Type-I and Type-II censoring schemes. Hybrid censoring scheme is the mixture of Type-I and Type-II censoring scheme. In this article, we consider the estimation of parameters of Marshall-Olkin exponential distribution based on Type-I Hybrid censored data. Both classical and Bayesian methodology have been discussed to estimate the model parameters. In classical set-up maximum likelihood estimators (MLEs) of the parameters have been obtained by using Newton-Raphson method and also by using Fisher information matrix 95% asymptotic confidence intervals are provided. In Bayesian set-up Lindley's approximation technique and Markov Chain Monte Carlo (MCMC) technique have been used to compute the Bayes estimators. Further, we have also provided highest posterior density (HPD) intervals of the parameters based on MCMC samples. To compare the performances of the estimators Monte Carlo simulation has performed and one data set is analysed for illustrative purpose of the study.

Keywords: Hybrid Censoring, Maximum likelihood estimator, Bayes estimator, Lindley's approximation, Markov Chain Monte Carlo Technique.

1 INTRODUCTION

In Reliability/engineering or medical sciences, generally observations are not completely known due to time and cost or inherent structure of the situations. Due to this cause censoring of the data can take place naturally. Therefore, the various censoring scheme can be classified by the experimenter depending upon the data obtaining processes. The two most common censoring schemes in life testing experiments are Type-I and Type-II censoring schemes and both censoring scheme have their own advantages and disadvantages. The Type-I and Type-II censoring schemes, however, do not allow removing surviving items at the times other than the termination time of the life test. This allowance, however, may be desirable when a compromise between reducing test time and an expectation of some extreme lifetimes in life test can be sought. Type-I censoring scheme controls the duration of the life test and the efficiency of the test may be too low due to number of failure items, whereas Type-II censoring scheme controls the efficiency of the test but time of the test is uncertain. The mixture of Type-I and Type-II censoring schemes, named as hybrid censoring scheme and this censoring schemes have been widely discussed in the literature see Epstin [1]. The hybrid censoring scheme is of two types namely Type-I hybrid and Type-II hybrid censoring scheme. In Type-I hybrid censoring, the test is terminated at a time $T_1 = min(X_{k:n}, T)$, where $X_{k:n}$ represents the failure time of the k^{th} item and T is the prefixed maximum allowable time of the test. In hybrid Type-II censoring, the test is terminated at a time $T_1 = max(X_{k:n}, T)$. It is clear that the test have at least k failure items in Type-II hybrid censoring scheme where as in Type-I hybrid censoring scheme, the test can never be reached beyond the time T. Type-I hybrid censoring scheme is quite useful in reliability acceptance plan in MIL-STD-781C [10]. Epstein [1] proposed two-sided confidence intervals for the parameter without any formal proof. Fairbanks et al. [2] modified proposition of Epstein [1] and suggested a simple methodology to obtain confidence intervals. Chen and Bhattacharya [3] have discussed the classical estimation for the parameters. In Bayesian framework Drapper and Guttmann [4] used this censoring scheme and proposed the Bayes estimation procedure using the gamma prior. Several authors have discussed about the hybrid censoring schemes, see, Gupta and Kundu [9] Ebrahimi [5],

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Kundu [6], Kundu and Pradhan [7], Gupta and Singh [16], Balakrishnan N, Kundu D [22], S. Dey, B. Pradhan [27], Kundu et al.[22], B. Al-zahrani, M. Gindwan [25], Maheshwari et al.[26] and M. K. Rastogi, Y. M. Tripathi [27]. Under Type-I hybrid censoring scheme, we have one of the following two types of censored data;
Case I: $\{X_{1:n} < X_{2:n} < \cdots < X_{k:n}\}$ if $X_{k:n} < T$
Case II: $\{X_{1:n} < X_{2:n} < \cdots < X_{m:n}\}$ if $m \leq r, X_{m+1:n} > T$

In this paper, we have considered the Type-I hybrid censored lifetime data when lifetime of each experimental unit follows Marshall-Olkin exponential distribution (MOED). The MOED was originally proposed by Marshall and Olkin in 1970 see [**?**]. This distribution is the generalization the exponential distribution. Firstly, G. Srinivasa Rao et al. have used this distribution for making reliability test plan with sampling point of view see [12]. Since this distribution contain shape and scale parameters and it has various shape of hazard rate for different values of shape parameter (α). Therefore, sometimes it seems to be a good alternative to the gamma distribution, Weibull distribution and other exponentiated family of distributions. It has increasing (decreasing) hazard function when $\alpha > 1 (\alpha < 1)$ respectively and has constant hazard rate for $\alpha = 1$. For other details about this distribution, we refer A. W. Marshall and I. Olkin [11] . The cumulative distribution function (c.d.f.) and probability density function (p.d.f) of MOED are given as;

$$F(x,\alpha,\lambda) = \frac{1-e^{-\lambda x}}{1-\bar{\alpha}e^{-\lambda x}} \quad ; x \geq 0,\ \alpha,\lambda > 0 \tag{1}$$

$$f(x,\alpha,\lambda) = \frac{\alpha\lambda e^{-\lambda x}}{\left(1-\bar{\alpha}e^{-\lambda x}\right)^2} \quad ; x \geq 0,\ \alpha,\lambda > 0 \tag{2}$$

where α is the shape parameter and λ is the scale parameter of the distribution and $\bar{\alpha} = 1-\alpha$.

The aim of this paper is to propose the classical and Bayesian estimation procedures for the unknown parameters under Type-I hybrid censoring scheme. It is observed that the maximum likelihood estimators can not be obtained directly, therefore, iterative procedure like Newton-Raphson method has been used to solve the non-linear equations. To obtain the Bayes estimators of the parameters using independent gamma prior for both shape (α) and scale (λ) parameters, it is observed that, the Bayes estimators can not be obtained in closed form. Therefore, we used the Lindely's approximation method to obtain the estimators of the parameters. Furthermore, we have also constructed 95% asymptotic intervals based on MLEs but we are unable to construct 95% highest posterior density (HPD) intervals using Lindely's method. The other important approximation like Markov Chain Monte Carlo (MCMC) is used to compute the Bayes estimators and corresponding highest posterior density (HPD) credible intervals of the parameters. We have compared the performances of the classical estimators with corresponding Bayes estimators by Monte Carlo simulations and proposed procedure is illustrated by one real data set .

The rest of the paper is organized as follows; In section 1, we described the model and the available data. The maximum likelihood estimators and asymptotic confidence intervals are provided in sections 2. Bayes estimators are discussed in subsection of 2. Section 3 provided the illustration of the proposed procedure by using real data set. Simulation results are presented in section 4. Finally, conclusions are given in section 5.

2 ESTIMATION OF THE PARAMETERS

2.1 Classical Inferences for the parameters

In this subsection, we have considered the maximum likelihood estimation of the parameters. Let us suppose that, $X_{1:n}, X_{2:n}, \cdots, X_{n:n}$ are n independent ordered lifetime failure observations in presence of Type-I hybrid censored samples from $MOED(\alpha,\lambda)$. Therefore, in this case the likelihood function for the considered cases are;
Case I:

$$L(\alpha,\lambda) = \frac{n!}{(n-k)!}\alpha^n\lambda^k \left\{ (1-\bar{\alpha}e^{-\lambda x_{k:n}})^{-(n-k)} e^{-\lambda\left[\sum_{i=1}^{k} x_i + (n-k)x_{k:n}\right]} \right\} \prod_{i=1}^{k}(1-\bar{\alpha}e^{-\lambda x_i})^{-2} \tag{3}$$

Case II:

$$L(\alpha,\lambda) = \frac{n!}{(n-m)!}\alpha^n\lambda^m \left\{ (1-\bar{\alpha}e^{-\lambda T})^{-(n-m)} e^{-\lambda\left[\sum_{i=1}^{m} x_i + (n-m)T\right]} \right\} \prod_{i=1}^{m}(1-\bar{\alpha}e^{-\lambda x_i})^{-2} \tag{4}$$

Thus combined likelihood can be written as;

$$L(\alpha,\lambda) = \frac{n!}{(n-r)!}\alpha^n\lambda^r \left\{ (1-\bar{\alpha}e^{-\lambda t})^{-(n-r)} e^{-\lambda\left[\sum\limits_{i=1}^{r} x_i + (n-r)t\right]} \right\} \prod_{i=1}^{r}(1-\bar{\alpha}e^{-\lambda x_i})^{-2} \tag{5}$$

where, r and t are defined as,

$$r = \begin{cases} k & for\ caseI \\ m & for\ caseII \end{cases}$$

and

$$t = \begin{cases} x_{k:n} & for\ caseI \\ T & for\ caseII \end{cases}$$

Now, the Log likelihood function can be expressed as;

$$\ln L(x|\alpha,\lambda) = n\ln\alpha + r\ln\lambda - \lambda\left[\sum_{i=1}^{r} x_i + (n-r)t\right] - 2\sum_{i=1}^{r}\ln(1-\bar{\alpha}e^{-\lambda x})$$

$$-(n-r)\ln(1-\bar{\alpha}e^{-\lambda t}) \tag{5.1}$$

Therefore, the MLEs of the parameter α and λ is the simultaneous solution of the following normal equations. But we observed that these expressions are not in closed form. Therefore, MLEs can be secured through iterative procedure. Here we suggest to use Newton Raphson (N-R) method.

$$\frac{n}{\alpha} - 2\sum_{i=1}^{r}\frac{e^{-\lambda x_i}}{(1-\bar{\alpha}e^{-\lambda x_i})} - \frac{(n-r)e^{-\lambda t}}{(1-\bar{\alpha}e^{-\lambda t})} = 0$$

and

$$\frac{r}{\lambda} - \left[\sum_{i=1}^{r} x_i + (n-r)t\right] - 2\sum_{i=1}^{r}\frac{\bar{\alpha}x_i e^{-\lambda x_i}}{(1-\bar{\alpha}e^{-\lambda x_i})} - \frac{(n-r)\bar{\alpha}te^{-\lambda t}}{(1-\bar{\alpha}e^{-\lambda t})} = 0$$

2.1.1 Interval Estimation

In this subsection, we will find the Fisher information matrix for constructing 95% asymptotic confidence interval for the parameters based on limiting s-normal distribution. The Fisher information matrix can be obtained by using equation (5.1). Thus we have

$$I(\hat{\alpha},\hat{\lambda}) = \begin{pmatrix} -\dfrac{\partial^2 \ln L}{\partial\alpha^2} & -\dfrac{\partial^2 \ln L}{\partial\alpha\partial\lambda} \\ -\dfrac{\partial^2 \ln L}{\partial\lambda\partial\alpha} & -\dfrac{\partial^2 \ln l}{\partial\lambda^2} \end{pmatrix}_{(\hat{\alpha},\hat{\lambda})} \tag{2.1.1}$$

where, all the corresponding derivatives of the matrix are presented in subsection of 2.2. The above matrix can be inverted to obtain the estimate of the asymptotic variance-covariance matrix of the MLEs and diagonal elements of $I^{-1}(\hat{\alpha},\hat{\lambda})$ provides asymptotic variance of α and λ respectively. Then by using large sample theory the two sided $100(1-\beta)\%$ approximate confidence interval for α can be constructed as

$$\hat{\alpha} \pm Z_{1-\beta/2}\sqrt{var(\hat{\alpha})}$$

and similarly, for λ the two sided $100(1-\beta)\%$ approximate confidence interval can be obtained as

$$\hat{\lambda} \pm Z_{1-\beta/2}\sqrt{var(\hat{\lambda})}$$

2.2 Bayesian Inferences for the parameters

In this section, we have obtained the Bayes estimators of the parameters α and λ based on Type-I hybrid censored data. In Bayesian analysis, we need to specify prior distribution for the parameters, therefore we consider two independent gamma prior such as, $gamma(a,b)$ as a prior of α and $gamma(c,d)$ considered as a prior of λ where a,b,c and d are the hyper-parameters and is non negative. The motivation of considering these prior is that, it is flexible in nature and mathematical ease. The important non-informative prior is Jeffreys prior. It is to be mentioned here that the above considered prior reduces in to non-informative prior by taking the values of hyper-parameters are zero.
Therefore, the joint prior for (α,λ) may be taken as ;

$$\pi(\alpha,\lambda) \propto \alpha^{a-1}\lambda^{c-1}e^{(-b\alpha-d\lambda)} \ ; \ a,b,c \ and \ d \geq 0 \tag{2.2.1}$$

Then by using equation (5) and (2.2.1) the joint posterior is given as

$$p(\alpha,\lambda|\underline{\mathrm{x}}) = K^{-1}\alpha^{n+a-1}\lambda^{r+c-1}e^{-[b\alpha+\lambda(T0+d)]}Q(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda \tag{2.2.2}$$

Thus, the Bayes estimators of α and λ under SELF are,

$$\hat{\alpha}_B = E(\alpha|\underline{\mathrm{x}},\lambda) = K^{-1}\int_{\alpha}\int_{\lambda}\alpha^{n+a}\lambda^{r+c-1}e^{-[b\alpha+\lambda(T0+d)]}Q(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda \tag{2.2.3}$$

$$\hat{\lambda}_B = E(\lambda|\underline{\mathrm{x}},\alpha) = K^{-1}\int_{\alpha}\int_{\lambda}\alpha^{n+a-1}\lambda^{r+c}e^{-[b\alpha+\lambda(T0+d)]}Q(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda \tag{2.2.4}$$

and where; $K, T0, Q(\alpha,\lambda)$ and $P(\alpha,\lambda)$ are interpreted as

$$K = \int_{\alpha}\int_{\lambda}\alpha^{n+a-1}\lambda^{r+c-1}e^{-[b\alpha+\lambda(T0+d)]}Q(\alpha,\lambda)P(\alpha,\lambda)d\alpha d\lambda$$

$$T0 = \sum_{i=1}^{r}x_i + (n-r)t, Q(\alpha,\lambda) = (1-\bar{\alpha}e^{-\lambda t})^{-(n-r)}$$

and

$$P(\alpha,\lambda) = \prod_{i=1}^{r}(1-\bar{\alpha}e^{-\lambda x_i})^{-2}$$

Thus the posterior distribution of (α,λ) takes a ratio of the two integrals. We may note that the above equation can not be reduced in a closed form and hence the evaluation of the posterior expectation for obtaining Bayes estimators of (α) and (λ) will be tedious. To overcome such difficulties, we propose to use Lindley's approximation and MCMC method to obtain Bayes estimators under squared error loss function.

Lindely's Approximation Method (Bayes 1) :

We consider the Lindley's approximation technique for the estimation of the (α) and (λ) . Consider that the posterior expectation is expressible in the form of ratio of integral as given below:

$$I(x) = E(\alpha,\lambda|\underline{\mathrm{x}}) = \frac{\int u(\alpha,\lambda)e^{L(\alpha,\lambda)+G(\alpha,\lambda)}d(\alpha,\lambda)}{\int e^{L(\alpha,\lambda)+G(\alpha,\lambda)}d(\alpha,\lambda)} \tag{6}$$

where,
$u(\alpha,\lambda)$= is a function of α and λ only
$L(\alpha,\lambda)$= Log- likelihood function
$G(\alpha,\lambda)$= Log of joint prior density
According to D. V. Lindley [13], if ML estimates of the parameters are available and n is sufficiently large then the above ratio of the integral can be approximated as:

$$\begin{aligned} I(x) = u(\hat{\alpha},\hat{\lambda}) + \frac{1}{2}[(\hat{u}_{\lambda\lambda}+2\hat{u}_{\lambda}\hat{p}_{\lambda})\hat{\sigma}_{\lambda\lambda} + (\hat{u}_{\alpha\lambda}+2\hat{u}_{\alpha}\hat{p}_{\lambda})\hat{\sigma}_{\alpha\lambda} + (\hat{u}_{\lambda\alpha}+2\hat{u}_{\lambda}\hat{p}_{\alpha})\hat{\sigma}_{\lambda\alpha} + (\hat{u}_{\alpha\alpha}+2\hat{u}_{\alpha}\hat{p}_{\alpha})\hat{\sigma}_{\alpha\alpha}] \\ + \frac{1}{2}[(\hat{u}_{\lambda}\hat{\sigma}_{\lambda\lambda}+\hat{u}_{\alpha}\hat{\sigma}_{\lambda\alpha})(\hat{L}_{\lambda\lambda\lambda}\hat{\sigma}_{\lambda\lambda}+\hat{L}_{\lambda\alpha\lambda}\hat{\sigma}_{\lambda\alpha}+\hat{L}_{\alpha\lambda\lambda}\hat{\sigma}_{\alpha\lambda}+\hat{L}_{\alpha\alpha\lambda}\hat{\sigma}_{\alpha\alpha}) + (\hat{u}_{\lambda}\hat{\sigma}_{\alpha\lambda}+\hat{u}_{\alpha}\hat{\sigma}_{\alpha\alpha}) \\ (\hat{L}_{\alpha\lambda\lambda}\hat{\sigma}_{\lambda\lambda}+\hat{L}_{\lambda\alpha\alpha}\hat{\sigma}_{\lambda\alpha}+\hat{L}_{\alpha\lambda\alpha}\hat{\sigma}_{\alpha\lambda}+\hat{L}_{\alpha\alpha\alpha}\hat{\sigma}_{\alpha\alpha})] \end{aligned} \tag{7}$$

where $\hat{\alpha}$ and $\hat{\lambda}$ are the MLE of α and λ respectively.

$$\hat{u}_\alpha = \frac{\partial u(\hat{\alpha},\hat{\lambda})}{\partial\hat{\alpha}}, \hat{u}_\lambda = \frac{\partial u(\hat{\alpha},\hat{\lambda})}{\partial\hat{\lambda}}, \hat{u}_{\alpha\lambda} = \frac{\partial u(\hat{\alpha},\hat{\lambda})}{\partial\hat{\alpha}\partial\hat{\lambda}}, \hat{u}_{\lambda\alpha} = \frac{\partial u(\hat{\alpha},\hat{\lambda})}{\partial\hat{\lambda}\partial\hat{\alpha}}, \hat{u}_{\alpha\alpha} = \frac{\partial^2 u(\hat{\alpha},\hat{\lambda})}{\partial\hat{\alpha}^2}, \hat{u}_{\lambda\lambda} = \frac{\partial^2, u(\hat{\alpha},\hat{\lambda})}{\partial\hat{\lambda}^2}$$

$$\hat{p}_\alpha = \frac{\partial G(\hat{\alpha},\hat{\lambda})}{\partial\hat{\alpha}}, \hat{p}_\lambda = \frac{\partial G(\hat{\alpha},\hat{\lambda})}{\partial\hat{\lambda}}, \hat{L}_{\alpha\alpha} = \frac{\partial^2 L(\hat{\alpha},\hat{\lambda})}{\partial\hat{\alpha}^2}, \hat{L}_{\lambda\lambda} = \frac{\partial^2 L(\hat{\alpha},\hat{\lambda})}{\partial\hat{\lambda}^2}, \hat{L}_{\alpha\alpha\alpha} = \frac{\partial^3 L(\hat{\alpha},\hat{\lambda})}{\partial\hat{\alpha}^3}, \hat{L}_{\alpha\alpha\lambda} = \frac{\partial^3 L(\hat{\alpha},\hat{\lambda})}{\partial\hat{\alpha}\partial\hat{\alpha}\partial\hat{\lambda}}$$

$$\hat{L}_{\lambda\lambda\alpha} = \frac{\partial^3 L(\hat{\alpha},\hat{\lambda})}{\partial\hat{\lambda}\partial\hat{\lambda}\partial\hat{\alpha}}, \hat{L}_{\lambda\alpha\lambda} = \frac{\partial^3 L(\hat{\alpha},\hat{\lambda})}{\partial\hat{\lambda}\partial\hat{\alpha}\partial\hat{\lambda}}, \hat{L}_{\alpha\alpha\lambda} = \frac{\partial^3 L(\hat{\alpha},\hat{\lambda})}{\partial\hat{\alpha}\partial\hat{\alpha}\partial\hat{\lambda}}, \hat{L}_{\alpha\lambda\lambda} = \frac{\partial^3 L(\hat{\alpha},\hat{\lambda})}{\partial\hat{\alpha}\partial\hat{\lambda}\partial\hat{\lambda}}, \hat{L}_{\lambda\alpha\alpha} = \frac{\partial^3 L(\hat{\alpha},\hat{\lambda})}{\partial\hat{\lambda}\partial\hat{\alpha}\partial\hat{\alpha}}$$

After substitution of $p(\alpha,\lambda|\underline{x})$ from (2.2.2) in above equation (6) then this integral must be reduces like Lindleys integral, where:

$u(\alpha,\lambda) = \alpha$

$L(\alpha,\lambda) = n\ln\alpha + r\ln\lambda - \lambda\left[\sum_{i=1}^{r} x_i + (n-r)t\right] - 2\sum_{i=1}^{r}\ln(1-\bar{\alpha}e^{-\lambda x_i}) - (n-r)\ln(1-\bar{\alpha}e^{-\lambda t})$

$G(\alpha,\lambda) = (a-1)\ln\alpha + (c-1)\ln\lambda - (b\alpha + d\lambda)$

it may verified that,

$u_\alpha = 1,\ u_{\alpha\alpha} = u_{\lambda\lambda} = u_{\alpha\lambda} = u_{\lambda\alpha} = 0,\ p_\alpha = \dfrac{a-1}{\alpha} - b,\ p_\lambda = \dfrac{c-1}{\lambda} - d$

$L_\alpha = \dfrac{n}{\alpha} - 2\sum_{i=1}^{r}\dfrac{e^{-\lambda x_i}}{(1-\bar{\alpha}e^{-\lambda x_i})} - \dfrac{(n-r)e^{-\lambda t}}{(1-\bar{\alpha}e^{-\lambda t})}$

$L_{\alpha\alpha} = \dfrac{-n}{\alpha^2} + 2\sum_{i=1}^{r}\dfrac{e^{-2\lambda x_i}}{(1-\bar{\alpha}e^{-\lambda x_i})^2} + \dfrac{(n-r)e^{-2\lambda t}}{(1-\bar{\alpha}e^{-\lambda t})^2},$

$L_{\alpha\lambda} = L_{\lambda\alpha} = \sum_{i=1}^{r}\dfrac{2x_i e^{-\lambda x_i}}{(1-\bar{\alpha}e^{-\lambda x_i})} + \sum_{i=1}^{r}\dfrac{2\bar{\alpha}x_i e^{-2\lambda x_i}}{(1-\bar{\alpha}e^{-\lambda x_i})^2} + \dfrac{(n-r)te^{-\lambda t}}{(1-\bar{\alpha}e^{-\lambda t})} + \dfrac{(n-r)\bar{\alpha}te^{-2\lambda t}}{(1-\bar{\alpha}e^{-\lambda t})^2}$

$L_{\alpha\alpha\alpha} = \dfrac{2n}{\alpha^3} - 4\sum_{i=1}^{r}\dfrac{e^{-3\lambda x_i}}{(1-\bar{\alpha}e^{-\lambda x_i})^3} - 2\dfrac{(n-r)e^{-3\lambda t}}{(1-\bar{\alpha}e^{-\lambda t})^3}$

$L_\lambda = \dfrac{r}{\lambda} - \left(\sum_{i=1}^{r} x_i + (n-r)t\right) - 2\sum_{i=1}^{r}\dfrac{x_i\bar{\alpha}e^{-\lambda x_i}}{(1-\bar{\alpha}e^{-\lambda x_i})} - \dfrac{2t\bar{\alpha}(n-r)e^{-\lambda x_i}}{(1-\bar{\alpha}e^{-\lambda t})}$

$L_{\lambda\lambda} = \dfrac{-r}{\lambda^2} + 2\sum_{i=1}^{r}\dfrac{x_i^2\bar{\alpha}e^{-\lambda x_i}}{(1-\bar{\alpha}e^{-\lambda x_i})} + 2\sum_{i=1}^{r}\dfrac{x_i^2\bar{\alpha}^2e^{-2\lambda x_i}}{(1-\bar{\alpha}e^{-\lambda x_i})^2} + \dfrac{2t^2(n-r)\bar{\alpha}e^{-\lambda t}}{(1-\bar{\alpha}e^{-\lambda t})} + \dfrac{2t^2(n-r)\bar{\alpha}^2e^{-2\lambda t}}{(1-\bar{\alpha}e^{-\lambda t})^2}$

$L_{\lambda\lambda\lambda} = \dfrac{2n}{\lambda^3} - 2\sum_{i=1}^{r}\dfrac{x_i^3\bar{\alpha}e^{-\lambda x_i}}{(1-\bar{\alpha}e^{-\lambda x_i})} - 6\sum_{i=1}^{r}\dfrac{x_i^3\bar{\alpha}^2e^{-2\lambda x_i}}{(1-\bar{\alpha}e^{-\lambda x_i})^2} - 4\sum_{i=1}^{r}\dfrac{x_i^3\bar{\alpha}^3e^{-3\lambda x_i}}{(1-\bar{\alpha}e^{-\lambda x_i})^3} - \dfrac{2t^3(n-r)\bar{\alpha}e^{-\lambda t}}{(1-\bar{\alpha}e^{-\lambda t})} - \dfrac{6t^3(n-r)\bar{\alpha}^2e^{-2\lambda t}}{(1-\bar{\alpha}e^{-\lambda t})^2} - \dfrac{4t^3(n-r)\bar{\alpha}^3e^{-3\lambda t}}{(1-\bar{\alpha}e^{-\lambda t})^3}$

$L_{\alpha\alpha\lambda} = L_{\lambda\alpha\alpha} = -4\sum_{i=1}^{r}\dfrac{x_i e^{-2\lambda x_i}}{(1-\bar{\alpha}e^{-\lambda x_i})^2} - 4\sum_{i=1}^{r}\dfrac{x_i\bar{\alpha}e^{-3\lambda x_i}}{(1-\bar{\alpha}e^{-\lambda x_i})^3} - \dfrac{4(n-r)te^{-2\lambda t}}{(1-\bar{\alpha}e^{-\lambda t})^2} - \dfrac{4(n-r)t\bar{\alpha}e^{-3\lambda t}}{(1-\bar{\alpha}e^{-\lambda t})^3}$

$L_{\alpha\lambda\lambda} = L_{\lambda\lambda\alpha} = -2\sum_{i=1}^{r}\dfrac{x_i^2e^{-\lambda x_i}}{(1-\bar{\alpha}e^{-\lambda x_i})} - 6\sum_{i=1}^{r}\dfrac{x_i^2\bar{\alpha}e^{-2\lambda x_i}}{(1-\bar{\alpha}e^{-\lambda x_i})^2} - 4\sum_{i=1}^{r}\dfrac{x_i^2\bar{\alpha}^2e^{-3\lambda x_i}}{(1-\bar{\alpha}e^{-\lambda x_i})^3} - \dfrac{2t^2(n-r)e^{-\lambda t}}{(1-\bar{\alpha}e^{-\lambda t})} - \dfrac{6t^2(n-r)\bar{\alpha}e^{-2\lambda t}}{(1-\bar{\alpha}e^{-\lambda t})^2} - \dfrac{4t^2(n-r)\bar{\alpha}^2e^{-3\lambda t}}{(1-\bar{\alpha}e^{-\lambda t})^3}$

If α and λ are orthogonal then $\sigma_{ij} = 0$ for $i \neq j$ and $\sigma_{ij} = \left(-\dfrac{1}{L_{ij}}\right)$ for $i = j$. After evaluation of all U-terms, L-terms, and p- terms at the point $(\hat{\alpha},\hat{\lambda})$ and using the above expression, the approximate Bayes estimator of α under SELF is,

$$\hat{\alpha}_B = \hat{\alpha} + \hat{u}_\alpha\hat{p_\alpha}\hat{\sigma}_{\alpha\alpha} + 0.5\left(\hat{u}_\alpha\hat{\sigma}_{\alpha\alpha}\hat{\sigma}_{\lambda\lambda}\hat{L}_{\alpha\lambda\lambda} + \hat{u}_\alpha\hat{\sigma}^2_{\alpha\alpha}\hat{L}_{\alpha\alpha\alpha}\right) \tag{8}$$

and similarly the Bayes estimate for λ under SELF is,

$u_\lambda = 1,\ u_{\alpha\alpha} = u_{\lambda\lambda} = u_{\alpha\lambda} = u_{\lambda\alpha} = 0$ and remaining L-terms and -terms will be same as above, thus we have,

$$\hat{\lambda}_B = \hat{\lambda} + \hat{u}_\lambda\hat{p_\lambda}\hat{\sigma}_{\lambda\lambda} + 0.5\left(\hat{u}_\lambda\hat{\sigma}^2_{\lambda\lambda}\hat{L}_{\lambda\lambda\lambda} + \hat{u}_\lambda\hat{\sigma}_{\alpha\alpha}\hat{\sigma}_{\lambda\lambda}\hat{L}_{\alpha\alpha\lambda}\right) \tag{9}$$

Markov Chain Monte Carlo method (Bayes 2) :

The expression of posterior distribution can not be expressed in any standard form therefore, numerical technique is needed for summarising its characteristics. In such a situation, the most appropriate MCMC methods namely Gibbs sampler and Metropolis-Hastings Algorithm can be effectively used to obtain the Bayes estimates and highest posterior density (HPD) credible intervals of the parameters. For more detail about MCMC method see [19], [20], [21] and [28]. For implementing the Gibbs algorithm, the full conditional posterior densities of α and λ are given by;

$$p_1(\alpha|\underline{x},\lambda) \propto \alpha^{n+a-1}e^{-b\alpha}Q(\alpha,\lambda)P(\alpha,\lambda) \tag{10}$$

and

$$p_2(\lambda|\underline{x},\alpha) \propto \lambda^{r+c-1}e^{-\lambda(T0+d)}Q(\alpha,\lambda)P(\alpha,\lambda) \tag{11}$$

where $T0, P(\alpha,\lambda), Q(\alpha,\lambda)$ are defined same as above.

The algorithm consist the following steps

- Set the initial values of α and λ say (α_0,λ_0)
- Set j=1
- Generate posterior sample for α and λ from (10) and (11) respectively.
- Repeat step 2, for all $j = 1,2,3,\cdots N$ and obtained $(\alpha_1,\lambda_1),(\alpha_2,\lambda_2),\cdots,(\alpha_N,\lambda_N)$

After obtaining the posterior samples the Bayes estimate of the parameters under SELF are the mean of the posterior samples. Therefore we have,

$$\hat{\alpha}_B \approx E(\alpha|\underline{x}) = \frac{1}{N}\sum_{j=1}^{N}\alpha_j$$

$$\hat{\lambda}_B \approx E(\lambda|\underline{x}) = \frac{1}{N}\sum_{j=1}^{N}\lambda_j$$

After extracting the posterior samples we can easily construct the HPD credible intervals for α and λ. Therefore for this purpose order $\alpha_1,\alpha_2,...,\alpha_N$ as $\alpha_1 < \alpha_2 < ... < \alpha_N$ and $\lambda_1,\lambda_2,\cdots,\lambda_N$ as $\lambda_1 < \lambda_2 < ... < \lambda_N$. Then $100(1-\beta)\%$ credible intervals of α and λ are

$$(\alpha_1,\alpha_{[N(1-\beta)+1]}),\cdots,(\alpha_{[N\beta]},\alpha_N)$$

and

$$(\lambda_1,\lambda_{[N(1-\beta)+1]}),\cdots,(\lambda_{[N\beta]},\lambda_N)$$

Here $[x]$ denotes the greatest integer less than or equal to x. Then, the HPD credible interval is that interval which has the shortest length.

3 REAL DATA ANALYSIS

In this section, to illustrate our discussed methodology, we have considered a data set representing the failure times of the release of software given in terms of hours with average life time be 1000 hours from the starting of the execution of the software. The considered real data set have taken from A. Wood [18].

We have checked the suitability of the considered distribution to this real data set over more popular life-time models namely exponential, exponentiated exponential and gamma distribution see Singh et al. see [17]. The empirical cdf plot for considered real data set is plotted in Figures 1 and one can easily conclude that the Marshall-Olkin exponential distribution gives better fits than the above four competitive distributions. For analysing of this data set under Type-I hybrid censoring scheme,we have considered six censoring schemes as;

$Scheme1 : R = 8, T = 5.0$, $Scheme2 : R = 10, T = 6.0$, $Scheme3 : R = 12, T = 6.0$, $Scheme4 : R = 14, T = 7.0$, $Scheme5 : R = 15, T = 8.0$ and $Scheme6 : R = 16, T = 9.0$ For all censoring schemes, the maximum likelihood estimates and the Bayes estimates along with 95% confidence intervals (CI) and HPD credible intervals are obtained and presented in Table 6. For obtaining the Bayes estimates for real data set, non-informative prior is considered.

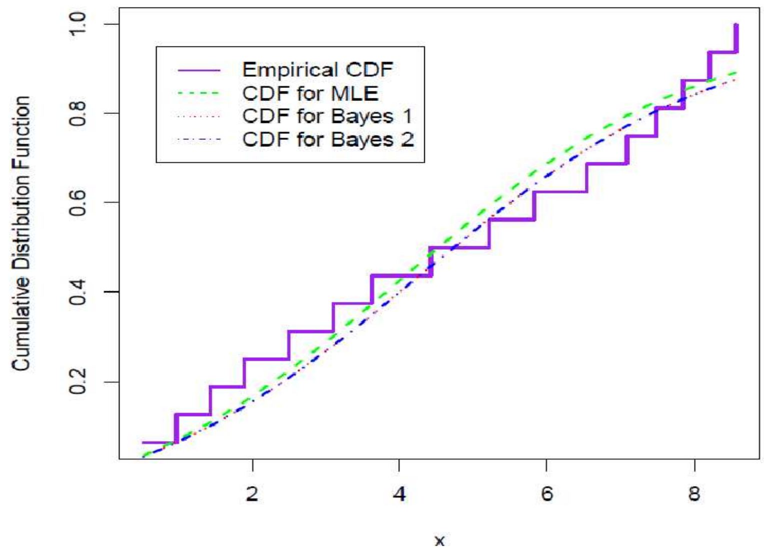

Fig. 1: The empirical cumulative distribution function (ECDF) plot for the considered real data set.

4 NUMERICAL COMPARISON

In previous section, we have obtained the estimators for the unknown parameters of the considered model and we notice that the the estimators are not in closed form. Thus, we numerically evaluate the risk (mean square error values) of all estimators and examined the performance of Bayes estimators with corresponding ML estimators on the basis of simulated samples from MOED. Since risk of the estimators are the function of n, r, T, α , λ and hyper parameters a, b, c and d. To study the performance of the estimators, we have generated a random sample of size (n=30,40 and 50) for fixed values of $\alpha = 3$ and $\lambda = 2$. In addition, we have considered the value of hyper parameter as $(a = b = c = d = 0)$ for non-informative prior (Prior 0) and $(a = 18, b = 6, c = 8, d = 4)$ for informative prior (Prior 1). In order to consider hybrid censoring scheme, we have chosen different combinations of the censoring parameters $(r[60\%, 80\%, 90\%],\ T[1.5, 2.5, 3.5])$. The simulation results are summarised in Tables 1-3.
From this extensive study, we have observed the following on the basis of Tables 1-3.

* The risk of the Bayes estimators are smaller than the risk of the maximum likelihood estimators under Prior 1. But the risk of Bayes estimators and ML estimators are quite similar under Prior 0 in all considered choices of n, R, T see Tables [1-3].
* The MSE's of the Bayes as well as ML estimators are decreases as percentage of r increases for given values of n, α and λ under both prior i.e Prior 1 and Prior 0.
* The both considered approximation method i.e. Linley's and MCMC works quite well but in between these two method we do not have any specific trend.
* The width of the HPD intervals are much lesser than the length of asymptotic intervals and width of the intervals under Prior 1 is less than Prior 0 see Table [4-5].

5 CONCLUSION

In this paper, we proposed the classical as well as Bayesian estimation of the parameters of Marshall-Olkin exponential distribution under Type-I hybrid censored data. The maximum likelihood estimates (MLE's) of the parameters have been obtained by using N-R method. The Bayes estimators of the parameters using Lindely's and MCMC method have been discussed. On the basis of comparison of the risk of the estimators, we observed that Bayes estimators performs better than ML estimators under both considered prior. It was also noted that, the length of the HPD intervals are smaller than the length of asymptotic interval of the parameters. From the discussion mentioned as above, we may conclude that the Bayes estimators obtained under Lindley or MCMC method can be recommended for their use.

Table 1: Estimates of the parameters and corresponding mean square error (MSEs) are coded under Prior 1 and Prior 0 when T=1.5

n	r	Parameter	MLE	Prior 1					Prior 0			
				MSE	Bayes1	MSE	Bayes2	MSE	Bayes1	MSE	Bayes2	MSE
30	18	α	2.303	1.278	2.932	0.239	2.479	0.585	2.784	0.618	2.492	0.736
		λ	1.756	0.328	1.733	0.203	1.833	0.156	1.856	0.215	1.840	0.227
	24	α	2.425	1.079	2.898	0.217	2.508	0.570	2.809	0.581	2.582	0.641
		λ	1.896	0.186	1.865	0.123	1.922	0.108	1.943	0.133	1.935	0.136
	27	α	2.708	0.742	2.872	0.201	2.686	0.417	2.817	0.571	2.568	0.642
		λ	2.236	0.172	2.142	0.089	2.201	0.106	2.025	0.129	1.998	0.132
40	24	α	2.407	1.115	2.810	0.174	2.522	0.575	2.751	0.583	2.532	0.674
		λ	1.780	0.272	1.761	0.195	1.834	0.147	1.856	0.186	1.845	0.194
	32	α	2.505	0.942	2.815	0.143	2.555	0.565	2.742	0.571	2.576	0.629
		λ	1.883	0.144	1.862	0.110	1.901	0.093	1.900	0.123	1.894	0.125
	36	α	2.843	0.594	2.848	0.119	2.789	0.345	2.860	0.515	2.671	0.546
		λ	2.261	0.156	2.187	0.096	2.223	0.105	2.030	0.106	2.012	0.108
50	30	α	2.399	1.123	2.727	0.250	2.500	0.641	2.781	0.618	2.604	0.668
		λ	1.760	0.263	1.749	0.205	1.811	0.153	1.873	0.269	1.866	0.175
	40	α	2.580	0.845	2.783	0.154	2.605	0.546	2.774	0.548	2.639	0.582
		λ	1.899	0.127	1.882	0.103	1.910	0.087	1.925	0.105	1.921	0.107
	45	α	2.905	0.491	2.879	0.060	2.847	0.296	2.902	0.508	2.749	0.516
		λ	2.258	0.132	2.200	0.090	2.224	0.094	2.031	0.100	2.017	0.101

Table 2: Estimates of the parameters and corresponding mean square error (MSEs) are coded under Prior 1 and Prior 0, when T=2.5

n	r	Parameter	MLE	Prior 1					Prior 0			
				MSE	Bayes1	MSE	Bayes2	MSE	Bayes1	MSE	Bayes2	MSE
30	18	α	2.382	1.163	2.496	0.515	2.538	0.685	2.836	0.589	2.438	0.744
		λ	1.790	0.279	1.777	0.229	1.811	0.180	1.873	0.239	1.856	0.230
	24	α	2.368	1.139	2.534	0.558	2.539	0.679	2.758	0.603	2.442	0.734
		λ	1.822	0.208	1.907	0.131	1.835	0.151	1.888	0.150	1.880	0.153
	27	α	2.416	1.094	2.606	0.441	2.503	0.665	2.727	0.566	2.452	0.727
		λ	1.825	0.177	1.899	0.117	1.835	0.132	1.896	0.129	1.877	0.133
40	24	α	2.375	1.155	2.764	0.597	2.484	0.580	2.463	0.702	2.544	0.680
		λ	1.772	0.261	1.852	0.166	1.818	0.144	1.765	0.234	1.841	0.174
	32	α	2.474	0.987	2.576	0.550	2.494	0.593	2.736	0.597	2.571	0.653
		λ	1.824	0.185	1.814	0.128	1.829	0.133	1.906	0.132	1.901	0.134
	36	α	2.546	0.836	2.668	0.392	2.552	0.581	2.757	0.562	2.585	0.616
		λ	1.865	0.130	1.850	0.119	1.865	0.103	1.902	0.121	1.887	0.113
50	30	α	2.417	1.147	2.776	0.581	2.490	0.633	2.478	0.692	2.599	0.638
		λ	1.780	0.254	1.865	0.167	1.815	0.146	1.773	0.233	1.856	0.172
	40	α	2.515	0.900	2.593	0.545	2.599	0.616	2.732	0.572	2.524	0.660
		λ	1.844	0.155	1.835	0.143	1.890	0.118	1.894	0.146	1.846	0.124
	45	α	2.543	0.831	2.642	0.368	2.634	0.578	2.772	0.540	2.546	0.617
		λ	1.880	0.125	1.908	0.093	1.897	0.096	1.867	0.116	1.879	0.101

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Table 3: Estimates of the parameters and corresponding mean square error (MSEs) are coded under under Prior 1 and Prior 0, when T=3.5

n	r	Parameter	MLE	Prior 1					Prior 0			
				MSE	Bayes1	MSE	Bayes2	MSE	Bayes1	MSE	Bayes2	MSE
30	18	α	2.394	1.162	2.927	0.236	2.494	0.653	2.801	0.613	2.507	0.723
		λ	1.798	0.321	1.761	0.193	1.833	0.199	1.863	0.225	1.846	0.235
	24	α	2.435	1.064	2.901	0.209	2.479	0.639	2.791	0.578	2.569	0.643
		λ	1.830	0.211	1.810	0.148	1.843	0.158	1.888	0.148	1.880	0.158
	27	α	2.458	1.012	2.909	0.199	2.537	0.563	2.767	0.577	2.540	0.656
		λ	1.859	0.181	1.839	0.129	1.885	0.113	1.891	0.142	1.871	0.146
40	24	α	2.364	1.193	2.790	0.192	2.452	0.553	2.746	0.608	2.527	0.699
		λ	1.759	0.301	1.743	0.183	1.851	0.184	1.863	0.176	1.795	0.202
	32	α	2.470	0.974	2.813	0.151	2.501	0.544	2.747	0.557	2.581	0.615
		λ	1.826	0.176	1.813	0.118	1.889	0.130	1.894	0.127	1.837	0.140
	36	α	2.480	0.945	2.817	0.144	2.540	0.536	2.745	0.539	2.575	0.598
		λ	1.848	0.147	1.836	0.118	1.870	0.099	1.898	0.127	1.884	0.109
50	30	α	2.682	0.604	2.892	0.047	2.703	0.379	2.789	0.582	2.610	0.633
		λ	1.920	0.149	1.882	0.110	1.919	0.121	1.872	0.157	1.864	0.163
	40	α	2.700	0.530	2.889	0.043	2.706	0.399	2.752	0.560	2.619	0.600
		λ	1.919	0.107	1.897	0.087	1.916	0.097	1.893	0.115	1.889	0.117
	45	α	2.678	0.580	2.867	0.053	2.681	0.462	2.757	0.564	2.618	0.603
		λ	1.928	0.095	1.910	0.079	1.926	0.089	1.900	0.108	1.889	0.110

Table 4: 95% asymptotic and HPD confidence Intervals when informative Prior (Prior 1) is considered.

T	n	R	Asymptotic Inetrvals						Bayes Intervals					
			α_L	α_U	Length	λ_L	λ_U	Length	α_L	α_U	Length	λ_L	λ_U	Length
1.5	30	18	0.000	5.298	5.298	0.000	3.790	3.790	1.726	3.258	1.532	1.296	2.385	1.089
		24	0.000	5.245	5.245	0.222	3.571	3.349	1.799	3.235	1.437	1.456	2.395	0.940
		27	0.106	5.311	5.206	0.615	3.858	3.243	1.949	3.445	1.496	1.693	2.716	1.023
	40	24	0.000	5.147	5.147	0.000	3.164	3.164	1.807	3.258	1.452	1.353	2.322	0.969
		32	0.025	4.986	4.961	0.338	3.427	3.089	1.890	3.237	1.347	1.495	2.314	0.819
		36	0.457	5.329	4.871	0.755	3.767	3.012	2.087	3.508	1.421	1.776	2.679	0.903
	50	30	0.000	4.081	4.081	0.000	3.005	3.005	1.835	3.184	1.348	1.376	2.253	0.877
		40	0.201	4.059	3.858	0.441	3.358	2.916	1.974	3.251	1.277	1.541	2.282	0.741
		45	0.527	4.283	3.756	0.841	3.675	2.834	2.185	3.524	1.340	1.821	2.633	0.812
2.5	30	18	0.000	5.436	5.436	0.000	3.833	3.833	1.808	3.200	1.392	1.369	2.305	0.936
		24	0.000	5.065	5.065	0.186	3.474	3.289	1.901	3.067	1.165	1.457	2.235	0.778
		27	0.000	4.946	4.946	0.341	3.377	3.036	1.810	3.286	1.477	1.441	2.336	0.895
	40	24	0.000	5.182	5.182	0.000	3.641	3.641	1.833	3.084	1.251	1.390	2.205	0.816
		32	0.004	4.936	4.932	0.300	3.353	3.052	1.975	3.032	1.057	1.499	2.179	0.680
		36	0.150	4.810	4.660	0.439	3.257	2.818	1.869	3.231	1.362	1.479	2.265	0.786
	50	30	0.000	4.793	4.793	0.296	3.607	3.311	1.997	3.228	1.230	1.514	2.139	0.625
		40	0.000	4.499	4.499	0.854	3.010	2.156	2.030	3.277	1.247	1.577	2.188	0.611
		45	0.000	4.353	4.353	1.015	2.840	1.825	2.006	3.351	1.344	1.668	2.293	0.624
3.5	30	18	0.000	5.366	5.366	0.000	3.796	3.796	1.863	2.972	1.109	1.396	2.188	0.792
		24	0.000	5.090	5.090	0.217	3.546	3.329	1.762	3.446	1.685	1.466	2.373	0.907
		27	0.138	5.410	5.272	0.636	3.885	3.249	1.905	3.642	1.737	1.765	2.694	0.929
	40	24	0.000	5.245	5.245	0.082	3.768	3.685	1.935	3.545	1.611	1.460	2.213	0.753
		32	0.000	5.194	5.194	0.727	3.134	2.407	1.984	3.531	1.547	1.541	2.324	0.783
		36	0.000	4.836	4.836	0.911	2.963	2.052	1.961	3.562	1.602	1.540	2.342	0.803
	50	30	0.000	4.771	4.771	0.275	3.565	3.290	2.082	3.336	1.254	1.596	2.243	0.647
		40	0.000	4.680	4.680	0.845	2.992	2.147	2.194	3.224	1.030	1.687	2.146	0.459
		45	0.000	4.408	4.408	1.010	2.846	1.836	2.213	3.152	0.939	1.725	2.128	0.402

Table 5: Asymptotic and HPD confidence interval for the parameters in the case of non-informative prior (Prior0)

T	n	R	Asymptotic Inetrvals						Bayes Intervals					
			α_L	α_U	Length	λ_L	λ_U	Length	α_L	α_U	Length	λ_L	λ_U	Length
1.5	30	18	0.000	7.906	7.906	0.000	4.068	4.068	1.281	3.734	2.453	1.207	2.471	1.265
		24	0.000	6.741	6.741	0.544	3.442	2.898	1.470	3.728	2.258	1.416	2.457	1.040
		27	0.000	6.499	6.499	0.726	3.417	2.690	1.341	3.834	2.493	1.413	2.584	1.171
	40	24	0.000	7.195	7.195	0.078	3.745	3.667	1.477	3.614	2.137	1.305	2.385	1.080
		32	0.000	6.107	6.107	0.715	3.157	2.442	1.622	3.557	1.935	1.459	2.330	0.871
		36	0.000	6.117	6.117	0.915	3.214	2.299	1.577	3.791	2.214	1.516	2.508	0.992
	50	30	0.000	6.823	6.823	0.282	3.553	3.271	1.640	3.587	1.947	1.388	2.346	0.958
		40	0.000	5.827	5.827	0.859	3.050	2.191	1.769	3.529	1.760	1.533	2.313	0.781
		45	0.000	5.874	5.874	1.039	3.078	2.039	1.752	3.776	2.024	1.579	2.456	0.877
2.5	30	18	0.000	8.039	8.039	0.000	4.084	4.084	1.316	3.785	2.468	1.212	2.499	1.287
		24	0.000	6.611	6.611	0.530	3.342	2.811	1.448	3.664	2.216	1.375	2.384	1.009
		27	0.000	6.125	6.125	0.744	3.129	2.385	1.364	3.676	2.312	1.354	2.400	1.046
	40	24	0.000	7.225	7.225	0.079	3.736	3.657	1.497	3.612	2.116	1.295	2.386	1.091
		32	0.000	6.077	6.077	0.722	3.163	2.440	1.622	3.547	1.925	1.465	2.338	0.872
		36	0.000	5.769	5.769	0.905	2.959	2.054	1.571	3.628	2.058	1.440	2.336	0.896
	50	30	0.000	6.813	6.813	0.284	3.535	3.252	1.650	3.566	1.916	1.374	2.340	0.967
		40	0.000	5.740	5.740	0.841	3.004	2.162	1.743	3.478	1.735	1.505	2.275	0.771
		45	0.000	5.493	5.493	1.017	2.848	1.830	1.715	3.576	1.862	1.500	2.297	0.797
3.5	30	18	0.000	7.953	7.953	0.000	4.078	4.078	1.298	3.739	2.441	1.203	2.488	1.285
		24	0.000	6.684	6.684	0.536	3.336	2.800	1.466	3.705	2.239	1.377	2.383	1.006
		27	0.000	6.217	6.217	0.748	3.114	2.366	1.387	3.735	2.348	1.353	2.392	1.039
	40	24	0.000	7.185	7.185	0.074	3.764	3.691	1.485	3.590	2.105	1.302	2.403	1.101
		32	0.000	6.107	6.107	0.720	3.141	2.421	1.624	3.561	1.937	1.456	2.322	0.867
		36	0.000	5.744	5.744	0.903	2.954	2.051	1.566	3.612	2.046	1.439	2.333	0.893
	50	30	0.000	6.842	6.842	0.285	3.548	3.263	1.658	3.583	1.924	1.380	2.350	0.970
		40	0.000	5.779	5.779	0.845	2.998	2.153	1.758	3.504	1.746	1.506	2.273	0.767
		45	0.000	5.466	5.466	1.011	2.837	1.826	1.704	3.559	1.856	1.493	2.286	0.793

Table 6: Estimates of the parameters and corresponding interval estimates based on real data set

Schemes	MLE		Bayes1		Bayes2		Asymptotic Intervals				HPD Intervals			
	α	λ	α	λ	α	λ	α_L	α_U	λ_L	λ_L	α_L	λ_U	λ_L	λ_U
I	4.0003	0.3633	4.4719	0.3304	3.4307	0.3232	0.0000	15.7828	0.0000	0.9341	0.9411	5.9781	0.1479	0.4744
II	3.4768	0.3255	3.7774	0.3042	3.1722	0.3067	0.0000	11.9853	0.0000	0.7272	1.4259	5.1270	0.1847	0.4282
III	3.4768	0.3255	3.7774	0.3042	3.0335	0.2981	0.0000	11.9853	0.0000	0.7272	1.0026	5.1290	0.1331	0.4159
IV	3.5085	0.3255	3.7777	0.3073	3.1917	0.3069	0.0000	11.3846	0.0000	0.6773	1.3937	5.1327	0.1954	0.4241
V	5.8424	0.4236	6.2052	0.4101	5.5050	0.4156	0.0000	16.6766	0.1331	0.7140	2.9559	8.2350	0.3151	0.5105
VI	8.6380	0.5009	9.1365	0.4898	8.1358	0.4926	0.0000	23.8329	0.2241	0.7778	4.5290	11.7542	0.4057	0.5960

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