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# Performance Assessment of Feasible Scheduling Multiprocessor Tasks Solutions by using DEA FDH method

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**Abstract:** In this paper, an attempt has been made to investigate how DEA FDH method based on linear programming can select one or more efficient scheduling solutions on multiprocessor tasks obtained by any heuristic algorithms through some feasible solutions for NP-complete problems. This article will consider the problem of scheduling multiprocessor tasks with multi–criteria, namely, minimizing total completion time (makespan) and minimizing the number of tardy tasks and shows that most efficient schedule(s) will be determined.

**Keywords**: DEA, FDH, NP-complete, makespan, Multiprocessor tasks, feasible solution, efficient schedule.

## INTRODUCTION

New science of DEA based on linear programming was introduced by Charnes, Cooper, and Rhodes (CCR) (1978). In this science the goal is decision making, so the decision making units (DMUs) are the principles. Actually, by consuming suitable input(s) and output(s), DMUs are formed. Every feasible solution of NP- complete problem can be considered as DMU within which one or more efficient solution must be identified. As a matter of fact its the weakness that convex DEA method or CCR does not observe inefficient solutions but the non-convex DEA method i.e. FDH presented earlier by Deprins et al.(1984) observes hidden inefficient and refers them to efficient solution set. There are many case studies that are accomplished by means of DEA method in DMUs. For example, Tulkens(1993) extended FDH and used in retail banking, courts, and urban transit. Also Kuosmanen and Post(2002) applied another kind of DEA-quadratic for evaluating economical efficiency and Pendharkar (2005) proposed a DEA based approach for data processing.

In computer science we can use DEA method for the assessment of NP-Complete problem solutions. NP-Complete are those problems that according to input parameter (n), there is no polynomial time algorithm for them(Cormen et al.,2009); for example traveler salesperson problem(Neopolitan and Naimipour,2009), kanapsack (0/1), and problem of scheduling multiprocessor tasks specially by adding some criteria to the problem like minimizing the mean of execution time, minimizing the mean weighted

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of execution time etc(Drozdowski et al,1999). The complete execution of NP problems wastes a very long time on computers even for years. To solve this problem we can use heuristic Algorithms. The heuristic algorithms produce feasible solutions but the goal is to select the most efficient ones. In this article, the problem of scheduling multiprocessor tasks in order to minimizing execution time (makespan) and minimizing a number of tardy tasks along with some feasible solution obtained by any heuristic algorithms are taken in to account. This paper shows that how DEA FDH method selects the most efficient solutions. In the second part we are going to present DEA FDH method and in the third part we will discuss the problem of scheduling multiprocessor tasks.So remarkable results will be shown in conclusion part.

### *2. Literature Review:*

DEA method is used for decision making. Suppose that we have $n$ decision making units (DMUs) and for each DMU we consume $p$ parameters as input(s) and $q$ parameters as output(s). For example $DMU_k$ is $(x_k, y_k)$ where $\mathbf{x}_k = (x_{1k}, \ldots, x_{pk})$ and $\mathbf{y}_k = (y_{1k}, \ldots, y_{qk})$. As CCR model (Amin and Hosseini-shirvani,2009) dose not observe some inefficient units we use FDH model. Non-convex FDH model is defined for $k$th DMU as below (Charnes et al,1978)

$$\begin{aligned}
\theta_k^* &= \min \theta \\
&s.t. \\
&-\sum_{j=1}^{n} x_{ij}\lambda_j + x_{ik}\theta \geq 0 \quad i = 1, \ldots, p \\
&\sum_{j=1}^{n} y_{rj}\lambda_j \geq y_{rk} \qquad r = 1, \ldots, q \\
&\lambda_j \geq 0, \lambda_j \in \{0,1\} \quad j = 1, \ldots, n
\end{aligned}$$

Despite CCR, in FDH model $\lambda_i$ is binary (0 or 1). Aforementioned model will rank DMUs according to their efficiencies. In the following section new application of DEA FDH on specific problem will be explained.

### *3. Assessment For Feasible Solutions On Scheduling Multiprocessor Tasks:*

Parallel processing has committed a very extensive growth in computer and industrial engineering(Drozdowski et al,1999). The most important parameter is its speed or performance. Therefore, the need for optimal parallel algorithm is thoughtful (Drozdowski,1996). The problem of scheduling multiprocessor tasks are denoted by $\alpha|\beta|\delta$ notation(Blazewicz et al.,1996; Veltman et al.,1990) where

α is processor's characteristics,

β is the time needed for execution,

and δ is multi-criteria.

For instance, $P2|pmtn, r_j|\sum c_j$ (Du et al,1988), $P2|size_j|\sum w_j c_j$ (Labetoulle et al,1984), $P|p_j = 1, r_j, d_j|\sum c_j$ (Simon, 1983) etc. The problem of $P2|pmtn, r_j|\sum c_j$ means that we have two identical uniform processors and task division is forbidden and all tasks have different release time. Consequently, the criterion is to minimize makespan. All problems aforesaid are NP-Complete (Drozdowski et al,1999). In the circumstance above, we have to use heuristic algorithms and the objective is to select the most efficient feasible solution(s). Assume that set $N$ includes $n$ tasks as $N = \{t_1, t_2, \dots, t_n\}$ which are to be executed on $p$ identical parallel processors. Every task may need one or more processors simultaneously. Our problem is $P2|size_j, r_j = 0, d_j|\sum c_j and \sum u_j$ where,

(i) Every $task_j$ has execution time $size_j$.
(ii) Tasks are independent.
(iii) Task division is forbidden (McNaughton, 1959).
(iv) All tasks are ready at time=0.
(v) Every $task_j$ has due date as $d_j$.

Objectives are to minimize makespan and the number of tardy tasks. Makespan is computed as:

$$makespan = C_{max}^{s} = \{C_j^s : j = 1, \dots, n\}$$

Whereas the number of tardy tasks is calculated as:

$$\mathrm{N}_t^S = \sum_{j=1}^{n} u_j^S$$

where,

$$u_j^S = \begin{cases} 1 & \text{if } c_j^S > d_j \\ 0 & \text{otherwise} \end{cases}$$

Suppose that we have two identical parallel processors with four tasks indicated in Table 1:

**Table 1**. Task Characteristics

| *Task No.* | $size_j$ | *due date* ( $d_j$) |
|---|---|---|
| $t_1$ | 1 | 2 |
| $t_2$ | 1 | 2 |
| $t_3$ | 2 | 3 |
| $t_4$ | 3 | 4 |

By means of four different heuristic algorithms four feasible schedules have been made as shown in Figure 1. Notice that tardy tasks are highlighted.

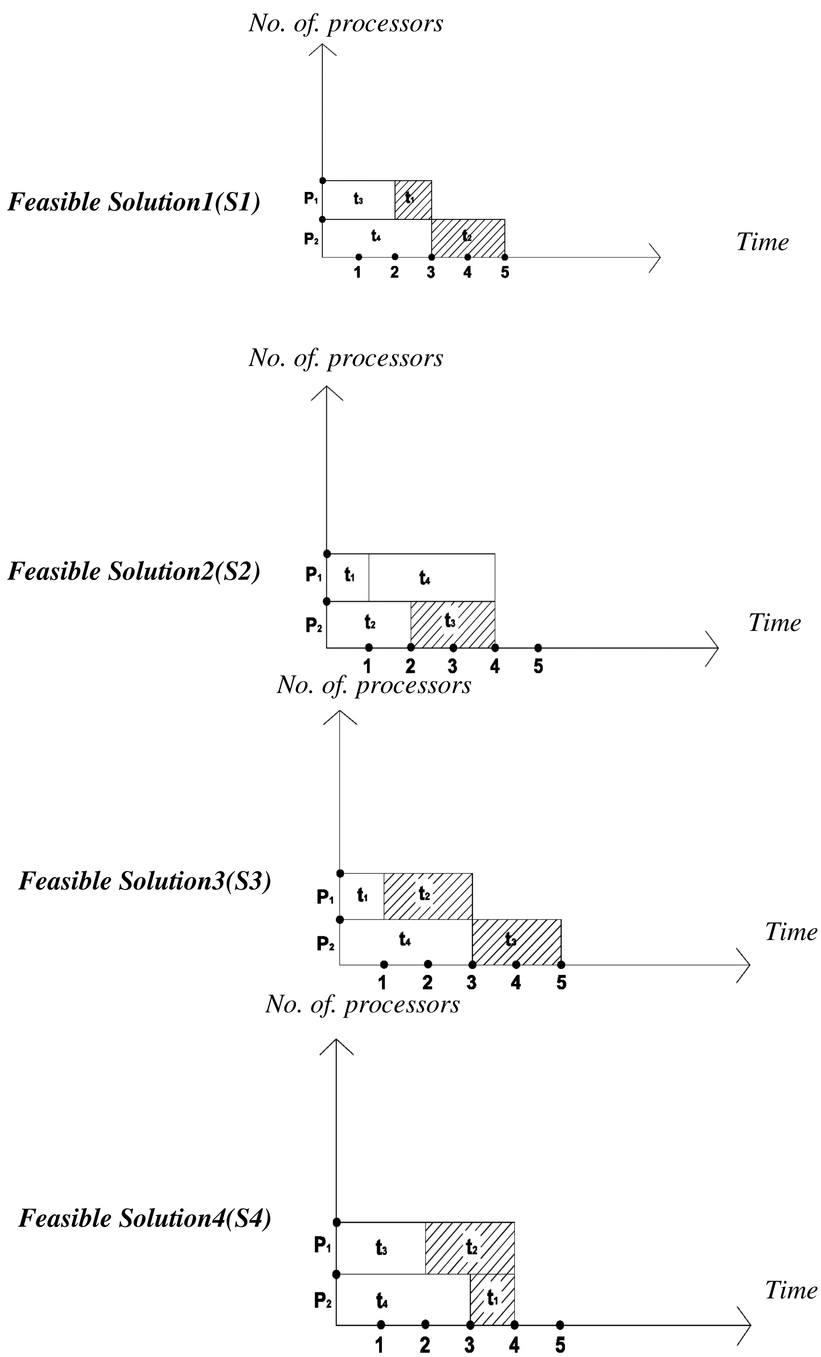


**Figure 1**. Feasible Schedules

Every feasible solution is DMU. Based on multi- criteria we can construct input(s) and output(s) for each DMU; for example we can consider two inputs like the number of tardy tasks as $I_1$ and makespan as $I_2$ whereas only one output is the number of tasks without tardiness as $O_1$. Every DMU traits for our problem are illustrated in Table 2:

**Table 2**.Inputs and Output for DEA

| *DMU No.* | *Tardy tasks ($I_1$)* | *Makespan ($I_2$)* | No. of tasks Without Tardiness ($O_1$) |
|---|---|---|---|
| Solution1(S1) | 2 | 5 | 2 |
| Solution2(S2) | 1 | 4 | 3 |
| Solution3(S3) | 2 | 5 | 2 |
| Solution4(S4) | 2 | 4 | 2 |

Now, among some feasible scheduling solution, we compute the efficiency of $DMU_4$ by using DEA FDH method as:

$$\begin{aligned}
\theta_4^* &= \min \theta \\
& s.t. \\
& -2\lambda_1 - \lambda_2 - 2\lambda_3 - 2\lambda_4 + 2\theta \geq 0 \\
& -5\lambda_1 - 4\lambda_2 - 5\lambda_3 - 4\lambda_4 + 4\theta \geq 0 \\
& 2\lambda_1 + 3\lambda_2 + 2\lambda_3 + 2\lambda_4 \geq 2 \\
& \lambda_1, \lambda_2, \lambda_3, \lambda_4 \in \{0,1\}
\end{aligned}$$

Regarding the above problem solution in DEA-solver provided by (Cooper et al,2006) the following optimal solution will be obtained:

$$\left(\lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_4^*, \theta_4^*\right) = (0,1,0,0,1)$$

It means that this method will consider second feasible scheduling solution (S2) as a reference set. If the above method is operated on the other units, all of them will converge to S2 as an optimal solution set.

***Conclusion:***

In this paper new application of DEA FDH method has been investigated, as though for NP – complete problems like scheduling multiprocessor tasks with multi-criteria feasible solutions given by heuristic algorithms are available and finally optimal solution have been identified by this method.

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