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Propagation of Waves in Micropolar Thermoelastic Cubic Crystals

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This paper concentrates on the study of propagation of waves in a homogenous isotropic micropolar thermoelastic plate possessing cubic symmetry subjected to stress free boundary conditions in context of Lord and Shulman (L-S) and Green and Lindsay (G-L) theories of thermoelasticity. The secular equations for homogeneous isotropic micropolar thermoelastic plate possessing cubic symmetry for symmetric and skew symmetric wave modes of propagation are derived. The amplitudes of displacement components, microrotation and temperature distribution are also computed during the symmetric and skew symmetric motion of the plate. Finally, in order to illustrate and verify the analytical developments, numerical solution of secular equation corresponding to stress free thermally insulated micropolar thermoelastic cubic crystal plates is carried out for magnesium crystal material.

Keywords: Micropolar thermoelastic plate, cubic symmetry, secular equations, phase velocity, symmetric and skewsymmetric amplitudes.

1 Introduction

As one knows, in the classical theory of thermoelasticity the velocity of heat propagation is assumed to be infinitely large. However, in studying dynamic thermal stresses in deformable solid bodies, when the inertia terms in the equations of motion cannot be neglected, one must take into account that heat propagates not with an infinite but with a finite velocity; a heat flow arises in the body not instantly but is characterized by a finite relaxation time.

Presently, there are at least two different generalizations of the classical theory of thermoelasticity. The first of them, Lord-Shulman's generalization [1] admits only one relaxation time constant; the other one, Green-Lindsay's generalization [2] is based on using two heat relaxation time constants. Both generalizations were developed as an attempt at explaining the paradox of the classical case that the heat propagation velocity is an infinite value.

The classical theory of elasticity is inadequate to represent the behavior of some modern engineering structures such as polycrystalline materials and materials with fibrous or coarse grain. The study of these materials requires incorporation of theory of oriented media. "Micropolar elasticity", termed by Eringen [3] is used to describe the deformation of elastic media with oriented particles. A micropolar continuum is a collection of interconnected particles in the form of small rigid bodies undergoing both translational and rotational motions. The force at a point of a surface element of bodies of these materials is completely characterized by a stress vector and a couple stress vector at that point.

Following various methods, the elastic fields of various loadings, inclusion and inhomogeneity problems, and interaction energy of point defects and dislocation arrangement have been discussed extensively in the past. Generally all materials have elastic anisotropic properties which mean the mechanical behavior of an engineering material is characterized by the direction dependence. However the three-dimensional study for an anisotropic material is much more complicated to obtain than the isotropic one, due to the large number of elastic constants involved in the calculation.

A wide class of crystals such as W, Si, Cu, Ni, Fe, Au, Al, etc., which are some frequently used substances, belong to cubic materials. The cubic materials have nine planes of symmetry the normals of which are on the three coordinate axes and on the coordinate planes making an angle $\pi/4$ with the coordinate axes. With the chosen coordinate system along the crystalline directions, the mechanical behavior of a cubic crystal can be characterized by four independent elastic constants A_1, A_2, A_3 and A_4.

To understand the crystal elasticity of a cubic material, Chung and Buessem [4] presented a convenient method to describe the degree of the elasticity anisotropy in a given cubic crystal. Later, Lie and Koehler [5] used a Fourier expansion scheme to calculate the stress fields caused by a unit force in a cubic crystal. Minagawa *et al.* [6] discussed the propagation of plane harmonic waves in a cubic micropolar medium. Kumar and Rani [7] studied time harmonic sources in a thermally conducting cubic crystal. However no attempt has been made to study source problems in micropolar cubic crystals. Kumar and Ailawalia [8] investigated elastodynamics of inclined loads in a micropolar cubic crystals. Kumar and Ailawalia [9] studied time harmonic sources at micropolar thermoelastic medium possessing cubic crystals with one relaxation time. Kumar and Ailawalia [10] discussed interactions due to mechanical/thermal sources in a micropolar thermoelastic medium possessing cubic crystals. Kumar and Ailawalia [11] presented interaction due to mechanical sources in micropolar cubic crystals. Kumar and Ailawalia [12] considered deformation due to time harmonic sources in micropolar thermoelastic medium possessing cubic symmetry with two relaxation times.

The present investigation is concerned to study the propagation of plane waves in an infinite homogeneous isotropic micropolar thermoelastic plate possessing cubic symmetry. The secular equations for different conditions of solutions have been deduced from the present one. Numerical solutions of the dispersion equations and amplitudes of displacement components, microrotation and temperature distribution for symmetric and skew symmetric modes are presented graphically.

2 Formulation of the Problem

We consider a homogeneous isotropic micropolar thermoelastic plate with cubic symmetry of thickness $2d$ initially at uniform temperature T_0. We take the origin of the coordinate system (x, y, z) on the middle surface of the plate. The $x - y$ plane is chosen to coincide with the middle surface of the plate and z-axis normal to it along the thickness as shown in Fig. 2.1.

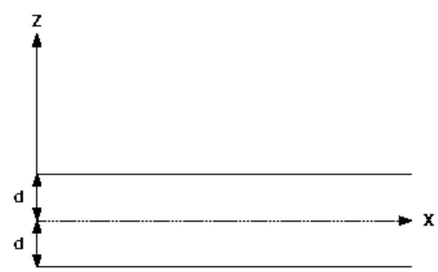

Figure 2.1: Geometry of the problem

If we restrict our analysis to plane strain problem parallel to $x - z$ plane with displacement vector $\vec{u} = (u_1, 0, u_3)$ and microrotation vector $\vec{\phi} = (0, \phi_2, 0)$, then the field equations and constitutive relations for micropolar thermoelastic solid with cubic symmetry in the absence of body forces, body couples and heat sources given by Minagawa *et al.* [6], Lord and Shulman [1], and Green and Lindsay [2] are

$$A_1\frac{\partial^2 u_1}{\partial x^2}+A_3\frac{\partial^2 u_1}{\partial z^2}+(A_2+A_4)\frac{\partial^2 u_3}{\partial x\partial z}-(A_3-A_4)\frac{\partial\phi_2}{\partial z}-\nu\frac{\partial}{\partial x}(1+\tau_1\frac{\partial}{\partial t})T=\rho\frac{\partial^2 u_1}{\partial t^2}, \tag{2.1}$$

$$A_1\frac{\partial^2 u_3}{\partial z^2}+A_3\frac{\partial^2 u_3}{\partial x^2}+(A_2+A_4)\frac{\partial^2 u_1}{\partial x\partial z}+(A_3-A_4)\frac{\partial\phi_2}{\partial x}-\nu\frac{\partial}{\partial z}(1+\tau_1\frac{\partial}{\partial t})T=\rho\frac{\partial^2 u_3}{\partial t^2}, \tag{2.2}$$

$$B_3(\frac{\partial^2\phi_2}{\partial x^2}+\frac{\partial^2\phi_2}{\partial z^2})+(A_3-A_4)(\frac{\partial u_1}{\partial z}-\frac{\partial u_3}{\partial x}-2\phi_2)=\rho j\frac{\partial^2\phi_2}{\partial t^2} \tag{2.3}$$

$$K^*\nabla^2 T=\rho C^*(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2})+\nu T_0(\frac{\partial}{\partial t}+\eta_0\tau_0\frac{\partial^2}{\partial t^2})(\frac{\partial u_1}{\partial x}+\frac{\partial u_3}{\partial z}), \tag{2.4}$$

$$t_{33}=A_1\frac{\partial u_3}{\partial z}+A_2\frac{\partial u_1}{\partial x}-\nu(1+\tau_1\frac{\partial}{\partial t})T, \tag{2.5}$$

$$t_{31}=A_3\frac{\partial u_1}{\partial z}+A_4\frac{\partial u_3}{\partial x}+(A_4-A_3)\phi_2, \tag{2.6}$$

$$m_{32}=B_3\frac{\partial \phi_2}{\partial z}. \tag{2.7}$$

where $A=A_1-A_2-A_3-A_4, B=B_1-B_2-B_3-B_4, \nu=(A_1+2A_2)\alpha_t$. $A_1,A_2,A_3,A_4,B_1,B_2,B_3,B_4$ are material constants, α_t is coefficient of linear thermal expansion, ρ is the density, j is the microinertia, t_{ij} and m_{ij} are the components of force stress and couple stress tensors respectively, K^* is the coefficient of thermal conductivity, C^* is specific heat at constant strain,τ_0 and τ_1 are thermal relaxation times, δ_{ij} is Kronecker delta. The comma notation denotes spatial derivatives.

We introduce the dimensionless quantities defined by the expressions

$$x'=\frac{\omega^* x}{c_1}, z'=\frac{\omega^* z}{c_1}, u_1'=\frac{\rho\omega^* c_1}{\nu T_o}u_1, u_3'=\frac{\rho\omega^* c_1}{\nu T_o}u_3, t'=\omega^* t, \phi_2'=\frac{\rho c_1^2}{\nu T_o}\phi_2, T'=\frac{T}{T_o},$$

$$\tau_o'=\omega^*\tau_o,\ \tau_1'=\omega^*\tau_1,\ t_{ij}'=\frac{1}{\nu T_o}t_{ij},\ m_{ij}'=\frac{\omega^* m_{ij}}{c_1\nu T_o},\ h'=\frac{c_1 h}{\omega^*},\ d'=\frac{\omega^* d}{c_1}, \tag{2.8}$$

where $c_1^2=A_1/\rho, \omega^*=\rho c_1^2 C^*/K^*$ and ω^* is the characteristic frequency of the medium.

Using Eq. (2.8) in Eqs. (2.1) - (2.4), and after suppressing the primes for convenience, we obtain

$$(d_1\frac{\partial^2}{\partial x^2}+d_2\frac{\partial^2}{\partial z^2})u_1+d_3\frac{\partial^2 u_3}{\partial x\partial z}-d_4\frac{\partial \phi_2}{\partial z}-\frac{\partial}{\partial x}(1+\tau_1\frac{\partial}{\partial t})T=\frac{\partial^2 u_1}{\partial t^2}, \tag{2.9}$$

$$(d_1\frac{\partial^2}{\partial z^2}+d_2\frac{\partial^2}{\partial x^2})u_3+d_3\frac{\partial^2 u_1}{\partial x\partial z}+d_4\frac{\partial \phi_2}{\partial x}-\frac{\partial}{\partial z}(1+\tau_1\frac{\partial}{\partial t})T=\frac{\partial^2 u_3}{\partial t^2}, \tag{2.10}$$

$$\nabla^2\phi_2-d_5(\frac{\partial u_3}{\partial x}-\frac{\partial u_1}{\partial z}+2\phi_2)=d_6\frac{\partial^2\phi_2}{\partial t^2}, \tag{2.11}$$

$$\nabla^2 T=(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2})+\epsilon(\frac{\partial}{\partial t}+\eta_0\tau_0\frac{\partial^2}{\partial t^2})(\frac{\partial u_1}{\partial x}+\frac{\partial u_3}{\partial z}), \tag{2.12}$$

where

$$d_1=\frac{A_1}{\rho c_1^2},\ d_2=\frac{A_3}{\rho c_1^2},\ d_3=\frac{A_2+A_4}{\rho c_1^2},\ d_4=\frac{A_3-A_4}{\rho c_1^2},$$

$$d_5=\frac{(A_3-A_4)c_1^2}{B_3\omega^*},\ d_6=\frac{\rho j c_1^2}{B_3},\ \epsilon=\frac{\nu^2 T_0}{\rho\omega^* K^*},\ \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}.$$

3 Formal Solution of the Problem

We assume the solution of Eqs. (2.9)–(2.12) of the type

$$(u_1, u_3, \phi_2, T) = \left[1, \bar{u_3}, \bar{\phi_2}, \bar{T}\right]\bar{u_1}e^{\iota\xi(x+mz-ct)} \tag{3.1}$$

where c = ω/ξ is the phase velocity of the waves, ω is the angular frequency and ξ is the wave number, m is an unknown parameter which signifies the penetration depth of the wave, $\bar{u_3}, \bar{\phi_2}, \bar{T}$ are the amplitude ratios of displacement u_3, microrotation ϕ_2 and temperature T to that of displacement u_1 respectively.

Using Eq. (3.1) in Eqs. (2.9)–(2.12), we obtain

$$\frac{m^2}{c^2} + a_1 + ma_2\bar{u_3} + ma_3\bar{\phi_2} + a_4\bar{T} = 0, \tag{3.2}$$

$$ma_5 + \frac{(m^2 + a_6)}{c^2}\bar{u_3} + a_7\bar{\phi_2} + ma_8\bar{T} = 0, \tag{3.3}$$

$$ma_9 - a_9\bar{u_3} - \Big(\frac{m^2+1}{c^2} + a_{10} + a_{13}\Big)\bar{\phi_2} = 0, \tag{3.4}$$

$$a_{11} + ma_{11}\bar{u_3} - \frac{m^2}{c^2}\bar{T} + a_{12}\bar{T} = 0. \tag{3.5}$$

The system of Eqs. (3.2)–(3.5) has a nontrivial solution if the determinant of coefficients of $(1, \bar{u_3}, \bar{\phi_2}, \bar{T})^T$ vanishes, which yields an algebraic equation relating m to c.

Solving the above equations, we obtain an eight degree equation of the form

$$m^8 + A'm^6 + B'm^4 + C'm^2 + D' = 0. \tag{3.6}$$

The roots of the Eq. (3.6) give four values of m^2. Using a computer program of Descard's method following Cardan's method, Eq. (3.6) leads to the following solution for displacements, microrotation and temperature:

$$(u_1, u_3, \phi_2, T) = \sum_{i=1}^{4}[E_i\cos\xi m_i z + F_i\sin\xi m_i z]\{1, r_i, l_i, t_i\}e^{\iota\xi(x-ct)}, \tag{3.7}$$

where

$$\begin{aligned}
&A' = A^*c^2 + 4, \qquad B' = B^*c^4 + 3A^*c^2 + 6,\\
&C' = C^*c^6 + 2B^*c^4 + 3A^*c^2 + 4, \quad D' = D^*c^8 + C^*c^6 + B^*c^4 + A^*c^2 + 1,\\
&A^* = [(a_{16} - a_{20} - k_0 + a_{21}) + c^2(a_8a_{20} - a_2a_5 + a_3a_9)],\\
&B^* = (k_0a_{20} - k_0a_{16} - a_{16}a_{20} - a_7a_9 - a_8a_{20} + a_2a_5 + a_{21}a_{16} - a_{21}a_{20} - k_0a_{21}\\
&\qquad - a_3a_9 + a_4a_{11}) + c^2(k_0a_2a_5 - a_8a_{20}a_{11} + a_2a_5a_{20} - a_2a_7a_9 + a_2a_8a_{11}\\
&\qquad + a_{21}a_{20}a_8 + a_3a_5a_9 - k_0a_3a_9 - a_3a_{16}a_9 - a_4a_5a_{11} + c^2a_3a_8a_9a_{11}),
\end{aligned}$$

$$
\begin{aligned}
C^* &= (a_8a_{20}a_{11} + k_0a_{16}a_{20} + k_0a_7a_9 - k_0a_2a_5 - a_2a_5a_{20} + a_2a_7a_9 - a_2a_8a_{11} \\
&\quad - a_3a_5a_9 + k_0a_3a_9 + a_3a_6a_9 - a_4a_{20}a_{11} + a_4a_5a_{11} + a_4a_6a_{11}) \\
&\quad + a_{21}(k_0a_{20} - k_0a_6 - a_{16}a_{20} - a_7a_9 - a_8a_{20}) \\
&\quad + c^2(k_0a_2a_7a_9 - k_0a_2a_5a_{20} - a_2a_8a_{20}a_{11} - a_{21}a_8a_{20}a_{11} + k_0a_3a_5a_9 \\
&\quad + k_0a_3a_{16}a_9 - a_3a_8a_9a_{20} - a_4a_7a_9a_{11} + a_4^2a_5a_{20}), \\
D^* &= k_0a_2a_5a_{20} - k_0a_2a_7a_9 + a_2a_8a_{20}a_{11} + a_{21}a_8a_{20}a_{11} + k_0a_{21}a_{16}a_{20} \\
&\quad + k_0a_{21}a_7a_9 + k_0a_3a_5a_9 + k_0a_3a_{16}a_9 + a_4a_5a_{20}a_{11} + a_4a_{16}a_{20}a_{11}, \\
a_1 &= \frac{(d_1 - c^2)}{c^2d_2}, \quad a_2 = \frac{d_3}{c^2d_2}, \quad a_3 = \frac{\iota d_4}{\xi c^2 d_2}, \quad a_4 = \frac{k_1}{cd_2}, \quad a_5 = \frac{d_3}{c^2d_1}, \\
a_6 &= \frac{(d_2 - c^2)}{c^2d_1}, \quad a_7 = -\frac{\iota d_4}{\xi c^2 d_1}, \quad a_8 = \frac{k_1}{cd_1}, \quad a_9 = \frac{\iota d_5}{\xi c^2}, \quad a_{10} = \frac{2d_5}{\xi^2c^2}, \\
a_{11} &= \iota\epsilon k_0'\xi, \quad a_{12} = \frac{-1 + c^2k_0}{\xi c^2}, \quad a_{13} = c^2d_6, \quad a_{16} = a_6 - \frac{1}{c^2}, \quad a_{20} = \frac{a_{13}}{c^2} - a_{10}, \\
a_{21} &= a_1 - \frac{1}{c^2}, \quad r_i = -\frac{\Delta_2}{\Delta_1}, \quad l_i = \frac{\Delta_3}{\Delta_1}, \quad t_i = -\frac{\Delta_4}{\Delta_1}, \\
\Delta_1 &= \frac{(-m_i^2 - 1 + c^2k_0)}{c^6}\Big[(m_i^2 + a)(-m_i^2 + b) + \frac{d_4d_5}{\xi^2d_1}\Big] - \frac{\iota m_i^2\epsilon k_0'k_1\xi^2(-m_i^2 + b)}{d_1c^4}, \\
\Delta_2 &= \frac{(-m_i^2 - 1 + c^2k_0)}{c^6}\Big[\frac{m_id_3(-m_i^2 + b)}{d_1} - \frac{m_id_4d_5}{\xi^2d_1}\Big] + \frac{m_ik_0'k_1\xi(-m_i^2 + b)}{d_1c^3}, \\
\Delta_3 &= \frac{\iota m_id_5(-m_i^2 - 1 + c^2k_0)}{\xi c^6}\Big[-(m_i^2 + a) - \frac{d_3}{d_1}\Big] - \frac{m_id_5\epsilon k_0'k_1\xi(m_i^2 + 1)}{d_1c^3}, \\
\Delta_4 &= \frac{\iota\epsilon\xi k_0'(-m_i^2 + b)}{c^4}\Big[-(m_i^2 + a) - \frac{d_3m_i^2}{d_1}\Big] + \frac{\iota d_4d_5\epsilon k_0'(m_i^2 + 1)}{d_1\xi c^4}, \\
a &= \frac{d_2 - c^2}{d_1}, \quad b = (-1 + c^2d_6 - \frac{2d_5}{\xi^2}), \\
k_0 &= \tau_0 + \iota\omega^{-1}, \quad k_0' = \eta_0\tau_0 + \iota\omega^{-1}, \quad k_1 = \tau_1 + \iota\omega^{-1}.
\end{aligned}
$$

3.1 Boundary conditions

We consider the following mechanical and thermal boundary conditions at the plate surfaces $z = \pm d$.

3.1.1 Mechanical conditions

The nondimensional mechanical boundary conditions at $z = \pm d$ are given as follows:

$$t_{33} = 0, \quad t_{31} = 0, \quad m_{32} = 0, \tag{3.8}$$

where

$$t_{33} = \sum_{i=1}^{4} E_i(-d_1\xi m_i r_i S_i + d_7\iota\xi C_i + \iota\omega k_1 t_i C_i) + \sum_{i=1}^{4} F_i(d_1\xi m_i r_i C_i + d_7\iota\xi S_i + \iota\omega k_1 t_i S_i)$$

$$t_{31} = \sum_{i=1}^{4} E_i(d_2\xi m_i S_i + d_8\iota\xi r_i C_i - d_4 l_i C_i) + \sum_{i=1}^{4} F_i(d_2\xi m_i C_i + d_8\iota\xi r_i S_i - d_4 l_i S_i)$$

$$m_{32} = \sum_{i=1}^{4} E_i(l_i\xi m_i S_i) + \sum_{i=1}^{4} F_i(l_i\xi m_i C_i),$$

$$S_i = \sin\xi m_i d, \quad C_i = \cos\xi m_i d, \quad i = 1, 2, 3, 4.$$

3.1.2 Thermal conditions

The thermal boundary conditions at $z = \pm d$ are given by

$$T_{,z} + hT = 0, \tag{3.9}$$

where h is the surface heat transfer coefficient. Here $h \to 0$ corresponds to thermally insulated boundaries and $h \to \infty$ refers to isothermal one.

4 Derivation of the secular equations

Substituting the values of u_1, u_3, ϕ_2, and T in the boundary conditions (3.8) and (3.9) on the surfaces $z = \pm d$ of the plate, we obtain

$$\sum_{i=1}^{4}\Big[\big((g_1 - g_{1i})C_i - g_{2i}S_i\big)E_i + \big((g_1 - g_{1i})S_i + g_{2i}C_i\big)F_i\Big] = 0,$$

$$\sum_{i=1}^{4}\Big[-g_{5i}S_iE_i + g_{5i}C_iF_i\Big] = 0,$$

$$\sum_{i=1}^{4}\Big[((g_1 - g_{1i})C_i + g_{2i}S_i)E_i + (-(g_1 - g_{1i})S_i + g_{2i}C_i)F_i\Big] = 0,$$

$$\sum_{i=1}^{4}\Big[g_{5i}S_iE_i + g_{5i}C_iF_i\Big] = 0,$$

$$\sum_{i=1}^{4}\Big[(g_{3i}C_i - g_{4i}S_i)E_i + (g_{3i}S_i + g_{4i}C_i)F_i\Big] = 0,$$

$$\sum_{i=1}^{4}\Big[(g_{3i}C_i + g_{4i}S_i)E_i + (-g_{3i}S_i + g_{4i}C_i)F_i\Big] = 0,$$

$$\sum_{i=1}^{4}\Big[(-g_{6i}S_i - ht_iC_i)E_i + (g_{6i}C_i + ht_iS_i)F_i\Big] = 0,$$

$$\sum_{i=1}^{4}\Big[(g_{6i}S_i + ht_iC_i)E_i + (g_{6i}C_i - ht_iS_i)F_i\Big] = 0, \tag{4.1}$$

where

$$g_1 = d_7\iota\xi, \quad g_{1i} = \iota\omega k_1 t_i, \quad g_{2i} = m_i r_i d_1, \quad g_{3i} = \iota\xi l_i d_8 - l_i d_4,$$
$$g_{4i} = m_i d_2, \quad g_{5i} = m_i l_i, \quad g_{6i} = m_i t_i, \quad i = 1,2,3,4.$$

The system of equations (4.1) has a nontrivial solution if the determinant of coefficients of E_i and F_i , $i = 1,2,3,4$ vanishes, which leads to a characteristic equation for the propagation of waves in the plate. The characteristic equation for the waves in this case, after applying lengthy algebraic reductions and manipulations of the determinant leads to the following secular equations

$$\begin{aligned} AT1[T_1T_3]^{\pm1} =& AT2[T_1T_2]^{\pm1} + AT3[T_1T_4]^{\pm1} + AT4[T_2T_3]^{\pm1} + AT5[T_2T_4]^{\pm1} \\ &+ AT6[T_3T_4]^{\pm1} \end{aligned} \tag{4.2}$$

where

$$\begin{aligned} &T_i = \tan\xi m_i d, \ i = 1,2,3,4. \\ &AT1 = (g_{53}g_{61} - g_{51}g_{63})\big[(g_1 - g_{12})g_{34} - (g_1 - g_{14})g_{32}\big], \\ &AT2 = (g_{52}g_{61} - g_{51}g_{62})\big[(g_1 - g_{13})g_{34} - (g_1 - g_{14})g_{33}\big], \\ &AT3 = (g_{51}g_{64} - g_{54}g_{61})\big[(g_1 - g_{13})g_{32} - (g_1 - g_{12})g_{33}\big], \\ &AT4 = (g_{53}g_{62} - g_{52}g_{63})\big[(g_1 - g_{11})g_{34} - (g_1 - g_{14})g_{31}\big], \\ &AT5 = (g_{54}g_{62} - g_{52}g_{64})\big[(g_1 - g_{13})g_{34} - (g_1 - g_{14})g_{33}\big], \\ &AT6 = (g_{54}g_{63} - g_{53}g_{64})\big[(g_1 - g_{11})g_{32} - (g_1 - g_{12})g_{31}\big]. \end{aligned}$$

Here the superscript $+1$ refers to skew symmetric and -1 refers to symmetric modes.

4.1 Special cases

4.1.1 Micropolar generalized thermoelastic plate with cubic symmetry and with one relaxation time (L-S theory)

In this case, $\tau_1 = 0, \tau_0 > 0$ and $\eta_0 = 1$.

4.1.2 Micropolar generalized thermoelastic plate with cubic symmetry and with two relaxation time (G-L theory)

In this case, $\tau_1 \geq \tau_0 > 0$ and $\eta_0 = 0$.

4.1.3 Micropolar elastic plate

In this case, $A_1 = \lambda + 2\mu + K,\ A_2 = \lambda,\ A_3 = \mu + K,\ A_4 = \mu,\ B_3 = \gamma.$

5 Amplitude of Displacements, Microrotation and Temperature Distribution

In this section the amplitudes of displacement components, microrotation and temperature distribution for symmetric and skew symmetric modes of plate waves are obtained as

$$\begin{aligned}
(u_1)_{sy}, (u_1)_{asy} &= \Sigma_{i=1}^{4}(E_i \cos\xi m_i z, F_i \sin\xi m_i z)e^{i\xi(x-ct)},\\
(u_3)_{sy}, (u_3)_{asy} &= \Sigma_{i=1}^{4} r_i(F_i \sin\xi m_i z, E_i \cos\xi m_i z)e^{i\xi(x-ct)},\\
(\phi_2)_{sy}, (\phi_2)_{asy} &= \Sigma_{i=1}^{4} l_i(F_i \sin\xi m_i z, E_i \cos\xi m_i z)e^{i\xi(x-ct)},\\
(T)_{sy}, (T)_{asy} &= \Sigma_{i=1}^{4} t_i(E_i \cos\xi m_i z, F_i \sin\xi m_i z)e^{i\xi(x-ct)}.
\end{aligned}$$

6 Example Results

For numerical computations, we take the following values of relevant parameters for micropolar medium with cubic symmetry:

$$\begin{aligned}
&A_1 = 19.6\times10^{10}Nm^{-2},\quad A_2 = 11.7\times10^{10}Nm^{-2},\quad A_3 = 5.6\times10^{10}Nm^{-2},\\
&A_4 = 4.3\times10^{10}Nm^{-2},\quad B_3 = 0.98\times10^{-09}N.
\end{aligned}$$

Micropolar parameters are

$$\begin{aligned}
&\rho = 1.74\times10^{3}Kgm^{-3},\quad \lambda = 9.4\times10^{10}Nm^{-2},\quad \mu = 4.0\times10^{10}Nm^{-2},\\
&K = 1.0\times10^{10}Nm^{-2},\quad \gamma = 0.779\times10^{-09}N,\quad j = 0.2\times10^{-19}m^2.
\end{aligned}$$

Thermal parameters are

$$\begin{aligned}
&\tau_0 = 6.131\times10^{-13}sec,\quad \tau_1 = 8.765\times10^{-13}sec,\quad \epsilon = 0.028,\quad T_0 = 298^oK,\\
&C^* = 1.04\times10^{3}JKg^{-1}deg^{-1},\quad K^* = 1.7\times10^{06}Jm^{-1}\sec^{-1}deg^{-1},\\
&\nu = 2.68\times10^{6}Nm^{-2}deg^{-1},\quad d = 0.01m.
\end{aligned}$$

A FORTRAN program has been developed for the solution of equation (4.2) to compute phase velocity c for different values of n by using the relations $\tan\theta = \tan(n\pi + \theta)$.

The nondimensional phase velocity of symmetric and skew symmetric modes of wave propagation in the context of L-S and G-L theories of thermoelasticity have been computed for various values of nondimensional wave number from dispersion equation (4.2) for stress

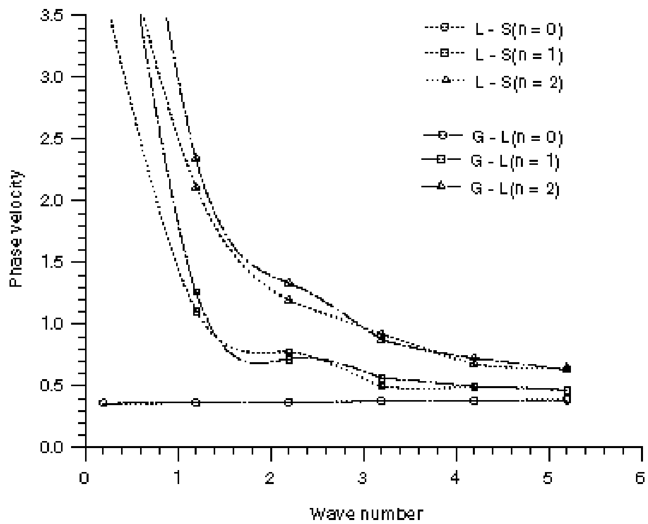

Figure 6.1: Variation of phase velocity of symmetric modes of wave propagation

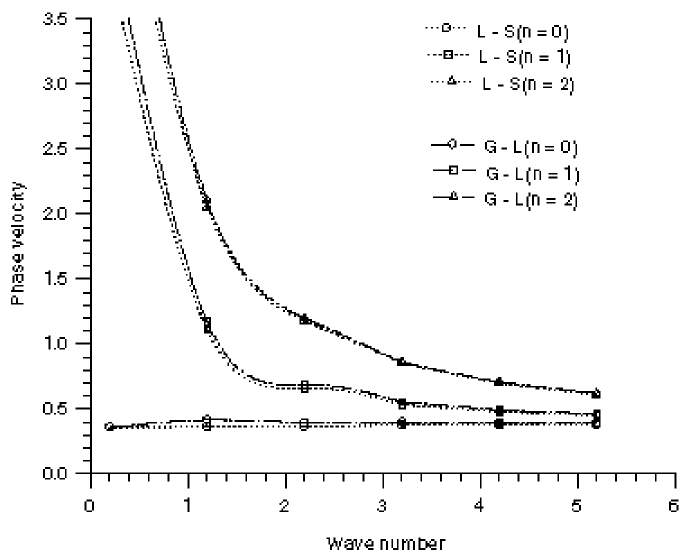

Figure 6.2: Variation of phase velocity of skewsymmetric modes of wave propagation

free thermally insulated micropolar thermoelastic plate with cubic symmetry and have been represented graphically for different modes ($n = 0$ to $n = 2$) in Figures 6.1 and 6.2. The dashed curves refer to L-S theory and broken-line curves correspond to G-L theory of thermoelasticity.

6.1 Phase velocity

The phase velocities of lowest mode of propagation, symmetric and skewsymmetric become dispersionless i.e. remain constant with variation in wave number. The phase velocities of higher modes of propagation, symmetric and skewsymmetric attain quite large values at vanishing wave number which sharply slashes down to become steady and asymptotic to the reduced Rayleigh wave velocity with increasing wave number.

It is observed for various symmetric modes of wave propagation from Fig.6.1 that (i) for lowest mode ($n = 0$), phase velocity profiles for L-S and G-L theory coincide; (ii) for $n = 1$, phase velocity for G-L theory is greater than that in the case of L-S theory for wave number $\xi d \leq 1.4$ and for wave number lying between 2.6 and 4.2; phase velocity for G-L theory is less than in case of L-S theory for wave number lying between 1.4 and 2.6; the phase velocity profiles in respect of L-S and G-L theory coincide for wave number $\xi d \geq 4.2$; (iii) for $n = 2$, phase velocity for G-L theory is greater than that in the case of L-S theory for wave number $\xi d \leq 3.0$ and $\xi d \geq 3.8$; phase velocity for G-L theory is less than in case of L-S theory for wave number lying between 3.0 and 3.8.

For skewsymmetric modes of wave propagation, we observe the following from Fig.6.2: (a) for modes $n = 0$ and $n = 1$, phase velocity for L-S theory is less than that in the case of G-L theory for wave number $\xi d \leq 5.2$; (b) for $n = 2$, phase velocity for L-S theory is slightly less than that in the case of G-L theory for wave number $\xi d \leq 3.2$ and phase velocity profiles in respect of L-S and G-L theories coincide for wave number $\xi d \geq 3.2$.

6.2 Amplitudes

Figs. 6.3–6.10 depict the variations of symmetric and skew symmetric amplitudes of displacements (u_1), (u_3), microrotation (ϕ_2) and temperature distribution (T) in the context of L-S and G-L theories of thermoelasticity for stress free thermally insulated boundary.

The displacement (u_1) and temperature distribution (T) of the plate is minimum at the centre and maximum at the surfaces for symmetric mode and zero at the centre and maximum at the surfaces for skewsymmetric mode as can be seen from Figs.6.3–6.4 and Figs.6.9–6.10. The microrotation (ϕ_2) of the plate is minimum at the centre and maximum at the surfaces for symmetric mode and zero at the centre, minimum at the bottom surface and maximum at the top surface of the plate for skewsymmetric mode as noticed from Figs.6.7–6.8. From Fig. 6.5 and Fig. 6.6, it is noticed that the values of the displacement (u_3) of the plate is zero at the centre and maximum at the surfaces for symmetric mode and maximum at the centre and minimum at the surfaces for skew symmetric mode. $(u_1)_{sym}$, $(u_1)_{asym}$, $(u_3)_{sym}$, $(u_3)_{asym}$, $(\phi_2)_{sym}$, $(\phi_2)_{asym}$, $(T)_{sym}$ and $(T)_{asym}$ correspond to the values of (u_1), (u_3), (ϕ_2) and (T) for symmetric and skew symmetric modes respectively.

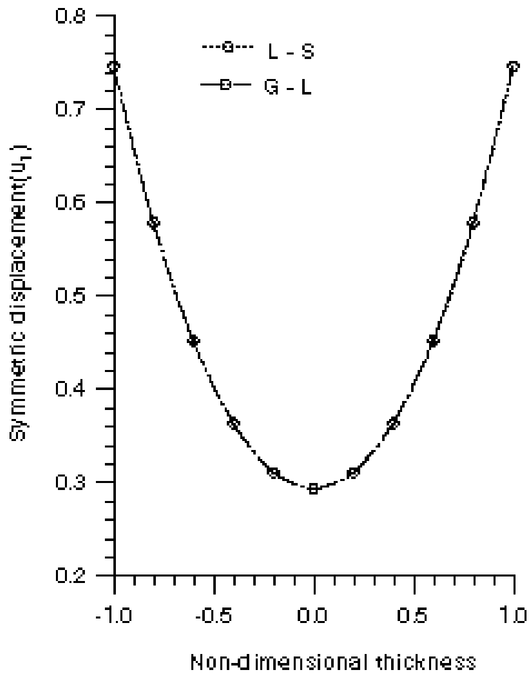

Figure 6.3: Amplitude of symmetric displacement u_1

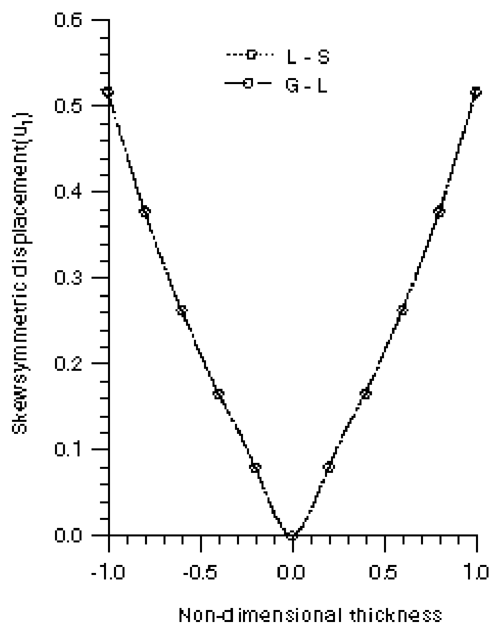

Figure 6.4: Amplitude of skewymmetric displacement u_1

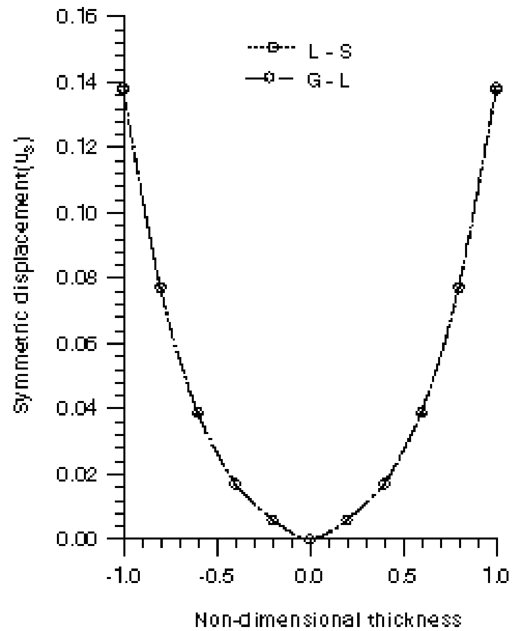

Figure 6.5: Amplitude of symmetric displacement u_3

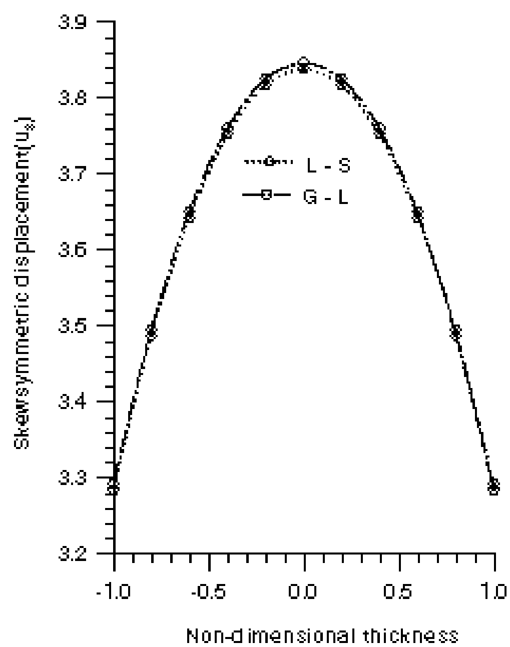

Figure 6.6: Amplitude of skewsymmetric displacement u_3

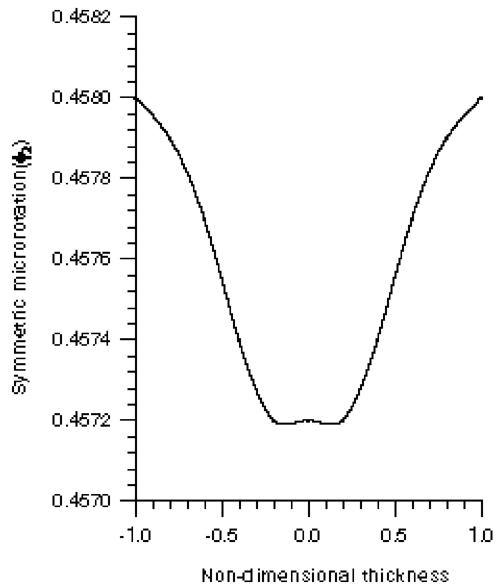

Figure 6.7: Amplitude of symmetric microrotation ϕ_2

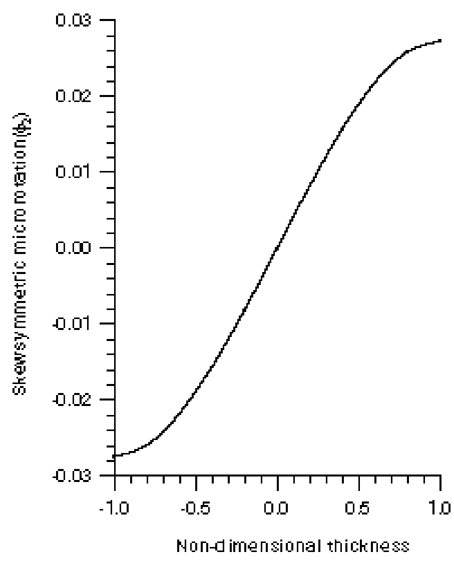

Figure 6.8: Amplitude of skewsymmetric microrotation ϕ_2

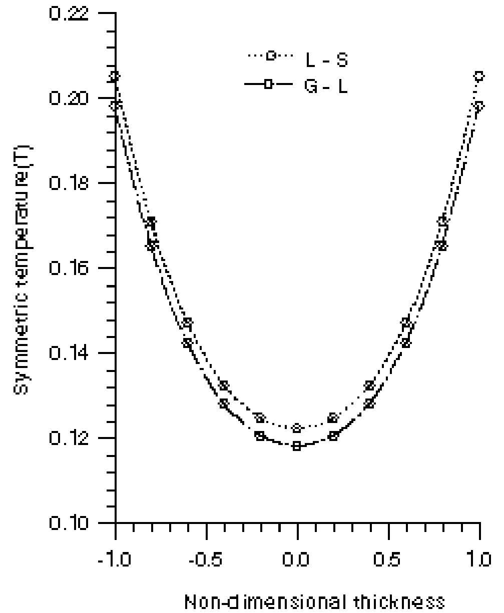

Figure 6.9: Amplitude of symmetric temperature T

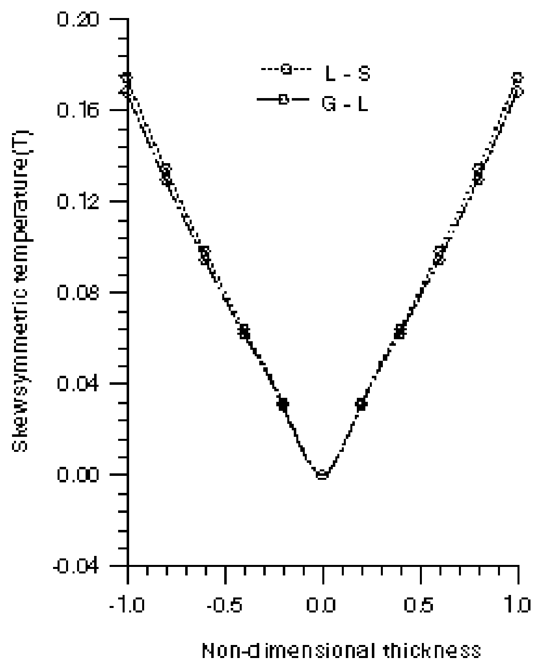

Figure 6.10: Amplitude of skewsymmetric temperature T

It is observed that the behavior and trend of variations of $(u_1)_{sym}$, $(\phi_2)_{sym}$ and $(T)_{sym}$ is same; whereas the behavior and trend of variations of $(u_1)_{asym}$, $(u_3)_{sym}$ and $(T)_{asym}$ is similar.

The values of the displacements of the plate in case of G-L theory are larger in comparison to L-S Theory for symmetric and skew symmetric modes, whereas the values of the temperature distribution of the plate in case of G-L theory are smaller in comparison to L-S Theory for symmetric and skew symmetric modes.

The values of the microrotation (ϕ_2) of the plate are the same in cases of L-S and G-L theories of thermoelasticity for symmetric and skew symmetric modes.

7 Conclusions

(i) The propagation of waves in a homogenous isotropic micropolar thermoelastic plate possessing cubic symmetry subjected to stress free boundary conditions has been studied in context of Lord and Shulman (L-S) and Green and Lindsay (G-L) theories of thermoelasticity.

(ii) The secular equations for homogeneous isotropic micropolar thermoelastic plate possessing cubic symmetry for symmetric and skew symmetric wave modes of propagation are derived.

(iii) The phase velocities of lowest mode of propagation, symmetric and skewsymmetric become dispersionless i.e. remain constant with variation in wave number. The phase velocities of higher modes of propagation, symmetric and skewsymmetric attain quite large values at vanishing wave number which sharply slashes down to become steady and asymptotic to the reduced Rayleigh wave velocity with increasing wave number.

(iv) The values of the displacements of the plate in case of G-L theory are larger in comparison to L-S theory, whereas the values of the temperature distribution of the plate in case of G-L theory are smaller in comparison to L-S Theory for symmetric and skew symmetric modes.

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