

Solving Fractional-Order Growth Model of Abalone Weight Using Variational Iteration Method

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Abstract: This research introduces a fractional-order growth model as an extension of the McKendrick model, which is analyzed using the Variational Iteration Method to predict the growth of abalone weight. The resulting solution yields a particular case of the Taylor series expansion. Simulations performed for several fractional orders indicate that increasing the fractional order leads to an increase in the resulting series values while simultaneously reducing the mean absolute error. Moreover, the results are illustrated by fractional order of 0.9.

Keywords: Fractional Calculus, Modified McKendrick Model, Variational Iteration Method, Taylor's series

1 Introduction

Abalone, a marine mollusk, is widely cultivated across several countries, including Indonesia, with notable production in regions such as Bali, Lombok, Sumbawa, Sulawesi, Maluku, and Papua. It is highly valued both as a food source, owing to its high nutritional value as protein-rich food, and as the production of attractive souvenirs made from its shells. This mollusk plays a vital role in coastal economies, supplying local demand, commercial markets, and international exports to regions such as Asia, Europe, and the United States. Consequently, abalone cultivation has expanded extensively to an industrial scale; however, constraints such as restricted seed supply and slow growth rates remain significant obstacles [1,2]. One of the crucial approaches to understanding abalone growth is the formulation of a mathematical model.

The concept of the growth model began in 1798 when Malthus claimed that the world's population would grow exponentially over time, eventually surpassing available resources. Since then, he introduced the exponential growth model using differential equation. The model provided accurate results which are indicated by some

studies including the growth of Taraba state population [3], predicting the growth of cell carcinoma and melanoma to analyze tumor development between patients [4], forecasting fatigue crack progression and remaining lifespan of mechanics of material [5].

According to Verhulst (1839), the model describes an initial phase of exponential growth driven by small population size and abundant resources, followed by a slowdown as the population increases and resource availability becomes constrained near the carrying capacity. The Verhulst model is well known as the logistic model. Furthermore, the growth model was developed in partial differential equations formed. The McKendrick equation is a well-established model that has provided a fundamental theoretical foundation for the development of mathematical modeling based on partial differential equations. McKendrick (1926) introduced an age-structured population growth model governed by a partial differential equation. In this context, $v(s, t)$ denotes the population density at age s and time t , i.e.

$$\frac{\partial v(s, t)}{\partial t} + \frac{\partial v(s, t)}{\partial s} = -\mu(s, t)v(s, t), \quad (1)$$

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accompanied by the initial condition

$$v(s, 0) = v_0(s), \text{ and } v(0, t) = \int_{s_1}^{s_2} \delta(s, t) v(s, t) ds, \quad (2)$$

where $\mu(s, t)$ denotes the mortality rate and $\delta(s, t)$ represents the fertility rate, both of which are time-dependent. This formulation is commonly referred to as the McKendrick equation.

Furthermore, fractional calculus is one of the fundamental theories that has played a significant role in the development of mathematical modeling concepts. Many studies have shown that mathematical models with fractional order are more popular than classical integer-order models because they produce more realistic results. For instance, fractional-order partial differential equation models have been shown to provide accurate solutions and predictions for the behavior of oil pollution in water, a phenomenon that cannot be adequately described using classical integer-order PDEs [6]. In addition, fractional-order PDEs have produced more precise results for systems involving anomalous diffusion processes compared to their integer-order counterparts [7]. Similarly, fractional PDEs have contributed to a more comprehensive understanding of complex fluid flow in confined nano-scale shale formations, thereby enabling more effective optimization of shale gas recovery than models based on integer-order PDEs [8]. These studies employed the Caputo operator due to its relevance to the fundamental theory of derivatives. The Caputo fractional derivative addresses some limitations of the Riemann–Liouville fractional derivative. For a constant function $f(x) = c$, the Riemann–Liouville fractional derivative is generally nonzero, whereas the Caputo fractional derivative is zero, consistent with the fundamental property of derivatives. [9].

Moreover, the McKendrick model was modified into a fractional-order partial differential equation by applying the Caputo operator to develop a fractional-order growth model for predicting abalone length, as presented in [10]. Let $u(s, t)$ denote the population density of individuals of age s at time t , and η represent the growth rate. The modified McKendrick model is expressed as follows.

$$\frac{\partial u(s, t)}{\partial t} + \frac{\partial^\beta u(s, t)}{\partial s^\beta} = \eta u(s, t), \quad (3)$$

with the initial condition

$$u(s, 0) = Le^{rs},$$

where β for $\beta \in (0, 1]$, L is defined as the initial length of the abalone, and r represents its intrinsic growth rate. The model was investigated through the Adomian decomposition method and successfully applied in estimating the abalone length growth.

On the other hand, the variational iteration method (VIM) is widely recognized as a semi-analytical approach for solving both linear and nonlinear ordinary as well as

partial differential equations. A key advantage of this method is its adaptability and its efficiency in handling nonlinear systems [11]. Extensive research has demonstrated its successful application across diverse classes of differential equations. Notably, VIM has been used to solve the Hamilton Jacobi Bellman equation [12], to quadratic optimal control problems involving linear partial differential equations [13], and to non isothermal reaction diffusion systems in spherical catalysts [14].

Furthermore, VIM has exhibited strong performance in obtaining rapidly convergent series solutions for optical bright and dark solitons in coupled nonlinear Schrödinger systems [15], resulting in accurate series form solutions for aggregation, aggregation breakage, and pure growth equations [16], and being successfully applied to various models such as the Riccati equation, the Lane Emden equation, and the SIR epidemic model with a constant vaccination rate [17]. Moreover, VIM has proven to be an efficient numerical approach for obtaining solutions to coupled Lane Emden Fowler type equations [18] and has provided accurate results in the analysis of nonzero-sum differential games [19].

The geometric and physical interpretations of classical integer-order models are well established, making them valuable tools for addressing practical problems across a wide range of scientific disciplines. In contrast, fractional-order integration and differentiation present a different situation, despite the rapid development of fractional calculus in both theoretical research and practical applications. Since the concept of differentiation and integration of arbitrary, non-integer orders was introduced, a universally accepted geometric or physical interpretation of these operations has remained elusive for several decades. This issue is widely recognized as an important open problem in the field of fractional calculus [20].

Consequently, a fractional-order growth model is proposed as an extension of the McKendrick model and is analyzed using VIM to predict the growth in abalone weight. The results are presented in both analytical and graphical forms, providing a clearer interpretation of the role of fractional-order derivatives in the proposed growth model.

The organization of the remainder of this paper is as follows. Section 2 reviews the theoretical preliminaries, Section 3 describes the variational iteration method, Section 4 presents the numerical implementation, and Section 5 concludes the study.

2 Preliminaries

This section outlines the basic notion of fractional integrals, fractional derivatives, and Ulam Hyers stability in a succinct manner.

Definition 1.[21] Let $h(t)$ be a continuous function on $[a, b]$. The Riemann Liouville fractional integral of order

β , denoted by I^β , is defined as:

$$I_s^\beta h(t) = \frac{1}{\Gamma(\beta)} \int_a^t (t-\tau)^{\beta-1} h(\tau) d\tau, \quad (4)$$

Definition 2.[22, 23] Let $h(t)$ be a continuous function on $[0, b]$. The Caputo fractional derivative of order β is defined by

$${}^C D^\beta h(t) = \int_a^t \frac{(t-\tau)^{n-\beta-1}}{\Gamma(n-\beta)} h^{(n)}(\tau) d\tau \text{ if } t > 0, \quad (5)$$

where $n \in \mathbb{N}$ is chosen such that $n-1 < \beta \leq n$.

The following basic properties are associated with the fractional integrals and derivatives of $\alpha, \beta \geq 0$ [24]:

1. $I^\alpha I^\beta h(t) = I^\beta I^\alpha h(t) = I^{\alpha+\beta} h(t)$,
2. $(D^\alpha D^\beta) h(t) = (D^\beta D^\alpha) h(t) = D^{(\alpha+\beta)} h(t) = D^{(\beta+\alpha)} h(t)$,
3. $D^\beta (gh) = D^\beta (g)h + gD^\beta (h)$
4. ${}^C D^\beta (ah(t) + bg(t)) = a {}^C D^\beta h(t) + b {}^C D^\beta g(t)$, a and b are constants.

Similarly, the fractional partial derivative is defined as follows:

Definition 3.[25] The function h has Caputo fractional partial derivatives of order β , defined as follows:

$$\frac{\partial^\beta h(s, t)}{\partial s^\beta} = \frac{1}{\Gamma(n-\beta)} \int_0^s (s-\tau)^{n-\beta-1} \frac{\partial^n h(\tau, t)}{\partial \tau^n} d\tau, \quad \beta \in (n-1, n) \quad (6)$$

and

$$\frac{\partial^\beta h(s, t)}{\partial t^\beta} = \frac{1}{\Gamma(n-\beta)} \int_0^t (t-\tau)^{n-\beta-1} \frac{\partial^n h(s, \tau)}{\partial \tau^n} d\tau, \quad \beta \in (n-1, n). \quad (7)$$

Additionally, the definitions of the Gamma and Beta functions, which are fundamental to fractional calculus, are presented below:

Definition 4.[26] The gamma function an improper integral denoted by $\Gamma(z)$, and it is defined as:

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt. \quad (8)$$

Definition 5.[27] The Beta function is defined by the two-parameter integral

$$B(z, v) = \int_0^1 \tau^{z-1} (1-\tau)^{v-1} d\tau, \quad (R(z) > 0, R(v) > 0). \quad (9)$$

An important identity connecting the Gamma and Beta functions is expressed as follows:

$$B(z, v) = \frac{\Gamma(z)\Gamma(v)}{\Gamma(z+v)}, \quad (10)$$

where z and v belong to the complex plane with $\Re(z) > 0$ and $\Re(v) > 0$.

Definition 6.[28] The Mittag Leffler function with a single parameter is defined as

$$E_\beta(z) = \sum_{m=0}^{\infty} \frac{z^m}{\Gamma(m\beta+1)}, \quad \beta > 0, z \in \mathbb{C}, \quad (11)$$

where the series is convergent and $\Gamma(\cdot)$ denotes the Gamma function. This series serves as a natural extension of the classical exponential function.

Moreover, Ulam-Hyers stability serves as an effective tool for assessing the stability of fractional-order differential equations. We now study the following fractional partial differential equation:

$${}^C D_t^\beta v(s, t) = h(x, t, v(s, t)), \quad \forall (s, t) \in D \quad (12)$$

and the following inequality

$$\| {}^C D_t^\beta v(s, t) - h(s, t, v(s, t)) \| < \varepsilon, \quad \forall (s, t) \in D. \quad (13)$$

Definition 7.[29] We say that Equation (12) possesses Ulam-Hyers stability if there exists a positive constant k such that for all $\varepsilon > 0$ and any $v \in C^1(D, \mathbb{R})$ satisfying inequality (13), there exists a corresponding solution $\omega \in C^1(D, \mathbb{R})$ of Equation (12) such that

$$\| v(s, t) - \omega(s, t) \| < \varepsilon k, \quad \forall (s, t) \in D. \quad (14)$$

3 Semi-analytical techniques

This part provides a brief overview of the fundamental techniques of the variational iteration method to solve fractional partial differential equations. The main steps of VIM include: determining the correctional function of the partial differential equation, identifying the Lagrange multiplier, and iterating the correctional function. Let us consider the following fractional partial differential equation [30]:

$$\frac{\partial v(s, t)}{\partial t} + \frac{\partial^\beta v(s, t)}{\partial s^\beta} + v(s, t) + (v(s, t))^2 = 0, \quad (15)$$

and the initial condition;

$$v(s, 0) = v_0(s),$$

where $n-1 < \beta < n, n \in \mathbb{N}$. Based on the variational iteration method, we provide the following correctional function:

$$\begin{aligned} v_{n+1}(s, t) = & v_n(s, t) \\ & + \int_0^t \lambda(\tau) \left[\frac{\partial v_n(s, t)}{\partial t} + \frac{\partial^\beta v_n(s, t)}{\partial s^\beta} \right. \\ & \left. + v_n(s, t) + (v_n(s, t))^2 \right] d\tau. \end{aligned} \quad (16)$$

where λ denotes a Lagrange multiplier identified using variational theory. By applying restricted variations to the nonlinear term, the Lagrange multiplier can be readily determined. Taking the above functional stationary and noting that $\delta\hat{w} = 0$,

$$\delta v_{n+1}(s, t) = \delta v_n(s, t) + \delta \int_0^t \lambda(\tau) \left(\frac{\partial v_n(s, t)}{\partial t} + \frac{\partial^\beta v_n(s, t)}{\partial s^\beta} \right) d\tau, \quad (17)$$

produces the following Lagrange multiplier $\lambda = -1$ for $0 < \beta \leq 1$ [31]. By inserting λ along with the initial condition into Equation (16), the solution of Equation (15) can be obtained by running iterations of the correctional function.

4 Numerical implementation

We examine the fractional growth model given in Equation (3) by applying VIM to estimate the growth of abalone weight. In this context, $w(s, t)$ represents the population density at age s and time t , while η represents the growth rate. The governing model can be represented as:

$$\frac{\partial v(s, t)}{\partial t} + \frac{\partial^\beta v(s, t)}{\partial s^\beta} = \eta v(s, t), \quad (18)$$

with the initial condition

$$v(s, 0) = Ae^{rs},$$

where $\beta \in (0, 1]$. Using VIM as defined in Section 3, the corresponding correction functional can be formulated as follows:

$$v_{n+1}(s, t) = v_n(s, t) + \int_0^t \lambda(\tau) \left[\frac{\partial v_n(s, \tau)}{\partial t} + \frac{\partial^\beta v_n(s, \tau)}{\partial s^\beta} - \eta v_n(s, \tau) \right] d\tau. \quad (19)$$

The successive approximation v_i , $i \geq 0$ can be determined by finding the Lagrange multiplier λ . The function $\tilde{v}_n$ is subject to a constraint, meaning that $\delta\tilde{v}_n = 0$. The next approximation $\tilde{v}_{n+1}$, corresponding to $n \geq 0$, can be derived from $v_n(s, t)$ through the application of the Lagrange multiplier, along with a chosen initial function $v(s, 0)$. Typically, the initial conditions $v(s, 0)$ and $v_t(s, 0)$ are used to determine the zeroth approximation v_0 . To derive the optimal value of the Lagrange multiplier λ , we proceed as follows:

$$\delta v_{n+1}(s, t) = \delta v_n(s, t) + \delta \int_0^t \lambda(\tau) \frac{\partial v_n(s, t)}{\partial \tau} d\tau = 0. \quad (20)$$

This leads to the stationary condition:

$$\lambda'(\tau) = 0 \text{ and } 1 + \lambda(\tau) = 0 \quad (21)$$

which provides $\lambda = -1$. Upon replacing the Lagrange multiplier in Equation (19), the following iterative scheme is obtained:

$$v_{n+1}(s, t) = v_n(s, t) - \int_0^t \left(\frac{\partial v_n(s, t)}{\partial t} + \frac{\partial^\beta v_n(s, t)}{\partial s^\beta} - \eta v_n(s, t) \right) d\tau. \quad (22)$$

Starting with the initial condition $v_0(s, t) = v(s, 0) = Ae^{rs}$, we derive the following series of successive approximations:

$$\begin{aligned} v_1(s, t) &= v_0(s, t) - \int_0^t \left(\frac{\partial v_0(s, t)}{\partial t} + \frac{\partial^\beta v_0(s, t)}{\partial s^\beta} - \eta v_0(s, t) \right) d\tau \\ &= Ae^{rs} - \int_0^t \left(\frac{\partial Ae^{rs}}{\partial t} + \frac{\partial^\beta Ae^{rs}}{\partial s^\beta} - \eta Ae^{rs} \right) d\tau \\ &= Ae^{rs} - A \left(r^\beta - \eta \right) e^{rs} t. \end{aligned} \quad (23)$$

Similarly, we get:

$$v_2(s, t) = Ae^{rs} - A \left(r^\beta - \eta \right) e^{rs} t + A \left(r^\beta - \eta \right)^2 e^{rs} \frac{t^2}{2} \quad (24)$$

and by continuing the process up to n iterations, the expression for $v_n(s, t)$ is obtained as follows:

$$v_n(s, t) = Ae^{rs} \left(1 - \left(r^\beta - \eta \right) t + \frac{\left(r^\beta - \eta \right)^2 t^2}{2!} - \dots + \frac{\left(r^\beta - \eta \right)^n t^n}{n!} \right). \quad (25)$$

Therefore, the solution associated with Equation (17) can be written as:

$$v(s, t) = \lim_{n \rightarrow \infty} v_n(s, t) = Ae^{rs} e^{(\eta - r^\beta)t}. \quad (26)$$

This solution is identical to that obtained using the ADM as presented in [10]. The solution is a continuous function and is convergent for $\eta - r^\beta < 0$, satisfying the conditions of Dirichlet's test as presented in [33]. This implies that the function satisfies the Ulam-Hyers stability condition. For more detail, we consider the implementation of Equation (25) for the growth of abalone. Specifically, we consider the following abalone weight growth data over a 24-month period from the Lombok Marine Aquaculture Center (LMAC), Indonesia [1].

Assume that w represents the weight of abalone and $\eta_i = \Delta w_i / \Delta t_i$ for $i = 0, 1, \dots, 23$, denotes the growth rate. Consequently, Equation (18) can be expressed as the following system.

$$\begin{aligned} \frac{\partial w(s, t)}{\partial t} + \frac{\partial^\beta w(s, t)}{\partial s^\beta} &= \eta_1 w(s, t) \\ \frac{\partial w(s, t)}{\partial t} + \frac{\partial^\beta w(s, t)}{\partial s^\beta} &= \eta_2 w(s, t) \\ &\vdots \\ \frac{\partial w(s, t)}{\partial t} + \frac{\partial^\beta w(s, t)}{\partial s^\beta} &= \eta_{23} w(s, t) \end{aligned}$$

Table 1: Abalone weight data

Month	w(gr)	Month	w(gr)	Month	w(gr)	Month	w(gr)
1	0.0279	7	6.0286	13	26.5905	19	58.3773
2	0.1999	8	8.4409	14	31.2768	20	64.3219
3	0.6228	9	11.2828	15	36.2486	21	70.3761
4	1.3738	10	14.5404	16	41.4749	22	76.5146
5	2.5057	11	18.1941	17	46.9250	23	82.7136
6	4.0516	12	22.2198	18	52.5689	24	88.9509

and the initial condition

$$w(s, 0) = A_i e^{rs},$$

where A_i denotes the weight of abalone in period i for $i = 1, 2, \dots, 23$ and r is an intrinsic or initial weight of abalone. Since this system is derived from Equation (17), its solution is written as

$$w(s, t) = A_i e^{rs} e^{(\eta_i - r^\beta)t}. \quad (27)$$

By substituting the parameter values into Equation (27) and performing the computations using MATLAB R2024a, the resulting values are presented in the following table.

Table 2: Abalone weight approximation using VIM

Month	η	$w_{0.5}$	$w_{0.6}$	$w_{0.7}$	$w_{0.8}$	$w_{0.9}$	$w_{1.0}$
1	0	0.0279	0.0279	0.0279	0.0279	0.0279	0.0279
2	0.8603	0.0574	0.0604	0.0625	0.0641	0.0652	0.0660
3	0.6790	0.3430	0.3606	0.3736	0.3829	0.3895	0.3942
4	0.5466	0.9361	0.9843	1.0195	1.0449	1.0630	1.0759
5	0.4517	1.8778	1.9746	2.0453	2.0961	2.1325	2.1582
6	0.3816	3.1930	3.3575	3.4776	3.5641	3.6259	3.6697
7	0.3279	4.8934	5.1456	5.3297	5.4622	5.5569	5.6240
8	0.2858	6.9806	7.3405	7.6030	7.7921	7.9272	8.0230
9	0.2519	9.4479	9.9350	10.2903	10.5463	10.7291	10.8587
10	0.2240	12.2821	12.9152	13.3772	13.7100	13.9476	14.1161
11	0.2008	15.4650	16.2622	16.8438	17.2628	17.5620	17.7742
12	0.1812	18.9746	19.9527	20.6664	21.1805	21.5476	21.8080
13	0.1644	22.7868	23.9614	24.8185	25.4359	25.8766	26.1893
14	0.1498	26.8755	28.2609	29.2718	30.0000	30.5198	30.8887
15	0.1372	31.2138	32.8229	33.9969	34.8426	35.4464	35.8747
16	0.1260	35.7746	37.6188	38.9643	39.9336	40.6256	41.1166
17	0.1161	40.5308	42.6201	44.1446	45.2428	46.0268	46.5830
18	0.1074	45.4558	47.7990	49.5087	50.7403	51.6195	52.2433
19	0.0995	50.5240	53.1285	55.0288	56.3977	57.3750	58.0684
20	0.0924	55.7108	58.5826	60.6781	62.1875	63.2652	64.0297
21	0.0860	60.9927	64.1368	66.4309	68.0835	69.2633	70.1003
22	0.0802	66.3477	69.7678	72.2633	74.0610	75.3443	76.2549
23	0.0749	71.7547	75.4536	78.1525	80.0966	81.4846	82.4693
24	0.0701	77.1948	81.1741	84.0776	86.1692	87.6624	88.7217

Let us consider that w_β denotes the predicted abalone weight of order β . By comparing Table 2 with the data in Table 1, the following errors are obtained.

Based on Table 3, the absolute mean errors for $w_{0.5}$, $w_{0.6}$, $w_{0.7}$, $w_{0.8}$, $w_{0.9}$ and $w_{1.0}$ are 4.4244, 3.0077, 1.9739, 1.2293, 0.6976 and 0.3205, respectively. This implies that as the fractional order increases, the mean absolute error decreases. Therefore, the results indicate that the fractional order $\beta = 0.1$ provides the closest agreement with the observed data, followed by the fractional order $\beta = 0.9$. For further illustration, the following graph is presented.

Table 3: Errors of abalone weight approximations using VIM

Age	$E_{0.5}$	$E_{0.6}$	$E_{0.7}$	$E_{0.8}$	$E_{0.9}$	$E_{1.0}$
1	0	0	0	0	0	0
2	0.1425	0.1395	0.1374	0.1358	0.1347	0.1339
3	0.2798	0.2622	0.2492	0.2399	0.2333	0.2286
4	0.4377	0.3895	0.3543	0.3289	0.3108	0.2979
5	0.6279	0.5311	0.4604	0.4096	0.3732	0.3475
6	0.8586	0.6941	0.5740	0.4875	0.4257	0.3819
7	1.1352	0.8830	0.6989	0.5664	0.4717	0.4046
8	1.4604	1.1005	0.8379	0.6489	0.5138	0.4179
9	1.8349	1.3478	0.9925	0.7365	0.5537	0.4241
10	2.2583	1.6252	1.1632	0.8304	0.5928	0.4243
11	2.7291	1.9319	1.3503	0.9313	0.6321	0.4199
12	3.2452	2.2671	1.5534	1.0393	0.6722	0.4118
13	3.8037	2.6291	1.7719	1.1546	0.7139	0.4012
14	4.4013	3.0159	2.0050	1.2768	0.7570	0.3881
15	5.0348	3.4257	2.2517	1.4059	0.8022	0.3739
16	5.7003	3.8561	2.5106	1.5413	0.8493	0.3583
17	6.3942	4.3049	2.7804	1.6822	0.8982	0.3420
18	7.1131	4.7699	3.0602	1.8286	0.9494	0.3256
19	7.8533	5.2488	3.3485	1.9796	1.0023	0.3089
20	8.6111	5.7393	3.6438	2.1344	1.0567	0.2922
21	9.3834	6.2393	3.9452	2.2926	1.1128	0.2758
22	10.1669	6.7468	4.2513	2.4536	1.1703	0.2597
23	10.9589	7.2599	4.5611	2.6169	1.2289	0.2443
24	11.7561	7.7768	4.8733	2.7817	1.2885	0.2292

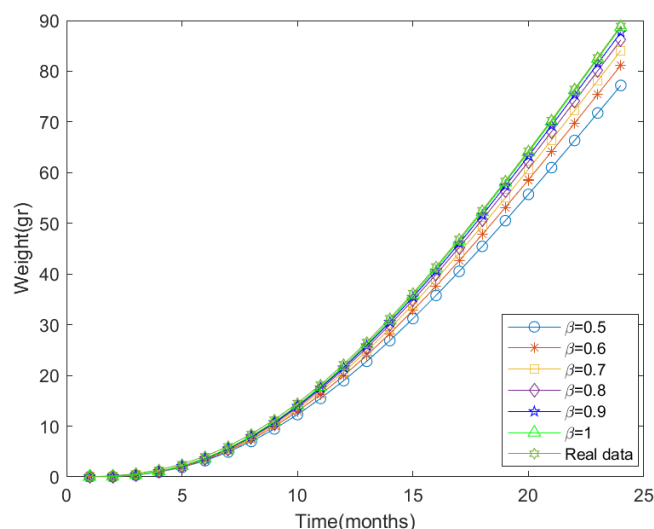
**Fig. 1:** Abalone weight approximations compared with the observed data

Figure 1 shows that as the fractional order increases, the resulting function values increase. This implies that the fractional order in the fractional growth model represents the exponent of the function's gradient. The graph corresponding to $\beta = 1$ exhibits the closest agreement with the observed data, followed by that for $\beta = 0.9$. As both the fractional- and integer-order models produce nearly equivalent results that are in close agreement with the observed data, the fractional-order

model is regarded as providing reliable and realistic predictions.

5 Conclusions

This study demonstrated that modifying the McKendrick model into a fractional-order growth model, with an exponential function as the initial condition, provides an accurate approach for predicting abalone weight growth. The proposed model was analyzed using the variational iteration method, which yielded a particular case of a Taylor series. By substituting the series with parameter values based on abalone weight growth data and simulating different fractional orders, the resulting approximations capture the underlying pattern of abalone weight growth. The results indicated that As the fractional-order increases, the mean absolute error decreases. This indicates that as the fractional order approaches unity, the resulting values exhibit greater accuracy and improved agreement with the observed data. For future research, the model will be extended to a two-fractional-order growth model, which will be analyzed using a fractional variational iteration method.

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