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A Finite Dimensional Control of the Dynamics of the Generalized Korteweg-de Vries Burgers Equation

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The paper deals with the finite dimensional control of the generalized Korteweg-de Vries Burgers (GKdVB) partial differential equation (PDE). A Karhunen-Loève Galerkin projection procedure is used to derive a system of ordinary differential equations (ODEs) that mimics the dynamics of the GKdVB equation. Using Lyapunov theory, it is shown that the highly nonlinear system of ODEs is stable. However, the simulation results indicate that the system of ODEs converges slowly to the origin. Therefore, two control schemes are proposed for the system of ODEs; the main objective of the controllers is to speed up the convergence to the origin. The first controller is a linear state feedback controller whereas the second controller is a nonlinear controller. It is proven that both controllers guarantee the asymptotic convergence of the states of the system of ODEs to zero. Simulation results indicate that the proposed control schemes work well.

Keywords: Generalized Korteweg-de Vries Burgers equation, Karhunen-Loève decomposition, linear control, nonlinear control.

1 Introduction

In this paper, we investigate the finite dimensional control of the dynamics of the generalized Korteweg-de Vries-Burgers (GKdVB) equation

$$\frac{\partial u}{\partial t} - \nu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^3 u}{\partial x^3} + u^\alpha \frac{\partial u}{\partial x} = 0, \;\; x \in (0, 2\pi), \; t \geq 0 \tag{1.1}$$

with periodic boundary conditions

$$u\,(0,t) = u\,(2\pi,t)\,, \qquad \frac{\partial u}{\partial x}\,(0,t) = \frac{\partial u}{\partial x}\,(2\pi,t)\,, \qquad \frac{\partial^2 u}{\partial x^2}\,(0,t) = \frac{\partial^2 u}{\partial x^2}\,(2\pi,t)\,, \tag{1.2}$$

and the initial condition

$$u\,(0,t) = f\,(x) = \sin(x), \tag{1.3}$$

where $\mu, \nu > 0$ and α is a positive integer.

When $\alpha = 1$ in Eq. (1.1), the GKdVB equation reduces to the Korteweg-de Vries Burgers (KdVB) equation which was used as a model for many physical phenomena and hydrodynamics processes. For instance, the KdVB was used as a model for long waves in shallow water [9] and as a model of unidirectional propagation of planar waves [18]. In the last few decades, the KdVB equation has been investigated analytically as well as numerically, see references [1, 4, 6–8, 15, 18, 20, 25]. If $\alpha = 1$ and $\nu = 0$, the GKdVB equation reduces to the Korteweg-de Vries (KdV) equation derived by Korteweg and de Vries as a model for waves propagating on the surface of a canal [19]. If $\alpha = 1$ and $\mu = 0$, the GKdVB equation reduces to the Burgers equation which models turbulent liquid flow through a channel [11].

Recently, a lot of work has been done to control the KdVB equation, the KdV equation and the Burgers equation. For example, Balogh and Krstić [4] investigated the control problem of the KdVB equation using boundary control. In their work, global stability of the solution in the L^2-sense and global stability in the H^1-sense were proved. Rosier [23, 24] worked on the the control problem of the KdV equation where an exact boundary control of the linear and nonlinear KdV equations was established in [23]; and the control was illustrated numerically in [24]. Smaoui [29, 30] and Smaoui *et al.* [33] considered the boundary and distributed control of the Burgers equation. A boundary control is used in [29] to show the exponential stability of the Burgers equation analytically as well as numerically. In [30], a system of ODEs was constructed to mimic the dynamics of the Burgers equation, then a state feedback control scheme was implemented on the system to show that the solution of the Burgers equation can be controlled to any desired state.

In this paper, the control problem of the GKdVB equation with periodic boundary conditions is investigated. A state feedback controller and a nonlinear controller are designed for the GKdVB equation. Simulation results are presented to illustrate the developed theory.

In section 2, numerical simulations of the GKdVB equation are obtained using the pseudo-spectral Fourier Galerkin method. Then, the Karhunen-Loève decomposition is used on the simulated data to extract the coherent structures of the equation for $\alpha = 2$. Next, we present the Galerkin projection method and apply it on the GKdVB equation for $\alpha = 2$; a system of ODEs that mimics the dynamics of the GKdVB equation is extracted. The data coefficients obtained are then used to approximate the numerical solution of the

GKdVB equation. The approximation is then compared to the numerical solution of the GKdVB equation. Section 3 gives the ODE dynamics which mimics the dynamics of the GKdVB and plots its numerical solution. Section 4 presents two control schemes to speed up the convergence of the GKdVB equation; numerical simulations are presented to illustrate the presented theory. Finally, some concluding remarks are given in section 5.

2 The Karhunen-Loève Galerkin Projection

2.1 The Karhunen-Loève decomposition

The Karhunen-Loève (K-L) decomposition is a very useful and powerful statistical technique that is used in many applications; it has many different names depending on the field where it is used in. It is known as the Hotelling transform in image processing [13,16], the principal component analysis (PCA) in pattern recognition and signal processing [17], the empirical component analysis in statistical weather prediction [21], the quasi-harmonic modes in biology [10], the factor analysis in psychology and economics [14], and the proper orthogonal decomposition (POD) or the singular value decomposition (SVD) in fluid dynamics [22, 26].

Among the many applications that utilize the Karhunen-Loève (K-L) decomposition, the K-L decomposition was used in the analysis of the two-dimensional Navier-Stokes (N-S) equation [2, 3, 28, 32], and in the study of flames [27, 31].

In this paper, we use the K-L decomposition on the numerically simulated data of the GKdVB equation given in (1) in order to extract the most energetic eigenfunctions or coherent structures that span the data set in an optimal way. Since the K-L decomposition is a well known procedure, we refer the reader to the references cited above for a detailed description of the technique.

The solution $u(x,t)$ of the GKdVB equation, obtained using a pseudo-spectral Galerkin method, can be written in terms of the eigenfunctions or coherent structures, $\psi_i^{'}s$, as

$$u\left(x,t\right)=\sum_{i=1}^{M}a_i\left(t\right)\psi_i\left(x\right), \tag{2.1}$$

where $a_i(t)$ are the data coefficients calculated from the projections of the sample vector solution onto an eigenfunction, i.e.,

$$a_i=\frac{<u,\psi_i>}{<\psi_i,\psi_i>}, \quad i=1,\dots,M$$

with M being the number of snapshots.

Figure 2.1 shows the solution $u\left(x,t\right)$ of the GKdVB equation as it evolves to its steady state solution. Applying the K-L decomposition on the numerical solution presented in

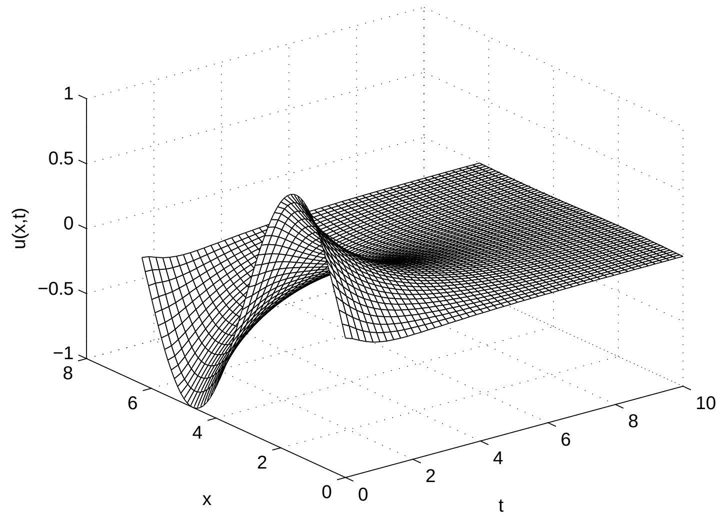

Figure 2.1: A 3-D landscape plot of the numerical solution of the GKdVB equation when $\alpha = 2$, $\nu = 0.5$, and $\mu = 0.01$.

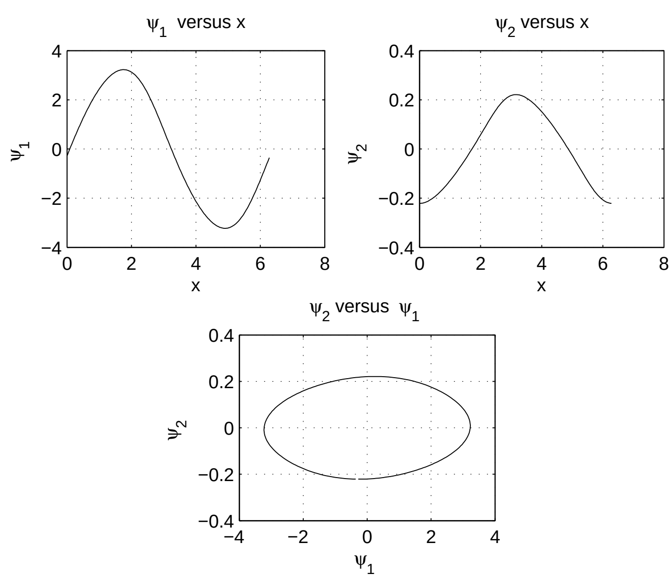

Figure 2.2: The eigenfunctions associated with the solution of the GKdVB equation when $\alpha = 2$, $\nu = 0.5$, and $\mu = 0.01$.

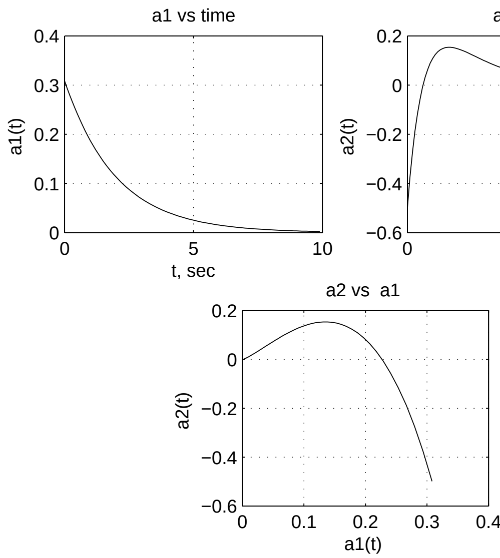

Figure 2.3: The data coefficients associated with the eigenfunctions of the solution of the GKdVB equation when $\alpha = 2$, $\nu = 0.5$, and $\mu = 0.01$.

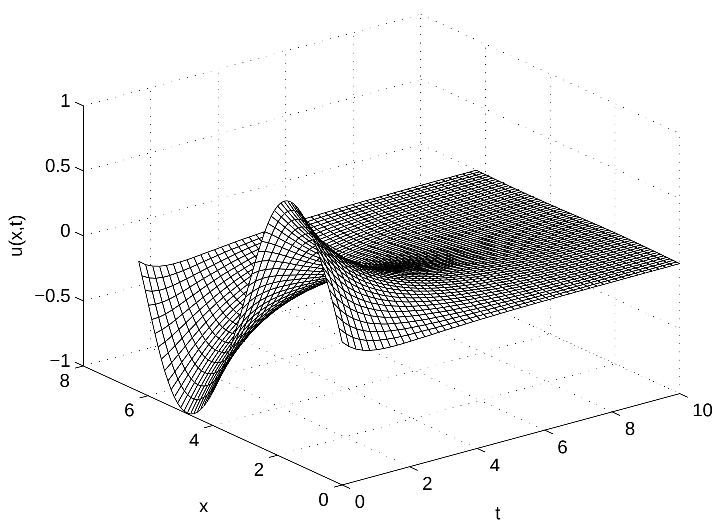

Figure 2.4: A 3-D landscape plot of the approximated solution of the GKdVB equation when $\alpha = 2$, $\nu = 0.5$, and $\mu = 0.01$.

Figure 2.1, two eigenfunctions capturing 99.98% of the energy were obtained; these eigenfunctions are plotted in Figure 2.2. The first eigenfunction captures 99.54% of the energy and the second one captures 0.44% of the energy. Figure 2.3 presents the corresponding data coefficients, and Figure 2.4 depicts the approximated solution of the GKdVB equation using the above eigenfunctions.

When comparing Figures 2.1 and 2.4, one can conclude that the K-L decomposition was able to capture the large scale dynamics of the GKdVB equation with only two eigenfunctions.

2.2 The Galerkin projection

The Galerkin projection is a method used to replace infinite dimensional continuous systems by finite dimensional ones [5, 12]. In this section, the Galerkin projection is used to extract a system of ODEs from the GKdVB equation. The system of ODEs can be solved to get an approximate solution of the original PDE. Thus, the approximation can be written in the following form:

$$u\left(x,t\right) \approx \sum_{i=1}^{K} a_i\left(t\right)\psi_i\left(x\right), \tag{2.2}$$

where $a_i\left(t\right)$ is the ith solution of the system of ODEs and can be computed in a way that minimize the residual error produced by the approximate solution above and $\psi_i\left(x\right)$ is the ith eigenfunction from the K-L decomposition; K was taken to be 2, since two eigenfunctions capture most of the dynamics of the GKdVB equation.

In order to extract the system of ODEs, we first write the original PDE as

$$\frac{\partial u}{\partial t} = D\left(u\right), \tag{2.3}$$

with given initial and boundary conditions, where "D" is the differential operator. Then, the system of ODEs is derived by projecting the normalized eigenfunctions, onto the PDE as

$$\dot{a}_i\left(t\right) = \left\langle D\Big(\sum_{i=1}^{K} a_i\left(t\right)\psi_i\left(x\right)\Big), \psi_i\left(x\right)\right\rangle,\ \ i = 1,\ldots,K \tag{2.4}$$

with initial condition

$$a_i\left(0\right) = \langle u\left(x,0\right), \psi_i\left(x\right)\rangle,\ \ i = 1,\ldots,K, \tag{2.5}$$

where $u\left(x,0\right)$ is obtained from the original PDE.

Using Eq.(2.2) and taking into account the two most energetic eigenfunctions, the GKdVB equation (1.1) with $\alpha = 2$

$$\frac{\partial u}{\partial t} = \nu\frac{\partial^2 u}{\partial x^2} - \mu\frac{\partial^3 u}{\partial x^3} - u^2\frac{\partial u}{\partial x},$$

becomes

$$\sum_{i=1}^{2} \dot{a}_i(t)\,\psi_i(x) = \nu \sum_{i=1}^{2} a_i(t)\,{\psi_i}''(x) - \mu \sum_{i=1}^{2} a_i(t)\,{\psi_i}'''(x) - \Big(\sum_{i=1}^{2} a_i(t)\,\psi_i(x)\Big)^2 \Big(\sum_{i=1}^{2} a_i(t)\,\psi_i'(x)\Big), \tag{2.6}$$

or

$$\sum_{i=1}^{2} \dot{a}_i(t)\,\psi_i(x) = \nu \sum_{i=1}^{2} a_i(t)\,{\psi_i}''(x) - \mu \sum_{i=1}^{2} a_i(t)\,{\psi_i}'''(x) - \sum_{i=1}^{2}\sum_{j=1}^{2} a_i^2(t)\,a_j(t)\,\psi_i^2(x)\,\psi_j'(x) - 2\sum_{i\neq j=1}^{2} a_i(t)\,a_j^2(t)\,\psi_i(x)\,\psi_j(x)\,\psi_j'(x)\,, \tag{2.7}$$

where $\dot{a}_i(t)$ is the derivative with respect to time and $\psi_i^{'}(x)$ is the derivative with respect to x. Now, taking the Euclidean inner product of the above equation with $\psi_k,\ k = 1, 2$ and using the orthogonality property of ψ's, we obtain the following system of ODEs

$$\dot{a}_k(t) = \nu \sum_{i=1}^{2} a_i(t)\,\langle \psi_k, {\psi_i}''\rangle - \mu \sum_{i=1}^{2} a_i(t)\,\langle \psi_k, {\psi_i}'''\rangle - \sum_{i=1}^{2}\sum_{j=1}^{2} a_i^2(t)\,a_j(t)\,\langle \psi_k, \psi_i^2\psi_j'\rangle - 2\sum_{i\neq j=1}^{2} a_i(t)\,a_j^2(t)\,\langle \psi_k, \psi_i\psi_j\psi_j'\rangle, \qquad (k = 1, 2). \tag{2.8}$$

The solution to the above ODE system can be obtained numerically using any ODE solver.

3 Stability of the ODE Dynamics of the GKdVB Equation

We have applied the above procedure on the GKdVB equation when $\alpha = 2$ and with the initial condition given by Eq.(1.3). The model of the obtained system of ODEs can be written as

$$\begin{aligned} \dot{a}_1 &= f_1(a_1, a_2), \\ \dot{a}_2 &= f_2(a_1, a_2), \end{aligned} \tag{3.1}$$

where

$$f_1(a_1,\ a_2) = -p_1 a_1^2 a_2 - p_2 a_1 a_2^2 - p_3 a_2^3 + p_4 \mu a_2 - p_5 \nu a_1 + p_6 \nu a_2, \tag{3.2}$$

$$f_2(a_1,\ a_2) = p_2a_1^2a_2 + p_3a_1a_2^2 + p_1a_1^3 - p_4\mu a_1 + p_6\nu a_1 - p_7\nu a_2 \tag{3.3}$$

with

$$p_1 = 0.4899793644, \quad p_2 = 0.0570998562, \quad p_3 = 0.5148487537,$$
$$p_4 = 1.0208303477, \quad p_5 = 1.0043594987, \quad p_6 = 0.0043221400,$$
$$p_7 = 1.0140806414, \quad \text{for } \alpha = 2,\ \mu = 0.01 \text{ and } \nu = 0.5.$$

Proposition 3.1. *The dynamics of the system of ODEs of the GKdVB equation given by (3.1) is asymptotically stable.*

Proof. Let the Lyapunov function candidate V be such that

$$V(a_1, a_2) = \frac{1}{2}a_1^2 + \frac{1}{2}a_2^2. \tag{3.4}$$

Taking the derivative of V with respect to time, it follows that

$$\begin{aligned}
\dot{V} &= a_1\dot{a}_1 + a_2\dot{a}_2 \\
&= (-p_1a_1^2a_2 - p_2a_1a_2^2 - p_3a_2^3 + p_4\mu a_2 - p_5\nu a_1 + p_6\nu a_2)a_1 \\
&\quad + (p_2a_1^2a_2 + p_3a_1a_2^2 + p_1a_1^3 - p_4\mu a_1 + p_6\nu a_1 - p_7\nu a_2)a_2 \\
&= -p_5\nu a_1^2 - p_7\nu a_2^2 + 2p_6\nu a_1a_2 \\
&= -p_5\nu a_1^2 - p_7\nu \left[(a_2 - \frac{p_6}{p_7}a_1)^2 - \frac{p_6^2}{p_7^2}a_1^2\right] \\
&= -(p_5 - \frac{p_6^2}{p_7})\nu a_1^2 - p_7\nu(a_2 - \frac{p_6}{p_7}a_1)^2 \\
&< 0 \qquad \text{for} \quad (a_1, a_2) \neq (0, 0).
\end{aligned} \tag{3.5}$$

Note that $\dot{V}$ is negative definite because it can be easily checked that $p_5 - p_6^2/p_7 > 0$, and p_7 and ν are positive. Therefore, the dynamics of the system of ODEs given by (3.1) is asymptotically stable. □

The system of ODEs given by (3.1) is simulated using the Matlab software. The initial conditions are chosen to be $a_1(0) = 10$ and $a_2(0) = -10$.

Figure 3.1 shows the simulation results of the system of ODEs. The coefficients $a_1(t)$ versus $a_2(t)$ when the time t is taken to be 10 seconds are plotted. Note that $a_1(t)$ and $a_2(t)$ take about 5 seconds to converge to zero. Therefore, it can be concluded that the dynamics of the ODEs of the GKdVB equation is asymptotically stable.

4 Design of Controllers for the GKdVB Equation

To speed up the convergence of the coefficients $a_1(t)$ and $a_2(t)$ to zero, we propose to add a forcing term to the system of equations. The forced system of ODEs can be written

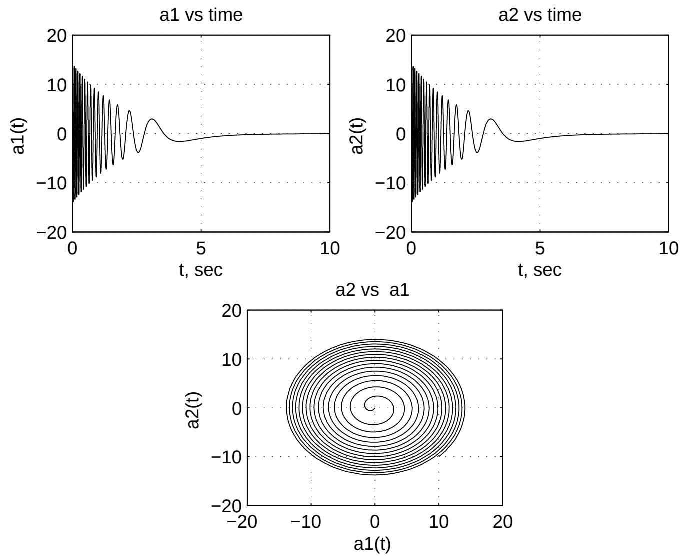

Figure 3.1: The profiles of the data coefficients $a_1(t)$ and $a_2(t)$ with no control.

as

$$\begin{aligned}\dot{a}_1 &= -p_1 a_1^2 a_2 - p_2 a_1 a_2^2 - p_3 a_2^3 + p_4 \mu a_2 - p_5 \nu a_1 + p_6 \nu a_2 + b_1 u = f_1(a_1, a_2) + b_1 u,\\ \dot{a}_2 &= p_2 a_1^2 a_2 + p_3 a_1 a_2^2 + p_1 a_1^3 - p_4 \mu a_1 + p_6 \nu a_1 - p_7 \nu a_2 + b_2 u = f_2(a_1, a_2) + b_2 u,\end{aligned} \tag{4.1}$$

where $u(t)$ is the forcing term (the input) of the system and b_1 and b_2 are multiplying factors.

It is desired to design control schemes such that the coefficients $a_1(t)$ and $a_2(t)$ converge to (0,0) as fast as possible.

4.1 Design of a state feedback controller

Let the design parameters k_1 and k_2 be chosen such that $k_1 b_2 = k_2 b_1$, $k_1 b_1 > 0$ and $k_2 b_2 > 0$.

Proposition 4.1. *The state feedback controller*

$$u(t) = -k_1 a_1(t) - k_2 a_2(t) \tag{4.2}$$

guarantees the asymptotic stability of the forced system of ODEs given in (4.1).

Proof. Let the Lyapunov function candidate V be such that

$$V(a_1, a_2) = \frac{1}{2} a_1^2 + \frac{1}{2} a_2^2. \tag{4.3}$$

Taking the derivative of V with respect to time, it follows that,

$$
\begin{aligned}
\dot{V} &= a_1\dot{a}_1 + a_2\dot{a}_2 \\
&= (-p_1a_1^2a_2 - p_2a_1a_2^2 - p_3a_2^3 + p_4\mu a_2 - p_5\nu a_1 + p_6\nu a_2 + b_1u)a_1 \\
&\quad +(p_2a_1^2a_2 + p_3a_1a_2^2 + p_1a_1^3 - p_4\mu a_1 + p_6\nu a_1 - p_7\nu a_2 + b_2u)a_2 \\
&= -p_5\nu a_1^2 - p_7\nu a_2^2 + 2p_6\nu a_1a_2 + (b_1a_1 + b_2a_2)u \\
&= -(p_5 - \frac{p_6^2}{p_7})\nu a_1^2 - p_7\nu(a_2 - \frac{p_6}{p_7}a_1)^2 + (b_1a_1 + b_2a_2)u \\
&\leq (b_1a_1 + b_2a_2)(-k_1a_1 - k_2a_2) \\
&= -b_1k_1a_1^2 - b_2k_2a_2^2 - (b_2k_1 + b_1k_2)a_1a_2 \\
&= -b_1k_1a_1^2 - b_2k_2\Big[a_2^2 + \frac{b_2k_1 + b_1k_2}{b_2k_2}a_1a_2\Big] \\
&= -b_1k_1a_1^2 - b_2k_2\Big[a_2 + \frac{b_2k_1 + b_1k_2}{2b_2k_2}a_1\Big]^2 + \frac{(b_2k_1 + b_1k_2)^2}{4b_2k_2}a_1^2 \\
&= -b_2k_2\Big[a_2 + \frac{b_2k_1 + b_1k_2}{2b_2k_2}a_1\Big]^2 + \frac{(b_2k_1 - b_1k_2)^2}{4b_2k_2}a_1^2 \\
&= -b_2k_2\Big[a_2 + \frac{b_2k_1 + b_1k_2}{2b_2k_2}a_1\Big]^2.
\end{aligned}
$$

Clearly $\dot{V} < 0$ for $(a_1, a_2) \neq (0, 0)$ and $\dot{V} = 0$ for $(a_1, a_2) = (0, 0)$. Therefore, $\dot{V}$ is negative definite, and V is a Lyapunov function for the system (4.1). Hence, the system of ODEs given by (4.1) is asymptotically stable. □

The forced system of ODEs given by (4.1) with the controller given by (4.2) is simulated using the Matlab ODE solver. The parameters b_1 and b_2 are taken to be $b_1 = b_2 = 1$. The initial conditions are chosen to be $a_1(0) = 10$ and $a_2(0) = -10$.

Figure 4.1 shows the simulation results of the forced system of ODEs when the state feedback controller is used. In Figure 4.1, the plots of the coefficients $a_1(t)$ and $a_2(t)$ versus time and the plots of the coefficients $a_1(t)$ versus $a_2(t)$ are shown for three different cases. The first case corresponds to gains $k_1 = k_2 = 3$; the second case, corresponds to gains $k_1 = k_2 = 10$; and the third case corresponds to gains $k_1 = k_2 = 20$. It is clear from the figure that the larger the gains of the controller, the faster the coefficients $a_1(t)$ and $a_2(t)$ converge to zero. However, the increase in the convergence speed comes at the expense of larger control actions.

4.2 Design of a nonlinear controller

Proposition 4.2. *Let W be a positive scalar. The nonlinear controller*

$$u(t) = -W sign(b_1a_1(t) + b_2a_2(t)) \tag{4.4}$$

guarantees the asymptotic stability of the forced system of ODEs given by (4.1).

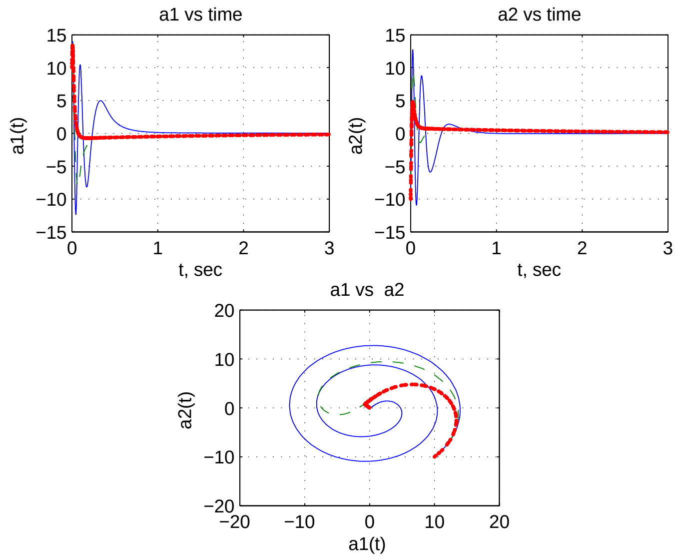

Figure 4.1: The profiles of the data coefficients $a_1(t)$ and $a_2(t)$ with the state feedback controller; the solid line corresponds to gains $k_1 = k_2 = 3$; the '- - -' line corresponds to gains $k_1 = k_2 = 10$; and the bold line corresponds to gains $k_1 = k_2 = 20$.

Proof. Let the Lyapunov function candidate V be such that

$$V(a_1, a_2) \quad = \quad \frac{1}{2}a_1^2 + \frac{1}{2}a_2^2. \tag{4.5}$$

Taking the derivative of V with respect to time, it follows that

$$\begin{aligned}
\dot{V} &= a_1\dot{a}_1 + a_2\dot{a}_2 \\
&= (-p_1a_1^2a_2 - p_2a_1a_2^2 - p_3a_2^3 + p_4\mu a_2 - p_5\nu a_1 + p_6\nu a_2 + b_1u)a_1 \\
&\quad +(p_2a_1^2a_2 + p_3a_1a_2^2 + p_1a_1^3 - p_4\mu a_1 + p_6\nu a_1 - p_7\nu a_2 + b_2u)a_2 \\
&= -p_5\nu a_1^2 - p_7\nu a_2^2 + 2p_6\nu a_1a_2 + (b_1a_1 + b_2a_2)u \\
&= -(p_5 - \frac{p_6^2}{p_7})\nu a_1^2 - p_7\nu(a_2 - \frac{p_6}{p_7}a_1)^2 + (b_1a_1 + b_2a_2)u \\
&\leq -W(b_1a_1 + b_2a_2)sign(b_1a_1 + b_2a_2).
\end{aligned}$$

Clearly $\dot{V} < 0$ for $(a_1, a_2) \neq (0, 0)$ and $\dot{V}(0, 0) = 0$. Therefore, $\dot{V}$ is negative definite and V is a Lyapunov function for the system (4.1) with controller 4.4. Thus, the controller given by (4.4) guarantees the asymptotic stability of the forced system given by (4.1). □

Remark 4.1. It can be shown that using the hyperbolic tangent instead of the sign function in (4.4) will also give good results (i.e., $u = -Wtanh(b_1a_1 + b_2a_2)$).

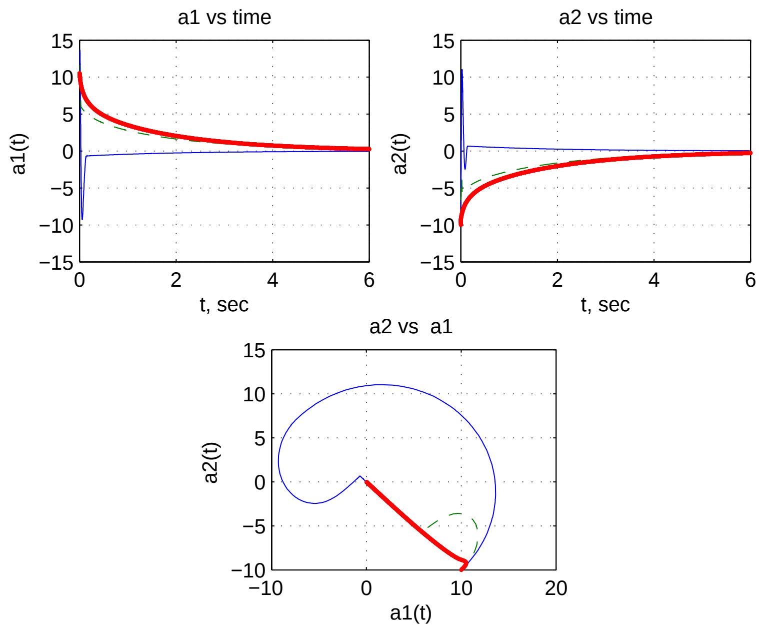

Figure 4.2: The profiles of the data coefficients $a_1(t)$ and $a_2(t)$ with the nonlinear controller, the solid line corresponds to $W = 100$; the '- - -' line corresponds to $W = 500$; and the bold line corresponds to $W = 1000$.

The forced system of ODEs given by (3.6) with the controller given by (4.4) is simulated using the Matlab ODE solver. The parameters b_1 and b_2 are taken to be $b_1 = b_2 = 1$. The initial conditions are chosen to be $a_1(0) = 10$ and $a_2(0) = -10$. The hyperbolic tangent is used in the simulations.

Figure 4.2 shows the simulation results of the system of ODEs when the proposed nonlinear controller is used. In Figure 4.2, the coefficients $a_1(t)$ and $a_2(t)$ versus time and the coefficients $a_1(t)$ versus $a_2(t)$ are shown for three cases corresponding to three different gains, $W = 100$, $W = 500$, and $W = 1000$. Hence, it is clear from the figure that these data coefficients are controlled to perform in a desired manner. Also, one can see from the figure that the larger the gain W of the controller, the faster the coefficients $a_1(t)$ and $a_2(t)$ converge to zero. Obviously, the larger the gain W the bigger the control actions are going to be.

Remark 4.2. The linear and nonlinear controllers are designed to drive the coefficients $a_1(t)$ and $a_2(t)$ to $(0, 0)$. By using proper change of variables, it can be easily shown that the steady states values of a_1 and a_2 can take any desired values.

5 Concluding Remarks

This paper addresses the control problem of the GKdVB equation. The numerical solutions of the nonlinear PDE is obtained using a pseudo-spectral Galerkin method. Then, coherent structures of the solutions are obtained using the K-L decomposition method. It is shown that for the case when $\alpha = 2$, only two eigenfunctions are necessary to capture the large scale dynamics of the equation. Applying the K-L Galerkin projection of the numerical solution data on the most energetic eigenfunctions results in a system of ODEs that mimics the dynamics of the original PDE. The obtained system of ODEs is stable but it converges slowly to $(0, 0)$. A linear and a nonlinear control schemes are proposed to speed up the convergence to the steady states. It is proven that both control schemes guarantee the asymptotic stability of the states of the forced system of ODEs. Moreover, numerical simulations show the effectiveness of proposed controllers.

It should be mentioned that, even though the paper concentrates on the case of $\alpha = 2$, similar results can be obtained for other integer values of α.

For future research, we will address the design of other controller such as adaptive and optimal control schemes to the GKdVB equation and for different integer values of α.

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