Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/200411

Forecasting Pharmaceutical Stock Prices in the Saudi Exchange Using Geometric and Fractional Brownian Motion Models

Anas Abbas[1,*] *and Mohammed Alhagyan*[2]

[1]Business Administration Department, College of Humanities and Science in Al Aflaj, Prince Sattam Bin Abdulaziz University, 11912, Saudi Arabia
[2]Mathematical Science Department, College of Sciences, United Arab Emirates University, UAE

Received: 12 Mar. 2026, Revised: 12 Jun. 2026, Accepted: 27 Jun. 2026
Published online: 1 Jul. 2026

Abstract: This study compares four continuous-time stochastic models for forecasting the daily closing prices of two pharmaceutical companies listed on the Saudi Exchange: Saudi Pharmaceutical Industries and Medical Appliances Corporation (SPIMACO) and Jamjoom Pharmaceuticals Factory Company (JAMJOOM). The models under consideration are geometric Brownian motion (GBM), geometric fractional Brownian motion (GFBM), GBM with stochastic volatility (SV-GBM), and GFBM with stochastic volatility (SV-GFBM). We choose these models to examine the efficiency of incorporating memory and stochastic volatility assumptions into GBM. Forecast accuracy is assessed by mean squared error (MSE) and mean absolute percentage error (MAPE). According to the two accuracy instruments, the SV-GFBM specification yields the smallest error values; however, the differences among competing models are modest. This result cannot be conclusive, so the evidence should be interpreted as exploratory and comparative. The contribution of the paper is methodological: it documents how long-memory and stochastic-volatility assumptions affect mean-path forecasts for two Saudi pharmaceutical stocks. This result has ensured the direct positive affection of incorporating the assumptions of long memory and stochastic volatility into the GBM model which agrees with some previous studies.

Keywords: Geometric Brownian motion, stochastic volatility, long memory, Pharma sector

1 Introduction

Forecasting stock prices remains a difficult applied problem because financial time series often exhibit noise, volatility clustering, and, in some markets, evidence of persistence or long-range dependence. In this context, Brownian-motion-based models remain useful benchmarks because they offer a transparent probabilistic structure and have been widely used in financial modelling. The present study focuses on the Saudi pharmaceutical sector and compares four related model specifications for the daily closing prices of Saudi Pharmaceuticals Industries and Medical Appliances Corporation (SPIMACO) and Jamjoom Pharmaceuticals Factory Co JAMJOOM.

The objective of the paper is to examine whether adding two modelling ingredients—long memory through fractional Brownian motion (FBM) and time-varying volatility through a stochastic-volatility (SV) component—improves forecast accuracy relative to the classical GBM benchmark. Further, to justify investment decisions or market policy to the pharma sector in KSA.

The literature motivating the comparison includes studies that use GBM or GFBM in forecasting and related financial applications, including Abbas and Alhagyan (2022, 2023), Alhagyan and Alduais (2020), Alhagyan and Yassen (2023), Alhagyan (2022), Rejichi and Aloui (2012), and Willinger et al. (1999). These studies suggest that persistence and stochastic volatility can be empirically relevant. The present manuscript contributes by comparing four benchmark specifications on two Saudi pharmaceutical stocks using the same general forecasting framework and by making the estimation window and holdout period explicit.

* Corresponding author e-mail: a.abbas@psau.edu.sa

2 Materials and Methods

The study follows a fixed experimental design. Models parameters are estimated from an in-sample period and then used to generate out-of-sample predictive trajectories during a subsequent testing period. The next section shows the models under consideration in this study.

2.1 Models under Study

Definition 1: *A Wiener Process or Brownian motion (BM) is a stochastic process B_t that satisfies the next conditions:*

i. B_t is a continuous function of time with $B_0 = 0$.
ii. $B_t - B_s$ and $B_v - B_u$ are independent for all $v > u > t > s$ (or B_t has independent increments).
iii. $B_t - B_s \sim N(0,\ t-s)$ for all $t > s$ (or B_t has normal increments).

Early studies (e.g., Sheldon, 1999) employed Brownian motion directly to model stock prices. However, this approach received significant criticism because Brownian motion allows for negative values, which is inconsistent with the non-negativity of asset prices. This limitation arises from the assumption that prices follow a normal distribution. To address this issue, geometric Brownian motion (GBM) was introduced as a non-negative alternative. Due to this property, GBM has become widely applicable in financial modeling. It is particularly useful for describing random price fluctuations over time and is commonly used for forecasting purposes.

Model 1: *The stochastic process S_t follows GBM if the next stochastic differential equation is satisfied (SDE):*

$$dS_t = \mu S_t\, dt + \sigma S_t\, dB_t, \tag{1}$$

where B_t is a BM, σ is a volatility and μ is a drift. The general solution of this SDE is given by

$$S_t = S_0 \exp\left\{\left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma B_t\right\}, \tag{2}$$

where S_0 is an initial value.

A defining feature of the classical GBM model is its assumption that price changes are independent of past movements. However, its widespread use in financial modeling has prompted many researchers to question this assumption, pointing to clear evidence of memory effects in financial time series (e.g., Painter 1998; Grau–Carles 2000; Rejichi and Aloui 2012; Alhagyan 2018, 2022; Alhagyan and Yassen 2023; Abbas and Alhagyan 2023; Kim et al. 2020; Han et al. 2020).

This growing body of research highlights the need to incorporate memory into the GBM framework. In response, an extended model—geometric fractional Brownian motion (GFBM)—was developed. GFBM replaces the standard Brownian motion in GBM with fractional Brownian motion (FBM), allowing the model to capture the dependence structure observed in real financial data.

Definition 2: *The fractional Brownian motion (FBM), $\{B_H(t)\}$, with Hurst parameter $H \in (0,1)$ is a centered Gaussian process whose paths are continuous with probability 1 and its distribution is defined by the covariance structure:*

$$E\left[B_H(t) B_H(s)\right] = \frac{1}{2}\left(t^{2H} + s^{2H} - |t-s|^{2H}\right). \tag{3}$$

FBM is a continuous Gaussian process with independent increments. The behavior of FBM is shaped by the way its increments relate to one another. Unlike standard BM, these increments are not independent. The correlation between the increments of FBM fluctuates consistently with self-similarity parameter H which is known as Hurst parameter. The value of H reveals the "memory" of the process: when $0.5 < H < 1$, FBM exhibits long-range dependence, meaning past trends tend to persist; when $H = 0.5$, it behaves like classical Brownian motion with no memory; and when $0 < H < 0.5$, it shows short-range dependence, where fluctuations tend to reverse quickly.

Model 2: *Let S_t be a stochastic process. Then S_t is said to follow a GFBM if it satisfies the following SDE:*

$$dS_t = \mu S_t\, dt + \sigma S_t\, dB_{H_1}(t), \tag{4}$$

where $B_{H_1}(t)$ is FBM, σ and μ are volatility and drift respectively. The solution of Equation (4) is given by

$$S(t) = S_0 \exp\left[\left(\mu - \frac{1}{2}\sigma^2 t^{2H_1 - 1}\right) t + \sigma B_{H_1}(t)\right], \tag{5}$$

where S_0 is an arbitrary initial value.

The GFBM model has proven to be a powerful tool across a wide range of financial applications. For instance, option pricing (Alhagyan 2018, Alhagyan et al. 2016a; 2016b, Misiran et al. 2010; 2012 and Kukush et al. 2005), index prices (Abbas & Alhagyan 2022; 2023, Alhagyan 2018, Xiao et al. 2015 and Alhagyan & Al-Duais 2020), value at risk (Alhagyan et al. 2021, Alhagyan 2018, and Wang et al. 2017), exchange rate (Alhagyan 2018, Alhagyan 2022, and Gözgör 2013), and mortgage insurance (Bardhan et al. 2006, Alhagyan 2018, Chen et al. 2013, and Alhagyan et al. 2021).

To make complex calculations more manageable, many researchers have traditionally assumed constant volatility (σ) in both GBM and GFBM. However, this simplification has been widely challenged. A growing body of work argues that volatility in real financial markets is far from constant and is better captured by stochastic volatility (SV) models (Stein, 1989; Bakshi et al., 2000; Aït-Sahalia and Lo, 1998). This shift in perspective led to the development of stochastic volatility models, where the fixed volatility term σ is replaced by a dynamic function $\sigma(Y_t)$. Here, Y_t follows its own stochastic differential equation, introducing an additional layer of randomness into the system. The strength of SV models lies in their ability to reflect time-varying market uncertainty—something constant-volatility models cannot adequately capture. By allowing both the underlying asset and its volatility to evolve randomly (and potentially in a correlated way), SV models provide a more realistic and flexible framework for understanding financial dynamics.

Model 3: *Let S_t be a stochastic process. Then S_t is said to follow GBM perturbed by SV if it satisfies the following SDE:*

$$dS_t = \mu S_t\, dt + \sigma(Y_t) S_t\, dW_{1t}, \tag{6}$$

where μ is mean of return, Y_t is a stochastic process, W_{1t} is a BM, and $\sigma(Y_t) = Y_t$ is a deterministic function. Let the dynamics of volatility Y_t be described by Ornstein–Uhlenbeck (OU) process which is the solution of the following SDE:

$$dY_t = \alpha(m - Y_t)\, dt + \beta\, dW_{2t}, \tag{7}$$

where α, β and m are constant parameters that represent mean reverting of volatility, volatility of volatility, and mean of volatility, respectively. W_{2t} is another BM which is independent from $B_{H_1}(t)$.

Model 4: *A stochastic process S_t is said to follow a GFBM perturbed by SV if it satisfies the following SDE:*

$$dS_t = \mu S_t\, dt + \sigma(Y_t) S_t\, dB_{H_1}(t), \tag{8}$$

where μ is mean of return, Y_t is a stochastic process, $B_{H_1}(t)$ is a FBM with Hurst index H_1, and $\sigma(Y_t) = Y_t$ is a deterministic function. Let the dynamics of volatility Y_t be described by fractional Ornstein–Uhlenbeck (FOU) process which is the solution of the following SDE:

$$dY_t = \alpha(m - Y_t)\, dt + \beta\, dB_{H_2}(t), \tag{9}$$

where α, β and m are constant parameters that represent mean reverting of volatility, volatility of volatility, and mean of volatility, respectively. $B_{H_2}(t)$ is another FBM which is independent from $B_{H_1}(t)$.

This study explores how introducing memory effects and stochastic volatility enhances the classical GBM framework. To do so, four models are examined: the standard GBM with neither memory nor stochastic volatility (Model 1); the GFBM with memory but constant volatility (Model 2); the GBM augmented with stochastic volatility but without memory (Model 3); and finally, the GFBM that incorporates both memory and stochastic volatility (Model 4). To evaluate their performance, these models are applied to forecast future index prices of two pharmaceutical companies listed on the Saudi Stock Exchange, providing a practical comparison of their effectiveness in real market conditions.

3 Forecasting Pharma Sector

The pharmaceutical sector in the Saudi Stock Exchange is represented by just two companies: Saudi Pharmaceutical Industries and Medical Appliances Corporation (SPIMACO) and Jamjoom Pharmaceuticals Factory Co. (JAMJOOM). In this study, historical stock price data for each company are analyzed individually to forecast their future price movements, allowing for a focused and company-specific evaluation of model performance. The empirical analysis uses daily closing prices obtained from the Saudi Exchange (Tadawul). The variable modelled in this study is the daily stock closing price for each company.

The performance of each model examined in this study is evaluated using two error metrics: mean squared error (MSE) and mean absolute percentage error (MAPE). Together, these measures provide a clear assessment of how accurately each model predicts the observed data.

$$\text{MSE} = \frac{\sum_{i=1}^{n}(Y_i - F_i)^2}{n}, \qquad \text{MAPE} = \frac{\sum_{i=1}^{n}\frac{|Y_i - F_i|}{Y_i}}{n}, \tag{10}$$

where F_i and Y_i represent forecast and actual price at day i, respectively, while n is the total number of forecasting days.

Table 1 provides the interval of accuracy judgment of forecasting method using MAPE according to Lawrence (2009).

Table 1: MAPE to judgment accuracy of forecasting method

Accuracy	MAPE
Highly accurate	MAPE $< 10\%$
Good accurate	$10\% \le$ MAPE $< 20\%$
Reasonable	$20\% \le$ MAPE $< 50\%$
Inaccurate	MAPE $\ge 50\%$

3.1 Forecasting SPIMACO

3.1.1 Description of historical data

The historical data performance is available online at https://www.saudiexchange.sa. The total working days are 120 beginning from 1st Jan. 2023 to 22nd Jun. 2023. Return series is considered in logarithm (i.e. $r_n = \ln(S_n/S_{n-1})$) to handle possible high volatility in the historical data. Figure 1 and Figure 2 show the close prices and its return series.

3.1.2 Forecasting and Evaluation

According to the data of SPIMACO at the first 120 working days of year 2023, all parameters involved in the four used models are computed by using Mathematica 10 software (See Table 2). Next, all these parameters are

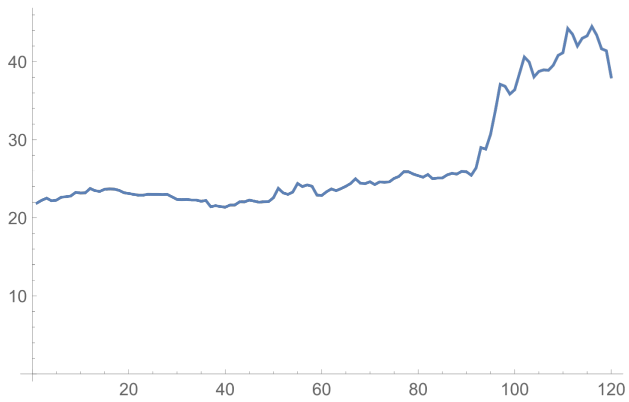

Fig. 1: Daily close price of SPIMACO

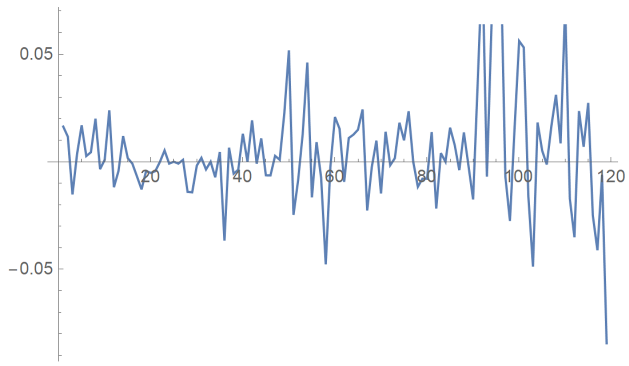

Fig. 2: Daily returns of SPIMACO

Table 2: Parameters value summary of SPIMACO

Parameter	Value
H_1: Hurst index of adjusted closed price	0.65527
H_2: Hurst index of daily volatility of closed price	0.65510
μ: mean of return	0.00464
β: volatility of volatility	0.00157
m: mean of daily volatility of log return	0.00065
α: mean reverting of daily volatility of log return	1.09153
Computed Volatility	
σ: Constant volatility	0.02565
$\sigma(Y_t)$: Stochastic volatility	0.07763

employed to calculate both constant volatility and stochastic volatility.

The parameters in Table 2 were used to forecast the closing prices for the next three months of 2023 (June, July, and September). The forecasted values were computed using four models: two GBM models and two GFBM models (see Definitions 2, 4, 5 and 6). The forecasted prices from the four models, along with the actual prices of SPIMACO, are presented in Table 3, while the accuracy levels of all models are reported in Tables 4 and 5.

The findings indicate that SV GFBM achieved the highest accuracy, as reflected by the smallest values of both MSE and MAPE. This result can be attributed to the presence of two sources of memory in SV GFBM, in addition to the stochastic volatility assumption. In contrast, GBM ranked last, showing the largest values of MSE and MAPE. This outcome is expected since the GBM model includes only one source of randomness and assumes constant volatility.

In general, the differences in accuracy rankings are small, as shown in Tables 4 and 5, where the MSE and MAPE values are very close across models. Furthermore, according to the Lawrence scale (Table 1), GBM, GFBM, SV GBM, and SV GFBM all demonstrate high predictive accuracy, as MAPE is less than 10% for all models.

These results support that models incorporating stochastic volatility provide more accurate forecasts of future stock prices than models with constant volatility. Moreover, long-memory models (i.e., GFBM) are more suitable than models without memory (i.e., GBM) for forecasting future stock prices. This finding is consistent with several empirical studies, including Abbas & Alhagyan (2022; 2023), Alhagyan & Yassen (2023), Alhagyan (2022), Willinger et al. (1999), Painter (1998), Rejichi & Aloui (2012), and Alhagyan & Alduais (2020).

Figure 3 illustrates the comparison between the actual closing prices and the forecasted closing prices.

4 Forecasting JAMJOOM

4.1 Description of historical data

The historical data performance is available online at https://www.saudiexchange.sa. The total working days are 48 beginning from 20th Jun. 2023 to 31st Aug. 2023. This period represents the first days of JAMJOOM company in trading in Saudi Stock Exchange Market. Return series is considered in logarithm (i.e. $r_n = \ln(S_n/S_{n-1})$) to handle possible high volatility in the historical data. Figure 4 and Figure 5 show the close prices and its return series.

4.2 Forecasting and Evaluation

According to the data of JAMJOOM at the first 48 working days of year 2023, all parameters involved in the four used models are computed by using Mathematica 10 software (See Table 6). Next, all these parameters are employed to calculate both constant volatility and stochastic volatility.

The parameters in Table 6 are exploited to forecast the close price of the next month of the year 2023 (September). The forecasted close price values were computed using four models; two GBM models and two GFBM models (see Definitions 2, 4, 5 and 6).

The forecasted prices of four models in addition to actual prices of JAMJOOM are shown in Table 7 and the accuracy levels of all models are listed in Tables 8 and 9.

Table 3: Actual and forecasted prices of SPIMACO using four models

Date	Actual	GBM	GFBM	SV GBM	SV GFBM	Date	Actual	GBM	GFBM	SV GBM	SV GFBM
02/07/23	38.55	38.603	38.509	38.681	38.498	15/08/23	39.3	38.592	38.634	38.670	38.826
03/07/23	42.4	38.574	38.513	38.654	38.509	16/08/23	40.2	38.590	38.596	38.669	38.711
04/07/23	43.15	38.571	38.624	38.655	38.858	17/08/23	39.7	38.605	38.530	38.690	38.506
05/07/23	41.15	38.492	38.531	38.564	38.555	20/08/23	40	38.663	38.552	38.745	38.563
06/07/23	41.25	38.651	38.540	38.727	38.586	21/08/23	38.3	38.617	38.520	38.691	38.472
09/07/23	39.95	38.560	38.608	38.639	38.797	22/08/23	38.35	38.574	38.585	38.651	38.666
10/07/23	40.95	38.587	38.592	38.666	38.743	23/08/23	38.15	38.730	38.506	38.803	38.429
11/07/23	40.9	38.598	38.609	38.677	38.795	24/08/23	38.05	38.621	38.648	38.694	38.854
12/07/23	40.9	38.546	38.484	38.620	38.414	27/08/23	37.8	38.584	38.675	38.664	38.935
13/07/23	40.45	38.622	38.454	38.703	38.314	28/08/23	37.95	38.622	38.611	38.703	38.733
16/07/23	39.75	38.559	38.554	38.644	38.623	29/08/23	38.05	38.634	38.595	38.716	38.694
17/07/23	40	38.602	38.534	38.692	38.556	30/08/23	38.35	38.550	38.597	38.621	38.686
18/07/23	39.3	38.579	38.528	38.659	38.547	31/08/23	37.6	38.565	38.650	38.640	38.853
19/07/23	39.65	38.557	38.652	38.633	38.911	03/09/23	37.8	38.584	38.617	38.661	38.750
20/07/23	39.3	38.566	38.490	38.643	38.419	04/09/23	37	38.542	38.633	38.626	38.799
23/07/23	39	38.557	38.609	38.631	38.786	05/09/23	36.7	38.666	38.527	38.733	38.477
24/07/23	41	38.591	38.526	38.660	38.521	06/09/23	35.9	38.628	38.662	38.703	38.874
25/07/23	40.4	38.591	38.621	38.668	38.822	07/09/23	35.95	38.627	38.569	38.700	38.609
26/07/23	40.15	38.565	38.676	38.651	38.985	10/09/23	35.7	38.676	38.533	38.760	38.497
27/07/23	39.9	38.553	38.620	38.632	38.803	11/09/23	36.3	38.591	38.622	38.667	38.758
30/07/23	39.75	38.508	38.499	38.585	38.434	12/09/23	35.45	38.506	38.610	38.577	38.724
31/07/23	39.65	38.633	38.494	38.712	38.418	13/09/23	34.75	38.611	38.575	38.685	38.608
01/08/23	39.7	38.618	38.568	38.694	38.642	14/09/23	34.45	38.659	38.602	38.735	38.698
02/08/23	39.6	38.604	38.647	38.677	38.884	17/09/23	34.5	38.616	38.521	38.690	38.436
03/08/23	39.35	38.735	38.546	38.818	38.570	18/09/23	34.95	38.534	38.616	38.614	38.732
06/08/23	38.75	38.599	38.609	38.674	38.764	19/09/23	34.45	38.669	38.688	38.735	38.947
07/08/23	36.25	38.615	38.598	38.692	38.730	20/09/23	34.2	38.634	38.676	38.704	38.905
08/08/23	38.4	38.628	38.567	38.710	38.633	21/09/23	34	38.574	38.549	38.650	38.518
09/08/23	37.95	38.594	38.584	38.666	38.682	25/09/23	31.5	38.586	38.579	38.650	38.610
10/08/23	38	38.566	38.670	38.636	38.941	26/09/23	32.35	38.515	38.567	38.597	38.585
13/08/23	38.1	38.561	38.538	38.640	38.536	27/09/23	33.45	38.568	38.551	38.648	38.522
14/08/23	39.8	38.471	38.643	38.548	38.844	28/09/23	35.3	38.644	38.592	38.709	38.647

Table 4: Accuracy ranking based on MSE (SPIMACO)

Rank	Model	MSE
1	SV GFBM	6.285
2	SV GBM	6.360
3	GFBM	6.382
4	GBM	6.393

Table 5: Accuracy ranking based on MAPE (SPIMACO)

Rank	Model	MAPE
1	SV GFBM	0.0533
2	SV GBM	0.0535
3	GFBM	0.0537
4	GBM	0.0538

Table 6: Parameters value summary of JAMJOO

Parameter	Value
H_1: Hurst index of adjusted closed price	0.65527
H_2: Hurst index of daily volatility of closed price	0.65510
α: mean reverting of daily volatility of log return	1.09153
μ: mean of return	0.00464
β: volatility of volatility	0.00157
m: mean of daily volatility of log return	0.00065
α: mean reverting of daily volatility of log return	1.09153
Computed Volatility	
σ: Constant volatility	0.02565
$\sigma(Y_t)$: Stochastic volatility	0.07763

Again, the findings show that SV GFBM ranked first in accuracy, as indicated by the smallest values of both MSE and MAPE. SV GFBM achieved this ranking because it incorporates two sources of memory in addition to the stochastic volatility assumption. Table 8 shows that SV models are more accurate than models under the constant volatility assumption, while Table 9 indicates that GFBM models outperform GBM models. In general, the differences in accuracy rankings are small, as seen in Tables 8 and 9, where both MSE and MAPE values are very close across models. Furthermore, according to the Lawrence scale (Table 1), GBM, GFBM, SV GBM, and SV GFBM all demonstrate high predictive accuracy, since MAPE is less than 10% for all models. These results suggest that models incorporating

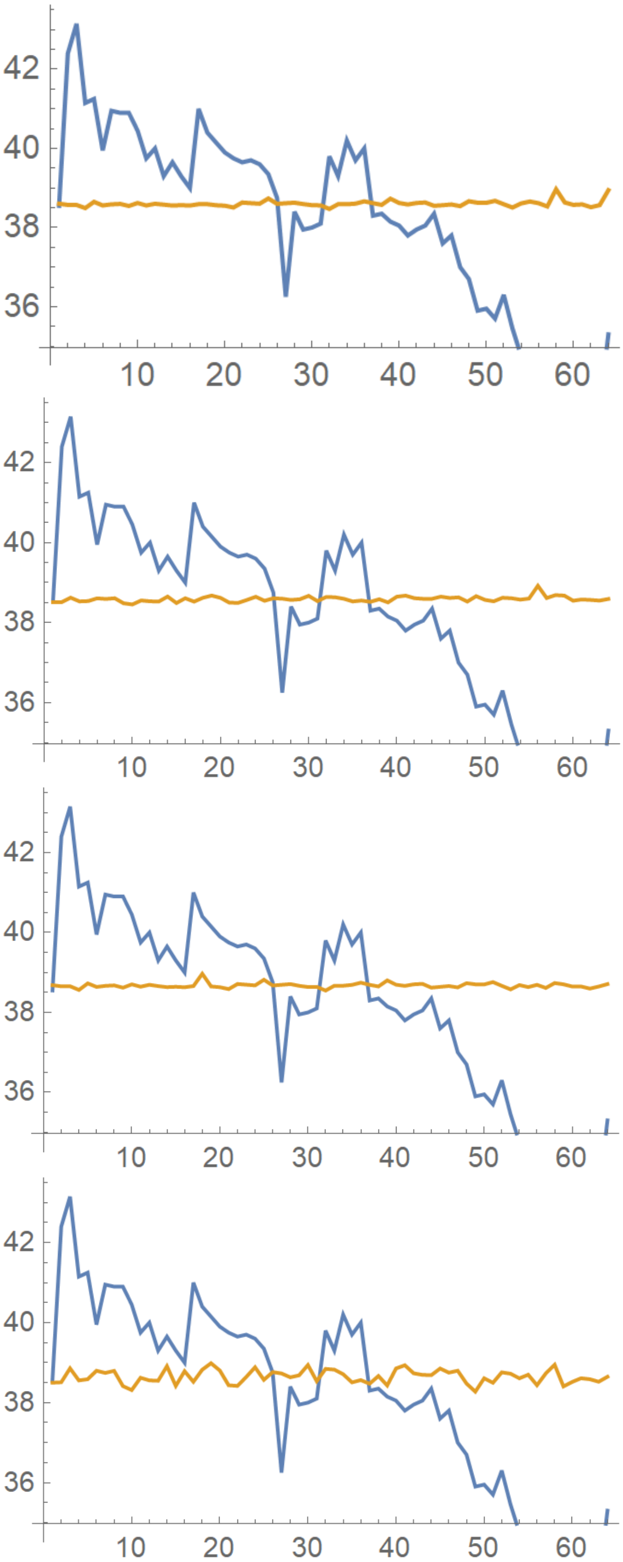

Fig. 3: Actual prices vs Forecast prices (SPIMACO).

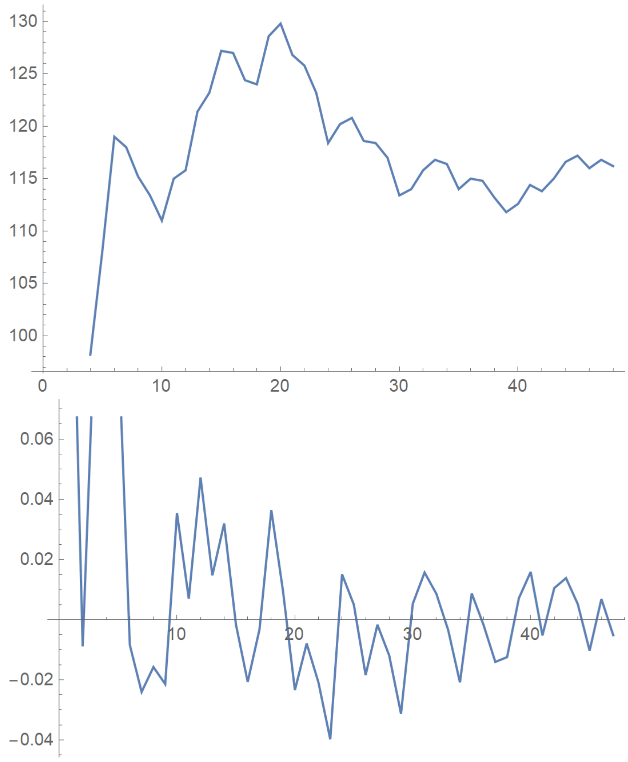

Fig. 4: Daily returns of JAMJOOM

Table 7: Actual and forecasted prices of JAMJOOM using four models

Date	Actual	GBM	GFBM	SV GBM	SV GFBM
03/09/23	113.8	114.031	113.927	114.253	114.076
04/09/23	116.4	113.825	114	114.073	114.2
05/09/23	118.4	114.094	113.679	114.314	113.665
06/09/23	115.8	113.567	113.742	113.807	113.768
07/09/23	114.4	113.902	113.98	114.126	114.152
10/09/23	112.8	113.69	114.033	113.921	114.232
11/09/23	114.8	113.979	114.108	114.231	114.344
12/09/23	113.2	114.01	113.716	114.258	113.72
13/09/23	112.6	113.941	114.001	114.161	114.176
14/09/23	114	113.504	114.088	113.735	114.309
17/09/23	113.4	114.087	113.996	114.328	114.159
18/09/23	113.4	113.584	113.951	113.789	114.09
19/09/23	117.4	113.927	113.93	114.152	114.042
20/09/23	115.6	113.812	114.049	114.026	114.231
21/09/23	115.4	114.019	114.196	114.251	114.468
25/09/23	115.2	113.881	113.823	114.122	113.87
26/09/23	112.2	113.939	113.637	114.169	113.563
27/09/23	114.6	114.007	113.962	114.271	114.085
28/09/23	115.8	113.879	113.675	114.113	113.623

Table 8: Accuracy ranking based on MSE (JAMJOOM)

Rank	Model	MSE
1	SV GFBM	3.084816
2	SV GBM	3.136117
3	GBM	3.215994
4	GFBM	3.271073

Table 9: Accuracy ranking based on MAPE (JAMJOOM)

Rank	Model	MAPE
1	SV GFBM	0.011493
2	GFBM	0.012104
3	GBM	0.012422
4	SV GBM	0.012565

long-memory stochastic volatility are more suitable for forecasting future stock prices than models with constant volatility or without memory. This finding is consistent with several empirical studies, including Abbas & Alhagyan (2022; 2023), Mansour & Alhagyan (2022), Alhagyan & Yassen (2023), Alhagyan (2022), Willinger et al. (1999), Painter (1998), Rejichi & Aloui (2012), and Alhagyan & Alduais (2020). Figure 6 illustrates the comparison between the actual closing prices and the forecasted closing prices of JAMJOOM.

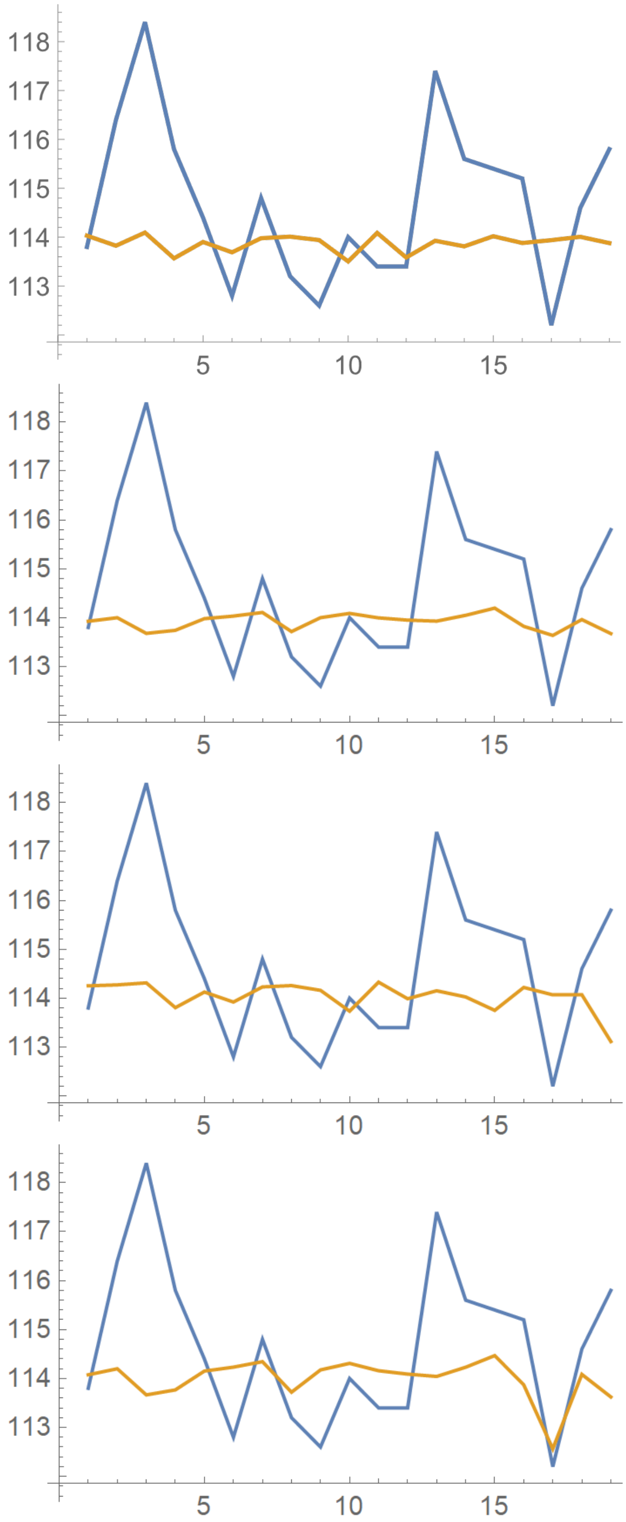

Fig. 5: Actual prices vs Forecast prices (JAMJOOM).

5 Conclusion

The stock price index of any company is considered a reflection of its economic growth and financial stability. Therefore, one of the main priorities of investors and stakeholders is to understand and predict the future direction of index prices. To achieve this, numerous models have been proposed, among which GBM is one of the most important.

In the literature, two main assumptions are associated with the volatility parameter in GBM: the constant volatility assumption and the stochastic volatility assumption. In addition, there are two assumptions regarding memory in financial time series: the absence of memory and the presence of memory. Consequently, four models arise: classical GBM (without memory and with constant volatility), SV GBM (without memory and with stochastic volatility), GFBM (with memory and constant volatility), and SV GFBM (with memory and stochastic volatility).

In this study, these models were applied in an empirical analysis to forecast future stock prices in the pharmaceutical sector of the Saudi Stock Exchange.

The results of the empirical study indicate that SV GFBM provides the best performance, as it yields the smallest values of MSE and MAPE. This finding highlights the positive impact of incorporating both long memory and stochastic volatility into the GBM framework. These results are consistent with previous empirical studies, including Abbas & Alhagyan (2022; 2023), Mansour & Alhagyan (2022), Alhagyan & Yassen (2023), Alhagyan (2022), Willinger et al. (1999), Painter (1998), Rejichi & Aloui (2012), and Alhagyan & Alduais (2020).

Overall, all models examined demonstrate high accuracy (MAPE $\leq$ 10%) and can therefore be effectively used as tools for forecasting index prices in real financial environments.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgement

The authors extend their appreciation to Prince Sattam Bin Abdulaziz University for funding this research work through the project number (IF2/PSAU/2023/01/25927).

Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] Abbas, A., & Alhagyan, M. (2022). The effect of incorporating memory and stochastic volatility into geometric Brownian motion in forecasting the performance of Tadawul all share Index (TASI). *Advances and Applications in Statistics*, **74**, 47–62.

[2] Abbas, A., & Alhagyan, M. (2023). Forecasting Exchange Rate of Sar/Cny By Incorporating Memory and Stochastic Volatility Into GBM Model. *Advances and Applications in Statistics*, **86**(1), 65–78.

[3] Alhagyan, M. (2022). The effects of incorporating memory and stochastic volatility into GBM to forecast exchange rates of Euro. *Alexandria Engineering Journal*, **61**(12), 9601–9608.

[4] Alhagyan, M. K. M. (2018). Modeling financial environments using geometric fractional Brownian motion model with long memory stochastic volatility. *Universiti Utara Malaysia*.

[5] Alhagyan, M., & Alduais, F. (2020). Forecasting the performance of Tadawul all share index (TASI) using geometric Brownian motion and geometric fractional Brownian motion. *Adv. Appl. Stat*, **62**(1), 55–65.

[6] Alhagyan, M., & Yassen, M. F. (2023). Incorporating stochastic volatility and long memory into geometric Brownian motion model to forecast performance of Standard and Poor's 500 index. *AIMS Mathematics*, **8**(8), 18581–18595.

[7] Alhagyan, M., Misiran, M. & Omar, Z. (2016a). Geometric fractional Brownian motion perturbed by fractional Ornstein-Uhlenbeck process and application on KLCI option pricing. *Open Access Library Journal*, **3**(08), 1.

[8] Alhagyan, M., Misiran, M. & Omar, Z. (2021). On effects of stochastic volatility and long memory towards mortgage insurance models: an empirical study. *Advances and Applications in Statistics*, **66**(2), 165–174.

[9] Alhagyan, M., Misiran, M., & Omar, Z. (2016b). Estimation of geometric fractional Brownian motion perturbed by stochastic volatility model. *Far East Journal of Mathematical Sciences*, **99**(2), 221.

[10] Bardhan, A., Karapandža, R., & Urošević, B. (2006). Valuing mortgage insurance contracts in emerging market economies. *The Journal of Real Estate Finance and Economics*, **32**(1), 9–20.

[11] Chen, C. C., Lin, S. K., & Chen, W. S. (2013). Mortgage insurance premiums and business cycle. Tunghai University working paper.

[12] Gözgör, G. (2013). The application of stochastic processes in exchange rate forecasting: Benchmark test for the EUR/USD and the USD/TRY. *Economic Computation and Economic Cybernetics Studies and Research*, **47**(2), 225–246.

[13] Grau-Carles, P. (2000). Empirical evidence of long-range correlations in stock returns. *Physica A: Statistical Mechanics and its Applications*, **287**(3), 396–404.

[14] Han, C., Wang, Y., & Xu, Y. (2020). Nonlinearity and efficiency dynamics of foreign exchange markets: evidence from multifractality and volatility of major exchange rates. *Economic research-Ekonomska istraživanja*, **33**(1), 731–751.

[15] Kim, K. H., Kim, N. U., Ju, D. C., & Ri, J. H. (2020). Efficient hedging currency options in fractional Brownian motion model with jumps. *Physica A: Statistical Mechanics and its Applications*, **539**, 122868.

[16] Kukush, A., Mishura, Y., & Valkeila, E. (2005). Statistical inference with fractional Brownian motion. *Statistical Inference for Stochastic Processes*, **8**, 71–93.

[17] Misiran, M. (2010). *Modeling and pricing financial assets under long memory processes* (Doctoral dissertation). Curtin University of Technology.

[18] Misiran, M., Zudi, L. U., Teo, K. L., & Grace, A. W. (2012). Estimating dynamic geometric fractional Brownian motion and its application to long-memory option pricing. *Dynamic Systems and Applications*, **21**(1), 49–66.

[19] Painter, S. (1998). Numerical method for conditional simulation of Levy random fields. *Mathematical Geology*, **30**(2), 163–179.

[20] Rejichi, I. Z., & Aloui, C. (2012). Hurst exponent behavior and assessment of the MENA stock markets efficiency. *Research in International Business and Finance*, **26**(3), 353–370.

[21] Sheldon, M. R. (1999). *An introduction to mathematical finance: Options and other topics*.

[22] Wang, X., Xie, D., Jiang, J., Wu, X., & He, J. (2017). Value-at-Risk estimation with stochastic interest rate models for option-bond portfolios. *Finance Research Letters*, **21**, 10–20.

[23] Willinger, W., Taqqu, M. S., & Teverovsky, V. (1999). Stock market prices and long-range dependence. *Finance and Stochastics*, **3**(1), 1–13.

[24] Xiao, W., Zhang, W., & Zhang, X. (2015). Parameter identification for the discretely observed geometric fractional Brownian motion. *Journal of Statistical Computation and Simulation*, **85**(2), 269–283.

Anas Abbas a faculty member in Department of Management at Prince Sattam Bin Abdulaziz University, Al Kharj, KSA. He is an accomplished researcher specializing in Human resources and financial Management. https://faculty.psau.edu.sa/a.abbas

Mohammed Alhagyan is a faculty member in Mathematical Science Department at United Arab Emirates University, Al Ain, Abu Dhabi, Emirates. He is an accomplished researcher specializing in financial mathematics and statistics. His ORCID ID: orcid.org/0000-0003-4090-6125