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Generalized Kinetic Approach to the Rabi Dynamics in the Jaynes-Cummings Model

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Abstract: We investigate the Rabi dynamics of the Jaynes-Cummings model within the framework of the generalized kinetic equation. Two physically different regimes of the atom-field interaction are considered. For the resonant interaction, the kinetic equation with finite temporal correlations is solved analytically, showing that the conventional Rabi oscillation naturally emerges from the time-dependent evolution of the system. In the weak-coupling off-resonant regime, an analytical kinetic equation is derived by assuming a weak atom-field interaction while retaining the kinetic description of the dynamics. The obtained solution demonstrates that the conventional Rabi oscillation is recovered only when a specific relation between the atom-field detuning and the coupling strength is satisfied. This condition arises naturally from the kinetic formulation, providing a complementary perspective on the coherent dynamics described by the conventional Schrödinger approach. The results show that the generalized kinetic equation not only reproduces the well-known dynamics of the Jaynes-Cummings model but also offers additional physical insight into the emergence of Rabi oscillations.

Keywords: Jaynes-Cummings model, Rabi oscillations, generalized kinetic equation, two-level atom

1 Introduction

The Jaynes-Cummings (JC) model is one of the fundamental exactly solvable models in quantum optics, describing the interaction between a two-level atom and a single quantized mode of the electromagnetic field [1]. Owing to its analytical solvability, the model has played a central role in understanding coherent atom-field interactions, cavity quantum electrodynamics, and quantum information processing [2,3,4,5,6]. One of its most prominent predictions is the Rabi oscillation [7,8], which is conventionally derived by solving the Schrödinger equation.

Although the Schrödinger formalism accurately describes the coherent evolution of the JC model, it does not explicitly identify the role of temporal correlations in the formation of the observed dynamics. The generalized kinetic equation (GKE) [9,10,11,12], on the other hand, provides a natural framework in which these correlations are incorporated directly into the evolution equation.

In this work, we apply the GKE [11,12] to the JC model and investigate the Rabi dynamics in two physically distinct regimes. First, for the resonant interaction, we solve the generalized kinetic equation with the full memory kernel and demonstrate that the conventional Rabi oscillation is naturally recovered. This result shows that coherent atom-field oscillations emerge directly from the temporal correlations incorporated in the memory kernel. Second, for the weak-coupling off-resonant regime, we derive a simplified kinetic equation under the assumption of weak atom-field interaction and obtain its analytical solution. The analysis reveals that the conventional Rabi oscillation survives only when a specific relation between the detuning and the atom-field coupling strength is satisfied. This condition arises naturally within the kinetic formalism and complements the conventional Schrödinger description by providing a kinetic interpretation of the observed dynamics.

These results demonstrate that the GKE is not merely an alternative mathematical formulation of the JC model, but also an effective theoretical framework for analysing the influence of memory effects and weak-coupling approximations on coherent quantum dynamics.

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2 The JC model

The JC model is one of the most fundamental models in quantum optics and describes the interaction between a two-level atom and a single quantized mode of the electromagnetic field [1]. Under the dipole and RWA, the Hamiltonian of the system can be written as

$$\hat{H} = \hbar\omega\hat{b}^{\dagger}\hat{b} + \hbar\omega_0\hat{S}^z + \hbar g\left(\hat{b}^{\dagger}\hat{S}^- + \hat{b}\hat{S}^+\right), \qquad (1)$$

where ω is the cavity-mode frequency, ω_0 is the atomic transition frequency, g is the atom-field coupling constant, and $\hat{b}^{\dagger},\hat{b}$ are the photon creation (annihilation) operators. The operators $\hat{S}^+,\hat{S}^-$, and $\hat{S}^z$ describe the dynamics of the two-level atom and satisfying the following commutation relations

$$[\hat{S}^{\pm},\hat{S}^z] = \mp\hat{S}^{\pm}, \quad [\hat{S}^+,\hat{S}^-] = 2\hat{S}^z.$$

We can make the JC model more convenient by specifying it as follows:

$$\hat{H} = \hat{H}(M) + \hat{H}(F) + \hat{H}(M,F), \qquad (2)$$

where M and F are denoted atom and field system, respectively.

The first term describes the energy of the single two-level atom system:

$$\hat{H}(M) = \hbar\omega_0\hat{S}^z.$$

A single mode cavity field Hamiltonian:

$$\hat{H}(F) = \hbar\omega\hat{b}^{\dagger}\hat{b}.$$

Interaction part of the system which two-level atom with single cavity field mode Hamiltonian:

$$\hat{H}(M,F) = \hbar g\left(\hat{b}^{\dagger}\hat{S}^- + \hat{b}\hat{S}^+\right).$$

3 The generalized kinetic equation

Let us begin to build the kinetic equation for a system with Hamiltonian (2). We introduce D_t, a statistical operator of the full system (M,F) with Hamiltonian (2), satisfying Liouville's equation:

$$i\hbar\frac{\partial}{\partial t}D_t(M,F) = [\hat{H}, D_t(M,F)],$$

with initial condition

$$D_{t_0}(M,F) = \rho_{t_0}(M)D_{t_0}(F).$$

Let us begin by assuming that the cavity field is initially prepared in a Fock (photon-number) state. The initial density operator of the field is therefore taken as

$$D_{t_0}(F) = |n\rangle\langle n|,$$

where $|n\rangle$ denotes the Fock state with exactly n is photon number.

Here $\rho_{t_0}(M)$ - initial density matrix of M system, $\mathrm{Tr}_{(M)}\rho_{t_0}(M) = 1$, and $\mathrm{Tr}_{(F)}D_{t_0}(F) = 1$. Suppose that interaction between M and F is weak.

Introduce the evolution operator $U(t,t_0)$ defined by the following equations

$$i\hbar\frac{\partial}{\partial t}U(t,t_0) = \hat{H}U(t,t_0), \qquad U(t_0,t_0) = 1,$$

and we have

$$-i\hbar\frac{\partial U^+(t,t_0)}{\partial t} = U^+(t,t_0)\hat{H}, \quad U^+(t_0,t_0) = 1,$$

where $U^+(t,t_0) = U^{-1}(t,t_0)$. As is well-known, by means of evolution operators for average dynamic value $\mathscr{U}(M,F)$ one can obtain the relation between the Schrödinger and the Heisenberg representations:

$$D_t(M,F) = U(t,t_0)D_{t_0}(M,F)U^{-1}(t,t_0).$$

It follows that

$$\langle\mathscr{U}_t\rangle = \mathrm{Tr}_{(M,F)}\{U^{-1}(t,t_0)\mathscr{U}(M,F)U(t,t_0)\}D_{t_0}(M,F). \qquad (3)$$

Evidently,

$$\mathscr{U}(M_t,F_t) = U^{-1}(t,t_0)\mathscr{U}(M,F)U(t,t_0) \qquad (4)$$

is Heisenberg representation.

Consider the operator $\hat{f}(M)$ that acts on eigenfunctions of the Hamiltonian (2) only via the variables of the M-system. The equations of motion for operator $\hat{f}(M)$ in the Heisenberg representation (4) are given by

$$i\hbar\frac{\partial}{\partial t}\hat{f}(M_t) = [\hat{f}(M_t),\hat{H}_t]. \qquad (5)$$

Using the method of eliminating boson variables and the average value of the dynamical quantity $\hat{f}(M)$ is defined as $\langle\hat{f}(M_t)\rangle = \mathrm{Tr}_{(M)}\hat{f}(M)\rho_t(M)$ developed in works [9,10,11,12], equation (5) can be transformed to the form

$$\begin{aligned}
&\mathrm{Tr}_{(M)}\left(\frac{\partial\hat{f}(M)}{\partial t} - \frac{[\hat{f}(M),\hat{H}(M)]}{i\hbar}\right)\rho_t(M)\\
&= g^2\int_{t_0}^{t} d\tau e^{-i\omega(t-\tau)}\\
&\times\mathrm{Tr}_{(M,F)}\{n\hat{S}^-(\tau)[\hat{f}(M_t),\hat{S}^+(t)]\\
&+(1+n)[\hat{S}^+(t),\hat{f}(M_t)]\hat{S}^-(\tau)\}D_{t_0}(M,F)\\
&+g^2\int_{t_0}^{t} d\tau e^{i\omega(t-\tau)}\\
&\times\mathrm{Tr}_{(M,F)}\{(1+n)\hat{S}^+(\tau)[\hat{f}(M_t),\hat{S}^-(t)]\\
&+n[\hat{S}^-(t),\hat{f}(M_t)]\hat{S}^+(\tau)\}D_{t_0}(M,F).
\end{aligned} \qquad (6)$$

Thus, the GKE (6) for JC model is obtained.

4 Solution of the GKE for JC model

In the previous section, we derived the GKE for the system when a two-level atom interact with single cavity field mode. In this section, we consider how the dynamics of single two-level atom evolve over time and how it relates to population inversion and quantum behaviour in some cases.

The Jaynes-Cummings Hamiltonian is given by (1). Since the total excitation number is conserved, the Hamiltonian can be diagonalized independently in each two-dimensional excitation subspace spanned by the basis states $\{|e,n\rangle, |g,n+1\rangle\}$, where $|e\rangle$ and $|g\rangle$ represent the excited and ground atomic states, respectively. Within each subspace, the Hamiltonian is reduced to an effective two-level form,

$$\hat{H}_n = \hbar\omega\left(n+\frac{1}{2}\right)\hat{I} + \hbar\Delta\hat{S}^z + \hbar g\sqrt{n+1}(\hat{S}^+ + \hat{S}^-),$$

where $\Delta = \omega_0 - \omega$ is the detuning, $\hat{I}$ denotes the identity operator in the two-dimensional invariant subspace.

Representing this effective Hamiltonian $\hat{H}_n$ in this basis yields the matrix form

$$\hat{H}_n = \hbar\begin{pmatrix} \omega n + \frac{\omega_0}{2} & g\sqrt{n+1} \\ g\sqrt{n+1} & \omega(n+1) - \frac{\omega_0}{2} \end{pmatrix}. \tag{7}$$

The corresponding eigenvalues are

$$E_\pm = \hbar\omega\left(n+\frac{1}{2}\right) \pm \frac{\hbar}{2}\sqrt{\Delta^2 + 4g^2(n+1)}.$$

Therefore, the energy splitting between the dressed states determines the generalized Rabi frequency,

$$\Omega_n = \sqrt{\Delta^2 + 4g^2(n+1)}. \tag{8}$$

In the resonant case $\Delta = 0$, this expression reduces to

$$\Omega_n = 2g\sqrt{n+1}, \tag{9}$$

which for the vacuum field $n = 0$ gives the vacuum Rabi frequency

$$\Omega_R = 2g.$$

For each excitation manifold, the state vector can be written as

$$|\psi(t)\rangle = c_e(t)|e,n\rangle + c_g(t)|g,n+1\rangle. \tag{10}$$

Substituting this expansion into the Schrödinger equation, yields the coupled amplitude equations

$$i\dot{c}_e = \frac{\Delta}{2}c_e + g\sqrt{n+1}c_g,$$

$$i\dot{c}_g = -\frac{\Delta}{2}c_g + g\sqrt{n+1}c_e.$$

Eliminating one of the amplitudes gives the second-order equation

$$\frac{d^2c_e}{dt^2} + i\Delta\frac{dc_e}{dt} + \left[g^2(n+1) + \frac{\Delta^2}{4}\right]c_e = 0,$$

whose solution oscillates with the generalized Rabi frequency Ω_n. For an atom initially prepared in the excited state,

$$c_e(0) = 1, \qquad c_g(0) = 0,$$

the amplitudes are

$$c_e(t) = \cos\left(\frac{\Omega_n t}{2}\right) - i\frac{\Delta}{\Omega_n}\sin\left(\frac{\Omega_n t}{2}\right),$$

$$c_g(t) = -i\frac{2g\sqrt{n+1}}{\Omega_n}\sin\left(\frac{\Omega_n t}{2}\right).$$

Therefore, the probabilities of finding the atom in the excited and ground states are

$$P_e(t) = |c_e(t)|^2 = 1 - \frac{4g^2(n+1)}{\Omega_n^2}\sin^2\left(\frac{\Omega_n t}{2}\right),$$

$$P_g(t) = |c_g(t)|^2 = \frac{4g^2(n+1)}{\Omega_n^2}\sin^2\left(\frac{\Omega_n t}{2}\right),$$

which satisfy $P_e(t) + P_g(t) = 1$. In the resonant case $\Delta = 0$,

$$P_e(t) = \cos^2\left(\frac{\Omega_n}{2}t\right), \quad P_g(t) = \sin^2\left(\frac{\Omega_n}{2}t\right). \tag{11}$$

In the resonant case $\Delta = 0$, the obtained population probabilities reduce to

$$P_e(t) = \cos^2(gt), \quad P_g(t) = \sin^2(gt),$$

which are in complete agreement with the well-known analytical solution of the standard JC Hamiltonian (1) for the first excitation manifold $\{|e,0\rangle, |g,1\rangle\}$.

4.1 Solution of the GKE for JC model with weak interaction

Consider the interaction between an atom system and field is weak [10,11,12]. Then we should make following approximation:

$$\hat{S}^\pm(\tau) = \hat{S}^\pm(t)e^{\mp i\omega_0(t-\tau)}. \tag{12}$$

Consider the case $\hat{f}(M_t) = \hat{S}^z(t)$. By using approximation (12), equation (6) is following form

$$\begin{aligned}
&\frac{\partial}{\partial t}\langle\hat{S}^z(t)\rangle \\
&= g^2\int_{t_0}^{t} d\tau e^{-i(\omega-\omega_0)(t-\tau)}\{n\langle\hat{S}^-(t)\hat{S}^+(t)\rangle \\
&-(1+n)\langle\hat{S}^+(t)\hat{S}^-(t)\rangle\} \\
&-g^2\int_{t_0}^{t} d\tau e^{i(\omega-\omega_0)(t-\tau)}\{(1+n)\langle\hat{S}^+(t)\hat{S}^-(t)\rangle \\
&-n\langle\hat{S}^-(t)\hat{S}^+(t)\rangle\}.
\end{aligned} \tag{13}$$

Then we obtain

$$\begin{aligned}\frac{\partial}{\partial t}\langle\hat{S}^z(t)\rangle &= -2g^2\int_0^t d\tau\cos[\Delta(t-\tau)] \\ &\times\left[2n\langle\hat{S}^z(t)\rangle+\langle\hat{S}^+(t)\hat{S}^-(t)\rangle\right]. \qquad (14)\end{aligned}$$

where $\Delta=\omega-\omega_0$ and $\langle\hat{S}^+(t)\hat{S}^-(t)\rangle = 1/2+\langle\hat{S}^z(t)\rangle$. After the integration in Eq. (14) transforms into

$$\frac{\partial}{\partial t}\langle\hat{S}^z(t)\rangle = -\frac{2g^2}{\Delta}\sin(\Delta t)\left(\frac{1}{2}+(2n+1)\langle\hat{S}^z(t)\rangle\right). \qquad (15)$$

$$\frac{1}{2}+(2n+1)\langle\hat{S}^z(t)\rangle = C_1 e^{c(t)}.$$

where

$$c(t) = \frac{2g^2(2n+1)}{\Delta^2}\cos(\Delta t).$$

Let us determine C_1 by using of initial condition. Consider the case when $t=0$ atom is in exited state $\langle\hat{S}^z(0)\rangle = \frac{1}{2}$. Then

$$C_1 = (n+1)e^{-\frac{2g^2(2n+1)}{\Delta^2}}.$$

Then

$$\langle\hat{S}^z(t)\rangle = \frac{1}{2n+1}\left((n+1)e^{\frac{2g^2(2n+1)}{\Delta^2}(\cos(\Delta t)-1)}-\frac{1}{2}\right)$$

It is insisted, that in the weak interaction the condition is $\Delta \gg g$. Then

$$e^{\frac{2g^2(2n+1)}{\Delta^2}(\cos(\Delta t)-1)} \approx 1+\frac{2g^2(2n+1)}{\Delta^2}(\cos(\Delta t)-1).$$

Then solution of equation (15) will be following form

$$\langle\hat{S}^z(t)\rangle = \frac{1}{2}\left(1+\frac{4g^2(n+1)}{\Delta^2}(\cos(\Delta t)-1)\right). \qquad (16)$$

You can see, solution (16) is satisfied initial condition when initial condition atom is exited state $\langle\hat{S}^z(0)\rangle = \frac{1}{2}$.

Let us obtain $I(t)$ intensity of atoms by using following relation

$$I(t) = -\hbar\omega_0\frac{d}{dt}\langle\hat{S}^z(t)\rangle.$$

Then the emission rate $(I(t)>0)$

$$I_{emit}(t) = \hbar\omega_0\frac{2g^2(n+1)}{\Delta}\sin(\Delta t), \qquad (17)$$

and the absorption rate $(I(t)<0)$

$$I_{abs}(t) = -\hbar\omega_0\frac{2g^2(n+1)}{\Delta}\sin(\Delta t). \qquad (18)$$

We can determine the probabilities $P_e(t)$ and $P_g(t)$ by using the solution (16) and following relation for single two-level atom system:

$$P_e(t) = \frac{1}{2}+\langle\hat{S}^z(t)\rangle, \quad P_g(t) = \frac{1}{2}-\langle\hat{S}^z(t)\rangle.$$

Then

$$P_g(t) = \frac{4g^2(n+1)}{\Delta^2}\sin^2\left(\frac{\Delta}{2}t\right) \qquad (19)$$

and

$$P_e(t) = 1-\frac{4g^2(n+1)}{\Delta^2}\sin^2\left(\frac{\Delta}{2}t\right). \qquad (20)$$

Then

$$P_g(t)+P_e(t) = 1. \qquad (21)$$

You can see from results (19) and (20), even weak coupling between atom-field system with framework JC model, the results with a small amplitude $\frac{4g^2(n+1)}{\Delta^2}$ are satisfied Rabi oscillation and normalisation of probability (21). Moreover from (16), (19) and (20) are matched standard Rabi oscillation results (11) for JC model when the following equality holds:

$$\frac{4g^2(n+1)}{\Delta^2} = 1. \qquad (22)$$

4.2 Solution of the GKE for JC model in the resonant case

We consider the case $\hat{f}(M_t) = \hat{S}^z(t)$. By using Eqs. (3) and (4), in Eq. (6) is following form

$$\begin{aligned}&\frac{\partial}{\partial t}\langle\hat{S}^z(t)\rangle \\ &= g^2\int_{t_0}^t d\tau e^{-i\omega(t-\tau)}\{n\langle\hat{S}^-(\tau)\hat{S}^+(t)\rangle \\ &-(1+n)\langle\hat{S}^+(t)\hat{S}^-(\tau)\rangle\} \\ &-g^2\int_{t_0}^t d\tau e^{i\omega(t-\tau)}\{(1+n)\langle\hat{S}^+(\tau)\hat{S}^-(t)\rangle \\ &-n\langle\hat{S}^-(t)\hat{S}^+(\tau)\rangle\}. \qquad (23)\end{aligned}$$

To evaluate the four correlation operators appearing in the GKE (23), such as $\langle\hat{S}^+(\tau)\hat{S}^-(t)\rangle$, $\langle\hat{S}^-(\tau)\hat{S}^+(t)\rangle$ and etc., we employ the Heisenberg equations of motion derived from the JC Hamiltonian (1).

Following the standard treatment of the resonant JC model [2], we employ the exact operator solution in the resonant case.

More generally, introducing the generalized Rabi frequency Ω_n, the result takes the form

$$\begin{aligned}\hat{S}^-(\tau) &= e^{-i\omega_0(\tau-t)}\Big[\hat{S}^-(t)\cos[\Omega_n(\tau-t)] \\ &+\frac{2i\hat{b}(t)\hat{S}^z(t)}{\sqrt{n+1}}\sin[\Omega_n(\tau-t)]\Big]. \qquad (24)\end{aligned}$$

This expression explicitly demonstrates that the two-time operator $\hat{S}^-(\tau)$ contains both the free atomic evolution and the coherent atom-field energy exchange responsible for the Rabi oscillations.

Then by using (24) calculating two-time correlators from the GKE (23), one can obtain following:

$$e^{-i\omega(t-\tau)}\langle\hat{S}^{-}(\tau)\hat{S}^{+}(t)\rangle = \cos[\Omega_n(\tau-t)]\langle\hat{S}^{-}\hat{S}^{+}\rangle_t + \frac{2i}{\sqrt{n+1}}\sin[\Omega_n(\tau-t)]\langle\hat{b}\hat{S}^{z}\hat{S}^{+}\rangle_t, \tag{25}$$

$$e^{-i\omega(t-\tau)}\langle\hat{S}^{+}(t)\hat{S}^{-}(\tau)\rangle = \cos[\Omega_n(\tau-t)]\langle\hat{S}^{+}\hat{S}^{-}\rangle_t + \frac{2i}{\sqrt{n+1}}\sin[\Omega_n(\tau-t)]\langle\hat{S}^{+}\hat{b}\hat{S}^{z}\rangle_t, \tag{26}$$

$$e^{i\omega(t-\tau)}\langle\hat{S}^{+}(\tau)\hat{S}^{-}(t)\rangle = \cos[\Omega_n(\tau-t)]\langle\hat{S}^{+}\hat{S}^{-}\rangle_t - \frac{2i}{\sqrt{n+1}}\sin[\Omega_n(\tau-t)]\langle\hat{S}^{z}\hat{b}^{\dagger}\hat{S}^{-}\rangle_t, \tag{27}$$

$$e^{i\omega(t-\tau)}\langle\hat{S}^{-}(t)\hat{S}^{+}(\tau)\rangle = \cos[\Omega_n(\tau-t)]\langle\hat{S}^{-}\hat{S}^{+}\rangle_t - \frac{2i}{\sqrt{n+1}}\sin[\Omega_n(\tau-t)]\langle\hat{S}^{-}\hat{b}^{\dagger}\hat{S}^{z}\rangle_t. \tag{28}$$

Here

$$\langle\hat{S}^{+}\hat{S}^{-}\rangle_t = \cos^2\left(\frac{\Omega_n}{2}t\right), \quad \langle\hat{S}^{-}\hat{S}^{+}\rangle_t = \sin^2\left(\frac{\Omega_n}{2}t\right),$$

$$\langle\hat{S}^{z}\hat{b}\hat{S}^{+}\rangle_t = -\frac{i\sqrt{n+1}}{4}\sin(\Omega_n t),$$

$$\langle\hat{S}^{+}\hat{b}\hat{S}^{z}\rangle_t = \frac{i\sqrt{n+1}}{4}\sin(\Omega_n t),$$

$$\langle\hat{S}^{z}\hat{b}^{\dagger}\hat{S}^{-}\rangle_t = -\frac{i\sqrt{n+1}}{4}\sin(\Omega_n t),$$

$$\langle\hat{S}^{-}\hat{b}^{\dagger}\hat{S}^{z}\rangle_t = \frac{i\sqrt{n+1}}{4}\sin(\Omega_n t).$$

By using these equalities from (25) to (28) in Eq. (23) when one can denote $t_0 = 0$, we obtain

$$\frac{\partial}{\partial t}\langle\hat{S}^{z}(t)\rangle = -g\sqrt{n+1}\sin(\Omega_n t). \tag{29}$$

From (29) one can obtain solution of the GKE for JC model which describe the evolution of the atomic inversion:

$$\langle\hat{S}^{z}(t)\rangle = \frac{1}{2}\cos(2g\sqrt{n+1}t). \tag{30}$$

This result describes the coherent exchange of excitation energy between the atom and the quantized cavity field. The oscillatory behaviour of the atomic inversion is a manifestation of the well-known Rabi oscillations characteristic of the JC model.

The probabilities of finding the atom in the excited and ground states can be expressed through the expectation value of the inversion operator as

$$P_e(t) = \cos^2(g\sqrt{n+1}t), \tag{31}$$

$$P_g(t) = \sin^2(g\sqrt{n+1}t). \tag{32}$$

Therefore, the atomic population oscillates periodically between the excited and ground states with the Rabi frequency. As time evolves, the excitation is coherently transferred to the cavity field and subsequently returns to the atom, producing a periodic exchange of energy.

The obtained probabilities satisfy the normalization condition

$$P_e(t) + P_g(t) = \cos^2(gt) + \sin^2(gt) = 1, \tag{33}$$

which expresses the conservation of total probability. Consequently, the atom is always found either in the excited state or in the ground state.

Obtained results demonstrating that the atomic inversion oscillates harmonically in time. These results coincide exactly with the standard solution of the JC model and confirm that the GKE correctly reproduces the coherent non-Markovian dynamics of the atom-field interaction.

5 Conclusion

In this work, we have investigated the Rabi dynamics of the JC model within the framework of the generalized kinetic equation. For the resonant interaction, the generalized kinetic equation with the full memory kernel was solved analytically, demonstrating that the conventional Rabi oscillation is naturally recovered. This result provides a kinetic interpretation of coherent atom-field oscillations and emphasizes the role of the memory kernel in the dynamical evolution of the system.

For the weak-coupling off-resonant regime, a simplified kinetic equation was derived under the assumption of weak atom-field interaction and solved analytically. The obtained solution shows that the conventional Rabi oscillation is recovered only when a specific relation between the atom-field detuning and the coupling strength is satisfied. This condition arises naturally within the kinetic formalism and provides a complementary physical interpretation of coherent atom-field dynamics.

Overall, the generalized kinetic equation provides a unified framework for describing the Rabi dynamics of the JC model under different physical conditions. In addition to reproducing the well-known resonant behaviour, it reveals the conditions governing coherent oscillations in the weak-coupling off-resonant regime. These results demonstrate that the GKE is a useful theoretical tool for investigating coherent light-matter interactions and may be extended to more complex quantum optical systems and quantum information theory.

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