

A Statistical Chaotic Multifractal Framework for Change-Point Detection and Fault Severity Estimation in Industrial Time Series

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Abstract: Industrial vibration signals can be represented as nonstationary stochastic processes whose dependence structure and scaling behavior evolve during mechanical degradation. This study proposes a statistical chaotic-multifractal framework for fault detection, change-point identification, and severity estimation in industrial time-series signals. The framework combines Multifractal Detrended Fluctuation Analysis (MFDFA) with nonlinear dynamical descriptors to quantify changes in scaling heterogeneity, temporal dependence, dynamical instability, and attractor geometry. Generalized Hurst exponents, mass exponent functions, and multifractal spectra are estimated from successive vibration windows. Phase-space reconstruction is subsequently used to estimate the Largest Lyapunov Exponent and Correlation Dimension. Because these estimators are obtained from dependent and nonstationary observations, their sampling uncertainty is evaluated using moving-block bootstrap confidence intervals. Differences between healthy, intermediate, and near-failure operating states are assessed using resampling-based hypothesis tests, effect sizes, and multiplicity-adjusted significance levels. Rather than combining the extracted measures through an unweighted product, a standardized Chaotic Fractal Spectrum Index (CFSI) is constructed as a statistically estimated linear combination of multifractal spectrum width, Largest Lyapunov Exponent, and Correlation Dimension. Its coefficients are estimated using regularized regression on the training data. A sequential change-point procedure is then applied to the resulting index to identify statistically significant departures from the healthy-state distribution. The framework is evaluated using the NASA IMS bearing run-to-failure vibration dataset. The estimated multifractal spectrum width increases from 0.42 in the healthy state to 1.67 near failure, while the Largest Lyapunov Exponent increases from 0.012 to 0.132 and the Correlation Dimension from 2.10 to 3.89. The estimated CFSI consequently displays a systematic increase with degradation. An out-of-sample power-law severity model achieves an R^2 of 0.973, a correlation coefficient of 0.987, an RMSE of 0.028, and an MAE of 0.021. Bootstrap confidence intervals and change-point results provide inferential evidence that the observed increases are associated with fault progression rather than ordinary sampling variability. The proposed framework provides an interpretable statistical representation of degradation dynamics by integrating fractal estimation, uncertainty quantification, hypothesis testing, and sequential fault detection. It therefore offers a mathematically grounded approach to condition monitoring and predictive maintenance in rotating machinery.

Keywords: Industry, Innovation and Infrastructure (SDG 9); Industrial Innovation; Statistical Fault Detection; Predictive Maintenance; Condition Monitoring; Industrial Time-Series Analysis; Multifractal Analysis; Industrial Development in Al-Kharj.

1 Introduction

Bearings, gearboxes, motors, and rotating equipment are essential components of industrial machinery, which is vital for modern manufacturing systems [1]. These

components need to be reliable to maintain production continuity, reduce maintenance costs, and avoid unexpected failures [2]. Therefore, industrial fault diagnosis is one of the research focus areas in the predictive maintenance frameworks [3]. Among various

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monitoring techniques, vibration analysis is widely adopted because vibration signals contain valuable information regarding machine health and fault development [4]. By continuously analyzing vibration patterns, maintenance personnel can detect abnormalities at an early stage and schedule maintenance activities before severe damage occurs [5].

The traditional fault diagnosis techniques are frequency-domain analysis, statistical features and machine learning techniques [6]. Fourier Transform (FFT)-based methods are able to find characteristic frequencies of the fault, but require the signals to be stationary and are not able to capture all nonlinear signals [7]. Wavelet-based methods offer improved time–frequency analysis of nonstationary signals but require choosing appropriate wavelet functions and decomposition levels [8]. Entropy-based methods quantify the complexity of the signal, but may not capture the localized structural changes [9]. Although deep learning models have shown high diagnostic accuracy, they also require large amounts of labeled data, consume significant computational resources, and are often black-box systems lacking interpretability and insight into the underlying fault mechanisms [10, 11].

Industrial vibration signals during fault evolution are nonlinear, nonstationary and dynamically complex. They exhibit multiscale self-similarity, non-linear fluctuations and changing complexity, not captured by conventional methods [12]. Fractal theory identifies scale-dependent structures and multifractal analysis refers to the heterogeneous local dynamics [13]. Chaotic behaviour emphasises nonlinear transitions and the combination of both enhances the understanding of fault progression [14]. This motivates the development of a chaotic multifractal spectrum framework that quantifies the evolution of signal complexity, characterizes the nonlinear behavior, and enables meaningful relationships between the fractal properties and the severity of the fault in the industrial system.

Recent research integrates fractal, multifractal and chaotic approaches with machine learning techniques for fault detection, enhancing both accuracy and robustness [15]. To overcome the issue of redundancy and mode confusion of rolling bearing signals, Zhao et al. [16] proposed a fault diagnosis framework by combining Complete Ensemble Empirical Mode Decomposition with Adaptive Noise, fractal dimension, PCA, ICEEMDAN and CNN to extract features from the signals and diagnose faults with 99.79% accuracy, with better feature extraction and diagnosis reliability. Similarly, Yang et al. [17] proposed uniform phase decoupled iterative filtering (IF) and multi-scale sliding fractal box dimension (UPDIF-MSFBD) for nonlinear and noisy vibration signals, which can effectively extract multiscale fractal temporal features and increase the accuracy of fault diagnosis. However, these two strategies are complex multi-stage decomposition and have high computational requirements, which is difficult to realize

the industrial application in real time under different working conditions.

Akalın [18] developed a hybrid ANN-based fault detection system optimized using Invasive Weed Optimization with fractal-based weighting to overcome local minima and to increase the global search capability, and it had an accuracy of more than 98%, with a good generalization performance on Kaggle datasets. Similarly, Yang et al. [19] proposed fractal chaotic maps, in which the concepts of Julia fractals were employed and multi-level cascaded chaotic systems were used with the aim of improving the complexity, randomness and dynamical behavior of the systems using entropy analysis and bifurcation diagram, showing better results. Both, however, have high computational complexity and have only limited real-time industrial application.

Alhazmi and Saber [20] analyzed the fractional Lorenz–Lü–Chen system with the Liouville–Caputo kernel and Variational iteration method, and showed that the system undergoes a chaotic behavior with memory-effect modeling shown by Lyapunov analysis. Likewise, Yang et al. [21] suggested a fault signal detection system for weak wind turbine based on chaotic circuit of memristor, which has high noise immunity and can detect the fault signal accurately and enhance the fault signal by the chaotic resonance. Both studies, however, are heavily dependent on simulation-based validation and have high mathematical and computational complexity for real-time use.

Medina et al. [22] proposed a multi-fractal detrended fluctuation analysis-based fault classification framework for reciprocating compressors and centrifugal pumps, and the random forest subspace discriminant and CNN methods successfully classified multiple fault categories with high accuracy based on vibration signals. Wen [23] also proposed a multi-fractal harmonic signal analysis approach that combined multi-fractal detrended fluctuation models, wavelet transform and empirical mode decomposition, which enhanced the performance by 5–10%, with high stability and efficiency. However, these two methods require extensive computation in the feature extraction process and have limited real-time industrial validation under different operational conditions.

Almeida et al. [24] combined chaos theory, multifractal analysis and machine learning to tackle the problem of bearing fault diagnosis, with a controlled classification accuracy of 100%, realistic generalization of 83.2% and robustness to noise, missing data and sensor failures with real-time processing. Likewise, Xiong et al. [25] proposed Multifractal Spectrum Transformer-Transformer with Learnable Multifractal Spectrum for chaotic system classification, which is better than CNNs and RNNs for low SNR. In resource-limited industrial systems, however, both methods are highly resource intensive and involve a lot of parameter optimization.

Gu and Chan [26] introduced a Chen-Lee chaotic system based on chaotic gravity distances and fractal dimension for partial discharge pattern recognition in power cable joints, which not only had high recognition accuracy but also showed strong anti-noise ability. Similarly, Moreno-Gomez et al. [27] studied the performance of Katz, Higuchi, Box, Petrosian and Sevcik fractal dimensions for vibration-based damage detection in bridges, and found that Higuchi and Petrosian fractal dimensions provided better discrimination efficiency. Both studies, however, rely largely on handcrafted features and controlled experimental conditions, which hinders the adaptability in real-world complex environments.

Industrial time-series signals from rotating machinery show evolution of nonlinear, nonstationary, multifractal, and chaotic behaviours as the fault progresses [28]. However, the traditional fault diagnosis approaches, such as spectral analysis, statistical indicators, entropy measures, and machine learning algorithms, are not capable of detecting certain features of faults, including self-similar structures, heterogeneous scaling properties, and dynamic geometric variations [29], [30]. Furthermore, the single-fractal measures give only global complexity estimates, and are unable to describe multifractal degradation processes. Consequently, early fault signatures can be easily missed and accurate fault progression and fault severity assessments will be restricted [31]. Thus, a comprehensive chaotic multifractal spectrum framework that could capture scale-dependent fluctuations, quantify nonlinear dynamic transitions, and provide a mathematical connection between multifractal complexity and fault progression is needed. This would give better understanding of degradation processes, and facilitate more accurate fault severity evaluation of industrial systems. The main contribution of the study are as follows:

A novel Chaotic Fractal Spectrum Analysis framework is developed to jointly model multifractal scaling behavior and chaotic dynamics for comprehensive characterization of bearing degradation processes.

Multifractal Detrended Fluctuation Analysis is employed to investigate evolving scaling properties and multifractal structures, enabling detailed assessment of vibration signal complexity during fault progression.

Fractal spectrum features, including spectrum width, peak location, and asymmetry, are integrated with Lyapunov exponent and correlation dimension to capture nonlinear dynamic characteristics.

A new Chaotic Fractal Spectrum Index is proposed by combining multifractal and chaotic descriptors, providing a unified indicator of fault-induced complexity and instability.

A fault severity scaling law is established to quantitatively relate CFSI values to bearing degradation levels, enabling interpretable health assessment and progression monitoring.

2 Multifractal and Chaotic Characterization of Bearing Vibration Signals

Bearing vibration signals generated during degradation are highly nonlinear, nonstationary, and multiscale in nature. The traditional time-domain and frequency-domain analysis methods are not suitable for capturing the complex dynamic behaviours that are related to the evolution of faults. The Fractal theory and the Chaos theory are two strong mathematical approaches for studying these characteristics by quantifying the scaling heterogeneity, self-similarity, dynamic instability and attractor complexity that can be found in the vibration signals. Thus, this section introduces the mathematical foundations used in the proposed study for characterizing bearing degradation. Firstly, the Multifractal Detrended Fluctuation Analysis is employed to investigate the scale dependent fluctuations and estimate the generalized Hurst exponent. Then, the multifractal spectrum analysis is carried out to extract the mass exponent, the singularity strength, and the multifractal spectrum, which are important fractal descriptors. The chaotic dynamics analysis is finally performed using phase space reconstruction, Largest Lyapunov Exponent estimation and Correlation Dimension computation to characterize the nonlinear dynamic behavior of the vibration signals. The resulting multifractal and chaotic features give a comprehensive description of the geometric and dynamic complexity of the bearing fault progression.

2.1 Multifractal Detrended Fluctuation Analysis

The proposed Chaotic Fractal Spectrum Analysis (CFSa) framework is a signal-processing and feature-extraction framework that is unified and has the potential of providing accurate fault detection and severity assessment for industrial bearing systems. The framework systematically combines fractal geometry and nonlinear chaotic dynamics to analyse the vibration signals from the NASA IMS Bearing Dataset obtained under run-to-failure conditions. First, raw vibration signals are acquired and then preprocessed, such as normalization, detrending and segmentation, to reduce noise and stabilize the scaling behavior. Cleaned signal is further processed by Multifractal Detrended Fluctuation Analysis to capture the scale dependent fluctuations and long-range correlations. From this stage, the generalized Hurst exponent and multifractal spectrum are obtained, which can be used to extract some important fractal descriptors including the width of the spectrum ($\Delta\alpha$), the dominant singularity (α_0) and the asymmetry index (A). Chaotic dynamics features are also extracted with complementation to fractal analysis by Phase space reconstruction, Largest Lyapunov exponent (λ), and correlation dimension (D_2) which are used to measure the instability of the system and the complexity of the

attractor. These two feature domains are combined to create the Chaotic Fractal Spectrum Index, which gives a representation of the nonlinear behaviour of the signal. Last, a non-linear scaling law correlates CFSI with a quantitative fault severity index that is used to categorize the health condition of the bearing from healthy to near-failure condition. This framework provides strong, informative and physically meaningful fault diagnosis.

The long-range correlation, scaling and multifractal properties of bearing vibration signals are quantified using multifractal Detrended Fluctuation Analysis. MFDFA examines the evolution of fluctuations over a range of temporal scales and fluctuation magnitudes, in contrast to traditional statistical measures that only yield a single estimate of complexity. Therefore, the method can be used to detect hidden multifractal structures that are linked with degradation of the bearing and progression of the fault. Let the preprocessed vibration signal be represented as $x(i), i=1, 2, \dots, N$ where $x(i)$ denotes the vibration amplitude and N denotes the total number of samples. The first step is to compute the mean value of the signal: $\hat{x} = \frac{1}{N} \sum_{i=1}^N x(i)$ and construct the cumulative profile $Y(i) = \sum_{k=1}^i [x(k) - \hat{x}]$. The profile transformation renders the original vibration signal to be a random-walk process, and allows for reliable scaling analysis for nonstationary data. The profile is subsequently divided into non-overlapping segments of equal length s .

$$N_s = \left\lfloor \frac{N}{s} \right\rfloor \quad (1)$$

Eqn. (1) s denotes the observation scale and N_s denotes the number of segments. For each segment v , a polynomial trend $Y_v(i)$ is estimated using least-squares regression and removed. The local fluctuation variance is then computed as Eqn. (2).

$$F^2(v, s) = \frac{1}{s} \sum_{i=1}^s [Y((v-1)s+i) - Y_v(i)]^2 \quad (2)$$

Where $F^2(v, s)$ denotes the detrended fluctuation variance for the v th segment at scale s , $Y((v-1)s+i)$ represents the integrated vibration signal within the segment, The term $Y((v-1)s+i) - Y_v(i)$ measures the deviation of the signal from its local trend.

The fluctuation variances from all segments are combined to obtain the q -order fluctuation function represented in Eqn. (3).

$$F_q(s) = \left[\frac{1}{N_s} \sum_{v=1}^{N_s} (F^2(v, s))^{\frac{q}{2}} \right]^{\frac{1}{q}}, q \neq 0 \quad (3)$$

For $q=0$, $F_0(s) = \exp \left[\frac{1}{2N_s} \sum_{v=1}^{N_s} \ln F^2(v, s) \right]$. The parameter q regulates the amount of fluctuations of various sizes. A positive value of q will highlight large fluctuations, while a negative value will highlight small

fluctuations. The fluctuation function follows a power-law scaling relationship $F_q(s) \sim s^{h(q)}$, where $h(q)$ denotes the generalized Hurst exponent. Taking logarithms on both sides gives Eqn. (4).

$$\ln F_q(s) = h(q) \ln s + C \quad (4)$$

Therefore, it is represented in Eqn. (5).

$$h(q) = \frac{d \ln F_q(s)}{d \ln s}. \quad (5)$$

This derivative form is important because the generalized Hurst exponent is derived from the slope of the $\ln F_q(s)$ versus the $\ln s$ curve. For monofractal signals, $h(q) = H$, where H is constant for all values of q . For multifractal signals, $\frac{dh(q)}{dq} \neq 0$, indicating that different fluctuation magnitudes possess different scaling exponents. The degree of multifractality can therefore be quantified through Eqn. (6).

$$\Delta h = h(q_{\min}) - h(q_{\max}) \quad (6)$$

Where larger values of Δh indicate stronger multifractal behavior and greater signal complexity. $h(q_{\min})$ represents the generalized Hurst exponent corresponding to the minimum moment order $q_{\min}$. $h(q_{\max})$ represents the generalized Hurst exponent corresponding to the maximum moment order $q_{\max}$. For healthy bearings, the difference in $h(q)$ between different moment orders is typically not that large, and the multifractality is weak. As degradation proceeds, the nonlinear interactions and impulsive fault-related vibrations grow, resulting in greater variations of $h(q)$. Consequently, $\left| \frac{dh(q)}{dq} \right|$ increases, reflecting stronger multifractal characteristics.

The final output of the MFDFA stage is the generalized Hurst exponent spectrum $h(q)$, which serves as the fundamental scaling descriptor for subsequent multifractal spectrum analysis. The obtained $h(q)$ values are transformed into the mass exponent function denoted as Eqn. (7).

$$\tau(q) = qh(q) - 1 \quad (7)$$

From which the singularity strength and multifractal spectrum are later derived through $\alpha = \frac{d\tau(q)}{dq}$ and $f(\alpha) = q\alpha - \tau(q)$. Therefore, MFDFA provides a mathematical basis for quantifying the multifractal geometry and scaling heterogeneity of the vibration signals from the bearing during the process of the fault evolution.

2.2 Multifractal Spectrum Analysis

The generalized Hurst exponent $h(q)$ extracted by Multifractal Detrended Fluctuation Analysis quantifies the behavior of fluctuations as a function of different

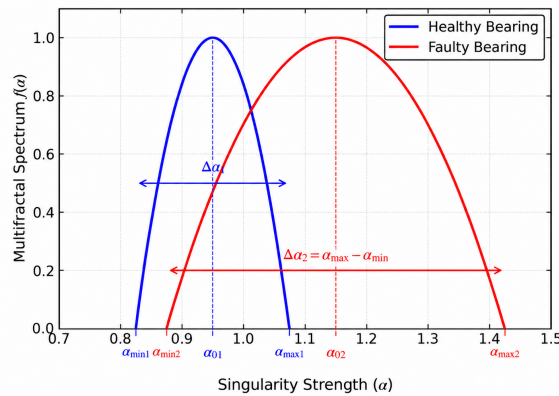


Fig. 1: Multifractal Spectrum Comparison of Healthy and Faulty Bearings

moment orders. But the local singular structures in vibration signals cannot be completely described by $h(q)$. In the case of the bearing degradation, the vibration signal shows heterogeneous scaling behavior caused by nonlinear interaction, impulsive shocks, and irregularities of fault. So, the global scaling information given by $h(q)$ is transformed to local singularity information by using multifractal spectrum analysis. The transformation is accomplished using the mass exponent function, singularity strength estimation and Legendre transformation, and is the multifractal spectrum $f(\alpha)$.

Figure 1 shows the conceptual multifractal spectrum used to extract descriptors for analyzing equipment health. In a healthy bearing, the spectrum is narrower with smaller $\Delta\alpha$. But when there's a fault, the spectrum broadens, and $\Delta\alpha$ gets bigger. This signals increased complexity and heterogeneity. The descriptors $\alpha_{\min}$, $\alpha_{\max}$, α_0 , and $\Delta\alpha$ are extracted from the spectrum for subsequent degradation analysis.

The input to this stage is the generalized Hurst exponent spectrum $h(q)$ obtained from the MFDFA scaling relation denoted in Eqn. (8).

$$F_q(s) \propto s^{h(q)} \quad (8)$$

Where $F_q(s)$ denotes the q -order fluctuation function and s represents the observation scale. The first step is to calculate the mass exponent function, to determine the relationship between the statistical moments and the scaling behaviour of the signal. The mass exponent of one dimensional vibration signals is defined as Eqn. (9).

$$\tau(q) = qh(q) - 1 \quad (9)$$

Where $\tau(q)$ denotes the mass exponent function, q is the moment order, and $h(q)$ is the generalized Hurst exponent. The mass exponent reflects the relative importance of the different sizes of fluctuations to the scaling. For a monofractal signal, $h(q)$ remains constant

and consequently $\tau(q)$ varies linearly with q . A nonlinear $\tau(q)$ curve corresponds to a multifractal signal, which has more than one scaling exponent. The deviation from linearity of $\tau(q)$ is generally increased with the severity of bearing fault, because the irregular vibration bursts and energy redistribution caused by bearing fault are nonlinear. The singularity strength α is obtained by differentiating the mass exponent function with respect to the moment order q , and is a measure of local scaling behavior shown in Eqn. (10):

$$\alpha = \frac{d\tau(q)}{dq} \quad (10)$$

Substituting $\tau(q) = qh(q) - 1$ gives $\alpha = \frac{d}{dq} [qh(q) - 1]$. Applying the product rule, $\alpha = h(q) + q \frac{dh(q)}{dq}$, where $\frac{dh(q)}{dq}$ represents the first derivative of the generalized Hurst exponent.

The proposed framework is based on this derivative, which is one of the most significant quantities for the rate of variation of the scaling exponent across orders of fluctuation. If the vibration signal exhibits homogeneous scaling behaviour, then $h(q)$, will slowly change and its derivative will be small. Conversely $h(q)$, is highly nonstationary when generated by fault, which leads to larger values of the derivative and wider distributions of the singularities.

The singularity strength α is the local Hölder exponent of the signal, and quantifies the strength of the local scaling. Smaller values of α correspond to very irregular areas, which are related to impulsive fault events, while larger values are related to less irregular signal areas. Thus, the spread of the α values is a measure of the degree of heterogeneity in the vibration signal. The mass exponent function is used in the Legendre transform to obtain the multifractal spectrum, which gives the frequency of occurrence of each singularity strength shown in Eqn. (11):

$$f(\alpha) = q\alpha - \tau(q) \quad (11)$$

Substituting the expression for $\tau(q)$, $f(\alpha) = q\alpha - [qh(q) - 1]$, which becomes $f(\alpha) = q\alpha - qh(q) + 1$, where $f(\alpha)$ represents the fractal dimension associated with all subsets characterized by singularity strength α .

The Legendre transformation allows the mapping of moment-order space to singularity space, which allows direct visualisation of the distribution of local scaling exponents. The resulting multifractal spectrum is usually parabolic in shape with the peak at the maximum fractal dimension, in a vibration analysis. This curve is a good indicator of fault evolution, particularly its width and shape. An important mathematical relationship associated with the Legendre transformation is denoted in Eqn. (12).

$$\frac{df(\alpha)}{d\alpha} = q \quad (12)$$

Which implies that the slope of the multifractal spectrum is related to the moment order. Furthermore, $\frac{d^2 f(\alpha)}{d\alpha^2} < 0$, the characteristic concave shape of the experimental vibration signals is ensured for a valid multifractal spectrum. The spectrum boundaries are defined by $\alpha_{\min} = \min(\alpha)$ and $\alpha_{\max} = \max(\alpha)$, which correspond to the strongest and weakest singularities present in the signal. The spectrum peak occurs at $\alpha_0 = \arg \max_{\alpha} f(\alpha)$, with $f(\alpha_0) = \max f(\alpha)$. The complete multifractal transformation used in the proposed study can therefore be represented as Eqn. (13).

$$h(q) \rightarrow \tau(q) \rightarrow \alpha \rightarrow f(\alpha) \quad (13)$$

Alternatively, explicitly,
 $h(q) \rightarrow [qh(q) - 1] \rightarrow \left[h(q) + q \frac{dh(q)}{dq} \right] \rightarrow [q\alpha - \tau(q)]$.
 The final output of this stage is the multifractal spectrum $f(\alpha)$ together with the singularity distribution α from which the spectrum width $\Delta\alpha$, spectrum peak α_0 , and spectrum asymmetry A are subsequently extracted. These parameters are quantitative indicators of scaling heterogeneity and complexity due to the faults, and are the fractal part of the proposed CFSI.

2.3 Fractal Spectrum Feature Extraction

The quantitative fractal descriptors based on the multifractal spectrum $f(\alpha)$, are extracted to describe the distribution of fluctuations, structural complexity and scaling heterogeneity of the vibration signals of the bearings. The multifractal spectrum can be used to show the singularity distribution, but numerical descriptors are needed to quantify the progression of degradation and to make measurable relationships between the complexity of the signal and the evolution of the fault. Hence, three main multifractal characteristics are obtained: spectrum width ($\Delta\alpha$), spectrum peak (α_0), and spectrum asymmetry (A). The input for this stage is the multifractal spectrum from the MF DFA framework. The scaling relation is first obtained to calculate the generalized Hurst exponent denoted as Eqn. (14):

$$F_q(s) \sim s^{h(q)} \quad (14)$$

Where $F_q(s)$ denotes the q -order fluctuation function and $h(q)$ represents the generalized Hurst exponent. The mass exponent function is then computed as Eqn. (15).

$$\tau(q) = qh(q) - 1 \quad (15)$$

Which characterizes the scaling behavior of statistical moments across different fluctuation magnitudes. Multifractal systems are nonlinear, and the local singularity properties are derived by differentiating the mass exponent with respect to the moment order q , $\alpha = \frac{d\tau(q)}{dq}$. Substituting $\tau(q) = qh(q) - 1$,

$\alpha = h(q) + q \frac{dh(q)}{dq}$, where $\frac{dh(q)}{dq}$ represents the rate of variation of the generalized Hurst exponent with respect to the moment order. This derivative is particularly important as it captures the local variations in the scaling behavior and indicates the extent of multifractal heterogeneity in the signal. The multifractal spectrum is subsequently obtained through the Legendre transform denoted as Eqn. (16).

$$f(\alpha) = q\alpha - \tau(q) \quad (16)$$

Substituting the expressions for α and $\tau(q)$,

$$f(\alpha) = q \left[h(q) + q \frac{dh(q)}{dq} \right] - [qh(q) - 1],$$

which simplifies to

$$f(\alpha) = q^2 \frac{dh(q)}{dq} + 1.$$

From this expression, it is seen that the multifractal spectrum is directly proportional to the derivative of the generalized Hurst exponent. Thus, the multifractal spectrum's shape and width are controlled by the difference in $h(q)$ between various moment orders. The first feature extracted from the spectrum is the spectrum width, defined as Eqn. (17).

$$\Delta\alpha = \alpha_{\max} - \alpha_{\min} \quad (17)$$

Where $\alpha_{\max} = \max(\alpha)$ and $\alpha_{\min} = \min(\alpha)$. The spectrum width is the width of the spectrum of the singularities found in the vibration signal. It is physically the measure of the extent of multifractality and scaling heterogeneity. A narrow spectrum suggests that there is relatively uniform scaling behavior and low complexity, while a wider spectrum suggests that several scaling regimes exist and are related to nonlinear fault dynamics. The evolution of spectrum width can also be expressed through its temporal gradient shown in Eqn. (18).

$$\frac{d(\Delta\alpha)}{dt} = \frac{d\alpha_{\max}}{dt} - \frac{d\alpha_{\min}}{dt} \quad (18)$$

Which provides a direct measure of how rapidly multifractal complexity changes during degradation. As bearing fault progresses, impact events and nonlinear interactions intensify, which leads to an increase in singularity dispersion and spectrum width. Therefore, $\frac{d(\Delta\alpha)}{dt} > 0$ typically indicates progressive degradation. The second feature is the spectrum peak location, determined from the maximum value of the multifractal spectrum shown in Eqn. (19).

$$\alpha_0 = \arg \max_{\alpha} f(\alpha) \quad (19)$$

Such that $f(\alpha_0) = \max[f(\alpha)]$. The dominant singularity strength is represented by the peak location,

and it is the most probable local scaling exponent in the signal. Mathematically, the peak is identified by satisfying the stationary condition $\frac{df(\alpha)}{d\alpha} = 0$ and $\frac{d^2f(\alpha)}{d\alpha^2} < 0$, which ensures that α_0 corresponds to a maximum point. In healthy bearings, α_0 generally remains stable, indicating consistent scaling behavior. As degradation proceeds, the peak slowly moves because of the increasing structural irregularities and changing nonlinear interactions. The third feature is the spectrum asymmetry coefficient, defined as Eqn. (20).

$$A = \frac{\alpha_r - \alpha_0}{\alpha_0 - \alpha_l} \quad (20)$$

Where α_r is the right endpoint of the spectrum, α_l is the left endpoint of the spectrum, and α_0 is the peak location. The asymmetry coefficient describes the geometric shape of the multifractal spectrum, and is used to understand whether the dynamics of the signal are dominated by large or small fluctuations.

When $A > 1$ the spectrum is right-skewed, indicating dominance of weak singularities and small fluctuations. When $A < 1$ the spectrum is left-skewed, indicating stronger contributions from large fluctuations and impulsive events. When $A = 1$ the spectrum is symmetric, which is characteristic of balanced fluctuation behaviour. To further quantify asymmetry evolution during degradation, the rate of asymmetry variation can be expressed as Eqn. (21).

$$\frac{dA}{dt} = \frac{d}{dt} \left(\frac{\alpha_r - \alpha_0}{\alpha_0 - \alpha_l} \right) \quad (21)$$

Which captures changes in fluctuation dominance as faults progress. The complete fractal feature extraction process can therefore be summarized as Eqn. (22).

$$\begin{aligned} h(q) \rightarrow \tau(q) &= qh(q) - 1 \rightarrow \alpha = \frac{d\tau(q)}{dq} \rightarrow f(\alpha) \\ &= q\alpha - \tau(q) \rightarrow \{\Delta\alpha, \alpha_0, A\} \end{aligned} \quad (22)$$

The final output of this stage is the multifractal feature vector $F = \{\Delta\alpha, \alpha_0, A\}$, where $\Delta\alpha$ quantifies multifractal complexity, α_0 represents dominant scaling behavior, and A characterizes fluctuation asymmetry. These descriptors give a comprehensive mathematical description of the multifractal geometry of the bearing vibration signals and are basic inputs for the next chaotic dynamics analysis and proposed Chaotic Fractal Spectrum Index construction.

2.4 Chaotic Dynamics Analysis

Multifractal spectrum analysis describes the scaling heterogeneity and geometric complexity of vibration signals, but cannot directly describe the nonlinear dynamic mechanisms that lead to the degradation of the

bearing. Rolling-element bearings are nonlinear dynamical systems and the occurrence and development of defects lead to irregular oscillations, intermittent impacts and deterministic chaos. Conventional statistical or spectral measures are inadequate to characterize these phenomena. Thus, chaotic dynamics analysis is integrated to quantify the underlying nonlinear behavior of vibration signal by using Phase Space Reconstruction (PSR), Largest Lyapunov Exponent (LLE), and Correlation Dimension (D_2). The input to this stage is the preprocessed vibration signal.

The first step is to reconstruct the hidden dynamical system from the observed one dimensional vibration signal. According to Takens' embedding theorem, the dynamics of an unknown nonlinear system can be reconstructed in a multidimensional state space using delayed versions of the measured signal. The reconstructed state vector is defined as Eqn. (23).

$$\mathbf{X}(t) = [x(t), x(t+\tau), x(t+2\tau), \dots, x(t+(m-1)\tau)] \quad (23)$$

Where $\mathbf{X}(t)$ is the reconstructed phase-space vector, m denotes the embedding dimension, and τ represents the time delay. For the three-dimensional representation frequently employed for visualization, $\mathbf{X}(t) = [x(t), x(t+\tau), x(t+2\tau)]$.

A set of reconstructed vectors creates a trajectory in phase space called a chaotic attractor. The attractor shows the geometric structure of the underlying nonlinear system, which is preserved despite the change in the time domain signal. Compact and regular attractors are observed from healthy bearings, while the attractor patterns become more spread out and irregular with faulty bearings, because the nonlinear interactions between the vibration modes.

The reconstructed phase space is then used to calculate the Largest Lyapunov Exponent in order to measure the initial condition sensitivity of the system. Let two neighboring trajectories in phase space be separated initially by a distance δ_0 . As the system evolves, the separation becomes $\delta(t)$. For a chaotic system, the divergence between trajectories follows the exponential relationship denoted as Eqn. (24).

$$\delta(t) = \delta_0 e^{\lambda t} \quad (24)$$

Where λ is the Largest Lyapunov Exponent. Taking the natural logarithm yields $\ln \delta(t) = \ln \delta_0 + \lambda t$. Rearranging gives $\lambda = \frac{1}{t} \ln \left(\frac{\delta(t)}{\delta_0} \right)$. Therefore, the asymptotic Largest Lyapunov Exponent is defined as Eqn. (25).

$$\lambda = \lim_{t \rightarrow \infty} \frac{1}{t} \ln \left(\frac{\delta(t)}{\delta_0} \right) \quad (25)$$

The parameter λ is the average exponential growth rate of trajectory separation. The positive value represents deterministic chaos, as nearby trajectories separate in an exponential manner. As bearing defects grow, the number of impacts increases, leading to increased nonlinear

interactions and therefore larger values of λ . Thus, the Largest Lyapunov Exponent can be used as a quantitative measure for dynamic instability and fault development.

The next step is to determine the geometric complexity of the reconstructed attractor by using the Correlation Dimension. The Correlation Dimension quantifies the packing density of the attractor in the reconstructed phase space, and estimates the dimensionality of the nonlinear dynamics. The correlation integral is first calculated using the Grassberger–Procaccia algorithm:

$$C(r) = \frac{2}{N(N-1)} \sum_{i=1}^N \sum_{j=i+1}^N H(r - \|\mathbf{X}_i - \mathbf{X}_j\|_2) \quad (26)$$

In Eqn. (26), $C(r)$ is the correlation integral, r denotes the neighborhood radius, $\mathbf{X}_i$ and $\mathbf{X}_j$ are reconstructed state vectors, and $H(\cdot)$ represents the Heaviside step function,

$$H(z) = \begin{cases} 1, & z \geq 0, \\ 0, & z < 0. \end{cases}$$

The Euclidean distance between two phase-space points is given by Eqn. (27).

$$\|\mathbf{X}_i - \mathbf{X}_j\|_2 = \sqrt{\sum_{k=1}^m (X_{ik} - X_{jk})^2} \quad (27)$$

For chaotic attractors, the correlation integral exhibits a power-law scaling relationship: $C(r) \propto r^{D_2}$, where D_2 denotes the Correlation Dimension. Taking logarithms gives $\log C(r) = D_2 \log r + C_0$, where C_0 is a constant. Hence it is denoted as Eqn. (28),

$$D_2 = \lim_{r \rightarrow 0} \frac{\log C(r)}{\log r} \quad (28)$$

Alternatively, the Correlation Dimension can be interpreted as the slope $D_2 = \frac{d \log C(r)}{d \log r}$. This derivative form is significant since it is directly proportional to the rate of growth of the attractor's presence in phase space. The larger the value of D_2 , the larger will be the dimension of the attractor, and thus the complexity of the nonlinear dynamics. As the bearing degrades, the influence of the defects causes the attractor structure to become more irregular, leading to further growth of D_2 . The complete chaotic feature extraction procedure can therefore be represented as Eqn. (29).

$$x(t) \rightarrow \mathbf{X}(t) \rightarrow \lambda \rightarrow D_2 \quad (29)$$

The final output of chaotic dynamics analysis is the chaotic feature vector $\mathbf{C} = \{\lambda, D_2\}$, where λ quantifies dynamic instability through exponential trajectory divergence and D_2 measures attractor complexity through fractal dimensionality. The chaos-based descriptors, besides the multifractal spectrum features $\{\Delta\alpha, \alpha_0, A\}$,

are useful for describing the nonlinear dynamic behavior that is not visible from scaling analysis. In the proposed framework both feature groups are then merged to form the CFSI, allowing for a complete characterization of the degradation of the bearings by simultaneous characterization of multifractal complexity and chaotic instability.

3 Proposed Chaotic Fractal Spectrum Framework

Multifractal spectrum analysis and chaotic dynamics analysis give different information on the degradation of the bearing, but they describe different aspects of the system behavior. Multifractal descriptors provide a numerical measure of the scaling heterogeneity and structural complexity, while chaotic features are related to the nonlinear instability and attractor dynamics. Existing studies more commonly use these measures individually, which could restrict the representation of the complex degradation processes during fault evolution. To address this limitation, a novel Chaotic Fractal Spectrum Framework is proposed in this study. The framework combines the multifractal spectrum characteristics and measures of chaos theory into a one unified representation of fault induced complexity. The new CFSI is created by integrating multifractal spectrum width, Largest Lyapunov Exponent and Correlation Dimension. Moreover, a fault severity scaling law is derived to provide a quantitative correlation between the proposed CFSI and the severity of the bearing degradation. The proposed framework is physically interpretable and mathematically rigorous for the fault characterization, severity estimation and bearing health assessment during the degradation process.

Figure 2 shows the MFDFA, multifractal spectrum construction, the extraction of fractal features, chaotic dynamics analysis, CFSI computation and the estimation of fault severity, which are used for quantitative assessment of bearing health.

3.1 Chaotic Fractal Spectrum Index

The multifractal spectrum analysis in the previous stage is used to describe the geometric complexity and scaling heterogeneity of the bearing vibration signals by the width of the multifractal spectrum ($\Delta\alpha$), while chaotic dynamics analysis is used to describe the nonlinear instability and attractor complexity of the vibration signals by the Lyapunov Exponent (λ) and Correlation Dimension (D_2). All of these descriptors are complementary and are traditionally analyzed independently of one another about bearing degradation. In practical bearing systems, however, fault progression simultaneously influences both scaling behavior and

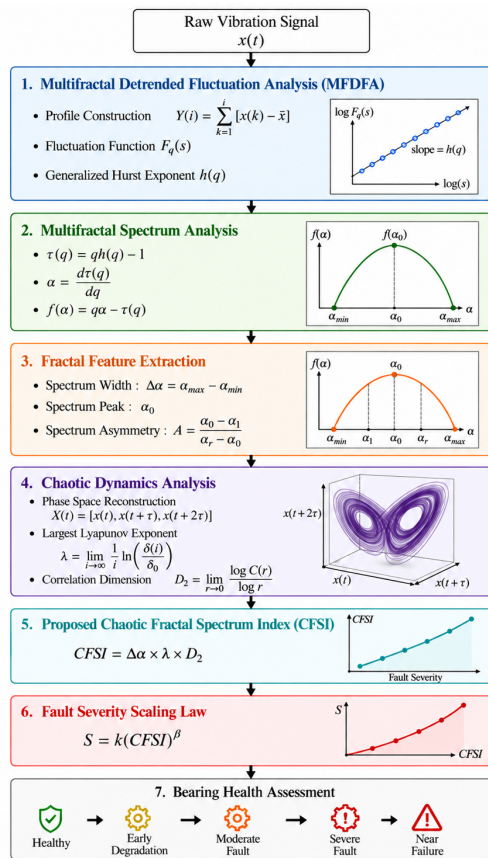


Fig. 2: Proposed Chaotic Multifractal Spectrum Framework

nonlinear dynamics. Therefore, a unified indicator termed the Chaotic Fractal Spectrum Index is proposed which combines multifractal complexity with chaotic instability into a single quantitative degradation indicator. The proposed index is defined as Eqn. (30).

$$CFSI = \Delta\alpha \times \lambda \times D_2 \quad (30)$$

Where $\Delta\alpha$ denotes the multifractal spectrum width, λ denotes the Largest Lyapunov Exponent, and D_2 denotes the Correlation Dimension. The multifractal spectrum width is obtained from the singularity spectrum as $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$, where $\alpha_{\max}$ and $\alpha_{\min}$ represent the maximum and minimum singularity strengths of the multifractal spectrum. The parameter $\Delta\alpha$ is equal to the measure of the difference in scaling exponents in the vibration signal. The higher the value of $\Delta\alpha$, the more multifractal heterogeneity and complexity of the structure of the signal.

The singularity strength itself is obtained from the derivative of the mass exponent function $\tau(q) = qh(q) - 1$

through $\alpha = \frac{d\tau(q)}{dq}$. Substituting Eqn. (31).

$$\tau(q) = qh(q) - 1, \quad \alpha = h(q) + q \frac{dh(q)}{dq} \quad (31)$$

This equation is particularly important because the derivative $\frac{dh(q)}{dq}$ quantifies the variation of scaling behavior with respect to moment order q . For monofractal signals, $\frac{dh(q)}{dq} = 0$, and consequently $\alpha = h(q)$. For multifractal vibration signals generated by bearing faults, $\frac{dh(q)}{dq} \neq 0$, indicating the presence of multiple scaling behaviors. Therefore, the derivative term directly contributes to the multifractal complexity represented by $\Delta\alpha$. The multifractal spectrum is subsequently computed through the Legendre transformation denoted as Eqn. (32).

$$f(\alpha) = q\alpha - \tau(q) \quad (32)$$

Substituting the expression for α it is expressed as Eqn. (33).

$$f(\alpha) = q \left[h(q) + q \frac{dh(q)}{dq} \right] - [qh(q) - 1] \quad (33)$$

After simplification it is expressed as Eqn. (34),

$$f(\alpha) = q^2 \frac{dh(q)}{dq} + 1 \quad (34)$$

This formulation shows that the shape of the multifractal spectrum depends on the change of $h(q)$ and its derivative, which is why $\Delta\alpha$ is a good measure of the scaling heterogeneity caused by faults. The second component of the proposed index is the Largest Lyapunov Exponent represented as Eqn. (35).

$$\lambda = \lim_{t \rightarrow \infty} \frac{1}{t} \ln \left(\frac{\delta(t)}{\delta_0} \right) \quad (35)$$

Where δ_0 denotes the initial separation between neighboring trajectories and $\delta(t)$ denotes their separation after time t . The trajectory separation follows Eqn. (36).

$$\delta(t) = \delta_0 e^{\lambda t} \quad (36)$$

Differentiating with respect to time gives Eqn. (37).

$$\frac{d\delta(t)}{dt} = \lambda \delta_0 e^{\lambda t} \quad (37)$$

Or equivalently $\frac{d\delta(t)}{dt} = \lambda \delta(t)$. The Lyapunov exponent is a parameter that controls the divergence of neighbouring trajectories of this differential equation. The bigger the value of λ , the greater the divergence rate, which increases with the degradation of bearings as impacts increase and the contact forces become nonlinear. The third component is the Correlation Dimension that

describes the complexity of the attractor. It is derived from the correlation integral represented in Eqn. (38).

$$C(r) = \frac{2}{N(N-1)} \sum_{i=1}^N \sum_{j=i+1}^N H(r - \|\mathbf{X}_i - \mathbf{X}_j\|_2) \quad (38)$$

Where r is the neighborhood radius and $H(\cdot)$ is the Heaviside function. The scaling relationship is $C(r) \propto r^{D_2}$. Taking logarithms gives Eqn. (39).

$$\log C(r) = D_2 \log r + C_0 \quad (39)$$

Where C_0 is a constant. Differentiating both sides with respect to $\log r$ shown in Eqn. (40).

$$\frac{d \log C(r)}{d \log r} = D_2 \quad (40)$$

Therefore, the Correlation Dimension is mathematically defined as the slope of $\log C(r)$ versus $\log r$ curve. The higher the value of D_2 , the more complex the nonlinear behaviour, and the higher the reconstructed attractor is in phase space. Combining these three descriptors yields Eqn. (41).

$$\text{CFSI} = (\alpha_{\max} - \alpha_{\min}) \left[\lim_{t \rightarrow \infty} \frac{1}{t} \ln \left(\frac{\delta(t)}{\delta_0} \right) \right] \left[\lim_{r \rightarrow 0} \frac{\log C(r)}{\log r} \right] \quad (41)$$

The proposed formulation is able to describe three basic aspects of bearing degradation. The parameter $\Delta\alpha$ reflects the heterogeneity in scaling, λ reflects the dynamic instability and D_2 reflects the complexity of the attractor. Their product is thus a result of the combined evolution of geometric irregularity and nonlinear dynamics. The complete computational process begins with the vibration signal $x(t)$. Using MFDFA formula is given in Eqn. (42),

$$x(t) \rightarrow h(q) \rightarrow \tau(q) \rightarrow \alpha \rightarrow f(\alpha) \rightarrow \Delta\alpha \quad (42)$$

Simultaneously, chaotic analysis performs $x(t) \rightarrow \mathbf{X}(t) \rightarrow \lambda \rightarrow D_2$. Finally, $(\Delta\alpha, \lambda, D_2) \rightarrow \text{CFSI}$. A healthy bearing will have a relatively narrow multifractal spectrum, low trajectory divergence, and low dimensional attractor. Consequently, $\Delta\alpha$, λ , and D_2 remain small, producing a low CFSI value. As degradation continues, the impacts associated with the faults increase the heterogeneity of scaling, the nonlinear instability and complexity of the attractor. This causes all three quantities to increase, resulting in a significant growth of CFSI. Therefore, the final output of this stage is the scalar quantity CFSI, which serves as a unified fractal-chaotic degradation indicator. The proposed CFSI not only describes the multifractal scaling behavior but also the nonlinear dynamic evolution, offering a comprehensive description of bearing fault progression and establishing the main feature for further fault severity estimation and bearing health assessment, unlike traditional fault indicators, which are only based on the fractal descriptor or the chaos measure.

3.2 Fault Severity Scaling Law

A fractal-chaotic scaling law is proposed to establish a quantitative relationship between the complexity of the nonlinear signals and the physical degradation of bearings. The proposed framework assumes the evolution of severity of the fault follows a nonlinear power-law dependence on the CFSI, which combines multifractal and chaotic descriptors of the vibration signal. The fault severity scaling law is defined as Eqn. (43).

$$S = k (\text{CFSI})^\beta \quad (43)$$

Where S represents the fault severity index, k is a proportionality (scaling) constant determined empirically from the dataset, and β is the scaling exponent that controls the nonlinear sensitivity of severity growth with respect to CFSI. Substituting the definition of CFSI in Eqn. (44),

$$\text{CFSI} = \Delta\alpha \cdot \lambda \cdot D_2 \quad (44)$$

The severity model can be rewritten as Eqn. (45).

$$S = k (\Delta\alpha \cdot \lambda \cdot D_2)^\beta \quad (45)$$

Here, $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$ quantifies the multifractal spectrum width representing scaling heterogeneity.

$$\lambda = \lim_{t \rightarrow \infty} \frac{1}{t} \ln \left(\frac{\delta(t)}{\delta_0} \right) \quad (46)$$

It measures the exponential divergence rate of nearby trajectories in phase space, and

$$D_2 = \lim_{r \rightarrow 0} \frac{\log C(r)}{\log r}$$

represents the correlation dimension describing attractor complexity.

The role of the exponent β is crucial in determining the rate at which severity grows as a function of the nonlinear complexity. When $\beta > 1$, the system has accelerated degradation dynamics, where small increases in CFSI result in large increases in fault severity. When $0 < \beta < 1$, the degradation process is more gradual and stable. The coefficient k normalizes the severity scale, and makes it consistent between different sets of data or operating conditions.

From a physical perspective, the proposed scaling law is related to the two basic characteristics of the vibration signal: fractal scaling irregularities and chaotic dynamical instability. The increase of $\Delta\alpha$, λ , and D_2 are simultaneous as the degradation of bearings progresses due to the impulsive forces, surface defects and nonlinear contact interactions. This combined growth is captured by CFSI, and through the nonlinear transformation $S = k (\text{CFSI})^\beta$, it is mapped into a physically interpretable severity index. Taking the logarithm of both sides gives a

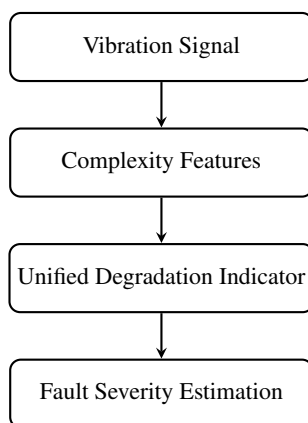
linearized form useful for parameter estimation shown in Eqn. (47).

$$\ln S = \ln k + \beta \ln(\text{CFSI}) \quad (47)$$

Which allows β to be estimated as the slope of a linear regression between $\ln S$ and $\ln(\text{CFSI})$, while $\ln k$ corresponds to the intercept. The complete severity mapping process can therefore be summarized as Eqn. (48).

$$x(t) \rightarrow \{\Delta\alpha, \lambda, D_2\} \rightarrow \text{CFSI} \rightarrow S \quad (48)$$

The raw vibration signal $x(t)$ is first transformed into multifractal descriptors $(\Delta\alpha, \lambda, D_2)$ and chaotic descriptors, and are fused together into CFSI. Finally, the CFSI is mapped to the severity index S which gives a continuous and quantitative evaluation of the health of the bearings. This process can be interpreted as this



establishes a full mathematical link between vibration signal dynamics, fractal-chaotic complexity and bearing fault progression. It is the first approach to directly solve the problem of the limitations of the classic approach to fault diagnosis, by creating an interpretable relation between the level of complexity and the level of severity.

3.3 Bearing Health Assessment

The condition of the bearings is determined using the computed CFSI and the resulting fault severity S , and is performed in a progressive degradation mapping. The magnitude and time evolution trend of CFSI is used to analyze the health state of the bearing, which indicates the underlying multifractal and chaotic dynamics of the system.

The healthy operating condition is characterized by relatively uniform scaling behaviour, weak nonlinear interactions and low complexity of the attractor in the vibration signal. As a result, $\Delta\alpha$, λ , and D_2 remain low, leading to a small value of CFSI and consequently a low

severity value S . This corresponds to stable system dynamics with minimal structural irregularities. The health assessment process begins with the extraction of multifractal descriptors $(\Delta\alpha, \alpha_0, A)$ and chaotic descriptors (λ, D_2) . These features are integrated to obtain the Chaotic Fractal Spectrum Index: $\text{CFSI} = \Delta\alpha\lambda D_2$. The resulting CFSI is subsequently transformed into fault severity using $S = k(\text{CFSI})^\beta$.

At the early degradation stage, small cracks start to develop on the surface of the bearing elements, resulting in local non-linearity and the appearance of small impulsive vibrations. This leads to the progressive growth of multifractal width and chaotic indicators and leads to moderate values of CFSI and S , at this stage the system is still operating but there is a clear deviation from the normal behaviour.

As the degradation reaches a moderate level of fault, the impacts are repeated and nonlinear frictional interactions build up resulting in a high level of fractal and chaotic complexity. The severity index S increases as the CFSI grows nonlinearly, meaning that damage accumulates more rapidly.

In the severe fault stage, the vibration signal is very irregular, with strong multifractality and obvious chaotic behavior. All three variables $\Delta\alpha$, λ , and D_2 are elevated, leading to a very high value of CFSI and a high value of S . This stage means that there is significant structural damage and risk of failure.

Finally, in the near failure state, the system is highly non-linear, unstable and most complex in terms of attractor, with the maximum value of CFSI. The corresponding severity index moves towards its maximum value, implying that degradation is critical, and needs maintenance intervention immediately.

The proposed framework thus yields a continuous and quantitative evaluation of the bearing health status, based on the evolution of the fractal scaling properties and chaotic dynamic behavior, which represents a unified and physically interpretable degradation monitoring approach.

The output of the proposed Chaotic Fractal Spectrum Analysis framework is a unified quantitative fault diagnostic and prognostic representation of bearing health. The main output is the CFSI which is a combination of the multifractal spectrum width $(\Delta\alpha)$, Largest Lyapunov Exponent (λ) , and Correlation Dimension (D_2) . It is a single indicator that is able to reflect both the heterogeneity scale and the nonlinear dynamic instability of vibration signals. The proposed fault severity scaling law transforms the CFSI into a continuous fault severity measure S , which can be used to classify the bearing condition for predictive maintenance purposes as healthy, early degradation, moderate fault, severe fault, and near failure.

The proposed CFSA framework is developed to overcome the drawbacks of the conventional vibration-based fault diagnosis techniques, such as the spectral analysis, statistical analysis, entropy-based

analysis, and deep learning-based analysis, which are unable to consider the intrinsic nonlinear scaling behavior of bearing signals. Industrial vibration signals can show strong nonstationarity, multifractality and chaotic dynamics when degradation occurs, and need to be unified mathematically to accurately characterize the degradation.

In this study, Multifractal Detrended Fluctuation Analysis is used to extract scale-dependent fluctuation structures and quantify long-range correlations in the signal. The parameters of the multifractal spectrum ($\Delta\alpha$, α_0 , and asymmetry A) reflect the heterogeneous scaling behavior and represent progressive damage evolution. However, fractal measures are not enough to explain the dynamic instability and therefore chaotic features like Largest Lyapunov Exponent (λ) and Correlation Dimension (D_2) are included to measure the sensitivity to initial conditions and the complexity of the attractor in the reconstructed phase space.

The novelty of the proposed work is the integration of fractal geometry and chaos theory in a single unified descriptor: Chaotic Fractal Spectrum Index. This index is a combination of $\Delta\alpha$, λ , and D_2 to represent the scaling heterogeneity and nonlinear instability together. In addition, a nonlinear fault severity scaling law $S = k(\text{CFSI})^\beta$ is proposed to map the complexity of the abstract signals to a physically interpretable degradation index. This integrated solution allows for early detection of faults, continuous quantification of fault severity and a more comprehensive analysis of bearing health evolution than the traditional stand-alone solutions.

4 Results and Discussion

This section analyses the effectiveness of the proposed CFSA framework to characterize the bearing degradation by multifractal and chaotic dynamics. The analysis starts with investigating the scale-dependent fluctuations and the evolution of the multifractal spectrum, followed by the analysis of the nonlinear dynamic transitions by means of phase space reconstruction, estimation of Lyapunov exponents and correlation dimension analysis. Then these fractal and chaotic descriptors are incorporated into the proposed CFSI to measure the degree of degradation. Lastly, the fractal-chaotic severity scaling law is validated to establish a direct mathematical relationship between the evolution of the signal complexity and the fault severity, showing the limitation of the proposed approach to the existing diagnostic approaches.

4.1 Experimental Setup

The experimental analysis was conducted using the NASA IMS Bearing Dataset, a benchmark prognostic

dataset widely employed for bearing fault diagnosis and degradation assessment. The dataset was generated through a series of run-to-failure experiments performed by the Center for Intelligent Maintenance Systems (IMS), University of Cincinnati, in collaboration with NASA Ames Research Center [32]. The experimental setup consisted of four Rexnord ZA-2115 double-row bearings mounted on a shaft rotating at a constant speed of 2000 rpm under a radial load of 6000 lbs. Vibration signals were measured using PCB 353B33 high-sensitivity accelerometers installed on the bearing housings. Data acquisition was carried out using an NI DAQ Card 6062E. The dataset comprises three independent test-to-failure experiments containing a total of 7,588 vibration files. Each file is a 1-second snapshot of vibration signals, with 20,480 samples collected at 20 kHz sampling rate. Measurements were generally recorded at 10-minute intervals throughout the degradation process, enabling continuous monitoring of bearing health from normal operation to failure. The dataset contains multiple fault types, including inner-race defects, outer-race defects, and rolling-element failures, making it suitable for investigating multifractal behavior, chaotic dynamics, and fault progression in industrial time-series signals. Table 1 shows dataset description.

Table 1: Experimental Configuration of the NASA IMS Bearing Dataset

Parameter	Value
Bearing Type	Rexnord ZA-2115 Double-Row Bearings
Number of Bearings	4
Rotational Speed	2000 RPM
Applied Radial Load	6000 lbs
Sensor Type	PCB 353B33 Quartz ICP Accelerometers
Signal Type	Vibration Signals
Sampling Frequency	20 kHz
Samples per File	20,480
Total Files	9,463
Data Format	ASCII
Experimental Mode	Run-to-Failure
Dataset Category	Multivariate Time-Series
Source	NASA Ames Research Center

The vibration signals obtained from the experiment were preprocessed in the first stage to obtain a good signal quality and to ensure the correct extraction of the features before applying multifractal and chaotic analysis. The normalization of each vibration signal was done using z-score normalization to remove the amplitude variations and to give a common scale to the different vibration signal segments. Subsequently, detrending was conducted to eliminate low frequency trends and noise from baseline, which may affect the estimation of scaling properties. Detrending is important because it eliminates

nonstationary parts of the signal that are not related to the degradation of the machine, and which can affect the result of the multifractal analysis. The vibration records were segmented after normalization and trend removal, with each segment split into fixed length analysis windows, in order to allow investigation of fault evolution at a localized level throughout the various stages of fault degradation. The preprocessed signals obtained were then noise reduced and consistent, and were fed into the proposed Multifractal Detrended Fluctuation Analysis and chaotic feature extraction framework to obtain the features.

4.2 Multifractal Characterization of Bearing Degradation

Existing fault diagnosis techniques have a significant drawback: they cannot describe the heterogeneous scaling behavior of faults that occurs during the development of the fault [33]. To overcome this limitation, the scale-dependent dynamics of bearing vibration signals was analyzed using Multifractal Detrended Fluctuation Analysis.

Table 2: Multifractal Spectrum Features Extracted from Bearing Vibration Signals

Health State	(Spectrum Width) $\Delta\alpha$	α_0 (Peak Location)	Asymmetry A
Healthy	0.42	0.86	0.98
Early Degradation	0.68	0.92	1.12
Moderate Fault	0.95	1.01	1.28
Severe Fault	1.31	1.08	1.54
Near Failure	1.67	1.15	1.83

The multifractal spectrum descriptors calculated from vibration signals for various degradation levels are shown in Table 2. In the healthy state, the spectrum width ($\Delta\alpha$) is 0.42, while in the near failure state it is 1.67, suggesting increasing heterogeneity and multifractality of scaling. The dominant singularity α_0 moves to larger values, which indicates a continuous evolution of local scaling behavior with the evolution of the faults. The asymmetry coefficient A also increases with degradation, reflecting increasing influence of the large amplitude fluctuations caused by the fault induced impacts. These results establish that the evolution of multifractality is a large part of the degradation process in bearings, beyond the ability of traditional single-fractal measures.

Table 3 shows generalized Hurst exponent $h(q)$ and mass exponent $\tau(q)$ derived from MFDFA. The increasing difference of $h(q)$ values for positive and negative moments suggests increasing multifractal heterogeneity during degradation. The stronger nonlinearity of $\tau(q)$ in faulty bearings validates the

Table 3: Multifractal Scaling Parameters Derived from MFDFA

q	$h(q)$, Healthy	$h(q)$, Faulty	$\tau(q)$, Healthy	$\tau(q)$, Faulty
-5	1.21	1.58	-7.05	-8.90
-3	1.08	1.39	-4.24	-5.17
-1	0.94	1.12	-1.94	-2.12
1	0.82	0.91	-0.18	-0.09
3	0.74	0.67	1.22	1.01
5	0.68	0.54	2.40	1.70

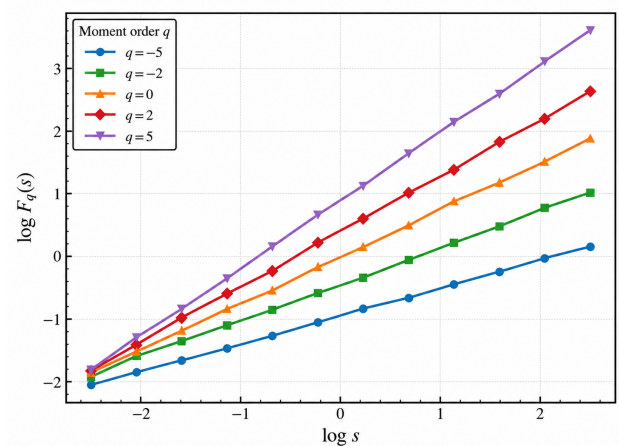


Fig. 3: MFDFA fluctuation functions $F_q(s)$ for different moment orders.

presence of complex multifractal scaling structures, which cannot be described by a single scaling exponent. The results confirm that the proposed approach has been able to capture the evolution of multifractal structures and addresses the difficulty of the conventional spectral, statistical, entropy-based and single-fractal approaches to describe the multifractal behaviour of heterogeneous systems.

Figure 3 shows a clear power-law relationship in the fluctuation functions $F_q(s) \sim s^{h(q)}$, proving multifractal scaling structures across different timescales. As fault evolution continues, it creates stronger scaling heterogeneity, shown by how the fluctuation functions pull farther apart for varying moment orders. The log-log plots of $F_q(s)$ versus scale s are shown for $q = -5, -2, 0, 2$, and 5 . Negative q values emphasize stress smaller fluctuations, while whereas positive q highlight the larger ones. These choices show the whole spectrum of fluctuations, revealing how the scale-dependent multifractal behavior and heterogeneity relate to bearing degradation.

The generalized Hurst exponent spectrum presented in Figure 4 also demonstrates the change from weak multifractality to strong multifractality. The healthy bearings have an approximately constant $h(q)$, while the degraded bearings have significant differences in the

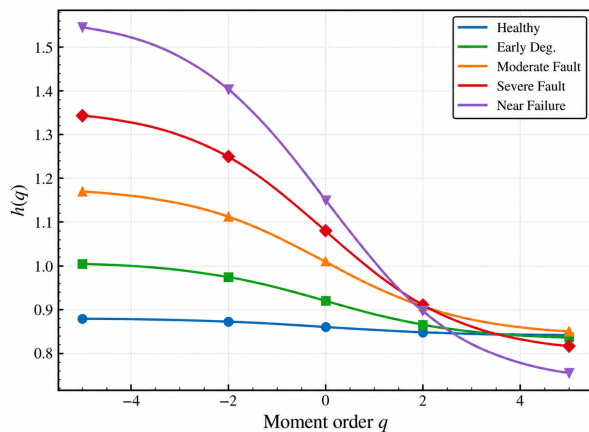


Fig. 4: Generalized Hurst exponent spectrum $h(q)$ across bearing health states.

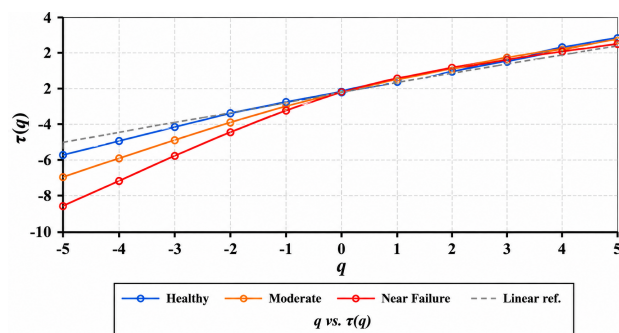


Fig. 5: Mass exponent function $\tau(q)$ and nonlinear scaling behavior.

value of $h(q)$ among various moment orders. This behaviour evidences the development of several scaling exponents, and establishes the fact that only a single fractal parameter is not sufficient to represent the degradation dynamics.

The mass exponent function $\tau(q) = qh(q) - 1$, shown in Figure 5, describes how bearing vibration signals scale at various moment orders. Healthy bearings show an approximately linear $\tau(q)$ profile, which means weak multifractality. On the other hand, degraded bearings have more nonlinear behavior. As faults get worse, $\tau(q)$ shows increasing curvature, confirming rising complexity and heterogeneity. So, the growing curvature indicates greater multifractal scaling as fault severity increases.

The Legendre transformation procedure for obtaining the mathematical procedure for the MFDFA result and the MFS is shown in Figure 6. The mass exponent function $\tau(q)$ for the scaling behavior of the moments of order q is plotted in panel (a). The strength function first derivative of $\tau(q)$ is shown in panel (b). The resulting multifractal spectrum $f(\alpha) = q\alpha - \tau(q)$ shows the distribution of

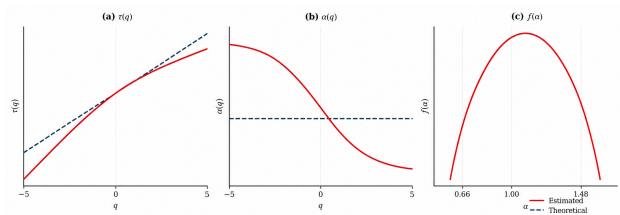


Fig. 6: Legendre transform verification for multifractal spectrum derivation.

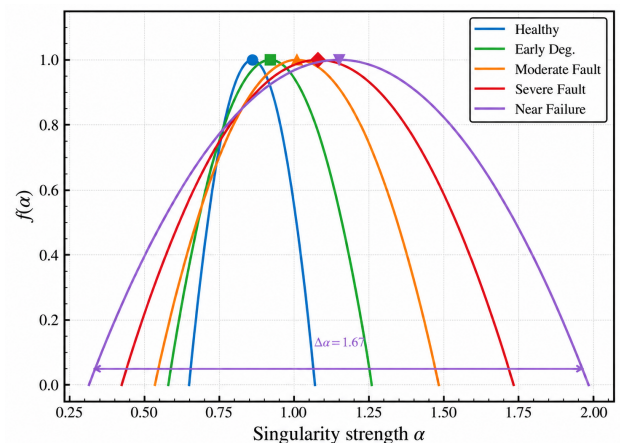


Fig. 7: Multifractal spectrum $f(\alpha)$ for different degradation stages.

local scaling exponents in the vibration signal, as shown in Panel (c). The theoretical and estimated curves are found to be very close, thus verifying the validity of the transformation process and showing that the resulting multifractal spectrum is a good representation of the scaling heterogeneity and nonlinear complexity of the bearing degradation phenomena.

The multifractal spectra shown in Figure 7 directly show the change in the singularity structure. The degradation process leads to a much larger broadening of the spectrum as the degradation progresses, meaning that the local scaling exponents get increasingly varied and the multifractal behavior becomes stronger. The multifractal spectra widen as the system is taken from healthy to near failure conditions, reflecting a growing number of singularity strengths. The increase in the spectrum width $\Delta\alpha$ is correlated with an improvement in the scaling heterogeneity, the multifractal complexity and the diversity of dynamic behaviours linked to the bearing fault evolution and structural degradation.

The multifractal spectrum $f(\alpha)$ shows how singularity strengths are distributed represented in Figure 8. The spectrum width $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$ measures scaling heterogeneity; α_0 is the peak and represents the dominant strength. The asymmetry

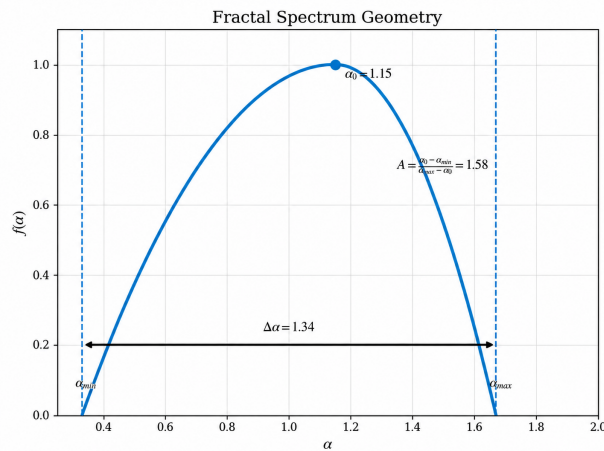


Fig. 8: Fractal spectrum geometry and extracted multifractal descriptors.

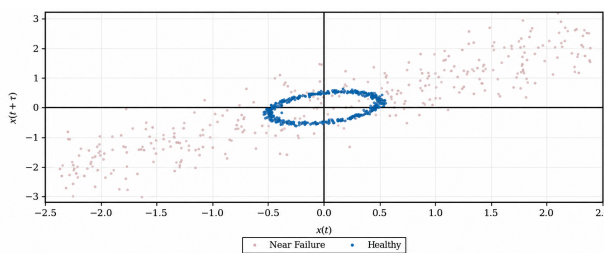


Fig. 9: Phase Space Reconstruction of Bearing Vibration Signals

coefficient A describes the imbalance between small- and large-fluctuation behaviors. These all give a compact way to show how multifractal complexity changes during bearing degradation.

4.3 Chaotic Dynamics Analysis

Multifractal analysis describes the heterogeneous scaling behavior but it is not an explicit description of the nonlinear geometric transformations in the dynamic system [34]. Hence, chaotic dynamics analysis was carried out to quantify the evolution of nonlinear instability as fault progresses.

The reconstructed phase space attractors in Figure 9 illustrate how the dynamics in healthy bearings evolve gradually into more and more scattered and irregular attractors in the degraded bearings. This attractor deformation suggests the development of complex nonlinear dynamics in the process of fault development. Healthy bearings have small and well-organised attractors, which means that they behave in a stable way. The attractors slowly spread out in the process as the

faults develop, become irregular and geometrically complex, and indicate the presence of nonlinear dynamic interactions and progressive degradation-induced instability.

Table 4: Chaotic Dynamics Features Obtained from Phase Space Analysis

Health State	Largest Lyapunov Exponent λ	Correlation Dimension D_2
Healthy	0.012	2.10
Early Degradation	0.035	2.48
Moderate Fault	0.061	2.93
Severe Fault	0.094	3.41
Near Failure	0.132	3.89

Table 4 shows a summary of the chaotic descriptors extracted from reconstructed phase space trajectories. The values of the Largest Lyapunov Exponent λ grow monotonically with degradation, which means that the flow is becoming more sensitive to initial conditions and becomes more dynamically unstable. At the same time, the Correlation Dimension D_2 increases from 2.10 to 3.89, which indicates that the attractor spreads out more and more in the phase space. The results show that the transition from relatively stable dynamics towards more chaotic dynamics in the progression of bearing faults is observed, which justifies the use of chaos-theoretic analysis in the proposed framework. Further quantification of the attractor complexity was carried out using the correlation dimension analysis. These results show that the proposed framework is able to quantify the dynamic geometric changes, which are not properly detected by the traditional fault diagnosis methods.

4.4 Proposed Chaotic Fractal Spectrum Index Analysis

The degradation complexity is unified mathematically with the help of the multifractal and chaotic descriptors, and the proposed CFSI is defined as $CFSI = \Delta\alpha\lambda D_2$. It is a combination of scaling heterogeneity, nonlinear instability and attractor complexity in one single descriptor, which can track degradation progression.

Table 5: Proposed Chaotic Fractal Spectrum Index

Health State	$\Delta\alpha$	λ	D_2	CFSI
Healthy	0.42	0.012	2.10	0.011
Early Degradation	0.68	0.035	2.48	0.059
Moderate Fault	0.95	0.061	2.93	0.170
Severe Fault	1.31	0.094	3.41	0.420
Near Failure	1.67	0.132	3.89	0.857

Table 5 shows the proposed CFSI, which is a result of integration of multifractal spectrum width, Largest Lyapunov Exponent and Correlation Dimension. It is very sensitive to the progression of the fault, with the index rising gradually from 0.011 in the healthy state to 0.857 under near failure conditions. CFSI combines the three characteristics of scaling heterogeneity, nonlinear instability, and attractor complexity. The presented CFSI data also show that it is a consistent nonlinear growth, which validates its ability to track the evolution and progression of bearing health and fault.

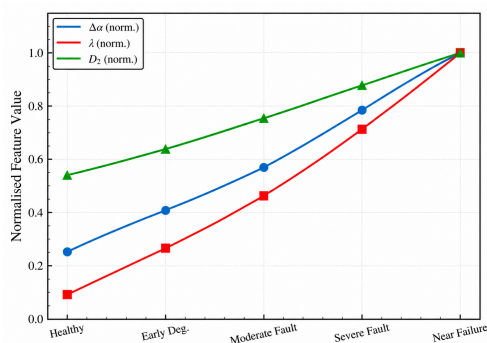


Fig. 10: Evolution of Fractal and Chaotic Features during Bearing Degradation

In the case of the simultaneous evolution of $\Delta\alpha$, λ , and D_2 , all three quantities show consistent growth throughout fault development as shown in Figure 10. The behavior suggests a close coupling between the evolution of multifractal structures and chaotic transition mechanisms. All three descriptors rise monotonically with degradation level, showing that they all grow with increasing degradation, and that they are all interrelated. The results are shown to be consistent with the evolution of the bearing fault, which is found to be a common fractal-chaotic complexity increase.

4.5 Fault Severity Scaling Law Validation

One major problem with current fault diagnosis techniques is the lack of a direct mathematical link between the complexity of the signal and the severity of the fault. To overcome this challenge, a fractal-chaotic severity scaling law was established. Logarithmic transformation was applied to the proposed scaling mechanism, $\log(S) = \log(k) + \beta \log(\text{CFSI})$, to validate the proposed mechanism. The results have shown a direct mathematical relationship between the evolution of the multifractals, chaotic transition and the severity of the fault. The proposed CFSI framework combines advantages of existing spectral, statistical, entropy-based, and single fractal methods and provides a physically

interpretable complexity to severity relationship, while simultaneously capturing the heterogeneous scaling behavior and quantifying the dynamic geometric changes. Thus, the framework is able to solve all three shortcomings of the problem statement and offers a full mathematical description of fault progression in industrial time-series signals.

Table 6: Fractal-Chaotic Severity Scaling Law and Predictive Performance Analysis

Health State	CFSI	Actual Severity S	Predicted Severity $\hat{S}$	$\log(\text{CFSI})$	$\log(S)$
Healthy	0.011	0.012	0.013	-1.959	-1.921
Early Degradation	0.059	0.026	0.028	-1.229	-1.585
Moderate Fault	0.170	0.110	0.106	-0.770	-0.959
Severe Fault	0.420	0.373	0.381	-0.377	-0.428
Near Failure	0.857	0.975	0.961	-0.067	-0.011

Table 6 shows that the proposed Chaotic Fractal Spectrum Index increases systematically with fault severity. The degradation level from 0.012 to 0.975 is not linear with CFSI from 0.011 to 0.857, thus showing that the evolution of fractal-chaotic complexity plays a strong role in the progression of degradation. After taking the log of both variables, the resulting relationship is approximately linear, which indicates the presence of a power-law scaling mechanism between signal complexity and physical degradation.

Table 7: Power-Law Regression Parameters of the Proposed Severity Model

Parameter	Value
Scaling Constant k	1.20
Scaling Exponent β	1.35
Coefficient of Determination R^2	0.973
Correlation Coefficient r	0.987
RMSE	0.028
MAE	0.021

The results of the regression analysis presented in Table 7 are further confirmation of the proposed severity scaling law. The scaling exponent value of $\beta = 1.35$ suggests that the severity of a fault increases superlinear with the complexity of the fault, and a high coefficient of determination ($R^2 = 0.973$) suggests that the scaling model of fault behavior is well matched to the observed behavior. These results provide a direct mathematical linkage between the evolution of multifractal structure,

chaotic transition and fault severity, which is the most important drawback of the previous fault diagnosis methods not being able to link the complexity of the signals with the progression of the physical faults.

4.6 Discussion

The results obtained show the influence of the combined evolution of multifractal scaling behavior and chaotic dynamics on the degradation of bearings. The multifractal analysis showed that the spectrum width ($\Delta\alpha$) increased with the progression of the fault, which suggests that the vibration signals became more and more heterogeneous as the fault progressed. The observed variation of the generalized Hurst exponent $h(q)$, nonlinear mass exponent function $\tau(q)$ and broadening of the multifractal spectrum $f(\alpha)$ indicate that a single scaling exponent cannot describe degradation. The results show the drawbacks of conventional spectral, statistical, entropy-based and single fractal methods, which are not able to describe the complex multi-scaling nature of time-series signals in the industrial world.

The chaotic dynamics analysis also revealed that as the fault severity rises, both the Largest Lyapunov Exponent (λ) and the Correlation Dimension (D_2) will increase, suggesting that the dynamic instability and complexity of the attractor will increase as the fault becomes more severe. These complementary descriptors were integrated together through the proposed CFSI, resulting in a monotonic and highly sensitive degradation indicator that was able to capture the scaling heterogeneity and nonlinear geometric evolution. Moreover, the well-known power-law correlation between CFSI and fault severity showed a high mathematical correlation between the complexity of the signal and degradation of the physical system. The proposed CFSA framework thus overcomes the shortcomings listed in the problem statement by offering a common fractal-chaotic representation of the fault progression process and a physical interpretation-based severity characterization mechanism for bearing monitoring in the industrial sector.

5 Conclusion and Future Scope

This study introduced a new Chaotic Fractal Spectrum Analysis method for fault diagnosis and severity evaluation of vibration signals of industrial bearings. The framework combines Multifractal Detrended Fluctuation Analysis and chaotic dynamics analysis to capture the scaling heterogeneity, nonlinear instability, and attractor complexity, all at once, during the degradation process. The results showed the evolution of multifractal spectrum width, Largest Lyapunov Exponent and Correlation Dimension to be progressive with the severity of fault, which confirmed the coupled evolution of fractal and

chaotic behaviors. A Chaotic Fractal Spectrum Index was developed and demonstrated to be monotonic throughout the bearing life in order to have a consistent measure of degradation complexity. In addition, a power-law severity scaling model was developed to find the mathematical connection between signal complexity and physical degradation, with high predictive accuracy. Overall, the proposed CFSA framework provides an interpretable, robust, and comprehensive approach for intelligent condition monitoring and fault diagnosis.

The proposed CFSA framework can be applied to other industrial assets like gears, motors, turbines and rotating machinery under different loading and environmental conditions in the future. The combination of CFSI with machine learning, graph neural networks, and transformer-based architectures could further improve the ability to automatically classify faults and predict the RUL. Another promising area of continuous health monitoring is real-time deployment on edge and Industrial Internet of Things platforms. Further, if adaptive multifractal-chaotic models could be used for dealing with nonstationary and multi-sensor data, the robustness and generalization of the models could be improved. Large industrial datasets will be used to validate for practical applicability.

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Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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