

An Alternating Diffusion-Clustering Framework for Image Segmentation: Stability and Convergence

Ali Abou El Qassime^{1,*}, Youness El Ansari² and Mehdi Abou El Qassime³

¹EEIS Lab, ENSET, Hassan II University, Mohammedia 20650, Morocco

²IRF-SIC Laboratory, Faculty of Sciences, Ibn Zohr University, Agadir 80000, Morocco

³Emines School of Industrial Management, LOT 660 43150, Ben Guerir, Morocco

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Abstract: We introduce a coupled mathematical model for image segmentation that relies on a bidirectional interaction between nonlinear diffusion and a discrete clustering operator. Rather than treating diffusion and segmentation separately, we couple them through a feedback mechanism: a Perona–Malik-type equation with a segmentation-dependent conductance interacts with a clustering step that is continuously regularized by the diffusion process. The resulting system defines a joint evolution where each component influences the other at every iteration. On the theoretical side, we prove existence and uniqueness for a linearized and regularized version of the diffusion subproblem with fixed segmentation. We also derive stability conditions for the explicit numerical scheme under a CFL-type constraint and study the convergence behavior of the iterative fixed-point algorithm. The model yields an unsupervised segmentation approach that preserves edges while enforcing spatial regularity, without requiring annotated data. Experiments on dermoscopic images achieve a Dice coefficient of 0.8672, a Jaccard index of 0.7714, and an overall accuracy of 0.9221, with particularly high specificity (0.9908) and precision (0.9808). These results demonstrate that the proposed coupled PDE-discrete system yields competitive performance while providing mathematical interpretability, offering a rigorous alternative to purely data-driven segmentation methods.

Keywords: Nonlinear diffusion; Image segmentation; K-means clustering; Perona-Malik equation; CFL stability; Coupled PDE system

1 Introduction

Over the years, image segmentation has been investigated from several mathematical perspectives, with variational methods, PDE-based models, and clustering techniques emerging as the three dominant directions. Variational approaches, including the classical Mumford–Shah model and related formulations [12,13], describe segmentation through the minimization of an energy functional. Such formulations seek a compromise between accurately representing image information and maintaining regular interfaces separating distinct regions. In parallel, PDE-based methods, notably nonlinear diffusion models introduced by Perona and Malik [14], provide powerful tools for edge-preserving smoothing and geometric regularization [15]. Statistical and clustering-based approaches, including K-means and mixture models, offer

complementary mechanisms for partitioning image domains based on intensity distributions [16,17].

Despite extensive developments in each of these directions, their interaction remains only partially understood. In most existing works, diffusion and segmentation are treated either sequentially or through loosely coupled optimization schemes.

A closer inspection of these hybrid frameworks reveals that the coupling between PDE-based regularization and discrete clustering is typically either static or unidirectional. Bae et al. [18] formulate multiclass segmentation as a convex variational problem on graphs, combining region and boundary terms within a max-flow optimization framework. While this yields global minimizers for certain combinatorial problems, the model remains an energy minimization on a fixed graph structure rather than a dynamic evolution where a PDE and a clustering operator mutually influence one another

* Corresponding author e-mail: ali.abou1-etu@etu.univh2c.ma

at each iteration. Shah et al. [19] integrate the Mumford Shah functional into the k-means clustering framework, thereby regularizing segment boundaries while retaining the computational simplicity of clustering. However, this approach is fundamentally a static energy coupling: the Mumford Shah term is used to derive a distance metric for k-means, but there is no temporal evolution or feedback from the segmentation to the diffusion process. Similarly, Gibou and Fedkiw [20] draw an explicit connection between level-set-based curve evolution and k-means preprocessing with nonlinear diffusion, proposing a fast hybrid algorithm. Yet their coupling is sequential rather than bidirectional: nonlinear diffusion serves merely as a preprocessing step to introduce a length scale, and the level-set evolution does not adapt its coefficients from the current clustering state. Consequently, the theoretical status of tightly coupled PDE-discrete systems particularly their stability, convergence, and well-posedness remains largely unsettled.

As a motivating application, we consider dermoscopic image segmentation, a challenging task characterized by low contrast, fuzzy boundaries, and the scarcity of annotated data. These constraints make unsupervised and mathematically interpretable approaches particularly relevant in this context.

Recent advances in supervised deep learning have achieved high segmentation accuracy on benchmark datasets [1,2,3,4,5]. However, these approaches rely heavily on large annotated datasets and often lack interpretability, which limits their applicability in data-scarce or clinically constrained environments. The creation of such resources demands considerable domain expertise and financial investment, which may not be feasible in many clinical or resource-constrained settings. In addition, inherent biases present in publicly available datasets exemplified by the widely used HAM10000 collection [6] further complicate the generalizability of these approaches. These issues collectively call attention to the growing necessity for robust unsupervised alternatives.

Several unsupervised segmentation methods have been proposed to overcome these limitations. For example, USL-Net [7] employs class activation maps coupled with adversarial learning to produce pseudo-labels. Zebari et al. [8] applied k-means clustering augmented with post-processing steps to address typical dermoscopic hurdles. Garcia-Arroyo et al. [9] developed a segmentation algorithm based on fuzzy pixel classification and histogram thresholding. Graph-based and entropy-driven methods have also shown efficacy, such as SLED [10] and saliency-based techniques [11].

While these unsupervised methods demonstrate promising empirical results, they often lack a rigorous mathematical foundation that could provide guarantees on stability, convergence, and well-posedness.

Despite these advances, a conspicuous gap remains in the theoretical landscape. Existing works that marry diffusion with clustering do so either

sequentially—nonlinear diffusion acts as a preprocessor, and clustering follows independently—or through a static energy coupling where the two components are optimized in alternation without mutual feedback at the iteration level. What has not been achieved, and what turns out to be mathematically subtle, is a genuine bidirectional feedback loop in which the segmentation field actively reshapes the diffusion conductance while the diffused image simultaneously guides the clustering. The difficulty is not merely computational: the K-means operator is discontinuous with respect to intensity perturbations, so the resulting alternating map falls outside the standard regularity assumptions of coupled parabolic systems. As a result, stability bounds, convergence criteria, and well-posedness guarantees for such tightly coupled PDE-discrete systems have remained largely unexplored.

We address this lacuna by introducing an alternating diffusion-clustering framework whose two components are locked in a sequential feedback mechanism. At every iteration, the conductance parameter adapts to the smoothed segmentation indicator, while the diffusion output regularizes the subsequent clustering step. We establish existence and uniqueness for the linearized diffusion subproblem with frozen segmentation, derive an explicit CFL stability condition for the standard five-point stencil, and monitor the iterative fixed-point behaviour through a pixel-wise indicator criterion.

To our knowledge, this constitutes the first mathematically grounded PDE-clustering hybrid that maintains a closed feedback loop without annotated training data.

The principal contributions of this work are fourfold:

1. **A rigorously defined alternating coupling framework:** Segmentation influences diffusion conductance through a feedback mechanism, while diffusion simultaneously regularizes the clustering structure, resulting in a bidirectional alternating iterative scheme where each component is updated sequentially based on the other's current state.
2. **Mathematical analysis of stability and convergence:** We analyze the proposed model through (i) existence and uniqueness analysis of a linearized and regularized diffusion subproblem, (ii) stability conditions of the explicit numerical scheme, and (iii) convergence behavior of the iterative fixed-point algorithm under practical assumptions.
3. **Computationally efficient implementation:** We develop a stable finite-difference discretization with adaptive parameter selection based on image statistics, ensuring robustness across diverse imaging modalities while maintaining computational tractability.
4. **Comprehensive validation in challenging medical imaging context:** We demonstrate the effectiveness of our framework on the segmentation of dermoscopic images a domain characterized by low contrast, blurred edges, and a lack of annotations by achieving

performance comparable to that of supervised methods while offering mathematical interpretability.

The article is structured as follows: Section 2 establishes the mathematical foundation and details the proposed coupled model with theoretical analysis. Section 3 presents the numerical implementation and experimental framework. Section 4 defines evaluation metrics and reports experimental results on dermatological datasets. Section 5 provides comparative analysis with state-of-the-art methods and discusses mathematical implications. Finally, Section 6 concludes with findings and future research directions.

2 Mathematical Background and Proposed Method

Addressing the challenge of biomedical image segmentation, we develop an approach that coordinates diffusion—as a segmentation-dependent smoothing mechanism—with clustering that provides structural feedback.

2.1 Connections to Classical PDE Models

Building upon classical diffusion theory [14,15] and variational clustering principles [21,16], our method introduces several key innovations while maintaining clear connections to established PDE-based segmentation models.

Key Innovations. Three properties distinguish the proposed framework from classical PDE-based approaches. The conductance function $K(S)$ depends on the smoothed boundary field $S_\sigma = G_\sigma * \mathbb{1}_{\{S=1\}}$: segmentation steers diffusion, and diffusion regularizes S_σ in return. Neither dependency is cosmetic. That mutual constraint is precisely what keeps the gradient-based coupling well-posed. Spatially, $K(S)$ responds to the local homogeneity of S_σ rather than to intensity gradients alone: stable interior regions receive stronger smoothing; zones where the segmentation label is uncertain receive less, so the transitions worth preserving are not eroded.

Parameter initialization is handled the same way. Initial values are read from image statistics, not set by hand, which is what allows the scheme to transfer across heterogeneous biomedical datasets without per-image tuning [29,28].

Relation to Perona–Malik Diffusion. These innovations extend several classical PDE-based segmentation approaches. When $\alpha = 0$ and $K(S) = K_0$ is constant, the proposed model reduces to the classical Perona–Malik equation [14]. The present framework generalizes this

approach by introducing segmentation-driven adaptation of the conductance parameter, addressing known limitations related to fixed parameter selection.

Connection to Mumford–Shah-Type Models. The regularization of the smoothed boundary field S_σ shares conceptual similarities with Mumford–Shah formulations [12], promoting piecewise smooth solutions while preserving sharp transitions. The Gaussian smoothing $S_\sigma = G_\sigma * \mathbb{1}_{\{S=1\}}$ corresponds to the Ambrosio–Tortorelli approximation of perimeter minimization, with σ controlling the interface width.

Comparison with Chan–Vese Models. Unlike Chan–Vese active contour models [13], which assume piecewise constant regions and rely on level-set evolution, the proposed approach allows for piecewise smooth regions through explicit clustering coupled with adaptive diffusion. This results in a fully coupled PDE-clustering framework rather than an alternating minimization of separate energies.

The resulting interaction establishes an iterative feedback mechanism in which segmentation guides diffusion conductance while diffusion reinforces boundary coherence. This design yields structurally consistent and noise-resilient partitions through an iterative feedback mechanism, and the explicit scheme subject to a CFL-type condition ensures practical implementability.

2.2 Coupled Diffusion-Segmentation Model

We propose a coupled system where diffusion and segmentation evolve jointly through mutual feedback. To ensure mathematical well-posedness while maintaining the bidirectional coupling philosophy, we introduce a smoothed version of the segmentation for regularization purposes.

Definition 1. *The system (1) is interpreted as an alternating iterative scheme rather than a continuously coupled PDE system. At each iteration, the segmentation operator $\mathcal{C}$ is applied to the updated image, and the resulting segmentation is used as a frozen coefficient in the diffusion equation. This alternating structure is the defining characteristic of the proposed framework; the continuous notation in (1) serves as a compact representation of the iterative coupling.*

Mathematical Formulation. The following formulation describes the alternating iterative scheme defined in Definition 1. The differential notation is used as a compact representation of the spatial operators applied at each discrete iteration; the time derivative $\partial u / \partial t$ is

understood in the sense of the explicit Euler update (2), not as a continuously evolving PDE.

$$\begin{cases} \frac{\partial u}{\partial t} = \nabla \cdot (g(|\nabla u|; \kappa(S_\sigma)) \nabla u) + \alpha \nabla \cdot (w(S_\sigma) \nabla S_\sigma), \\ S = \mathcal{C}(u), \quad S_\sigma = G_\sigma * \mathcal{K}_{\{S=1\}} \end{cases} \quad (1)$$

where:

- $u(x, y, t) : \Omega \times [0, \infty) \rightarrow \mathbb{R}$ is the evolving image intensity
 - $S(x, y) : \Omega \rightarrow \{1, \dots, K\}$ is the segmentation map
 - $g(|\nabla u|; K) = \frac{1}{1 + (|\nabla u|/K)^2}$ is the Perona-Malik conductance
 - $\kappa(S_\sigma) = K_0(1 + \beta f(S_\sigma))$ is the segmentation-adaptive conductance, where $f(S_\sigma)$ quantifies local structural homogeneity:
- $$f(S_\sigma)(x, y) = 1 - \frac{|\nabla S_\sigma(x, y)|}{\max_{(x, y) \in \Omega} |\nabla S_\sigma(x, y)|}.$$
- $S_\sigma = G_\sigma * \mathcal{K}_{\{S=1\}}$ is the smoothed lesion indicator, with G_σ a Gaussian kernel of standard deviation $\sigma = 1.5$ pixels
 - $w(S_\sigma) = \frac{1}{1 + |\nabla S_\sigma|_{\varepsilon_{\text{reg}}}}$ where $|\nabla S_\sigma|_{\varepsilon_{\text{reg}}} = \sqrt{|\nabla S_\sigma|^2 + \varepsilon_{\text{reg}}}$. This term should be interpreted as a regularization-induced forcing computed from the previous segmentation iterate.
 - $\mathcal{C}$ is the K-means clustering operator: $\mathcal{C}(u)(x, y) = \arg \min_k \|u(x, y) - \mu_k\|^2$
 - $K_0, \beta, \alpha > 0, \varepsilon > 0, \sigma > 0$ are parameters

Model Derivation. The model is derived through the following steps:

1. **Nonlinear diffusion:** Starting from isotropic diffusion $\partial_t u = \Delta u$, we introduce the Perona-Malik conductance $g(|\nabla u|; K)$ to obtain edge-preserving smoothing.
2. **Segmentation-adaptive conductance:** To incorporate semantic guidance, the conductance parameter is modulated by the smoothed segmentation field S_σ via $\kappa(S_\sigma) = K_0(1 + \beta f(S_\sigma))$, where $f(S_\sigma)$ measures local homogeneity based on the gradient magnitude of S_σ .
3. **Segmentation via clustering:** The segmentation is obtained from the evolving image using the K-means operator $\mathcal{C}$, establishing the first direction of coupling.
4. **Regularization via smoothed boundary field:** To impose spatial coherence while ensuring mathematical well-posedness, we regularize the smoothed field $S_\sigma = G_\sigma * \mathcal{K}_{\{S=1\}}$. The Gaussian convolution ensures S_σ is differentiable, allowing proper computation of ∇S_σ for the regularization term $\alpha \nabla \cdot (w(S_\sigma) \nabla S_\sigma)$.

5. Bidirectional coupling: Combining these elements yields the coupled system (1), which implements continuous feedback: segmentation guides diffusion through $K(S)$ (which depends on S_σ), while diffusion regularizes segmentation boundaries through $w(S_\sigma) \nabla S_\sigma$.

Remark (Mathematical Necessity of Smoothing). The term $\nabla \cdot (w(S_\sigma) \nabla S_\sigma)$ would be mathematically problematic for a piecewise-constant segmentation S , as ∇S is not defined in the classical sense at discontinuity boundaries. The convolution $S_\sigma = G_\sigma * \mathcal{K}_{\{S=1\}}$ transforms the binary segmentation into a differentiable field $S_\sigma \in C^\infty(\Omega)$, making ∇S_σ a well-defined classical gradient. This approach aligns with the Ambrosio-Tortorelli approximation of perimeter minimization in variational methods.

Remark (Expanded Form). Substituting the definitions into (1) gives the explicit form:

$$\frac{\partial u}{\partial t} = \nabla \cdot \left(\frac{1}{1 + \left(\frac{|\nabla u|}{\kappa(S_\sigma)} \right)^2} \nabla u \right) + \alpha \nabla \cdot \left(\frac{1}{1 + |\nabla S_\sigma|_{\varepsilon_{\text{reg}}}} \nabla S_\sigma \right),$$

with $S = \mathcal{C}(u)$ and $S_\sigma = G_\sigma * \mathcal{K}_{\{S=1\}}$, clearly showing the mutual dependence between diffusion and segmentation.

2.3 Numerical Implementation

Discrete Scheme. The continuous model (1) is discretized using an explicit finite difference scheme:

$$u^{t+1} = u^t + \lambda \left[\nabla \cdot (g(|\nabla u^t|; \kappa(S_\sigma^t)) \nabla u^t) + \alpha \nabla \cdot (w(S_\sigma^t) \nabla S_\sigma^t) \right], \quad (2)$$

where $S^t = \mathcal{C}(u^t)$ and $S_\sigma^t = G_\sigma * \mathcal{K}_{\{S^t=1\}}$.

Convergence is monitored using:

$$\Delta S^t = \frac{1}{|\Omega|} \sum_{(i,j) \in \Omega} \mathbb{I}\{S_{i,j}^{t+1} \neq S_{i,j}^t\}, \quad (3)$$

where $\mathbb{I}\{\cdot\}$ is the indicator function, and iterations stop when $\Delta S^t < \delta$, with $\delta > 0$ a prescribed convergence threshold.

Stability Analysis.

Lemma 1 (Stability Condition). The explicit scheme (2) is stable under the CFL condition

$$\lambda \leq \frac{1}{4} \frac{h^2}{\|g\|_\infty}, \quad (4)$$

where h is the uniform spatial grid spacing and $\|g\|_\infty = \sup g(|\nabla u|; K(S)) = 1$. For the standard 5-point Laplacian stencil in two dimensions, the constant is $C = 1/4$.

Proof. For fixed S_σ , the diffusion operator $L(u) = \nabla \cdot (g \nabla u)$ discretized with central differences on a uniform Cartesian grid yields a symmetric matrix whose eigenvalues are bounded by:

$$8\|g\|_\infty/h^2;$$

this bound is sharp for the 5-point stencil in two dimensions.

The explicit Euler update $u^{t+1} = u^t + \lambda L(u^t)$ is stable provided the spectral radius condition: $\lambda \cdot (8\|g\|_\infty/h^2) \leq 2$ is satisfied, which gives $\lambda \leq h^2/(4\|g\|_\infty)$.

Hence $C = 2/8 = 1/4$.

The feedback term $\alpha \nabla \cdot (w(S_\sigma) \nabla S_\sigma)$ depends only on the frozen field S_σ and acts as a source term during the diffusion step; it therefore does not enter the linear stability bound of the explicit scheme.

2.4 Theoretical Analysis of a Linearized Diffusion Subproblem

We now analyze a regularized and linearized version of the diffusion equation when the segmentation field is kept fixed. In particular, we prove existence and uniqueness of a weak solution. This result provides a theoretical justification for the continuous component of the model and serves as a building block for the full coupled system.

Proposition 1 (Existence and uniqueness for the diffusion substep with frozen segmentation). *Let $S_\sigma \in C^\infty(\Omega)$ be fixed and assume that the diffusion coefficient $g(\cdot; K(S)) \in L^\infty(\Omega)$ satisfies*

$$0 < g_{\min} \leq g(x) \leq g_{\max} < \infty,$$

for almost every $x \in \Omega$, with homogeneous Neumann boundary conditions. Then for any initial condition $u^0 \in L^2(\Omega)$ and any source term $f \in L^2(\Omega)$ defined by

$$f = \alpha \nabla \cdot (w(S_\sigma) \nabla S_\sigma),$$

the linearized parabolic problem

$$\frac{\partial u}{\partial t} = \nabla \cdot (g(x) \nabla u) + f,$$

admits a unique weak solution

$$u \in L^2(0, T; H^1(\Omega)) \cap L^\infty(0, T; L^2(\Omega)),$$

for any $T > 0$, provided the compatibility condition $\int_\Omega f \, dx = 0$ holds. In practice, this condition is satisfied due to the normalization of S_σ .

Proof. For fixed S_σ , the coefficient $g(x)$ is bounded and uniformly elliptic by assumption.

To verify that $f \in L^2(\Omega)$, we note that $S_\sigma \in C^\infty(\Omega)$ implies $\nabla S_\sigma \in C^\infty(\Omega; \mathbb{R}^2)$. Since $w(S_\sigma)$ is smooth and bounded, we have $w(S_\sigma) \nabla S_\sigma \in C^\infty(\Omega; \mathbb{R}^2) \subset H^1(\Omega; \mathbb{R}^2)$. Therefore $f = \alpha \nabla \cdot (w(S_\sigma) \nabla S_\sigma) \in L^2(\Omega)$ by definition of the divergence operator on H^1 .

The result then follows from the classical theory of linear parabolic equations with Neumann boundary conditions (see Evans [26], Chapter 7). Existence and uniqueness are obtained via Galerkin approximation and energy estimates.

This result provides theoretical justification for each individual diffusion step of Algorithm 1, but does not extend to the full nonlinear coupled system, whose analysis is discussed in the following paragraph.

Implications for the full nonlinear coupled system. Proposition 1 settles the well-posedness of each individual diffusion step once the segmentation field is held constant. The situation changes markedly when one considers the alternating iteration defined in Algorithm 1. In the full system, the diffusion coefficient $g(|\nabla u|; K(S))$ depends on the gradient of the evolving image, while the segmentation map S is refreshed by the clustering operator $\mathcal{C}$. The latter is not a smooth mapping: it partitions the domain according to the nearest centroid, and an arbitrarily small perturbation of u in a Sobolev topology may displace the Voronoi boundaries and flip the label of a pixel. Because $\mathcal{C}$ lacks the Lipschitz or Hölder regularity usually required by standard coupled parabolic theory, the classical machinery for quasilinear systems does not apply directly.

This observation has a concrete theoretical consequence. Although each step of the alternating scheme is locally justified—the diffusion update is stable under the CFL condition of Lemma 1, and the K-means step terminates in finitely many iterations for fixed intensities—the global convergence of the map $(u^t, S^t) \mapsto (u^{t+1}, S^{t+1})$ is not covered by existing PDE theory. One cannot rule out, in principle, periodic or even chaotic behaviour of the sequence of segmentations, even though no such pathology was ever encountered in our experiments.

Still, partial analytical control of the full dynamics is not out of reach. First, uniform bounds in $L^\infty(\Omega)$ are inherited from the explicit scheme. Under the CFL condition, the diffusion term acts as a convex combination of neighbouring values because $g > 0$ everywhere; meanwhile the feedback forcing $\alpha \nabla \cdot (w(S_\sigma) \nabla S_\sigma)$ remains bounded in L^∞ since S_σ is a smooth mollification of an indicator. Consequently, $\|u^t\|_{L^\infty}$ stays uniformly bounded provided the initial image u^0 is bounded. Second, energy dissipation is available for the diffusion sub-problem: for frozen S , the flux $g(|\nabla u|; K(S)) \nabla u$ derives from a convex potential whenever $|\nabla u|$ remains in the regime where the Perona–Malik weight is monotone, so the associated weighted Dirichlet energy is non-increasing. Third, at the discrete level, the K-means update necessarily lowers (or leaves unchanged) the within-cluster sum of squares. These three monotonicity properties suggest that the combined dynamics, despite the lack of a global

convergence proof, operates in a controlled regime where energy fluctuations are bounded. A rigorous treatment of the fully nonlinear coupled system would demand new analytical tools for parabolic equations driven by discontinuous, non-local set-valued operators; we leave this challenging question for future investigation.

2.5 Algorithm and Parameter Adaptation

The complete algorithm proceeds as follows:

Algorithm 1: Coupled Diffusion-Clustering Segmentation

Input: Input image u^0 , parameters $K_0, \beta, \alpha, \lambda, \delta, \varepsilon_{\text{reg}}$,
 $\sigma = 1.5$, number of clusters K

Output: Final segmentation S^*

Initialize: $t \leftarrow 0, S^0 \leftarrow \mathcal{C}(u^0)$ // Initial
 K-means clustering

repeat

Smoothing step: $S_\sigma^t \leftarrow G_\sigma * \mathbb{1}_{\{S^t=1\}}$;

Diffusion step: Update u^{t+1} via $u^{t+1} = u^t +$
 $\lambda [\nabla \cdot (g(|\nabla u^t|; \kappa(S_\sigma^t)) \nabla u^t) + \alpha \nabla \cdot (w(S_\sigma^t) \nabla S_\sigma^t)]$;

Clustering step: $S^{t+1} \leftarrow \mathcal{C}(u^{t+1})$;

Parameter adaptation: Update
 $\kappa(S_\sigma^{t+1}) = K_0 (1 + \beta f(S_\sigma^{t+1}))$;

Convergence check: Compute
 $\Delta S^t = \frac{1}{|\Omega|} \sum_{(i,j) \in \Omega} \mathbb{I}\{S_{i,j}^{t+1} \neq S_{i,j}^t\}$;

$t \leftarrow t + 1$;

until $\Delta S^t < \delta$ or $t > t_{\text{max}}$;

return S^t

Remark(Convergence monitoring). The iterative scheme does not admit a general convergence proof in the nonlinear case, due to the non-smooth nature of the clustering operator $\mathcal{C}$. In practice, convergence is monitored through the criterion $\Delta S^t < \delta$. Since K-means clustering converges in a finite number of steps for fixed image intensities (Lloyd's theorem [16]), and the diffusion step is contractive under the CFL condition, the sequence (ΔS^t) is observed to decrease monotonically in all experiments.

Remark(Parameter Selection). The parameters are adapted from image statistics:

- K_0 : Set to the p -th percentile of $|\nabla u^0|$ (typically $p = 90$)
- β : $\beta = \sigma^2 / \sigma_{\text{max}}^2$, where σ^2 is image variance
- α : $\alpha = \text{mean}(|\nabla u^0|) / \text{mean}(|\nabla S_\sigma^0|)$
- λ : Chosen as $0.8 \times Ch^2$ for stability margin, with C from (4)
- σ : Fixed at 1.5 pixels (Gaussian smoothing width)
- δ : Convergence threshold,
- ε_{reg} : Regularization parameter in $|\nabla S_\sigma|_{\varepsilon_{\text{reg}}}$.

This data-dependent initialization ensures robustness across imaging modalities.

3 Implementation of the Proposed Method and Experimental Framework

In this section, we describe the computational implementation of the proposed framework and the experimental setup used to evaluate its performance on dermoscopic image data. The experimental framework is structured around three core components: (i) characterization of the dataset employed, (ii) statistical validation to assess the reliability of the method, and (iii) a detailed description of the algorithmic pipeline. Together, these elements provide a comprehensive overview of how the theoretical model is translated into a working segmentation system.

3.1 Dataset and Statistical Study

The experimental validation in this study is based on the ISIC 2017 dataset, a commonly recognized benchmark in dermoscopic image analysis. This collection comprises skin lesion images that have been meticulously annotated by clinical experts, covering a range of malignancy types such as Melanoma, Basal Cell carcinoma, and Squamous cell Carcinoma. Due to its diversity, rigorous expert validation, and public availability [22,6,23], the dataset offers a suitable platform for evaluating unsupervised segmentation approaches, which must contend with heterogeneous lesion appearances and varying imaging conditions.

To guarantee that our assessment reflects real world clinical diversity, we randomly sampled images from this database, spanning the full spectrum of lesion morphologies, sizes, and pigmentation patterns. This selective yet representative subset allows us to capture the inherent challenges encountered in practical dermatological practice while maintaining computational tractability.

A sample size of 50 images was chosen to balance statistical reliability with computational feasibility. This number provides a representative subset of the dataset while keeping the computational cost manageable, ensuring that performance metrics are stable and comparable across images.

The use of $N = 50$ images reflects a deliberate trade-off between statistical reliability and the computational cost of the coupled PDE-clustering iteration. From a statistical perspective, $n = 50$ is the conventional boundary where sample means begin to exhibit approximate normality under the central limit theorem; this permits stable estimation of the mean Dice coefficient and its standard error without requiring the heavy-tailed corrections necessary for very small cohorts. To verify that the reported figures are robust with respect to the particular draw, we repeated the full experimental protocol on several independent random subsets of sizes $N = 50$, $N = 70$, and $N = 100$ extracted from the ISIC

2017 pool. The resulting mean Dice coefficients, Jaccard indices, and overall accuracies differed by less than 2% across these cohort sizes, with the standard deviations of the mean metrics remaining below 0.015. This empirical concordance indicates that the segmentation statistics have stabilized and that the selected panel of 50 images yields representative results.

3.2 Preprocessing

Prior to segmentation, standard dermoscopic preprocessing steps were applied to reduce acquisition artifacts and normalize image characteristics. These include border removal, hair artifact suppression, grayscale conversion, and adaptive contrast enhancement using established techniques such as morphological filtering and CLAHE [24,27].

These operations aim to improve image homogeneity and contrast, thereby facilitating the numerical stability of the diffusion process and enhancing clustering reliability. Finally, all images were resized to a fixed resolution to ensure consistent discretization of the PDE model across the dataset.

We emphasize that preprocessing is not the focus of this work; rather, it serves to provide standardized inputs for evaluating the proposed diffusion-clustering coupling framework.

The preprocessing stage is intentionally kept minimal and standard, in order to isolate the contribution of the proposed coupled PDE-clustering model.

3.3 Dynamic Diffusion-Clustering Segmentation

Implementing the theoretical formulation requires its computational realization; we now describe how the proposed model is implemented as a dynamic iterative process. Our implementation transforms the diffusion-clustering coupling model into a dynamic iterative system. To maintain theoretical fidelity and numerical stability, every iteration strictly adheres to the equations laid out in the mathematical model.

The process starts with adaptive nonlinear diffusion (Equation 2), where adaptive conduction $K(S^t) = K_0(1 + \beta f(S_\sigma^t))$ governs intensity evolution $u(x, y, t)$. The Perona-Malik conductance function enables selective smoothing of homogeneous regions while protecting edges. Conductance modulation by the smoothed segmentation field embeds segmentation feedback within diffusion dynamics. The clustering operator $\mathcal{C}$ then applies K-means to partition the image into K intensity-based regions, yielding updated segmentation map S^{t+1} that identifies key lesion and background intensity modes.

The smoothed segmentation field

$$S_\sigma^{t+1} = G_\sigma * \mathbb{1}_{\{S^{t+1}=1\}}$$

is regularized through the feedback term in Equation 2.

This regularized version is then used to update two key components: the conductance field

$$K(S^{t+1}) = K_0(1 + \beta f(S_\sigma^{t+1}))$$

and the feedback weight $w(S_\sigma^{t+1})$, which together prepare the diffusion process for the next iteration. The cycle repeats until the convergence criterion in Equation 3 is satisfied, ensuring that the final output is both numerically stable and semantically coherent.

Through this iterative feedback mechanism, diffusion stabilizes intensity gradients, thereby facilitating more coherent clustering. In our implementation, we employ explicit finite-difference discretization with time step λ satisfying the Courant-Friedrichs-Lewy (CFL) condition (Equation 4) [25,15]. The model parameters were tuned according to the adaptive selection rules described in Section 2 and include baseline conductance K_0 , modulation factor β , feedback weight α , time step λ , regularization parameter ϵ_{reg} , and smoothing width $\sigma = 1.5$ pixels.

The complete segmentation workflow is visually summarized in Figure 1, which illustrates the bidirectional feedback loop between diffusion and clustering. Algorithmically, the process follows the detailed pseudo-code provided in Algorithm 1 (Section 2).

3.4 Post-processing

To improve the quality of the segmentation, we add a post-processing step to remove false positives, smooth irregular boundaries, and ensure anatomical plausibility. Designed as an adaptive and robust module, it operates on the binary mask obtained from the diffusion clustering loop. The following three operations constitute this stage:

–**Edge Artifact Removal:** Dark borders frequently appear in dermoscopic images due to microscope optics and acquisition settings, leading to erroneous segmentation along edges. Our approach detects any connected region touching the image border and eliminates it, removing these acquisition induced false positives.

–**Clinical Annotation Removal:** Dermatologists sometimes add artificial markers such as pen marks, calibration grids, or color reference charts to highlight areas of interest or standardize color assessment. These clinical annotations create segmented areas that are unrelated to the rest of the image. We address this by identifying and removing these elements.

–**Morphological Reconstruction:** We use morphological reconstruction to fill internal holes within lesion regions and smooth irregular boundaries through hole-filling and opening-closing operations. These refinements increase compactness, continuity, and anatomical plausibility of the final lesion contour.

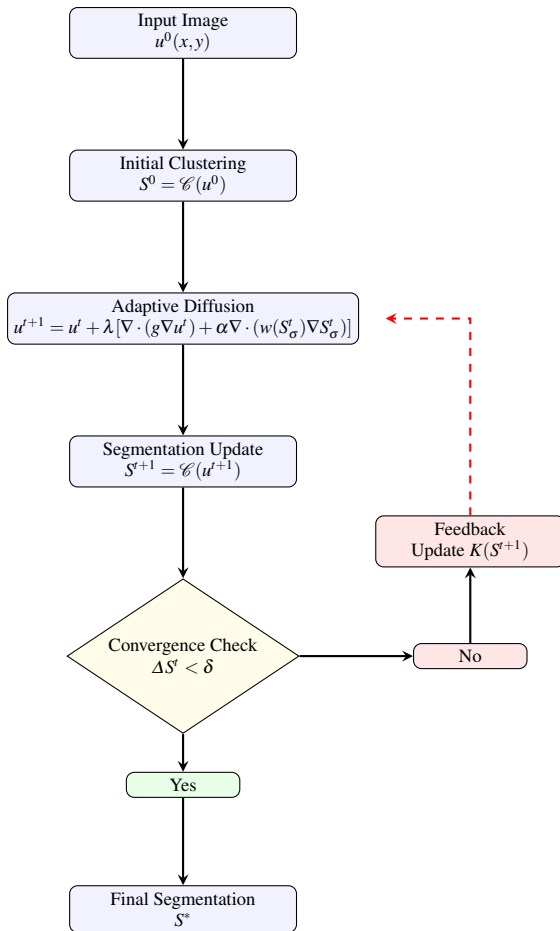


Fig. 1: Flowchart of the proposed method.

These post-processing stages enhance both spatial consistency and structural plausibility in the final segmentation mask. By reducing false positives and retaining anatomical accuracy along with fine contour details, the proposed method's performance is then assessed in Section 4 via statistical metrics and visual inspection on dermatological data.

4 Statistical and Visual Evaluation of the Proposed Method

In our experiments, we used a random sampling of dermoscopic images, each accompanied by ground truth masks annotated by experts. Figure 2 illustrates segmentation outputs for five representative examples together with their ground truth, allowing for direct visual comparison. We also propose to validate the performance of our proposal using the segmentation quality measurement metrics defined below.

4.1 Evaluation metrics

Segmentation performance is assessed using standard metrics widely adopted in the literature, including Accuracy, Precision, Recall (Sensitivity), Specificity, Dice coefficient, and Jaccard index. These measures respectively evaluate global classification accuracy, false positive control, lesion detection capability, background discrimination, and overlap quality between predicted and ground-truth regions.

Since these metrics are well-established, we omit their explicit definitions and focus instead on their collective interpretation.

In addition to these classical measures, we introduce a composite indicator based on the polygon area (PA), which integrates multiple performance criteria into a single scalar quantity. Given normalized metric values $\{r_i\}_{i=1}^n$, the polygon area is defined as

$$PA = \frac{1}{2} \sum_{i=1}^n r_i r_{i+1} \sin\left(\frac{2\pi}{n}\right), \quad (5)$$

where indices are taken cyclically.

This metric provides a global assessment of segmentation performance by capturing both the magnitude and balance of the individual criteria. In particular, it penalizes configurations where some metrics are high while others remain low, thereby favoring methods that achieve uniformly strong performance across all evaluation dimensions.

5 Results and Discussion

5.1 Experimental Evaluation

Our experimental evaluation examines segmentation performance using several statistical criteria, offering an overview of accuracy and stability. Quantitative results are summarized in Table 1, while the distributions of metric scores and the overall performance balance are visualized in Figures 3 and 4, respectively.

Quantitative assessment: Table 1 summarizes measurement results across the data samples through mean metrics. The Dice coefficient mean of **0.8672** indicates reliable boundary delineation, while Precision (**0.9808**) and Specificity (**0.9908**) values reflect minimal false detections. Sensitivity (**0.7867**) and Jaccard index (**0.7714**) show satisfactory lesion coverage with variability consistent with dermoscopic diversity. Overall Accuracy of **0.9221** further indicates consistent performance across imaging conditions.

Boxplot interpretation: As seen in Figure 3, the accuracy, specificity, and Dice scores show narrow distributions, indicating consistent segmentation behavior. This stability is maintained across the heterogeneous image set. Slight variations in sensitivity and Jaccard

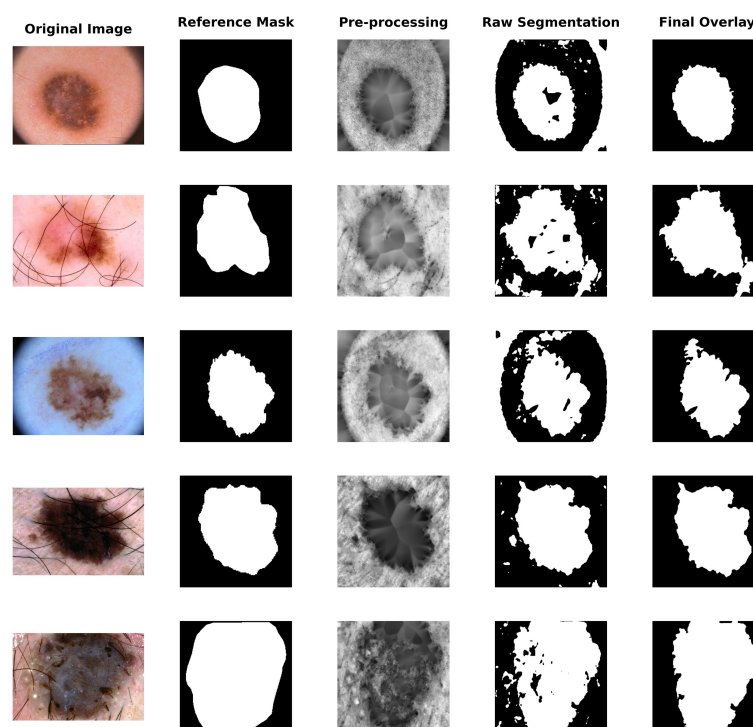


Fig. 2: Qualitative comparison between predicted and ground-truth masks.

Table 1: Mean segmentation performance metrics.

Metric	Mean
Dice	0.8672
Precision	0.9808
Sensitivity	0.7867
Jaccard	0.7714
Accuracy	0.9221
Specificity	0.9908

index are expected given the inherent variability in lesion appearance. We note that the method achieves accuracy levels comparable to those of supervised approaches.

Polygon-based evaluation of segmentation metrics: Mean performance metrics are visualized as a radar chart in Figure 4, revealing a relatively symmetric polygon with all values exceeding 0.75. Larger vertices for Precision, Accuracy, and Specificity indicate strong performance in false positive reduction and overall classification reliability. The polygon area $PA \approx 2.00$ quantitatively captures this balanced performance profile across evaluation dimensions.

Statistical analyses and graphical representations converge on a segmentation profile characterized by high accuracy, stable performance, and well-balanced indicators. This profile indicates that the method is robust and reproducible on heterogeneous dermoscopic images,

thus highlighting its potential for automated lesion delineation.

5.2 Comparative Analysis

To benchmark our results, we compare the performance of the proposed method against recent works that also report segmentation outcomes on the ISIC 2017 dermoscopic dataset. This comparison is conducted under consistent experimental settings to ensure a meaningful evaluation. A summary of this analysis, covering seven state-of-the-art supervised and unsupervised approaches, is presented in Table 2.

Unlike purely data-driven supervised methods that often operate as “black boxes,” our approach provides mathematical interpretability through the PDE framework, enabling clinicians to understand the segmentation rationale. While Zebari et al. [8] use standard K-means, our bidirectional coupling dynamically adjusts diffusion based on segmentation, explaining the superior specificity crucial for clinical false-positive reduction. This trade-off—slightly lower sensitivity but markedly higher specificity—makes the method particularly suitable for diagnostic applications where false positives carry significant clinical consequences.

Our method achieves a Dice coefficient of 0.8672, outperforming most unsupervised techniques and

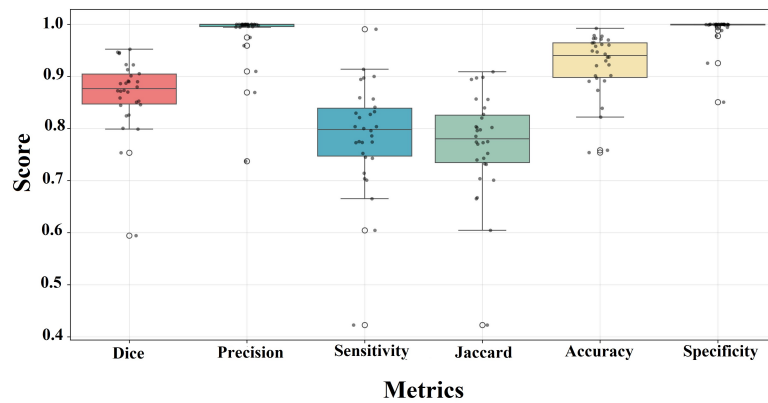


Fig. 3: Distribution of segmentation metrics across test images.

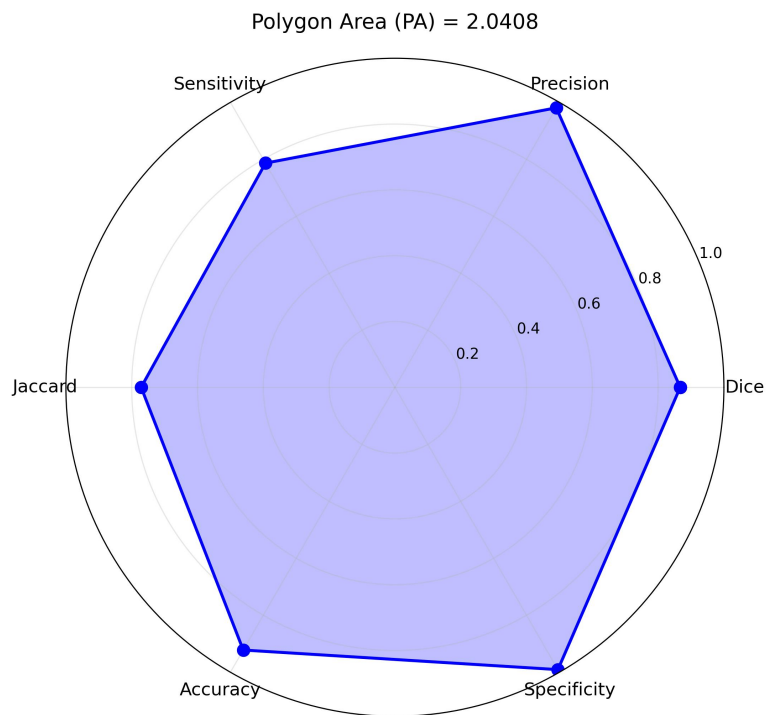


Fig. 4: Radar chart visualization of mean segmentation metrics with polygon area (PA) calculation.

Table 2: Comparative segmentation performance on ISIC 2017 dataset.

Method	Accuracy	Sensitivity	Specificity	Dice	Jaccard	Precision
Zeng et al. (2023) [10]	0.8881	0.7796	0.9558	0.7390	0.6445	–
Zhou et al. (2023) [11]	0.9440	0.7910	–	0.8780	–	0.8410
Zebari et al. (2022) [8]	0.9000	0.8933	0.9100	0.9112	0.8571	–
Sarwar et al. (2024) [2]	0.9500	0.9420	0.9670	0.9300	0.8750	–
Garcia et al. (2019) [9]	0.8840	0.8690	0.9230	0.7600	0.6650	–
Li et al. (2024) [7]	0.9050	0.8880	0.9370	0.8050	0.6850	–
Shan et al. (2020) [4]	0.9371	0.8382	0.9865	0.8456	0.7634	–
Proposed method	0.9221	0.7867	0.9908	0.8672	0.7714	0.9808

remaining competitive with several supervised alternatives. The exceptional specificity (0.9908) and precision (0.9808) values highlight the method's strong ability to reduce false positives, a feature of particular clinical interest for diagnostic reliability. The overall precision of 0.9221 further confirms the consistency of segmentation under heterogeneous imaging conditions.

Balanced performance includes competitive sensitivity (0.7867) and Jaccard index (0.7714), indicating stable and reliable segmentation across test cases. In conclusion, these results demonstrate that our segmentation framework offers performance competitive with several supervised approaches while providing mathematical transparency, making it potentially useful in clinical settings where labeled data is scarce or difficult to obtain.

Mathematical interpretation of the results. From a mathematical perspective, the observed segmentation performance can be directly related to the structure of the coupled PDE–clustering system. In particular, the high specificity values indicate that the regularization term $\alpha \nabla \cdot (w(S_\sigma) \nabla S_\sigma)$ effectively suppresses spurious regions by enforcing spatial coherence in the segmentation field. This term acts as a diffusion process on the smoothed indicator function, promoting compact and well-structured regions while penalizing isolated artifacts.

Conversely, the sensitivity of the method is influenced by the segmentation-adaptive conductance $K(S)$, which controls the strength of diffusion in relation to local structural homogeneity. The parameter β governs this interaction: larger values increase the influence of segmentation feedback, enhancing boundary adherence but potentially reducing robustness to noise, while smaller values favor smoother solutions at the cost of finer detail detection.

The balance between these competing effects explains the observed trade-off between sensitivity and specificity. This interplay highlights the role of the bidirectional coupling as a mechanism for controlling segmentation behavior through mathematically interpretable parameters.

Role of the coupled dynamics. The iterative interaction between diffusion and clustering can be interpreted as a fixed-point process in which each operator regularizes the other. Diffusion reduces noise and stabilizes intensity distributions, thereby facilitating clustering, while clustering provides structural information that modulates the diffusion coefficients. This mutual dependence leads to a self-consistent evolution, in contrast with classical alternating schemes where each component is optimized independently.

Such a coupled dynamics provides a possible explanation for the stability observed in the experimental results, as the feedback mechanism prevents the

accumulation of local inconsistencies and promotes convergence toward structurally coherent segmentations.

Implications for mathematical modeling. The proposed framework suggests a broader class of hybrid systems combining continuous PDE dynamics with discrete operators. From a theoretical standpoint, this raises new questions regarding the analysis of coupled continuous–discrete systems, including existence, uniqueness, and convergence properties of the full nonlinear model.

In particular, the presence of the clustering operator $\mathcal{C}$ introduces a non-smooth and non-local component into the evolution, which cannot be directly handled by classical PDE theory. This highlights the need for new analytical tools to rigorously study such systems, positioning the proposed model as a first step toward a more general theory of PDE–discrete couplings.

These observations reinforce the idea that the effectiveness of the proposed method is not only empirical but also rooted in its mathematical structure.

6 Conclusion

This paper introduced a PDE-discrete iterative framework for image segmentation that couples nonlinear diffusion with a clustering operator through a bidirectional feedback mechanism. In this formulation, diffusion evolves under a Perona–Malik-type conductance modulated by the current segmentation, while the segmentation is updated through a discrete clustering step driven by the diffused image.

From a theoretical perspective, we established existence and uniqueness results for the linearized regularized diffusion subproblem under a fixed segmentation field, and derived a CFL-type stability condition for the explicit numerical scheme. The analysis of the full nonlinear iterative coupling, which involves a non-smooth discrete operator, remains an open problem.

From a computational viewpoint, the proposed method provides an unsupervised segmentation strategy that preserves edges while enforcing spatial regularity, without requiring annotated data. Experimental results on dermoscopic images demonstrate competitive performance, particularly in terms of specificity and precision, while maintaining a balanced sensitivity–specificity trade-off governed by the model parameters.

The framework is currently limited to K-means clustering and two-dimensional image domains. Extensions to more general clustering strategies, including fuzzy and spectral methods, as well as to volumetric data, constitute natural directions for future research. Another promising direction is the integration of the proposed PDE-discrete coupling into hybrid learning architectures while preserving interpretability.

More broadly, the proposed model contributes to the development of hybrid systems combining continuous PDE dynamics with discrete iterative operators, highlighting new mathematical challenges in the analysis of coupled nonsmooth evolution processes.

In summary, the framework provides a consistent PDE-discrete coupling strategy that integrates nonlinear diffusion with clustering-based segmentation in a unified iterative scheme.

References

- [1] Manzoor, K., Gilal, N. U., Agus, M., and Schneider, J., Dual-stage segmentation and classification framework for skin lesion analysis using deep neural network, *Digital health* **11**, 20552076251351858 (2025).
- [2] N.Sarwar, A. Irshad, Q. H. Naith, K.D. Alsufiani, and F. A. Almallki, Skin lesion segmentation using deep learning algorithm with ant colony optimization, *BMC medical informatics and decision making* **24**(1), 265 (2024).
- [3] W. Shen, Z. Peng, X. Wang, H. Wang, J. Cen, D. Jiang and Q. Tian, A survey on label-efficient deep image segmentation: Bridging the gap between weak supervision and dense prediction., *IEEE transactions on pattern analysis and machine intelligence* **45**(8), 9284-9305 (2023).
- [4] P. Shan, Y. Wang, C. Fu, W. Song and J. Chen, Automatic skin lesion segmentation based on FC-DPN, *Computers in Biology and Medicine* **123**, 103762 (2020).
- [5] A. Gharawi, M.D. Alahmadi and L. Ramaswamy, Self-supervised skin lesion segmentation: An annotation-free approach, *Mathematics* **11**(18), 3805 (2023).
- [6] P. Tschandl, C. Rosendahl and H. Kittler, The HAM10000 dataset, a large collection of multi-source dermatoscopic images of common pigmented skin lesions, *Scientific data* **5**(1), 1–9 (2018).
- [7] X. Li, B. Peng, J. Hu, C. Ma, D. Yang and Z. Xie, USL-Net: Uncertainty self-learning network for unsupervised skin lesion segmentation, *Biomedical Signal Processing and Control* **89**, 105769 (2024).
- [8] N.A. Zebari and E. Tenekeci, Skin Lesion Segmentation Using K-means Clustering with Removal Unwanted Regions, *Adiyaman Univ. J. Eng. Sci.* **9**(18), 519–529 (2022).
- [9] J.L. Garcia-Arroyo and B. Garcia-Zapirain, Segmentation of skin lesions in dermoscopy images using fuzzy classification of pixels and histogram thresholding, *Computer methods and programs in biomedicine* **168**, 11–19 (2019).
- [10] G. Zeng, H. Peng, A. Li, Z. Liu, C. Liu, P.S. Yu and L. He, Unsupervised skin lesion segmentation via structural entropy minimization on multi-scale superpixel graphs, in *2023 IEEE International Conference on Data Mining (ICDM)*, pp. 768–777, IEEE, (2023).
- [11] X. Zhou, T. Tong, Z. Zhong, H. Fan and Z. Li, Saliency-CCE: exploiting colour contextual extractor and saliency-based biomedical image segmentation, *Computers in Biology and Medicine* **154**, 106551 (2023).
- [12] D. Mumford and J. Shah, Optimal approximation by piecewise smooth function and associated variational problems, *Communications on pure and applied mathematics* **42**(5), 577–685 (1989).
- [13] T.F. Chan and L.A. Vese, Active contours without edges, *IEEE Transactions on image processing* **10**(2), 266–277 (2001).
- [14] P. Perona and J. Malik, Scale-space and edge detection using anisotropic diffusion, *IEEE Transactions on pattern analysis and machine intelligence* **12**(7), 629–639 (1990).
- [15] J. Weickert, Anisotropic diffusion in image processing, *European Consortium for Mathematics in Industry*, (1998).
- [16] J. MacQueen, Multivariate observations, in *Proceedings of the fifth Berkeley symposium on mathematical statistics and probability*, Vol. 1, pp. 281–297, Oakland, CA, USA, (1967).
- [17] G.J. McLachlan and D. Peel, Finite mixture models. Hoboken, NJ: John Wiley & Sons, **10**, 0471721182, (2000).
- [18] BAE, Egil et MERKURJEV, Ekaterina, Convex variational methods on graphs for multiclass segmentation of high-dimensional data and point clouds. *Journal of Mathematical Imaging and Vision* **58**, p. 468–493 (2017).
- [19] N. Shah, D. Patel and P. Franti, k-Means image segmentation using Mumford Shah model, *Journal of Electronic Imaging* **30**, 063029 (2021).
- [20] F. Gibou and R. Fedkiw, A fast hybrid k-means level set algorithm for segmentation, in *Proc. 4th Annu. Hawaii Int. Conf. Statistics and Mathematics*, pp. 281–291 (2005).
- [21] D.L. Pham, C. Xu and J.L. Prince, Current methods in medical image segmentation, *Annual review of biomedical engineering* **2**(1), 315–337 (2000).
- [22] N.C. Codella et al., Skin lesion analysis toward melanoma detection: A challenge at the 2017 international symposium on biomedical imaging (isbi), hosted by the international skin imaging collaboration (isic), in *2018 IEEE 15th International Symposium on Biomedical Imaging (ISBI 2018)*, pp. 168–172, IEEE, (2018).
- [23] B. Cassidy, C. Kendrick, A. Brodzicki, J. Jaworek-Korjakowska and M. H. Yap, Analysis of the ISIC image datasets: Usage, benchmarks and recommendations, *Medical image analysis*, **75**, 102305, (2026).
- [24] L. Wu, J. Zhuang, W. Chen and et. all, Data augmentation based on multiple oversampling fusion for medical image segmentation, *PloS one* **17**, e0274522 (2022).
- [25] M.Ceccarelli, V. De Simone and A. Murli, Well-posed anisotropic diffusion for image denoising, *IEE Proceedings-Vision, Image and Signal Processing*, **149**, 244–252 (2002).
- [26] L.C. Evans, *Partial differential equations*, **19**, American mathematical society, 2022.
- [27] K. Zuiderveld, “Contrast limited adaptive histogram equalization,” in *Graphics Gems IV*, pp. 474–485, Academic Press, 1994.
- [28] M. Jahanian, A. Karimi, N. Eraghi, F. Zarafshan, Adaptive clustering for medical image analysis using the improved separation index, *Scientific Reports*, **15** pp. 28191, (2025).
- [29] M. Kowdiki, V. Deshmukh, A. Khaparde, Advantages and challenges of metaheuristic algorithms in biomedical image processing: a systematic review, *Multimedia Tools and Applications*, **85** pp. 522, (2026).

Ali ABOU EL QASSIME received the PhD degree in Applied Mathematics, Statistics and Signal Processing at the National School of Applied Sciences and Technology (ENSET) Mohammedia, Hassan II University of Casablanca, laboratory of Signals, Distributed Systems and Artificial Intelligence (SSDIA), Morocco. His main areas of research are: applied mathematics in general, and in particular the study of chaotic systems and their applications, statistics, and signal processing.

Youness EL ANSARI is an Associate Professor with Habilitation at the Higher School of Technology (EST), Agadir, Morocco, and is affiliated with the IRFSIC Laboratory at the Faculty of Sciences, Agadir. He received his Ph.D. in Stochastic Modelling and Statistics in 2020 from the Faculty of Sciences, Sidi Mohamed Ben Abdellah University, where he conducted his research within the Laboratory of Computer Science, Modeling and Systems, Department of Mathematics. He also holds an Engineering Diploma in Statistics and Business Intelligence from the Faculty of Sciences and Techniques, Abdelmalek Essaadi University. His research interests focus on stochastic dynamic systems in financial markets and stochastic epidemic modeling.

Mehdi ABOU EL QASSIME received the Ph.D. degree in Applied Mathematics from Ecole Polytechnique. His research lies at the intersection of machine learning, optimal control, and optimization, with a particular focus on the development of mathematical and computational methods for data-driven decision-making and dynamical systems.