

Strictly Alternating Three-Player Prisoner's Dilemma Game with Different Memory-Length

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Abstract: The study of cooperation through the Prisoner's Dilemma has yielded fundamental insights, yet real-world dilemmas often involve multiple participants interacting in turn and with differing cognitive capacities, such as varying memory lengths. To address how these factors jointly shape cooperation, this work introduces a three-player alternating Prisoner's Dilemma model. Contests are analyzed between distinct strategy classes that extend Tit-for-Tat logic under different memory capacities. Through evolutionary simulations supported by graphical and numerical analysis, the most successful strategies and the highest achievable payoffs are identified, revealing the direct impact of memory length on outcomes. Furthermore, the role of player relatedness in modulating strategic behavior and performance within this complex environment is examined.

Keywords: Iterated Prisoner's Dilemma (IPD), Markov Transition Matrix, Memory, Relatedness, Strictly Alternating, Tit-For-Tat (TFT), Three-Player

1 Introduction

Game theory is a collection of concepts for using mathematical methods to analyze conflict models. The strategic interaction between strategically active rational agents can be analyzed formally using this method [1]. One approach that is more applicable to the behavioral and biological sciences is repetitive game theory. It provides the primary framework for understanding how individuals may nevertheless work together [2,3].

There are common models in game theory, such as in biology and economics, including Hawk-Dove and others. The Prisoner's Dilemma in iterated games, however, is the most important paradigm. It demonstrates why two reasonable people would not work together, even if it appeared to be advantageous to them. One can use the Prisoner's Dilemma to make decisions in a variety of daily life situations.

Moreover, games can be classified into two categories: simultaneous games [1,3] and alternating games [4,5]. Moreover, there exist two categories of alternating games: randomly alternating and strictly alternating. We will discuss everything related to these two types and compare them, but this research will focus on the second type (Alternating).

1.1 Prisoner's Dilemma Games for Two-Payer

The prisoner's dilemma is a sophisticated illustration of reciprocal assistance. There are two motions for both players: C and D [1,2]. When both players cooperate (C), they each receive $\mathcal{R}$ (reward); when they both defect (D), they each receive $\mathcal{P}$ (penalty), which is less than $\mathcal{R}$. Defectors will obtain the largest payment $\mathcal{T}$ (the temptation to defect), whereas cooperators will receive the lowest $\mathcal{S}$ (Sucker's) if one is defective and the opponent cooperates. Defection is, of course, the only viable

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option for a single round. If the game (repeated prisoners' dilemma game) is played again, something can change. Four outward transitions result from this: both players cooperate (C, C), the first player cooperates only (C, D), the second player cooperates only (D, C), and both players defect (D, D).

Therefore, there are 16 different strategies made out of 2^4 probability transition arrows denoted by $S_0, S_1, \dots, S_{15}$. Where it is a plan that instructs the player on the best move to make in any scenario. These strategies can be categorized by (y_1, y_2, y_3, y_4) of ones and zeros. Here $y_i = 1$ or 0 ; if the player plays C or D , respectively. For instance, $S_{10} = (1, 0, 1, 0)$, which is known as the Tit-For-Tat (TFT) strategy [1], represented as in Fig. 1

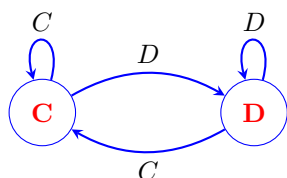


Fig. 1: $S_{10} = (1, 0, 1, 0)$

✓ Simultaneous Case

In this case, the two players make decisions simultaneously without being aware of each other's decision as represented in Table 1

Table 1: Representation of Units in Two-Player Simultaneous Games

Number of rounds	$2k - 1$	$2k$
Player I	C	D
Player II	C	C

$\underbrace{\hspace{10em}}$
unit $2k - 1$

$\underbrace{\hspace{10em}}$
unit $2k$

Additionally, the payoff matrix for every Prisoner's Dilemma game round [1] is provided by

$$\begin{matrix} & C & D \\ \begin{matrix} C \\ D \end{matrix} & \begin{bmatrix} \mathcal{R} & \mathcal{S} \\ \mathcal{T} & \mathcal{P} \end{bmatrix} \end{matrix}, \quad (1)$$

where

$$2\mathcal{R} > \mathcal{T} + \mathcal{S} \text{ and } \mathcal{T} > \mathcal{R} > \mathcal{P} > \mathcal{S}. \quad (2)$$

Assume that the player use approach $\theta = (\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4)$ and the opponent uses strategy ϵ with probability $\epsilon = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4)$; (where ϑ_i and ϵ_i are the probability of player I and player II, respectively, for playing C after state i). The Markov chain transition matrix (M_S) is provided by [1,3,4].

$$M_S = \begin{bmatrix} \vartheta_1\epsilon_1 & \vartheta_1(1-\epsilon_1) & (1-\vartheta_1)\epsilon_1 & (1-\vartheta_1)(1-\epsilon_1) \\ \vartheta_2\epsilon_3 & \vartheta_2(1-\epsilon_3) & (1-\vartheta_2)\epsilon_3 & (1-\vartheta_2)(1-\epsilon_3) \\ \vartheta_3\epsilon_2 & \vartheta_3(1-\epsilon_2) & (1-\vartheta_3)\epsilon_2 & (1-\vartheta_3)(1-\epsilon_2) \\ \vartheta_4\epsilon_4 & \vartheta_4(1-\epsilon_4) & (1-\vartheta_4)\epsilon_4 & (1-\vartheta_4)(1-\epsilon_4) \end{bmatrix} \quad (3)$$

As seen in the accompanying (3), if we wish to convert state $\mathcal{R}$ to $\mathcal{T}$, player I will alter his choice and play D after (C, C) , whereas player II will insist on his choice and play C after (C, C) . Consequently, $(1 - \vartheta_1)\epsilon_1$ is the probability of the transition from state $\mathcal{R}$ to $\mathcal{T}$.

✓ Alternating Case

In this case, a player makes a decision initially, and the other player follows suit after learning of the first player's decision as represented in Table 2

Table 2: Representation of Units in Two-Player Alternative Games

Number of rounds	$2k - 2$	$2k - 1$	$2k$	$2k + 1$
Player I	C		D	
Player II		C		C

$\underbrace{\hspace{10em}}$
unit $2k - 2$

$\underbrace{\hspace{10em}}$
unit $2k$

$\underbrace{\hspace{10em}}$
unit $2k - 1$

One of the players with influence over the result is the leader of each round. The leader receives α_1 points and the adversary obtains β_1 points if he chooses C . Additionally, if the leader chooses D , the adversary receives β_2 and the leader receives α_2 .

The leader's choice D is superior to C in a single round. When the profit is $\beta_1 - \beta_2$ and the loss is $\alpha_2 - \alpha_1$, we obtain

$$0 < \alpha_2 - \alpha_1 < \beta_1 - \beta_2 \quad (4)$$

We take into account that the players alternately lead for two rounds in a row. If both play C , they

obtain $\alpha_1 + \beta_1$, represented by $\mathcal{R}$. Additionally, if both play D , they both receive $\alpha_2 + \beta_2$, represented by $\mathcal{P}$. However, if one plays C and the opponent D , the collaborator party benefits $\alpha_1 + \beta_2$, which we designate by $\mathcal{S}$, and the deviant gets $\alpha_2 + \beta_1$, which is represented by $\mathcal{T}$. Consequently, we get

$$2\mathcal{R} > \mathcal{T} + \mathcal{S} \text{ and } \mathcal{T} > \mathcal{R} > \mathcal{P} > \mathcal{S} \quad (5)$$

The payoff condition for simultaneous Prisoner's Dilemma games is inequality (2). However, the reward condition in the alternating Prisoner's Dilemma games will be the same as in (5) and (7).

$$\mathcal{T} + \mathcal{S} = \mathcal{R} + \mathcal{P} \quad (6)$$

Additionally, the Markov chain transition matrix for simultaneous games is represented by the matrix M_S , which is displayed in (3). Player I will alter his choice, and Player II will insist on his choice, as transforming state $\mathcal{R}$ to $\mathcal{T}$ entails changing from (C, C) to (D, C) . Consequently, $(1 - \vartheta_1)\varepsilon_1$ is the transformation probability from state $\mathcal{R}$ to $\mathcal{T}$. However, as demonstrated [5], the Markov chain transition matrix for alternating games M_A is different.

$$M_A = \begin{bmatrix} \vartheta_1\varepsilon_1 & \vartheta_1(1-\varepsilon_1) & (1-\vartheta_1)\varepsilon_2 & (1-\vartheta_1)(1-\varepsilon_2) \\ \vartheta_2\varepsilon_3 & \vartheta_2(1-\varepsilon_3) & (1-\vartheta_2)\varepsilon_4 & (1-\vartheta_2)(1-\varepsilon_4) \\ \vartheta_3\varepsilon_1 & \vartheta_3(1-\varepsilon_1) & (1-\vartheta_3)\varepsilon_2 & (1-\vartheta_3)(1-\varepsilon_2) \\ \vartheta_4\varepsilon_3 & \vartheta_4(1-\varepsilon_3) & (1-\vartheta_4)\varepsilon_4 & (1-\vartheta_4)(1-\varepsilon_4) \end{bmatrix} \quad (7)$$

As seen in the accompanying (7), if we wish to convert state $\mathcal{R}$ to $\mathcal{T}$, player I will alter his choice and play D after (C, C) , whereas player II will insist on his choice and play C after (C, D) . Consequently, $(1 - \vartheta_1)\varepsilon_2$ is the probability of the transition from state $\mathcal{R}$ to $\mathcal{T}$.

1.2 Prisoner's Dilemma Games for Three-Payer

If we have three players playing the Prisoner's Dilemma game, this game will be called three-player Prisoner's Dilemma (3P-IPD). We consider an iterated game that consists of repeating the game infinitely with a probability of 1. Generally speaking, each participant in the iterated Prisoner's Dilemma games has two options, C and D . So, we have eight outcomes $[CCC, CCD, CDC, CDD, DCC, DCD, DDC, DDD]$. We assume that the game is symmetric, so the outcomes decrease from eight to six combinations $[(C, (C, C)), (C, (C, D)), (C, (D, D)), (D, (C, C)), (D, (C, D)), (D, (D, D))]$ which are defined by the player's payoff $\mathcal{R}, \mathcal{K}, \mathcal{S}, \mathcal{T}, \mathcal{L}$ and $\mathcal{P}$.

Therefore, there are 64 different strategies denoted by $S_0, S_1, \dots, S_{63}$ which can be categorized by $(y_1, y_2, y_3, y_4, y_5, y_6)$ of ones and zeros. Here $y_i = 1$ or 0; if the player plays C or D , respectively. For instance, S_{36}, S_{38}, S_{52} , and S_{54} , which are known as TFT 1, TFT 2, TFT 3, and TFT 4 strategies [3], which are represented as in Fig. 2

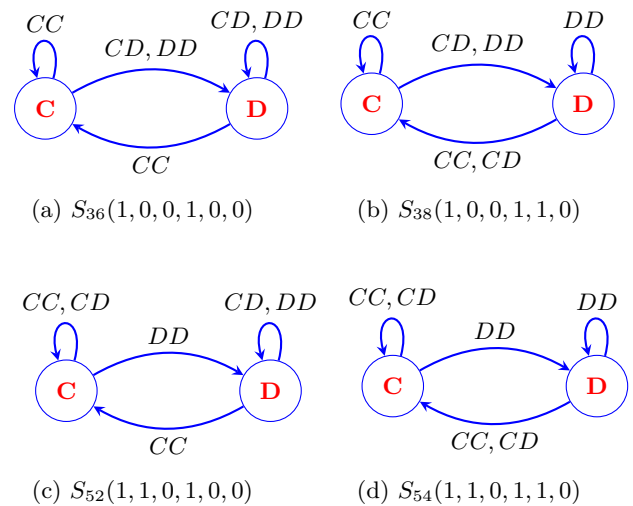


Fig. 2: Automates of TFT strategies

✓ Simultaneous Case

This case is represented in Table 3

Table 3: Representation of Units in Three-Player Simultaneous Games

Number of rounds	$3k - 1$	$3k$
Player I	C	D
Player II	C	C
Player III	D	D

$\underbrace{\hspace{10em}}_{\text{unit } 3k - 1} \quad \underbrace{\hspace{10em}}_{\text{unit } 3k}$

Additionally, (3P-IPD)'s payoff matrix [3] is provided by

$$\begin{matrix} & \begin{matrix} CC & CD & DD \end{matrix} \\ \begin{matrix} C \\ D \end{matrix} & \begin{bmatrix} \mathcal{R} & \mathcal{K} & \mathcal{S} \\ \mathcal{T} & \mathcal{L} & \mathcal{P} \end{bmatrix} \end{matrix}, \quad (8)$$

where

$$\mathcal{S} < \mathcal{P} < \mathcal{K} < \mathcal{L} < \mathcal{R} < \mathcal{T}. \quad (9)$$

Assuming that player I plays with probability $\theta = (\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5, \vartheta_6)$, player II with probability $\epsilon = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5, \epsilon_6)$, and player III with $\rho = (\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5, \varrho_6)$; (where ϑ_i , ϵ_i and ϱ_i are the probability of player I, player II and player III respectively for playing C after state i). Consequently, the three-player Markov transition matrix [6] is provided by $M_S =$

$$\begin{pmatrix} \vartheta_1 \epsilon_1 \varrho_1 & a_1 [\epsilon_1 (1 - \varrho_1) + (1 - \epsilon_1) \varrho_1] \\ \vartheta_2 \epsilon_2 \varrho_4 & \vartheta_2 [\epsilon_2 (1 - \varrho_4) + (1 - \epsilon_2) \varrho_4] \\ \vartheta_3 \epsilon_5 \varrho_5 & \vartheta_3 [\epsilon_5 (1 - \varrho_5) + (1 - \epsilon_5) \varrho_5] \\ \vartheta_4 \epsilon_2 \varrho_2 & \vartheta_4 [\epsilon_2 (1 - \varrho_2) + (1 - \epsilon_2) \varrho_2] \\ \vartheta_5 \epsilon_3 \varrho_5 & \vartheta_5 [\epsilon_3 (1 - \varrho_5) + (1 - \epsilon_3) \varrho_5] \\ \vartheta_6 \epsilon_6 \varrho_6 & \vartheta_6 [\epsilon_6 (1 - \varrho_6) + (1 - \epsilon_6) \varrho_6] \end{pmatrix}$$

$$\begin{array}{ll} a_1 (1 - \epsilon_1) (1 - \varrho_1) & (1 - \vartheta_1) \epsilon_1 \varrho_1 \\ \vartheta_2 (1 - \epsilon_2) (1 - \varrho_4) & (1 - \vartheta_2) \epsilon_2 \varrho_4 \\ \vartheta_3 (1 - \epsilon_5) (1 - \varrho_5) & (1 - \vartheta_3) \epsilon_5 \varrho_5 \\ \vartheta_4 (1 - \epsilon_2) (1 - \varrho_2) & (1 - \vartheta_4) \epsilon_2 \varrho_2 \\ \vartheta_5 (1 - \epsilon_3) (1 - \varrho_5) & (1 - \vartheta_5) \epsilon_3 \varrho_5 \\ p_6 (1 - \epsilon_6) (1 - \varrho_6) & (1 - \vartheta_6) \epsilon_6 \varrho_6 \end{array}$$

$$\begin{pmatrix} (1 - \vartheta_1) [\epsilon_1 (1 - \varrho_1) + (1 - \epsilon_1) \varrho_1] & (1 - \vartheta_1) (1 - \epsilon_1) (1 - \varrho_1) \\ (1 - \vartheta_2) [\epsilon_2 (1 - \varrho_4) + (1 - \epsilon_2) \varrho_4] & (1 - \vartheta_2) (1 - \epsilon_2) (1 - \varrho_4) \\ (1 - \vartheta_3) [\epsilon_5 (1 - \varrho_5) + (1 - \epsilon_5) \varrho_5] & (1 - \vartheta_3) (1 - \epsilon_5) (1 - \varrho_5) \\ (1 - \vartheta_4) [\epsilon_2 (1 - \varrho_2) + (1 - \epsilon_2) \varrho_2] & (1 - \vartheta_4) (1 - \epsilon_2) (1 - \varrho_2) \\ (1 - \vartheta_5) [\epsilon_3 (1 - \varrho_5) + (1 - \epsilon_3) \varrho_5] & (1 - \vartheta_5) (1 - \epsilon_3) (1 - \varrho_5) \\ (1 - \vartheta_6) [\epsilon_6 (1 - \varrho_6) + (1 - \epsilon_6) \varrho_6] & (1 - \vartheta_6) (1 - \epsilon_6) (1 - \varrho_6) \end{pmatrix} \quad (10)$$

What about the Alternating Case?. Most earlier research, whether with memory one or two, has looked at the simultaneous Iterated Prisoner's Dilemma game for two players in [1, 7] and three players in [3, 8, 6]. Additionally, the impact of the players' relationships on the strategies' behavior was examined in [3, 6, 9, 10, 11]. Then, a few recent studies examined games only for the two-player alternating prisoner's dilemma either for memory one or two [4, 5, 12]. Furthermore, if there is relatedness between players, either for memory one or two [13].

In our paper, we present the first study of a strictly alternating iterated Prisoner's Dilemma game for three players, and we construct a Markov transition matrix for this study. Furthermore, we show what happens to the players' payoffs if there is a relationship between the three players.

2 Transition Matrix

In this section, we will discuss a three-player strictly alternating iterated Prisoner's Dilemma game where players play as in Table 4

We will put the construction for the Markov transition matrix for a three-player strictly

Table 4: Representation of Units in Three-Player Alternative Games

Number of rounds						$3k-3$	$3k-2$	$3k-1$	$3k$	$3k+1$	$3k+2$
Player I	C			D							
Player II				C				C			
Player III							D				D
						$\underbrace{\hspace{10em}}$ unit $3k-3$			$\underbrace{\hspace{10em}}$ unit $3k$		
						$\underbrace{\hspace{10em}}$ unit $3k-2$					
									$\underbrace{\hspace{10em}}$ unit $3k-1$		

alternating iterated Prisoner's Dilemma game. Generally speaking, each participant in three-person Iterated Prisoner's Dilemma games has two options, C and D . So, we have 2^3 outcomes $[CCC, CCD, CDC, CDD, DCC, DCD, DDC, DDD]$. In a symmetric case for three-player Iterated Prisoner's Dilemma games, we can reduce the eight outcomes to six outcomes. It doesn't matter to the first player if the second player plays C and the third plays D , or if the second player plays D and the third plays C . All he cares about is that he is playing with two players, one of them plays C and the other plays D . I.e., the outcomes become $[CCC, CCD, CDD, DCC, DCD, DDD]$. Considering player I to be a probability player $\theta = (\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4, \vartheta_5, \vartheta_6)$, player II with probability $\epsilon = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5, \epsilon_6)$ and player III with $\rho = (\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5, \varrho_6)$. Consequently, the Markov transition matrix for a strictly alternating three-player game will be provided by $M_A =$

$$\begin{pmatrix} \vartheta_1 \epsilon_1 \varrho_1 & \vartheta_1 [\epsilon_1 (1 - \varrho_1) + (1 - \epsilon_1) \varrho_2] \\ \vartheta_2 \epsilon_2 \varrho_4 & \vartheta_2 [\epsilon_2 (1 - \varrho_4) + (1 - \epsilon_2) \varrho_5] \\ \vartheta_3 \epsilon_5 \varrho_4 & \vartheta_3 [\epsilon_5 (1 - \varrho_4) + (1 - \epsilon_5) \varrho_5] \\ \vartheta_4 \epsilon_1 \varrho_1 & \vartheta_4 [\epsilon_1 (1 - \varrho_1) + (1 - \epsilon_1) \varrho_2] \\ \vartheta_5 \epsilon_2 \varrho_4 & \vartheta_5 [\epsilon_2 (1 - \varrho_4) + (1 - \epsilon_2) \varrho_5] \\ \vartheta_6 \epsilon_5 \varrho_4 & \vartheta_6 [\epsilon_5 (1 - \varrho_4) + (1 - \epsilon_5) \varrho_5] \end{pmatrix}$$

$$\begin{array}{ll} \vartheta_1 (1 - \epsilon_1) (1 - \varrho_2) & (1 - \vartheta_1) \epsilon_2 \varrho_2 \\ \vartheta_2 (1 - \epsilon_2) (1 - \varrho_5) & (1 - \vartheta_2) \epsilon_3 \varrho_5 \\ \vartheta_3 (1 - \epsilon_5) (1 - \varrho_5) & (1 - \vartheta_3) \epsilon_6 \varrho_5 \\ \vartheta_4 (1 - \epsilon_1) (1 - \varrho_2) & (1 - \vartheta_4) \epsilon_2 \varrho_2 \\ \vartheta_5 (1 - \epsilon_2) (1 - \varrho_5) & (1 - \vartheta_5) \epsilon_3 \varrho_5 \\ \vartheta_6 (1 - \epsilon_5) (1 - \varrho_5) & (1 - \vartheta_6) \epsilon_6 \varrho_5 \end{array}$$

$$\begin{pmatrix} (1 - \vartheta_1) [\epsilon_2 (1 - \varrho_2) + (1 - \epsilon_2) \varrho_3] & (1 - \vartheta_1) (1 - \epsilon_2) (1 - \varrho_3) \\ (1 - \vartheta_2) [\epsilon_3 (1 - \varrho_5) + (1 - \epsilon_3) \varrho_6] & (1 - \vartheta_2) (1 - \epsilon_3) (1 - \varrho_6) \\ (1 - \vartheta_3) [\epsilon_6 (1 - \varrho_5) + (1 - \epsilon_6) \varrho_6] & (1 - \vartheta_3) (1 - \epsilon_6) (1 - \varrho_6) \\ (1 - \vartheta_4) [\epsilon_2 (1 - \varrho_2) + (1 - \epsilon_2) \varrho_3] & (1 - \vartheta_4) (1 - \epsilon_2) (1 - \varrho_3) \\ (1 - \vartheta_5) [\epsilon_3 (1 - \varrho_5) + (1 - \epsilon_3) \varrho_6] & (1 - \vartheta_5) (1 - \epsilon_3) (1 - \varrho_6) \\ (1 - \vartheta_6) [\epsilon_6 (1 - \varrho_5) + (1 - \epsilon_6) \varrho_6] & (1 - \vartheta_6) (1 - \epsilon_6) (1 - \varrho_6) \end{pmatrix} \quad (11)$$

As seen in the accompanying (11), if we wish to convert state $\mathcal{R}$ to $\mathcal{T}$, player I will alter his choice

and play D after (C, C, C) , while players II and III will insist on their choice and play C after (C, C, D) . Consequently, $(1 - \vartheta_1)\varepsilon_2\varrho_2$ is the probability of the transition from state $\mathcal{R}$ to $\mathcal{T}$.

3 Monitor Reactions of Tit-For-Tat Strategies

In this section, we will study the reactions of Tit-For-Tat Strategies. For example, player I (S_{38}) is pitted against player III (S_{38}) alongside player II (S_{38}) in two cases (memory-one case and memory two case).

3.1 Memory-One

• Seq. 1: If the three players begin with C . Thus, this sequence has a one-round repetition period with $\mathcal{R}$ payoffs. It yields an average reward of $\mathcal{R}$ (Regime R_1).

$S_{38} :$	C	$\underline{C}$	C	C	C	
$S_{38} :$	$\underline{C}$	C	C	C	C	$\mathcal{R}$
$S_{38} :$		$\underline{C}$	C	C	C	C

• Seq. 2: Player II begins with D , but players I and III begin with C . Consequently, this sequence has a one-round repetition time with $\mathcal{P}$ payoffs. It yields an average reward of $\mathcal{P}$ (Regime R_2).

$S_{38} :$	C	$\underline{D}$	D	D	D	
$S_{38} :$	C	$\underline{D}$	D	D	D	$\mathcal{P}$
$S_{38} :$		D	$\underline{D}$	D	D	D

• Seq. 3: Player III begins with D , but players I and II begin with C . As a result, this sequence has a two-round repetition period with $(\mathcal{K}, \mathcal{L})$ payoffs. Our technique yields an average reward of $\frac{\mathcal{K}+\mathcal{L}}{2}$ (Regime R_3).

$S_{38} :$	C	$\underline{D}$	C	D	C	
$S_{38} :$	D	$\underline{C}$	D	C	D	$\frac{\mathcal{K}+\mathcal{L}}{2}$
$S_{38} :$		$\underline{C}$	$\underline{D}$	C	D	C

• Seq. 4: Player II begins with D , but players I and III begin with C . Therefore, this sequence's repetition time is one round, and payoffs are $\mathcal{P}$ as in seq. 2.

$S_{38} :$	C	$\underline{D}$	D	D	D	
$S_{38} :$	D	$\underline{D}$	D	D	D	$\mathcal{P}$
$S_{38} :$		$\underline{D}$	D	D	D	D

• Seq. 5: If player I begins with D , but players II and III begin with C . Thus, this sequence's repetition time is one round, and the payoffs are $\mathcal{R}$ as in seq. 1.

$S_{38} :$	D	$\underline{C}$	C	C	C	
$S_{38} :$	C	$\underline{C}$	C	C	C	$\mathcal{R}$
$S_{38} :$		$\underline{C}$	C	C	C	C

• Seq. 6: Players I and II begin with D , but player III begins with C . Consequently, this sequence has a four-round repetition time, payoffs of $(\mathcal{K}, \mathcal{L})$ as in Seq. 3.

$S_{38} :$	D	$\underline{C}$	D	C	D	
$S_{38} :$	C	$\underline{D}$	C	D	C	$\frac{\mathcal{L}+\mathcal{K}}{2}$
$S_{38} :$		$\underline{D}$	$\underline{C}$	D	C	D

• Seq. 7: Players I and III begin with D , but player II begins with C . Consequently, this sequence's repetition duration is one round, with payoffs $\mathcal{R}$ as in Seq. 1 and Seq. 5.

$S_{38} :$	D	$\underline{C}$	C	C	C	
$S_{38} :$	D	$\underline{C}$	C	C	C	$\mathcal{R}$
$S_{38} :$		C	$\underline{C}$	C	C	C

• Seq. 8: If all three players begin with D . As a result, this sequence has a one-round repetition time, with payoffs $\mathcal{P}$ as in Seq. 2 and Seq. 4.

$S_{38} :$	D	$\underline{D}$	D	D	D	
$S_{38} :$	$\underline{D}$	D	D	D	D	$\mathcal{P}$
$S_{38} :$		$\underline{D}$	D	D	D	D

The three techniques (regimes) in the preceding eight sequences are referred to as R_1 , R_2 , and R_3 . Under the possibility of error (playing C instead of D or vice versa), let's create a little random noise. Only implementation errors will be examined in this

investigation the we get probability Matrix as

$$\begin{matrix} & R_1 & R_2 & R_3 \\ \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} & \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/2 & 1/2 & 0 \end{bmatrix} \end{matrix} \quad (12)$$

By computing the left eigenvectors for the eigenvalue 1, we get

$$-\frac{2}{3}\ell_1 + \frac{1}{3}\ell_2 + \frac{1}{2}\ell_3 = 0, \quad (13)$$

$$\frac{1}{3}\ell_1 - \frac{2}{3}\ell_2 + \frac{1}{2}\ell_3 = 0, \quad (14)$$

$$\frac{1}{3}\ell_1 + \frac{1}{3}\ell_2 - \ell_3 = 0, \quad (15)$$

The eigenvector ℓ is obtained by solving the linear system of the preceding equations from (13) to (15) using the equation $\ell_1 + \ell_2 + \ell_3 = 1$.

$$\ell = (\ell_1, \ell_2, \ell_3) = (3/8, 3/8, 1/4) \quad (16)$$

We can now obtain the payoff values by

$$\begin{aligned} E(S_{38}, S_{38}, S_{38}) &= \ell_1.R_1 + \ell_2.R_2 + \ell_3.R_3 \\ &= 1/8 [3\mathcal{R} + 3\mathcal{P} + \mathcal{K} + \mathcal{L}] \end{aligned} \quad (17)$$

Equation (17) states that $(3, 1, 0, 0, 1, 3)$ is the payoff vector $\Pi = (\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6)$. Examining every TFT strategies using the same methodology. As a result, we got Tables 4 to 7 below.

Table 5: Player I's payoff against Player III when Player II using (S_{36}) approach

	S_{36}	S_{38}	S_{52}	S_{54}
S_{36}	(0, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 1)
S_{38}	(1, 0, 0, 0, 0, 2)	(1, 0, 0, 0, 0, 2)	(1, 0, 0, 0, 0, 2)	(1, 0, 0, 0, 0, 2)
S_{52}	(0, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 1)
S_{54}	(1, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 0)	(1, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 0)

Table 6: Player I's payoff against Player III when Player II using (S_{38}) approach

	S_{36}	S_{38}	S_{52}	S_{54}
S_{36}	(0, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 1)
S_{38}	(1, 0, 0, 0, 0, 2)	(3, 1, 0, 0, 1, 3)	(1, 0, 0, 0, 0, 1)	(2, 0, 0, 0, 0, 1)
S_{52}	(0, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 0)
S_{54}	(1, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 0)	(1, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 0)

Table 7: Player I's payoff against Player III when Player II using (S_{52}) approach

	S_{36}	S_{38}	S_{52}	S_{54}
S_{36}	(0, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 1)
S_{38}	(1, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 1)
S_{52}	(1, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 1)
S_{54}	(1, 0, 0, 0, 0, 0)	(1, 0, 0, 0, 0, 0)	(1, 0, 0, 0, 0, 0)	(1, 0, 0, 0, 0, 0)

Table 8: Player I's payoff against Player III when Player II using (S_{54}) approach

	S_{36}	S_{38}	S_{52}	S_{54}
S_{36}	(0, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 1)
S_{38}	(1, 0, 0, 0, 0, 1)	(2, 0, 0, 0, 0, 1)	(2, 0, 0, 0, 0, 1)	(2, 0, 0, 0, 0, 1)
S_{52}	(1, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 1)	(1, 0, 0, 0, 0, 0)	(1, 0, 0, 0, 0, 0)
S_{54}	(1, 0, 0, 0, 0, 0)	(1, 0, 0, 0, 0, 0)	(1, 0, 0, 0, 0, 0)	(1, 0, 0, 0, 0, 0)

3.2 Memory-Two

• Seq. 1: Assuming that the three players begin with C in the first and the second round. Thus, this sequence has a two-round repetition period with $\frac{\mathcal{R}+\mathcal{R}}{2}$ payoffs. It yields an average reward of $\mathcal{R}$ (Regime R_1).

$$\begin{aligned} S_{38} : & C & C & C & C & C \\ S_{38} : & C & \underline{C} & C & C & C & \frac{\mathcal{R}+\mathcal{R}}{2} \\ S_{38} : & \underline{C} & \underline{C} & C & C & C \end{aligned}$$

• Seq. 2: Suppose that the three players begin with C in the first round. But in the second round, player III begins with D , and players I and II begin with C . As a result, this sequence has a two-round repetition period with $(\mathcal{K}, \mathcal{L})$ payoffs. Yields an average reward of $\frac{\mathcal{K}+\mathcal{L}}{2}$ (Regime R_2).

$$\begin{aligned} S_{38} : & C & C & C & \underline{D} & C \\ S_{38} : & C & C & D & \underline{C} & \underline{D} & \frac{\mathcal{K}+\mathcal{L}+\mathcal{K}+\mathcal{L}}{4} \\ S_{38} : & C & D & C & \underline{D} & \underline{C} \end{aligned}$$

• Seq. 3: Player III begins with D and players I and II begin with C in the first round. But in the second round, player II begins with D , although players I and II begin with C .

• Seq. 4: Player III begins with D and players I and II begin with C in the first round,. But in the second round, players II and III begin with D , although player I begins with C .

• Seq. 5: Player II begins with D and players I and III begin with C in the first round. But in the second round, player I begins with D , although players II and III begin with C .

• Seq. 6: Player II begins with D and players I and III begin with C in the first round. But in the second round, players I and III begin with D , although player II begins with C .

• Seq. 7: Players II and III begin with D and player I begins with C in the first round. But in the second round, player I and II begin with D , although players III begin with C .

• Seq. 8: Players II and III begin with D and player I begins with C in the first round. But in the second round, the three players play D .

• Seq. 9: Player I begins with D and players II and III begin with C in the first round. But in the second round, the three players play C .

• Seq. 10: Player I begins with D and players II and III begin with C in the first round. But in the second round, player III begins with D , although players I and II begin with C .

• Seq. 11: Player III begins with D and players I and II begin with C in the first round. But in the second round, player II begins with D , although players I and III begin with C .

• Seq. 12: Player III begins with D and players I and II begin with C in the first round. But in the second round, player II begins with D , although players I and III begin with C .

• Seq. 13: Player III begins with D and players I and II begin with C in the first round. But in the second round, player II begins with D , although players I and III begin with C .

• Seq. 14: Player III begins with D and players I and II begin with C in the first round. But in the second round, player II begins with D , although players I and III begin with C .

• Seq. 15: Player III begins with D and players I and II begin with C in the first round. But in the second round, player II begins with D , although players I and III begin with C .

All sequences from 3 to 15 give $\frac{\kappa+\mathcal{L}}{2}$ payoffs as seq. 2

• Seq. 16: If all three players begin with D in the first and the second rounds. As a result, this sequence has a two-round repetition time, with payoffs $\mathcal{P}$.

$$\begin{array}{cccccc} S_{38} : D & D & D & D & D & \\ S_{38} : & D & \underline{D} & D & D & D & \frac{\mathcal{P}+\mathcal{P}}{2} \\ S_{38} : & \underline{D} & \underline{D} & D & D & D & \end{array}$$

The three techniques (regimes) in the preceding eight sequences are referred to as R_1 , R_2 , and R_3 . Under the possibility of error (playing C instead of D or vice versa), let's create a little random noise. Only implementation errors will be examined in this investigation the we get probability Matrix as

$$\begin{array}{c} R_1 \ R_2 \ R_3 \\ \begin{bmatrix} R_1 & 0 & 1 & 0 \\ R_2 & 0 & 1 & 0 \\ R_3 & 0 & 1 & 0 \end{bmatrix} \end{array} \quad (18)$$

By computing the left eigenvectors for the eigenvalue 1, we get

$$\ell_1 = 0, \quad (19)$$

$$\ell_1 + \ell_2 + \ell_3 = \ell_2, \quad (20)$$

$$\ell_3 = 0, \quad (21)$$

The eigenvector ℓ is obtained by solving the linear system of the preceding equations from (19) to (21) using the equation $\ell_1 + \ell_2 + \ell_3 = 1$.

$$\ell = (\ell_1, \ell_2, \ell_3) = (0, 1, 0) \quad (22)$$

We can now obtain the payoff values by

$$\begin{aligned} E(S_{38}, S_{38}, S_{38}) &= \ell_1 \cdot R_1 + \ell_2 \cdot R_2 + \ell_3 \cdot R_3, \\ &= 1/2 [\mathcal{K} + \mathcal{L}] \end{aligned} \quad (23)$$

Equation (23) states that $(0, 1, 0, 0, 1, 0)$ is the payoff vector $\Pi = (\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6)$. Examining every TFT strategies using the same methodology. As a result, we got Tables 8 to 11 below.

Table 9: Player I's payoff against Player III when Player II using (S_{36}) approach

	S_{36}	S_{38}	S_{52}	S_{54}
S_{36}	(0, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 1)
S_{38}	(0, 0, 0, 0, 0, 1)	(0, 1, 0, 0, 1, 1)	(0, 0, 0, 0, 0, 1)	(1, 2, 0, 0, 2, 2)
S_{52}	(0, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 1)
S_{54}	(0, 0, 0, 0, 0, 1)	(0, 1, 0, 0, 1, 1)	(0, 0, 0, 0, 0, 1)	(3, 1, 0, 0, 1, 1)

Table 10: Player I's payoff against Player III when Player II using (S_{38}) approach

	S_{36}	S_{38}	S_{52}	S_{54}
S_{36}	(0, 1, 0, 0, 1, 2)	(0, 1, 0, 0, 1, 0)	(0, 1, 0, 0, 1, 3)	(1, 0, 0, 0, 0, 4)
S_{38}	(0, 1, 0, 0, 1, 0)	(0, 1, 0, 0, 1, 0)	(1, 2, 0, 0, 2, 0)	(0, 1, 0, 0, 1, 0)
S_{52}	(0, 2, 0, 0, 2, 1)	(0, 1, 0, 0, 1, 0)	(3, 2, 0, 0, 2, 0)	(3, 1, 0, 0, 1, 0)
S_{54}	(0, 2, 0, 0, 2, 1)	(0, 1, 0, 0, 1, 0)	(3, 1, 0, 0, 1, 0)	(2, 1, 0, 0, 1, 0)

Table 11: Player I's payoff against Player III when Player II using (S_{52}) approach

	S_{36}	S_{38}	S_{52}	S_{54}
S_{36}	(0, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 1)	(5, 3, 0, 0, 3, 5)	(3, 1, 0, 0, 1, 3)
S_{38}	(0, 0, 0, 0, 0, 1)	(1, 1, 0, 0, 1, 0)	(10, 5, 0, 0, 5, 12)	(1, 0, 0, 0, 0, 0)
S_{52}	(5, 2, 0, 0, 2, 7)	(1, 1, 0, 0, 1, 1)	(1, 1, 0, 0, 1, 1)	(1, 1, 0, 0, 1, 1)
S_{54}	(8, 5, 0, 0, 5, 14)	(1, 0, 0, 0, 0, 0)	(8, 5, 0, 0, 5, 14)	(1, 0, 0, 0, 0, 0)

Table 12: Player I's payoff against Player III when Player II using (S_{54}) approach

	S_{36}	S_{38}	S_{52}	S_{54}
S_{36}	(1, 1, 0, 0, 1, 3)	(1, 1, 0, 0, 1, 0)	(1, 0, 0, 0, 0, 0)	(3, 1, 0, 0, 1, 0)
S_{38}	(0, 2, 0, 0, 2, 1)	(1, 1, 0, 0, 1, 0)	(1, 0, 0, 0, 0, 0)	(1, 0, 0, 0, 0, 0)
S_{52}	(1, 0, 0, 0, 0, 0)	(1, 0, 0, 0, 0, 0)	(1, 0, 0, 0, 0, 0)	(1, 0, 0, 0, 0, 0)
S_{54}	(1, 0, 0, 0, 0, 0)	(1, 0, 0, 0, 0, 0)	(1, 0, 0, 0, 0, 0)	(1, 0, 0, 0, 0, 0)

4 Results

Comparing the payoff of the strategies resulting from competition, assigning numerical values to the payoffs, in addition to studying the dominance of the strategies to identify the strongest strategy during the competition.

4.1 Numerical Application

We now employ a few numerical values ($\mathcal{S} = 0$, $\mathcal{P} = 1$, $\mathcal{K} = 3$, $\mathcal{L} = 5$, $\mathcal{R} = 9$, and $\mathcal{T} = 10$) in two cases (memory-one case and memory-two case, respectively).

4.1.1 Memory-one

As a result, we illustrate the optimal TFT strategy using a few graphs, as seen in Fig. 3.

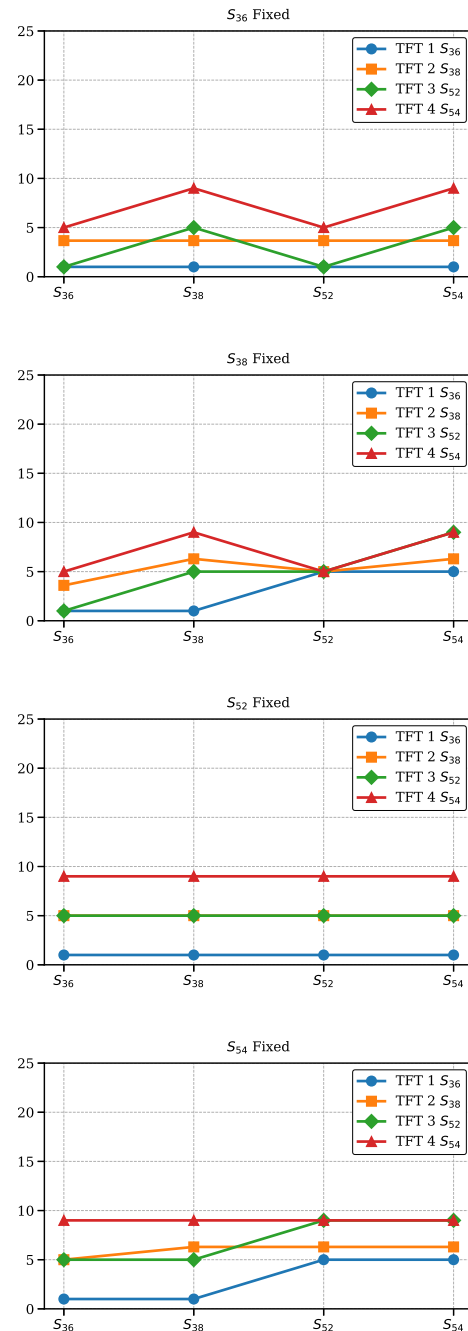


Fig. 3: Payoff via Memory-One Competition

Fig. 3 shows that, when player II (S_{36}) is fixed, TFT 4 S_{54} is the best against all other TFT and it has the highest payoff against S_{38} and S_{54} . But when player II (S_{38}) or player II (S_{54}), TFT 4 S_{54} is the best against S_{36} and S_{38} , and equivalent to S_{52} against S_{52} and S_{54} . Moreover, when player II (S_{52}) is fixed, TFT 4 S_{54} is the best against all other TFT, and it has the same payoff against all TFT strategies also, S_{38} and S_{52} are equivalent against all TFT strategies. Finally,

we can say that S_{54} is the Best TFT strategy against all other TFT strategies.

4.1.2 Memory-Two

As a result, we illustrate the optimal TFT strategy using a few graphs, as seen in Fig. 4.

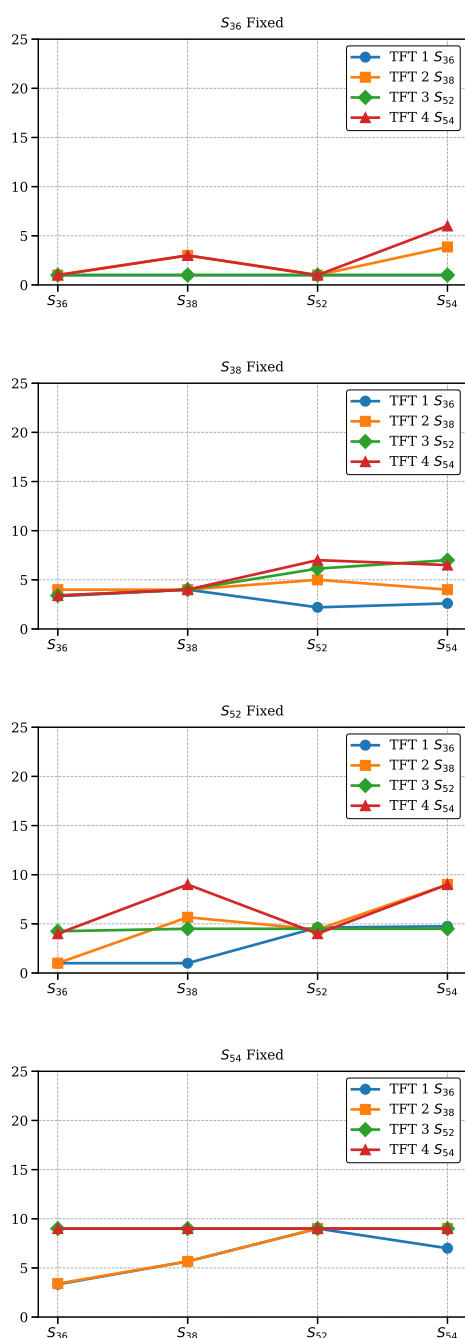


Fig. 4: Payoff Via Memory-Two Competition

Fig. 4 shows that, when player II (S_{36}) is fixed, TFT 4 S_{54} is the best against S_{38} and S_{54} and has the highest payoff against itself, but all strategies are equivalent against S_{36} and S_{52} . When player II (S_{38}) is fixed, TFT 4 S_{54} is the best against S_{52} but S_{38} is the best against S_{36} and S_{52} the best against S_{54} , but all strategies are equivalent against S_{38} . Moreover, when player II (S_{52}) is fixed, TFT 4 S_{54} is the best against S_{38} and S_{54} but S_{52} is the best against S_{36} and S_{52} . Furthermore, when player II (S_{54}) is fixed, TFT 4 S_{54} , TFT 3 S_{52} and S_{36} are the best and equivalent against all other strategies except against S_{54} , TFT 4 S_{54} , TFT 3 S_{52} only are the best and equivalent.

Now, what is the noticeable difference resulting from the change in memory length?

We see a change in payoffs. Although in the case of memory-one, strategy TFT 4 S_{54} was superior in all cases, against all competitors, and against all Tit-For-Tat strategies, in the case of memory-two, strategy TFT 4 S_{54} declined in some cases and was outperformed in these cases by strategy TFT 2 S_{38} and TFT 3 S_{52} .

4.2 The Domination

The dominance of the Tit-For-Tat strategies in two cases (memory-one case and memory-two case) will be covered in this section. If both $a_{kl} > a_{ll}$ and $a_{kk} > a_{lk}$, where a_{ll} , a_{lk} , a_{kl} , and a_{kk} are components of the payoff matrix, then S_l is outcompeted by S_k . We can write $S_l << S_k$ if S_k outcompetes the strategy S_l .

4.2.1 Memory-one

Based on Tables 4–7, the dominance of TFT strategies is as follows:

Table 13: A list of strategies outcompeting S_l

	Player II: S_{36} fixed	Player II: S_{38} fixed	Player II: S_{52} fixed	Player II: S_{54} fixed
$S_{36} <<$	S_{38}, S_{54}	S_{38}, S_{54}	S_{38}, S_{54}	S_{38}, S_{52}, S_{54}
$S_{38} <<$	S_{54}	S_{54}	S_{54}	S_{52}, S_{54}
$S_{52} <<$	S_{54}	—	S_{54}	—
$S_{54} <<$	—	—	—	—

From the dominance table, we observe that TFT 4 (S_{54}) is the best and strongest strategy of all,

dominating all other strategies and not being dominated by any other strategy. TFT 1 (S_{36}) is the weakest strategy, dominated by most strategies and not dominating any of them.

4.2.2 Memory-Two

Based on Tables 8–11, the dominance of TFT strategies is as follows:

Table 14: A list of strategies outcompeting S_l

	Player II: S_{36} fixed	Player II: S_{38} fixed	Player II: S_{52} fixed	Player II: S_{54} fixed
$S_{36} <<$	—	S_{52}, S_{54}	S_{38}, S_{54}	S_{54}
$S_{38} <<$	—	—	S_{54}	—
$S_{52} <<$	—	—	—	—
$S_{54} <<$	—	—	—	—

Again, from the dominance table, we observe that TFT 4 (S_{54}) is the best and strongest strategy of all, dominating all other strategies and not being dominated by any other strategy. TFT 1 (S_{36}) is the weakest strategy, dominated by most strategies and not dominating any of them.

5 Impact Relatedness

The recurring prisoner's dilemma alternating game, in which the players have a relationship, is examined in this section. Maynard Smith suggested a straightforward method for studying games between relatives for the Hawk-Dove game [14,15].

Studying the games between relatives can be done in two ways. Grafen's [15] personal fitness approach modifies a player's fitness by making it possible for him to meet a co-player who uses the same strategy as him more frequently than other players in the population. The "inclusive fitness" approach multiplies the player's payoff by the payoff of his teammate. The recurring prisoner's dilemma that finite state automata play is studied using the inclusive fitness approach, which is prone to a slight inaccuracy [14].

For a two-player game, we will employ the relatedness as described in [9,16,17]. Given that the

two-player Prisoner's Dilemma reward matrix with relatedness is

$${}^C_C \begin{bmatrix} \mathcal{R}(1+\wp) & \mathcal{S} + \mathcal{T}\wp \\ \mathcal{T} + \mathcal{S}\wp & \mathcal{P}(1+\wp) \end{bmatrix}. \quad (24)$$

where

$$0 \leq \wp \leq 1 \quad (25)$$

We present the ideas of relationships between three actors in this part. Consequently, if $\wp_1$ represents the relatedness factor between player I and player II, then $\wp_2$ represents the relatedness factor between player I and player III. Consequently, the three-player Prisoner's Dilemma payoff matrix with relatedness will be provided by

$$\begin{matrix} & \begin{matrix} CC & CD \end{matrix} \\ \begin{matrix} C \\ D \end{matrix} & \begin{pmatrix} \mathcal{R}(1+\wp_1+\wp_2) & \mathcal{K}(1+\wp_1)+\wp_2\mathcal{T} \\ \mathcal{T}+\mathcal{K}(\wp_1+\wp_2) & \mathcal{L}(1+\wp_2)+\wp_1\mathcal{S} \end{pmatrix} \end{matrix}$$

$$\begin{matrix} & \begin{matrix} DC & DD \end{matrix} \\ \begin{matrix} C \\ D \end{matrix} & \begin{pmatrix} \mathcal{K}(1+\wp_2)+\wp_1\mathcal{T} & \mathcal{S}+\mathcal{L}(\wp_1+\wp_2) \\ \mathcal{L}(1+\wp_1)+\wp_2\mathcal{S} & \mathcal{P}(1+\wp_1+\wp_2) \end{pmatrix} \end{matrix}, \quad (26)$$

where

$$0 \leq \wp_1 \leq 1 \quad (27)$$

$$0 \leq \wp_2 \leq 1 \quad (28)$$

When we reduce the outcomes to six, the payoff matrix with relatedness will be given by

$${}^C_C \begin{bmatrix} \mathcal{R}(1+\wp_1+\wp_2) & \frac{\mathcal{K}(2+\wp_1+\wp_2)+\mathcal{T}(\wp_1+\wp_2)}{2} & \mathcal{S}+\mathcal{L}(\wp_1+\wp_2) \\ \mathcal{T}+\mathcal{K}(\wp_1+\wp_2) & \frac{\mathcal{L}(2+\wp_1+\wp_2)+\mathcal{S}(\wp_1+\wp_2)}{2} & \mathcal{P}(1+\wp_1+\wp_2) \end{bmatrix}, \quad (29)$$

where

$$0 \leq \wp_1 \leq 1 \quad (30)$$

$$0 \leq \wp_2 \leq 1 \quad (31)$$

5.1 Memory-One

In Tables 4:7, the impact of relatedness is now applied through five scenarios of the values of $\wp_1$ and $\wp_2$ according to (30) and (31).

- Case 1: For $\wp_1 = 1$ and $\wp_2 = 1$, “Perfect Relationship” from both sides, we get

- Case 2: For $\wp_1 = 0.9$ and $\wp_2 = 0.9$, “Strong Relationship” from both sides, we get

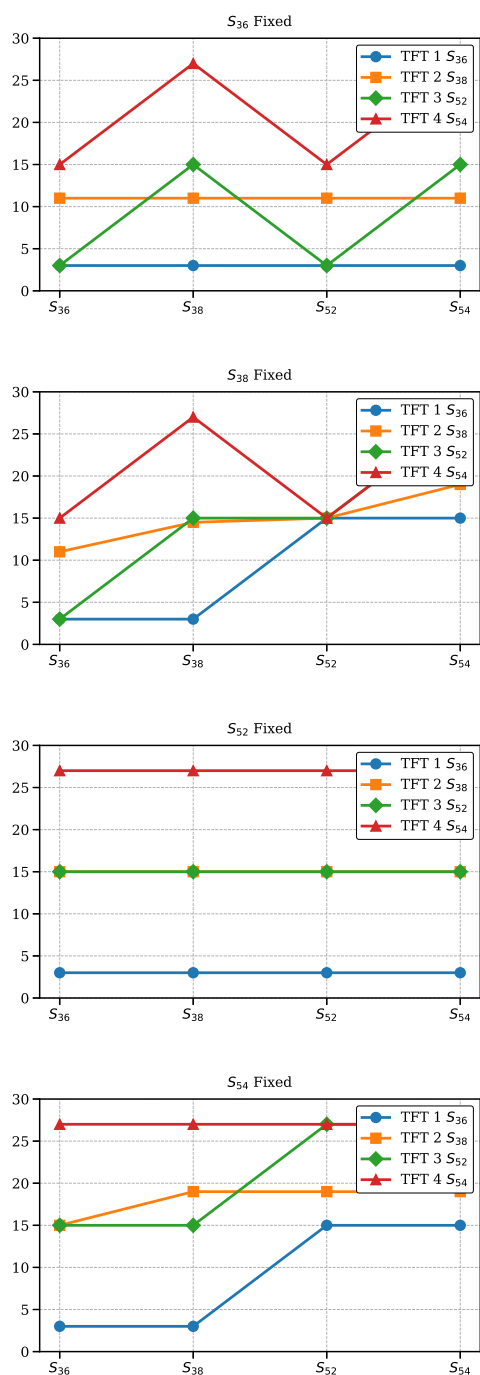


Fig. 5: “Perfect Relationship” in Memory-One Case

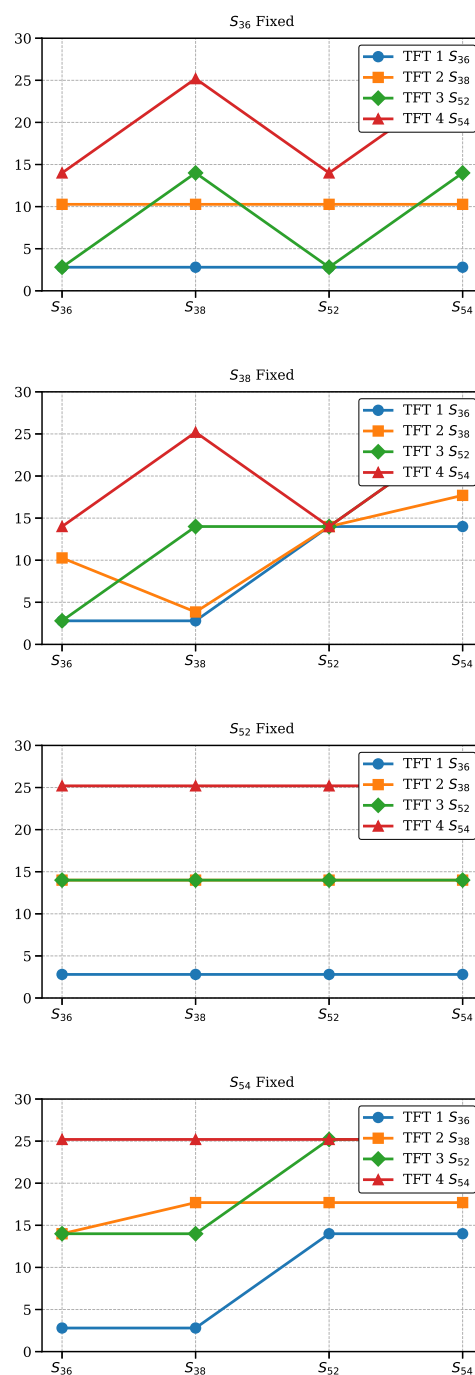


Fig. 6: “Strong Relationship” in Memory-One Case

- Case 3: For $\wp_1 = 0.1$ and $\wp_2 = 0.1$, “Weak Relationship” from both sides, we get

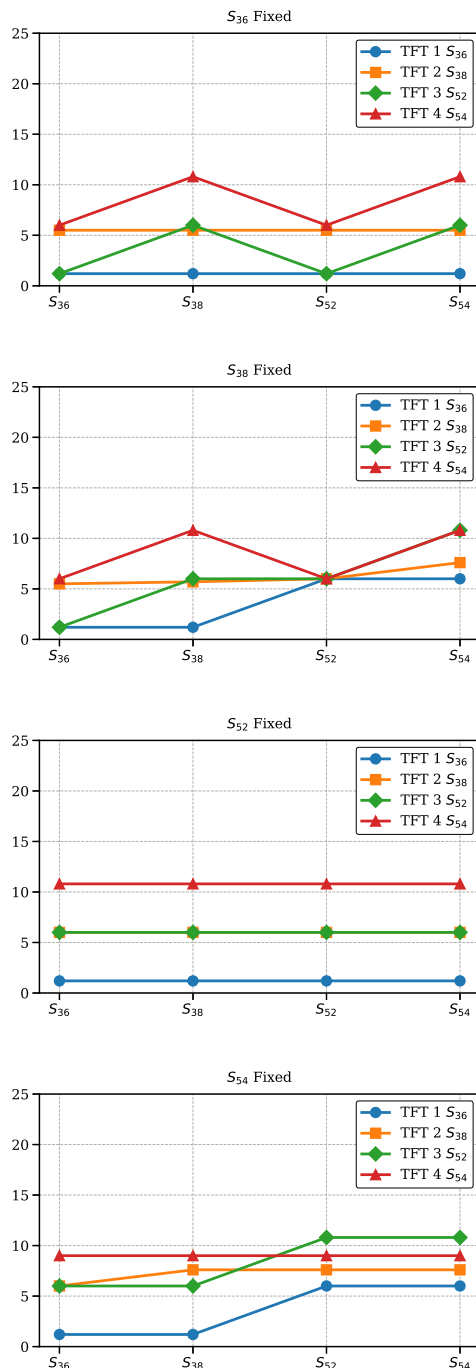


Fig. 7: “Weak Relationship” in Memory-One Case

- Case 4: For $\wp_1 = 1$ and $\wp_2 = 0$, “Perfect Relationship” from one side and “No Relationship” from the other side,
 - Case 5: For $\wp_1 = 0.9$ and $\wp_2 = 0.1$, “Strong Relationship” from one side and “Weak Relationship” from the other side,
- the two cases yield the same results as we get

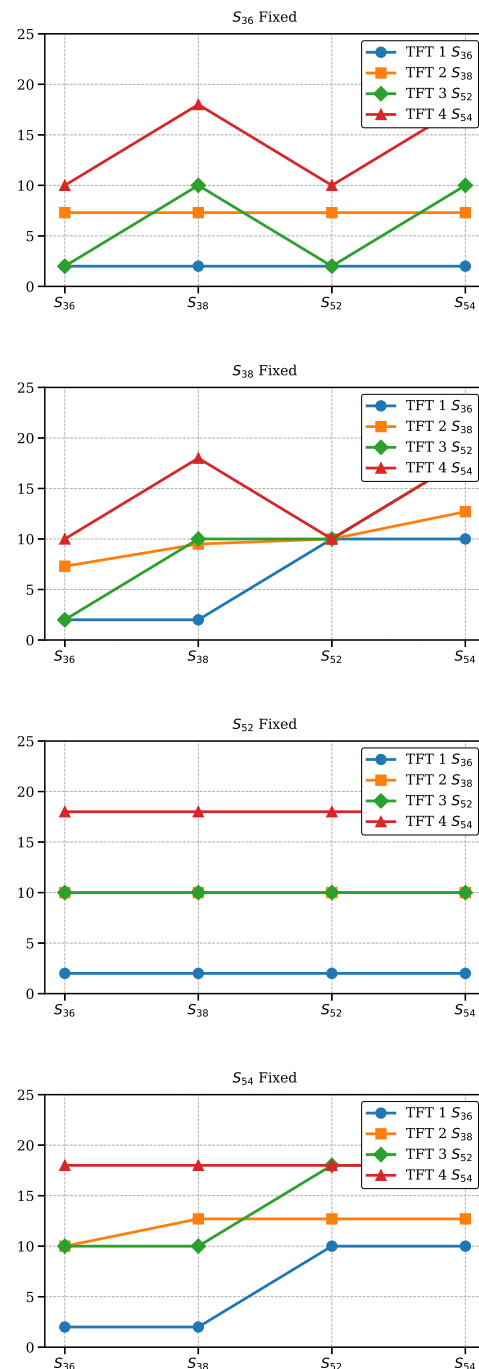


Fig. 8: “Mixed Relationship” in Memory-Two Case

5.2 Memory-Two

- Case 1: For $\wp_1 = 1$ and $\wp_2 = 1$, “Perfect Relationship” from both sides, we get

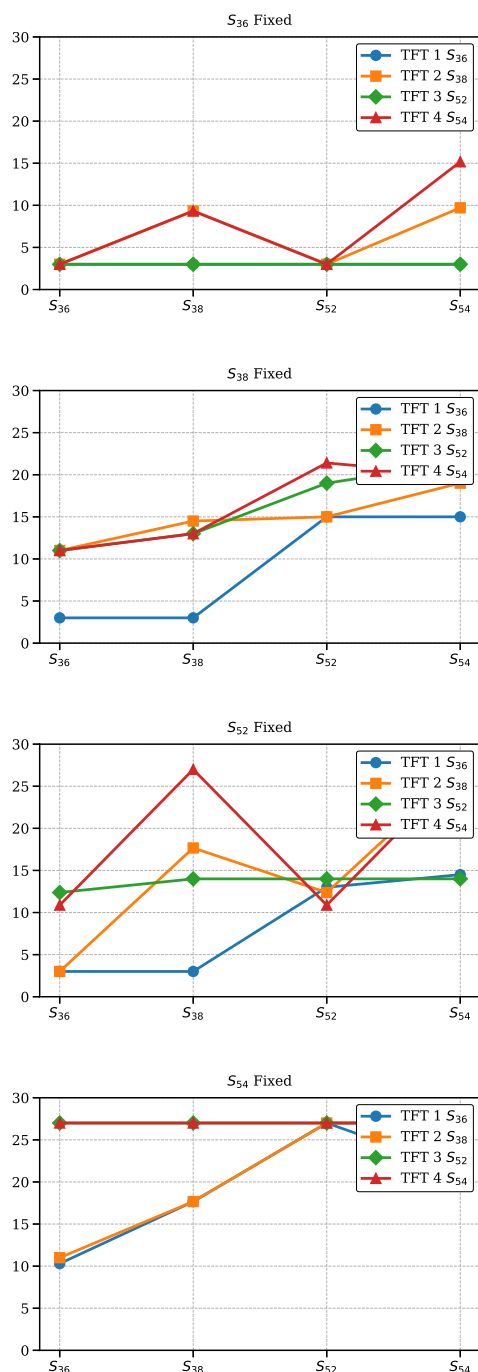


Fig. 9: “Perfect Relationship” in Memory-Two Case

- Case 2: For $\wp_1 = 0.9$ and $\wp_2 = 0.9$, “Strong Relationship” from both sides, we get

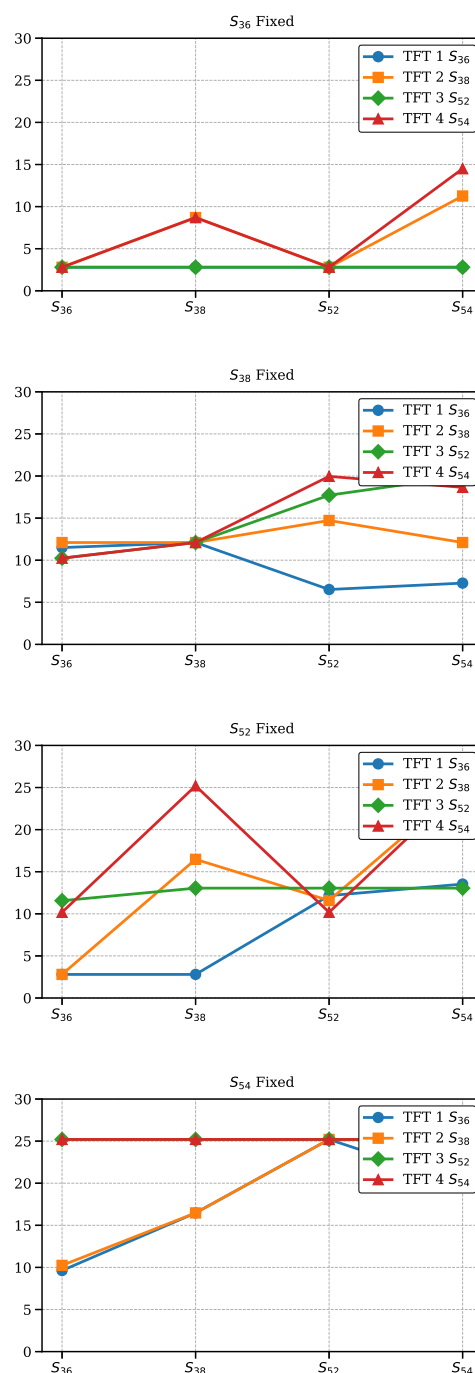


Fig. 10: “Strong Relationship” in Memory-Two Case

- Case 3: For $\wp_1 = 0.1$ and $\wp_2 = 0.1$, “Weak Relationship” from both sides, we get

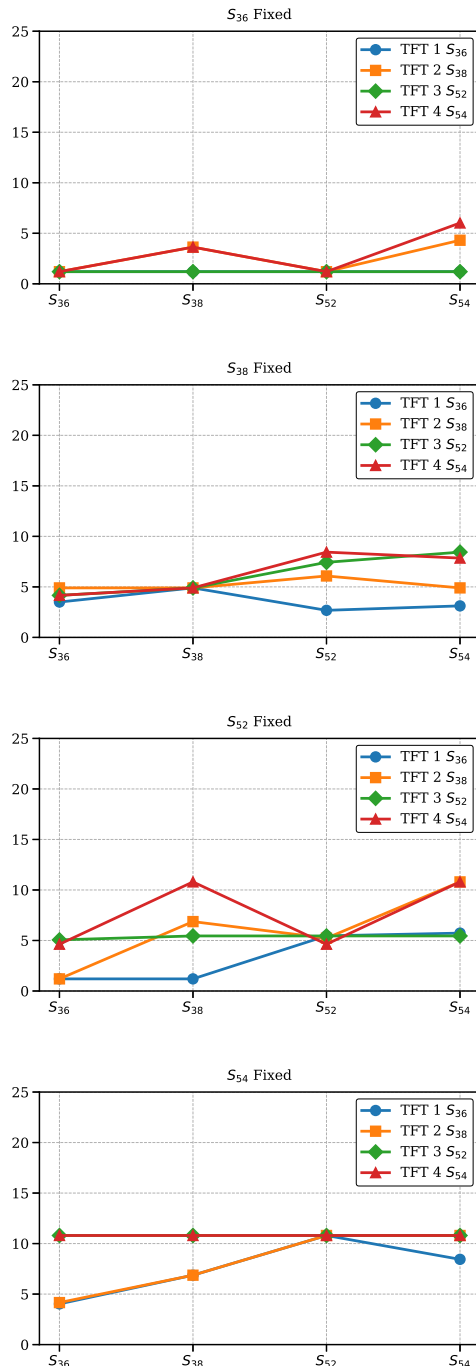


Fig. 11: “Weak Relationship” in Memory-Two Case

- Case 4: For $\wp_1 = 1$ and $\wp_2 = 0$, “Perfect Relationship” from one side and “No Relationship” from the other side,
 - Case 5: For $\wp_1 = 0.9$ and $\wp_2 = 0.1$, “Strong Relationship” from one side and “Weak Relationship” from the other side,
- the two cases yield the same results as we get

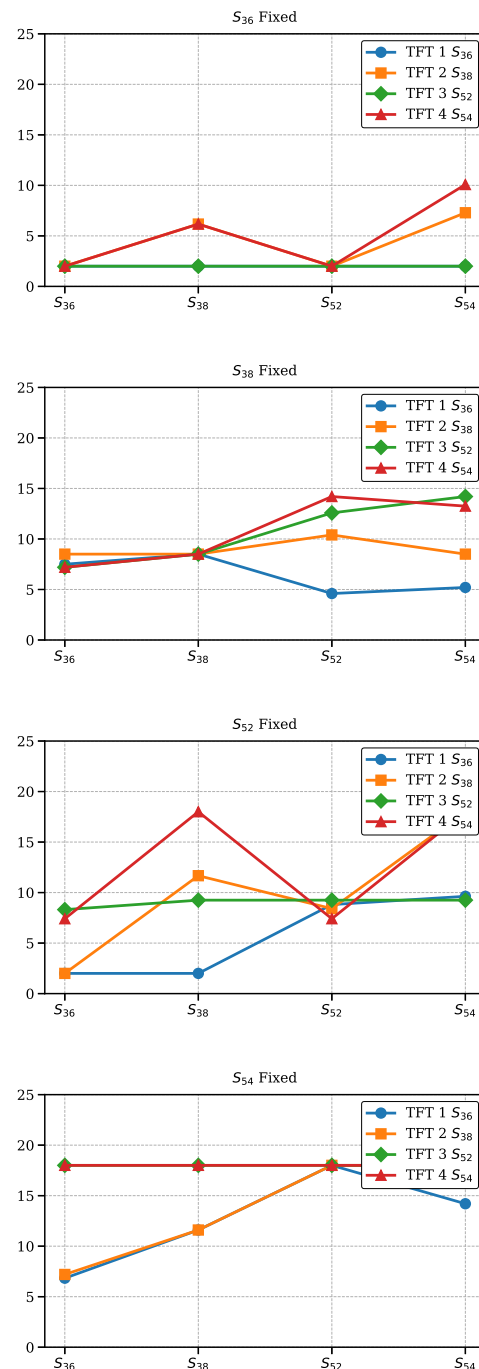


Fig. 12: “Mixed Relationship” in Memory-Two Case

6 Conclusion

A version of the Prisoner's Dilemma game was presented using a strict three-player alternating model. We introduced this concept and studied the competition between several specific strategies characterized by "Tit-For-Tat" strategies. We then presented the best strategy with the highest payoff using specific numerical values. Furthermore, we discussed the impact of relatedness between players on the behavior of the strategies and their payoffs.

Moreover, we conducted this study in two cases: one with memory-one and the other with memory-two. We then determined the effect of memory length on the behavior of the strategies and their payoffs resulting from the competition.

After studying the behavior of the "Tit-For-Tat" strategies against each other, It was found that strategy TFT 4 (S_{54}) was the best and strongest strategy overall, outperforming all other strategies and remaining completely not dominated by any other strategy. Strategy TFT 1 (S_{36}) was the weakest strategy, being susceptible to most other strategies and unable to outperform even a single one.

Data Availability

The authors declare that the data supporting the findings of this study are available within the paper

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Conflicts of Interest

Regarding the publishing of this paper, the authors state that they have no conflicts of interest.

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