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# Enhancing Building Environmental Simulation through Climatic Variable Forecasting Using ARIMA and VAR Models with Box–Cox Pre-Processing

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**Abstract:** Accurate forecasting of climatic variables is vital for reliable building-environment simulations and energy-performance analysis, so this study compares univariate and multivariate time-series approaches—ARIMA and ninth-order VAR—using Box–Cox transformation to stabilize variance and improve homogeneity. After transformation, series are tested for stationarity and subjected to diagnostic validation; ARIMA models are fitted to individual variables while the VAR model captures dynamic interrelationships, with model selection guided by information criteria and forecasting evaluated by error metrics and graphical inspection. Empirical results from northern Iraq show Box–Cox preprocessing improves data stability and parameter estimation for both approaches, ARIMA delivers strong forecasts for single variables, and VAR captures inter-variable dependencies but does not consistently outperform univariate models. Overall, the findings underscore the value of appropriate preprocessing and the need to weigh inter-variable correlations when choosing forecasting methods for building simulations, offering a practical framework to improve climatic inputs for sustainable building design.

**Keywords:** Building Environmental Simulation; Climatic Variable Forecasting; ARIMA; Vector Autoregression (VAR); Box–Cox Transformation; Time Series Modelling; Building Performance Simulation; Climate Data Analysis.

## 1. Introduction

Climatic variables such as temperature (T), rainfall (R), and humidity (H) exhibit complex temporal dynamics influenced by atmospheric circulation patterns, hydrological cycles, and seasonal variability. Accurate modelling and forecasting of these variables are essential for climate prediction and for supporting decision-making across sectors including agriculture, water resource management, environmental planning, and increasingly, building environmental simulation and energy performance analysis. Reliable climate inputs are particularly important for building performance models, as simulation accuracy strongly depends on the quality and temporal consistency of climatic datasets used to represent external environmental conditions (Wilks, 2019). Consequently, robust statistical approaches capable of capturing the temporal structure of climatic variables are necessary to enhance the reliability of simulation-based analyses.

Time series modelling has long been a fundamental methodology in climate and environmental research because climatic processes often evolve through strong autocorrelation, seasonality, and stochastic fluctuations (Shumway & Stoffer, 2017). Among the most widely applied approaches is the Autoregressive Integrated Moving Average (ARIMA) framework introduced by Box and Jenkins (1976). ARIMA models have been extensively used for forecasting individual climatic variables due to their ability to capture temporal dependence structures within a single time series. Empirical studies have shown that univariate ARIMA models can achieve high forecasting accuracy when the underlying dynamics of a climatic variable are largely self-driven (Hyndman &

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Athanasopoulos, 2021; Munir et al., 2020).

An important prerequisite for reliable time series modelling is ensuring that the statistical properties of the data satisfy model assumptions such as variance stability and approximate normality. Climatic datasets often exhibit heteroscedasticity and skewed distributions, which can negatively affect model estimation and forecasting performance. To address these issues, variance-stabilizing power transformations are commonly applied during data preprocessing. Among these, the Box–Cox transformation, originally proposed by Box and Cox (1964), has been widely used to stabilize variance and improve distributional properties of time series data. Extensions such as the Yeo–Johnson transformation allow similar adjustments for datasets containing both positive and negative values (Yeo & Johnson, 2000). Previous studies have demonstrated that applying these transformations can improve parameter estimation, facilitate model identification, and enhance forecasting accuracy in climatic time series modelling (Chen & Lee, 1997; Guerrero, 1993; Atkinson et al., 2021). Applications of Box–Cox transformations in climate research have shown improvements in ARIMA model identification and forecasting performance, particularly in the presence of strong seasonal patterns or heteroscedasticity (Proietti & Lütkepohl, 2013; Othman & Mohammed Ali, 2023).

While univariate models focus on the internal dynamics of individual variables, climatic variables are inherently interconnected. Temperature influences atmospheric moisture levels, humidity affects precipitation processes, and rainfall can in turn modify temperature through evaporative cooling mechanisms. Such interdependencies suggest that modelling climatic variables jointly may provide additional predictive insights. Multivariate time series approaches, particularly Vector Autoregression (VAR) models, provide a flexible framework for capturing dynamic interactions among multiple variables simultaneously (Lütkepohl, 2005). Empirical studies have shown that multivariate models can improve forecasting performance when strong cross-correlations exist among climatic variables. Applications of VAR models have been reported in forecasting temperature and humidity interactions (Adeyemi et al., 2017), rainfall dynamics (Ogallo et al., 2020), and joint temperature–rainfall systems (Cebrián et al., 2019).

Recent research suggests that the relative forecasting performance of univariate and multivariate approaches depends largely on the strength and stability of cross-variable correlations within the climatic system (Živković et al., 2022). In situations where variables evolve largely independently, univariate models may perform equally well or better than multivariate alternatives. Conversely, when strong interdependencies exist, multivariate frameworks can capture additional information embedded in cross-variable interactions. Emerging studies have also indicated that applying appropriate transformations prior to multivariate modelling may enhance the detection of inter-variable relationships and improve forecasting accuracy (Othman & Mohammed Ali, 2025).

Beyond parametric modelling frameworks, recent advances in climate time series analysis have explored nonparametric and functional approaches to capture nonlinear dynamics. Methods such as kernel regression, local polynomial smoothing, and functional K-nearest neighbour forecasting have been proposed to model complex climatic processes that may not conform to standard parametric assumptions (Ferraty & Vieu, 2006; Shang, 2017). In these contexts, variance-stabilizing transformations remain important preprocessing steps, as they can improve smoothing performance and reduce estimation bias (Raymaekers & Rousseeuw, 2021). Applications of functional time series modelling to temperature curves and seasonal climate patterns have demonstrated that transformation-based preprocessing can improve the estimation of periodic structures and short-term forecasting performance (He & Zheng, 2018).

Despite these methodological advances, relatively few studies have undertaken systematic comparisons of univariate and multivariate time series approaches within a consistent preprocessing framework for multiple climatic variables. Much of the existing literature focuses on individual climatic indicators, specific modelling approaches, or transformation techniques in isolation. As a result, the practical question of whether multivariate modelling consistently provides forecasting advantages over well-specified univariate models remains insufficiently addressed.

In response to this research gap, the present study investigates the forecasting performance of univariate ARIMA and multivariate VAR models for key climatic variables—temperature, rainfall, and humidity—within a unified preprocessing framework based on the Box–Cox transformation. The objective is to evaluate whether incorporating cross-variable interactions through VAR modelling improves forecasting accuracy relative to individual ARIMA models when both approaches are applied under comparable statistical conditions. By applying identical preprocessing procedures and model evaluation criteria, the study provides a transparent comparison of the predictive capabilities of these modelling strategies.

The empirical analysis is conducted using climatic observations from northern Iraq, a region characterized by pronounced seasonal variability and complex climatic interactions. By evaluating model structure, diagnostic adequacy, and forecasting performance for standardized climatic datasets, the study aims to identify the circumstances under which multivariate modelling provides practical forecasting benefits. The findings contribute to the methodological literature on climatic time series forecasting and offer practical insights for improving climate input generation used in building environmental simulation and sustainable building design.

The remainder of this paper is organized as follows. Section 2 presents the theoretical framework underlying variance-stabilizing transformations and the formulation of ARIMA and VAR time series models. Section 3 describes the empirical methodology, including data description, preprocessing procedures, model estimation, and forecasting evaluation. Section 4 discusses the results and examines the methodological and practical implications of univariate and multivariate forecasting approaches for climatic time series and their potential application in building environmental simulation contexts.

## 2. Theoretical Background and Model Specification

Reliable forecasting of climatic variables is a fundamental prerequisite for improving the accuracy of building environmental simulation models, which depend heavily on high-quality climatic input data. In simulation environments, external weather conditions—particularly temperature, rainfall, and humidity—directly influence predictions of building energy consumption, indoor environmental performance, and thermal comfort. Consequently, the statistical quality of climatic time series used for forecasting plays a critical role in enhancing the reliability of simulation outcomes.

Within time series analysis, the Box–Jenkins modelling framework (Box & Jenkins, 1976) provides a systematic methodology for developing forecasting models based on historical observations. A key preliminary step in this framework involves preparing the data so that the underlying statistical assumptions required for model estimation are satisfied. In particular, the observed series must exhibit variance stability and an appropriate distributional form to ensure reliable model identification and parameter estimation. For many climatic datasets, raw observations often display heteroscedasticity, skewness, or irregular scaling, which may compromise the validity of standard time-series models if not addressed during preprocessing.

To overcome these challenges, statistical transformations are commonly applied prior to model estimation. In the context of climatic time series forecasting, transformations serve an operational role by regularizing the statistical properties of the data and improving the suitability of the series for subsequent modelling procedures. This preprocessing step becomes particularly important when forecasting climate variables that will later be used as inputs in building environmental simulations, where inaccuracies in climate forecasting can propagate through simulation models and reduce predictive reliability.

Among the available transformation techniques, the Box–Cox transformation (BCT) introduced by Box and Cox (1964) remains one of the most widely applied approaches for variance stabilization and scale adjustment in time series analysis. The Box–Cox transformation provides a flexible parametric framework capable of reducing heteroscedasticity and improving the approximate normality of the data distribution. By transforming the original series into a more statistically stable form, BCT facilitates more accurate model identification and enhances parameter estimation in both univariate and multivariate time-series models.

In the present study, the Box–Cox transformation is employed as a data preprocessing step prior to the implementation of ARIMA and VAR forecasting models. The transformation helps to regularize the climatic time series and ensures that the statistical assumptions underlying the modelling procedures are more closely satisfied. This approach enables a consistent analytical framework for comparing univariate ARIMA models, which capture the temporal dynamics of individual climatic variables, with multivariate Vector Autoregression (VAR) models, which account for potential interactions among multiple climatic variables.

The Box–Cox transformation is mathematically defined as:

$$g_\lambda(y) = \begin{cases} \dfrac{y^\lambda - 1}{\lambda} & \lambda \neq 0 \\ ln\ y & \lambda = 0 \end{cases} \qquad (1)$$

When a series is transformed as $Z = g_\lambda(y)$, the likelihood of the original data can be expressed through the likelihood of the transformed values and the derivative of the transformation. Box and Cox showed that the maximum log-likelihood for a sample $y = (Y_1, Y_2, \dots, Y_n)$ takes the general form $Ln\, L_{max}(\lambda, y) = -\,(n/2)\, log\, \sigma^2(\lambda) + log\, J(\lambda, y)$**,** where $\sigma^2(\lambda)$ is the variance of the transformed series and $J(\lambda, y)$**,** is the Jacobian term. Maximizing this expression yields the optimal transformation parameter (Box & Cox, 1964).

In this study, the climatic variables T, R, and H are denoted by $Y_T, Y_R$ and $Y_H$. Because climatic series often exhibit heteroscedasticity, skewness, and departures from Gaussian behavior, each series is transformed at the outset using the BCT to obtain stabilized versions $Z_T = g_{\lambda_T}(Y_T)$, $Z_R = g_{\lambda_R}(Y_R)$, and $Z_H = g_{\lambda_H}(Y_H)$. Applying a transformation before model identification follows the Box–Jenkins principle that variance stabilization and distributional regularity are prerequisites for meaningful ARIMA and VAR modeling (Box & Jenkins, 1976). By operating on the transformed scale, both univariate and multivariate structures rest on more stable stochastic foundations, improving estimation efficiency and diagnostic accuracy.

After transformation, the next step is to assess the stationarity of each series. Because a single unit-root test may be sensitive to short-run dynamics or structural variation, three complementary procedures are applied: the Augmented Dickey–Fuller test (ADFT) (Dickey & Fuller, 1979), the Phillips–Perron test (PPT) (Phillips & Perron, 1988), and the Kwiatkowski–Phillips–Schmidt–Shin test (KPSST) (Kwiatkowski et al., 1992). Where non-stationarity is detected, the transformed series is differenced until it satisfies the standard stationarity conditions. In the multivariate context, the same set of tests is applied to the vector $(Z_{T,t}, Z_{R,t}, Z_{H,t})^T$, and if the three series appear integrated of the same order, the Johansen cointegration test is used to detect long-run equilibrium relationships among them (Johansen, 1988, 1991). The combination of these tests ensures a robust characterization of the dynamic order of the climatic system.

### 2.1 Univariate ARIMA Modeling

Once stationarity is established, a univariate $ARIMA(p,d,q)(p,d,q)(p,d,q)$ model is constructed for each transformed series. The $ARIMA$ representation $\Phi(B)(1-B)^d Z_t = \theta(B)\varepsilon_t$ where $B$ is the backshift operator, $\Phi(B)$ and $\theta(B)$ are the autoregressive and moving-average polynomials of orders $p$ and $q$, respectively, $(1-B)^d$ denotes the differencing operator, and $\varepsilon_t$ is a white-noise innovation term (Box & Jenkins, 1976). Identification of $p$ and $q$ is guided by the sample autocorrelation function (ACF), partial autocorrelation function (PACF), and information criteria such as the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). Parameters are estimated by maximizing the conditional Gaussian log-likelihood of the form

$$\ln L = (-n/2)\ln(\sigma^2) - (1/2\sigma^2)\sum e_t^2 + C,$$

where $e_t$ denotes the filtered residuals and C denotes a constant that does not depend on the parameters of the ARIMA model and therefore does not affect the maximization of the likelihood (Hamilton, 1994).

Model adequacy is assessed using the Ljung–Box Q-test for residual independence (Ljung & Box, 1978), the Jarque–Bera test for normality (Jarque & Bera, 1987), and ARCH-LM tests for heteroscedasticity (Engle, 1982). A univariate $ARIMA$ model is retained when residuals exhibit white-noise behavior, ensuring that the dynamic specification reflects genuine structure rather than noise.

### 2.2 VAR Modeling

To capture the joint dynamics of the transformed climatic variables, the vector

$$\boldsymbol{Z}_t = (Z_{T,t}, Z_{R,t}, Z_{H,t})^T$$

is modeled using a Vector Autoregressive process of order $p$. The VAR representation,

$$\boldsymbol{Z}_t = \sum_{i=1}^{p} A_i \boldsymbol{Z}_{t-i} + + \boldsymbol{u}_t ,$$

where $A_1, \dots, A_p$ are coefficient matrices and $\boldsymbol{u}_t$ is a vector of innovations with covariance matrix $\sum_u$. This formulation allows each component of the climatic system to depend not only on its own past but also on the lagged values of the other transformed variables, thereby capturing cross-variable interactions between T, R, and H (Lütkepohl, 2005). Before estimating the model, the appropriate lag order $p$ is determined by fitting preliminary VAR specifications and comparing their information criteria. The AIC, the BIC, the Hannan–Quinn Criterion (HQC), and the Final Prediction Error (FPE) are used jointly to balance statistical goodness of fit with model parsimony. These criteria penalize unnecessary increases in lag order and identify the specification that provides the optimal trade-off between fit and complexity (Wei, 2019). Once the lag order is selected, the coefficient matrices $A_1, \dots, A_p$ are estimated using ordinary least squares applied to each equation of the system. Because all equations share identical regressors, OLS estimation is asymptotically equivalent to Gaussian maximum likelihood (Hamilton, 1994). Letting the residuals be defined as $\boldsymbol{u}_t(\lambda) = \boldsymbol{Z}_t(\lambda) - A_1\boldsymbol{Z}_{t-1}(\lambda) - \cdots - A_p\boldsymbol{Z}_{t-p}(\lambda)$, the Gaussian log-likelihood of the VAR model (up to an additive constant) is expressed as

$$\ln L = \frac{-n}{2}\, ln|\Sigma_u| - \frac{1}{2}\sum_{t=1}^{n} \boldsymbol{u}_t' \Sigma_u^{-1} \boldsymbol{u}_t \; + C \qquad (2)$$

where $\Sigma_u$ denotes the covariance matrix of the innovation vector and C is a constant independent of the model parameters. Substituting the OLS estimate $\hat{\Sigma}_u$ into this expression yields the concentrated log-likelihood, $\ln L_C = (-n/2)\, ln|\hat{\Sigma}_u| + C$, which serves as a convenient criterion for comparing alternative VAR specifications. In the present study, estimation proceeds conditional on the transformed data; the Box–Cox parameters are obtained separately at the univariate level and are treated as fixed preprocessing inputs rather than being re-estimated within the multivariate system.

In the post-estimation phase, a series of diagnostic tests are implemented to assess whether the VAR model is sufficiently effective. System-wide Portmanteau tests are employed to uncover any residual vector serial correlation, while multivariate Jarque–Bera statistics assess the normality of the innovations (Doornik & Hansen, 2008). Multivariate ARCH effects are assessed for conditional heteroscedasticity. A major prerequisite for valid inference and forecasting incorporates examining dynamic stability by evaluating the unit circle eigenvalues of the VAR companion matrix. The VAR framework is appropriately termed stable if all eigenvalues are strictly in the unit circle, indicating an insufficient magnitude of the shocks to dissipate over time and well-behaved multi-step forecasts.

## 3. Empirical Analysis and Results: Climate Data from Northern Iraq

Building environmental simulation models rely heavily on accurate climatic input data to generate reliable predictions of building performance, energy demand, and indoor environmental conditions. Consequently, improving the forecasting accuracy of key climatic variables such as temperature, rainfall, and humidity represents an important step toward enhancing the robustness of simulation-based analyses. In line with the objective of this study—to enhance building environmental simulation through improved climatic variable forecasting—this section presents the empirical implementation of the theoretical framework described in the previous sections.

Following the discussion of variance-stabilizing transformations and time-series modelling approaches, this section applies the proposed methodological framework to climatic observations from northern Iraq. The empirical analysis focuses on preparing and modelling three key climatic variables—temperature (T), rainfall (R), and humidity (H)—which are commonly used as environmental inputs in building performance simulations. Particular attention is given to the role of Box–Cox preprocessing, which is applied to stabilize variance and improve the statistical properties of the climatic time series prior to model estimation.

The analysis begins with a detailed description of the dataset and the application of the Box–Cox transformation, followed by a series of diagnostic tests to evaluate the suitability of the transformed series for time-series modelling. These preliminary steps ensure that the statistical assumptions required for reliable model identification and parameter estimation are satisfied. Establishing these conditions is essential for producing credible forecasts that can ultimately improve the reliability of climate-driven building environmental simulations.

Once the preprocessing and diagnostic stages are completed, the study proceeds to estimate and compare two forecasting frameworks: univariate ARIMA models, which capture the internal temporal dynamics of individual climatic variables, and a multivariate VAR model, which accounts for potential interactions among the variables. The comparative evaluation focuses on model adequacy, parameter estimation, and forecasting performance, assessed through statistical error measures and graphical analysis of forecast trajectories.

By applying both modelling approaches under a consistent preprocessing framework, the empirical analysis provides a transparent assessment of whether multivariate modelling offers measurable forecasting advantages over univariate methods in the context of climatic variable prediction. The results presented in the following subsections therefore provide important insights into the suitability of different time-series modelling strategies for generating reliable climate inputs that can support more accurate building environmental simulation and sustainable building design analysis.

### 3.1 Data Description, Transformation, and Preliminary Diagnostics

The empirical inquiry analyzes monthly climatic time series for T, R, and H utilizing data from a meteorological station in northern Iraq. The dataset ultimately comprises $n = 192$ observations per variable, which encompasses a 16-year time frame. The original time-series data were acquired from the World Bank Climate Knowledge Portal (https://climateknowledgeportal.worldbank.org) and were imported into the statistical package R for preprocessing, transformation, and time series model implementation. As a first step, the raw time series for the primary climatic variables will be visually assessed by time-series plots (Figure 1).

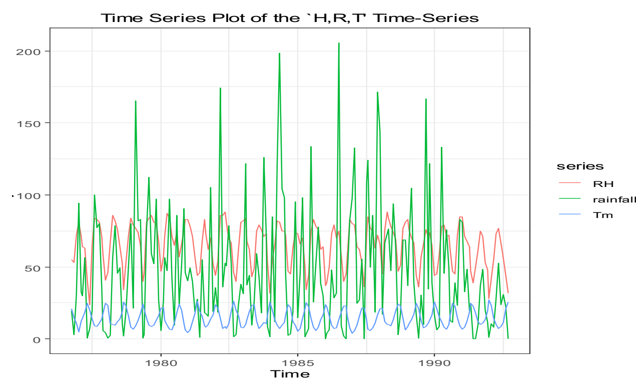



**Fig. 1:** Time–series graphs of the raw climatic variables: T, R, and H.

The graphs show evidence of strong seasonal variation and considerable intra-annual variation and noticeable differences in dispersion across the observed climatic time series. Therefore, this initial visual analysis confirms that there is a multi-scale temporal structure and multi-scale variability present in these time series, which justifies the use of variance-stabilizing transformation in advance of formal time-series modeling.

Formal unit-root tests are then applied to assess stationarity. Specifically, the ADFT and PPT are used to test the null hypothesis of a unit root against a stationary alternative, while the KPSST is used to test the null of stationarity against a unit-root alternative (Dickey & Fuller, 1979; Phillips & Perron, 1988; Kwiatkowski et al., 1992). Table 1 reports the corresponding test statistics and p-values for both the original and Box–Cox transformed series.

**Table 1:** Unit-root and stationarity results for the original and transformed climatic series

| Series | Optimal $\lambda$ | ADFT | | PPT | | KPSST | |
|---|---|---|---|---|---|---|---|
| | | Stat. | p-value | Stat. | p-value | Stat. | p-value |
| $Y_{T,t}$ | 1.00 | -8.58 | 0.01 | -67.18 | 0.01 | 0.04 | 0.1 |
| $Z_{T,t}$ | 0.40 | -6.39 | 0.01 | -66.49 | 0.01 | 0.04 | 0.1 |
| $Y_{R,t}$ | 1.00 | -7.34 | 0.01 | -70.58 | 0.01 | 0.26 | 0.1 |
| $Z_{R,t}$ | 1.33 | -7.23 | 0.01 | -70.00 | 0.01 | 0.26 | 0.1 |
| $Y_{H,t}$ | 1.00 | -5.93 | 0.01 | -154.35 | 0.01 | 0.16 | 0.1 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $Z_{H,t}$ | 1.10 | -5.84 | 0.01 | -157.69 | 0.01 | 0.16 | 0.1 |

The results in Table 1 show a consistent pattern for all three climatic variables: for both the original and transformed data, the ADFT and PPGT p-values are below the 5% significance level, while the KPSS p-values exceed 0.05. This combination—rejection of the unit-root null in ADFT/PPT and non-rejection of the stationarity null in KPSST—indicates that $T, R,$ and $H$ can be treated as stationary processes both before and after applying the BCT. In other words, the transformation is not introduced to induce stationarity, because the unit-root tests already support stationarity in the original domain; instead, the BCT is used to improve the distributional properties of the series.

To regularize the marginal distributions, each climatic variable is transformed using the BCT, with the power parameter $\lambda$ estimated by maximizing the log-likelihood under approximate Gaussianity.

Because the stationarity tests are re-computed on the transformed series, the conclusions in Table 1 can be interpreted as more reliable in the transformed domain: by reducing heavy-tail behavior and variance heterogeneity, the BCT mitigates violations of test assumptions and yields test statistics that are less sensitive to outliers. Subsequent tables and figures summarize the impact of this preprocessing on model specification and diagnostics. Table 2 reports the information-criterion values (AIC, HOQ, BIC, and FPE) used to select the ARIMA and VAR orders.

**Table 2:** Information-Criterion Lag Selection: (a) Original Series and (b) BCT-Transformed Series

| Series | AIC | HOQ | BIC | FPE |
|---|---|---|---|---|
| Original Series | 9 | 8 | 8 | 9 |
| BCT-Transformed Series | 9 | 8 | 7 | 9 |

Across all four information criteria, the selected lag orders are concentrated between 7 and 9 in both the original and the Box–Cox transformed series, indicating a consistent temporal structure across the climatic variables. The observed congruence between the varying criteria indicates a relative stability of dynamic dependence and that the chosen lag orders reflect a satisfactorily parsimonious parameterization that appropriately accounts for serial dependence.

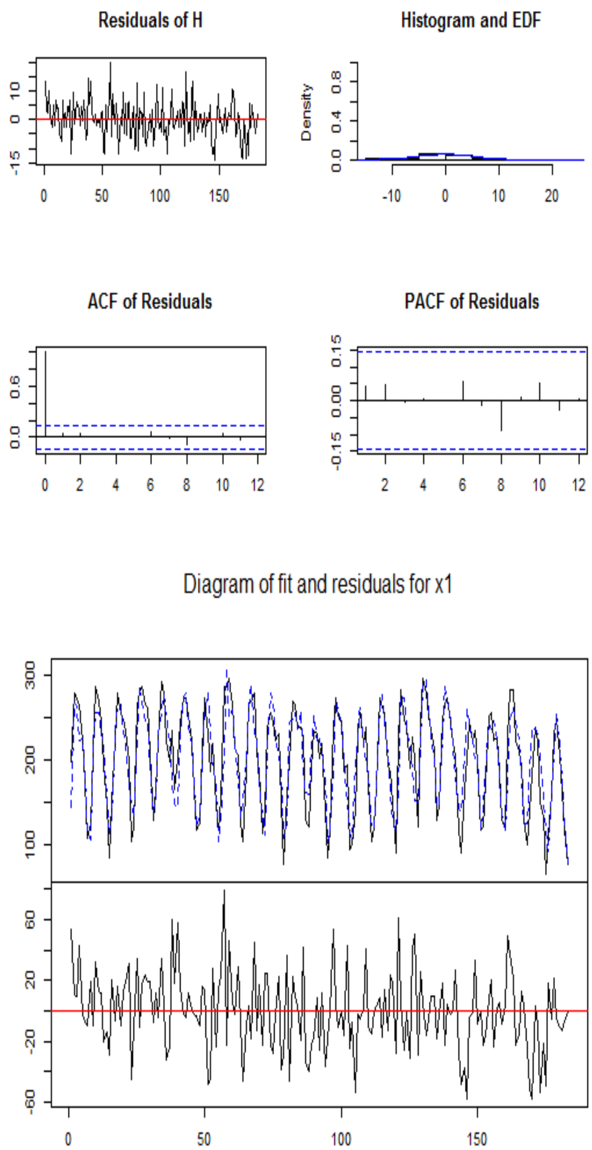
After specifying optimal lag orders, the next stage will entail estimating the univariate ARIMA and multivariate VAR models based on the transformed climatic series, followed by an assessment of model fit via standard model diagnostics, including examination of residual characteristics, autocorrelation, and so on., normality, and heteroscedasticity tests. The corresponding results are summarized in Table 3and illustrated in Figures 2, 3 and 4.

**Table 3:** Comparison of Autocorrelation, Heteroscedasticity, and Normality Diagnostics for $VAR(9)$ under Original and Box–Cox Transformed Data

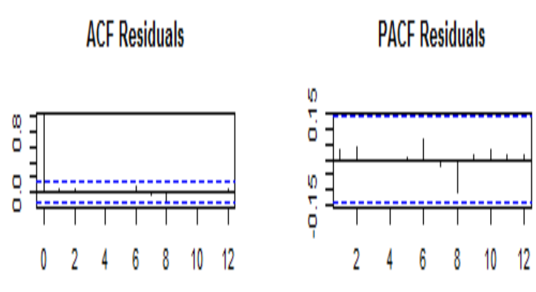
| Indicators | Original Data | | Transformed data | |
|---|---|---|---|---|
| | Stat. | p-value | Stat. | p-value |
| PT-Autocorrelation | 92.06 | 0.01 | 90.95 | 0.01 |
| ARCH-Heteroscedasticity | 143.25 | 0.98 | 130.59 | 0.99 |
| Jarque–Bera - Normality | 69.98 | 0.00 | 88.37 | 0.00 |
| Kurtosis | 37.42 | 0.00 | 47.73 | 0.00 |
| Skewness | 32.56 | 0.00 | 40.65 | 0.00 |

Table 3 reports the diagnostic statistics for the $VAR(9)$ specification estimated on both the original and Box–Cox transformed series. The Portmanteau statistic yields p-values around 0.01, indicating borderline evidence of residual autocorrelation at conventional significance levels. However, the residual ACF and PACF plots in Figures 2–4 do not reveal any systematic serial structure, suggesting that the remaining dependence is of limited practical relevance for forecasting. The ARCH–LM statistics show very large p-values ($\approx 0.99$), providing strong support for homoscedastic residual behavior in both the original and transformed models.

Regarding distributional properties, the Jarque–Bera test strongly rejects normality for both versions of the VAR model, and the skewness and kurtosis statistics also deviate from their Gaussian benchmarks. The conclusion is to be expected in weather applications. The typical joint modeling of variables with fundamentally different marginal distributions, T, R, and H, will produce multivariate residuals that do not normally distribute. The transformations with the BCT is successful in improving normality when testing each univariate series separately, however, once the variables are placed into a consolidated VAR system, improved univariate normality does not imply joint normality. Since the VAR residuals must be viewed as an overall error process that aggregates cross-dependent internal dynamics caused by the variables in the system, the newly embedded (cross-dependent) overall error process inherits heavy-tailed or asymmetrical components produced by the more irregular series, namely R, even when we preprocessing the univariate variables to achieve improved normality overall. Yet overall, the diagnostic evidence presented indicates that $VAR(9)$ specification meets the major adequacy conditions of temporal modeling related to no substantial serial dependence and stable variance, and that evidence of departures from joint normality reflect common distributional idiosyncrasies associated with climatic variables and not mis-specification of the modeling process. Therefore, the results support the forecasting evaluation portion of the research process, which is presented in the next rotational mantle section of the dissertation.

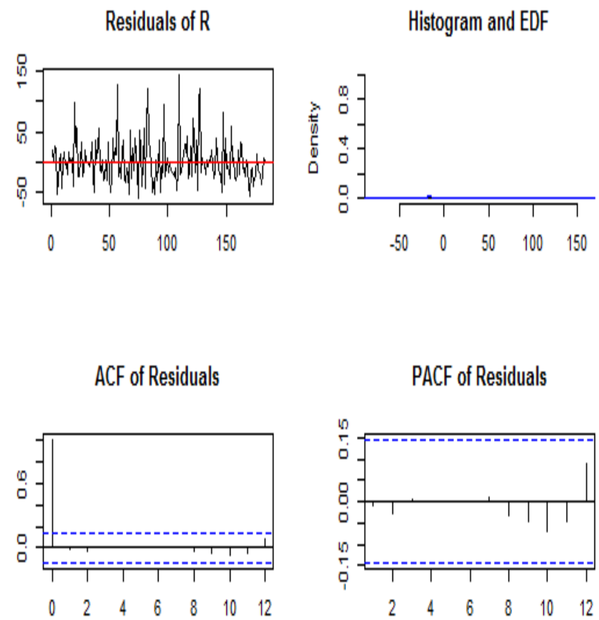



(a)

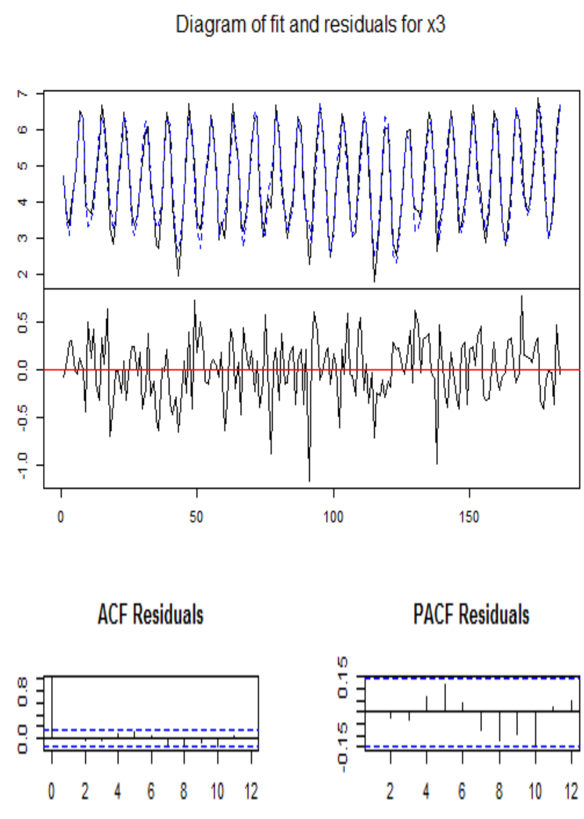



(b)

**Fig. 2:** Residual Plots for H: (a) Original Series and (b) BCT-Transformed Series




(a)




(b)

**Fig. 3:** Residual Plots for T: (a) Original Series and (b) BCT-Transformed Series

Figures 2-4 offer supplementary graphical diagnostics of the residuals of both the univariate models and the multivariate models. In every climatic variable, the residual series do not show observable signs of serial structure and in addition, the Box-Cox transformed models appear smoother and more homoscedastic than the untransformed models. The figures provide evidence to support the formal tests, and collectively confirm that the estimated models effectively capture the temporal dynamics of the climate variables of interest.

Now that we have concluded that the transformed series satisfy the main adequacy conditions, we will next proceed to estimate the univariate ARIMA models for each climate variable, before we model the multivariate VAR system.

### 3.2 Univariate and Multivariate Model Estimation

Building on the preliminary diagnostics presented in section 3.1, the transformed climatic series are now used to estimate both the univariate ARIMA specifications and the multivariate VAR model. The BCT stabilized the variance and improved the distributional properties of the T, R, and H series, providing a consistent basis for dynamic modeling. As shown earlier in Figures 2–4, the autocorrelation structure of the transformed series exhibits a rapid decay, indicating that relatively low-order AR and MA components are sufficient for capturing short-term persistence. These features support the use of parsimonious ARIMA models for the individual series and motivate the parallel estimation of a multivariate VAR structure to account for cross-variable interactions.

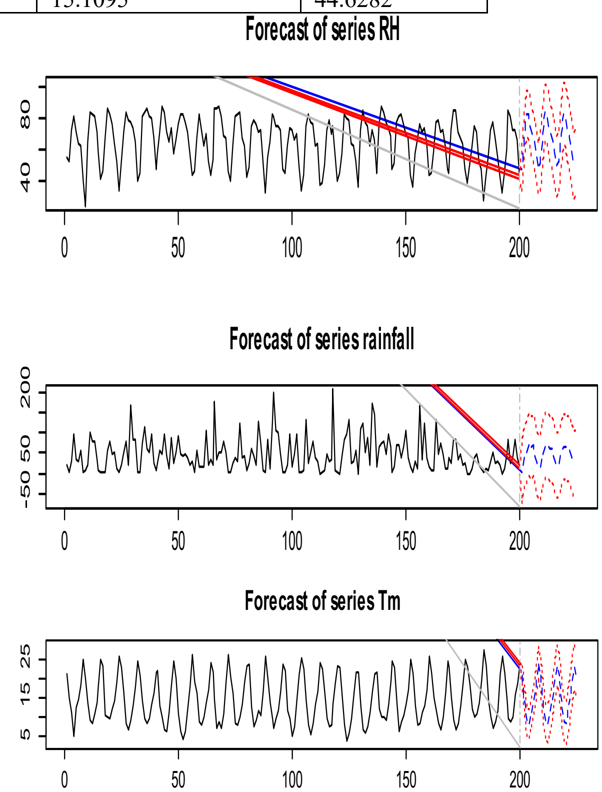
The model orders were selected using standard information criteria, balancing goodness of fit against model parsimony. Once the ARIMA orders and the VAR lag length (nine lags) were determined, the parameters of both modeling frameworks were estimated by maximum likelihood. The results, including the estimated coefficients and the mean squared errors (MSE) before and after the BCT, are summarized below.

**Table 4:** Estimated Parameters and MSE of the Univariate ARIMA and Multivariate VAR Models for the Box–Cox Transformed Climatic Series (H, R, T).

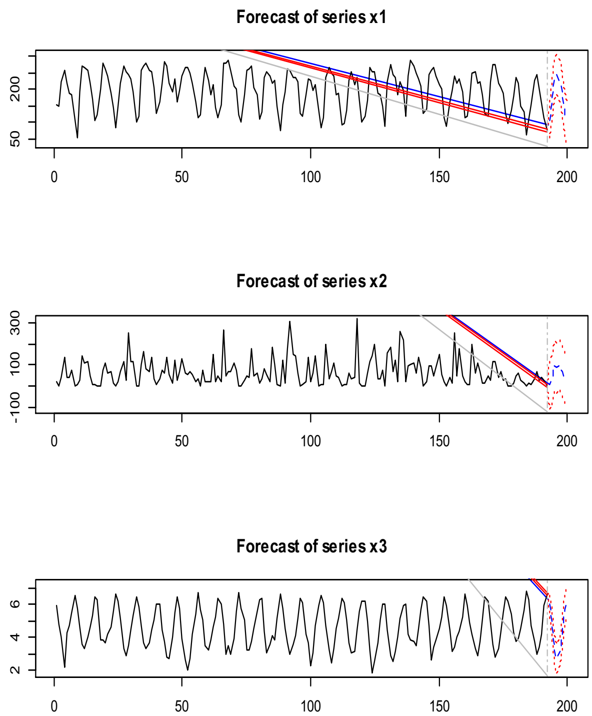
| Series | Original Data $\lambda_H = \lambda_R = \lambda_T = 1$ | | Transformed Data $\lambda_H = 1.3, \lambda_R = 1.1,\ \lambda_T = 0.4$ | |
|---|---|---|---|---|
| | $ARIMA$ (1,0,0)(1,1,1)8 | $VAR(9)$ | $ARIMA$ (1,0,0)(1,1,1)8 | $VAR(9)$ |
| T | 2.2405 | 3.2624 | 2.1668 | 3.033 |
| R | 366.388 | 497.190 | 379.5328 | 536.5995 |
| H | 15.6396 | 49.2073 | 15.1095 | 44.6282 |

Table 4 provides direct numerical evidence for the comparative performance of the univariate and multivariate models. The univariate ARIMA specifications exhibit MSE that are consistently lower than the $VAR(9)$ model, both prior to and after the BCT was applied. Although the multivariate structure is supposed to take advantage of the cross-variable dependencies in the forecasts, the fairly mild correlations among T, R and H limit most of the forecasting capabilities that VAR could provide. Conversely, the transformation achieved even more significant improvements in the univariate framework, where each of series is benefitting individually from variance stabilization and more favorable marginals. This reinforces the conclusion that univariate modeling is the better forecasting option for this dataset, compared to multivariate modeling.

To explore the forecasting implications of the multivariate structure, one-step-ahead predictions from the $VAR(9)$ model are compared for original series and transformed series. The figure below illustrates the forecast paths for T, R, and H with both series sets.




(a)




(b)

**Figure 4.** Comparative Forecasts from the $VAR(9)$ Framework: Original Data (a) versus Box–Cox Transformed Data (b).

The forecasts created from the adjusted series are more aligned with the observed series, especially during the periods of large variability, thus demonstrating the usefulness of variance stabilization and enhancing the predictive performance of the multivariate series. The estimation results led to the comparative analysis discussed in the subsequent section, where the follow-up time, critical for preventing a loss of power, was unit level.

## 4. Discussion and Implications

The primary objective of this research was to evaluate the forecasting performance of univariate and multivariate time-series models for key climatic variables—temperature (T), rainfall (R), and humidity (H)—within a unified preprocessing framework based on the Box–Cox transformation. This objective is directly aligned with the broader aim of the study: enhancing building environmental simulation through improved forecasting of climatic variables, which serve as critical inputs for building performance modelling and energy analysis.

In accordance with the Box–Jenkins modelling framework (Box & Jenkins, 1976), the Box–Cox transformation (BCT) was applied as a preprocessing step to stabilize variance and improve the distributional characteristics of the climatic time series prior to model estimation. The transformation was not intended as a theoretical innovation but rather as a practical procedure designed to ensure that the statistical assumptions required for reliable time-series modelling were satisfied. Such preprocessing is particularly important when climatic datasets are used for simulation-based applications, where inaccuracies in statistical modelling may propagate through environmental simulation systems.

The empirical findings indicate that although the original climatic time series exhibited stationarity according to the Augmented Dickey–Fuller (ADF), Phillips–Perron (PP), and KPSS tests, the application of the Box–Cox transformation significantly improved their marginal distributional properties by reducing skewness and heavy-tailed behaviour. This improvement was not merely cosmetic; the transformation resulted in more stable parameter estimates and improved residual diagnostics across the estimated models. These results are consistent with earlier findings by Guerrero (1993) and Atkinson et al. (2021), which demonstrated that power transformations can enhance the reliability of statistical inference in climatic and environmental time series.

The results further reveal that univariate ARIMA models exhibited strong forecasting performance following Box–Cox preprocessing, producing lower forecast errors compared with models estimated using the original untransformed data. These findings support the conclusions of Othman and Mohammed Ali (2023), who demonstrated that variance-stabilizing transformations can significantly improve ARIMA-based forecasting of temperature series. In addition, the persistent autoregressive behaviour observed in the temperature and humidity series supports the argument advanced by Hyndman and Athanasopoulos (2021) that when a variable's dynamics are largely self-driven, well-specified univariate models can outperform more complex modelling frameworks. The present study provides empirical evidence for this claim through the observed reductions in mean squared error (MSE) and improved diagnostic performance.

In contrast, the multivariate VAR(9) model was designed to capture the dynamic interactions among the three climatic variables. While the VAR framework successfully identified common seasonal patterns and structural relationships within the climatic system, the level of cross-variable correlation among T, R, and H was only moderate. As a result, the forecasting performance of the VAR model did not consistently exceed that of the univariate ARIMA models. These findings align with previous empirical research by Adeyemi et al. (2017) and Cebrián et al. (2019), which suggests that multivariate time-series models tend to outperform univariate models only when strong and stable inter-variable relationships exist.

Another notable observation concerns the distributional properties of the VAR residuals. Despite the application of the Box–Cox transformation, the residuals from the VAR model did not fully satisfy normality assumptions. This outcome reflects the inherent complexity of multivariate climatic systems, where variables may follow different marginal distributions. Similar findings have been reported by Živković et al. (2022), who noted that multivariate climate models often retain irregular distributional features even after transformation. The results of the present study therefore provide empirical confirmation of these observations.

Taken together, these findings highlight an important methodological implication: multivariate models should not automatically be assumed to outperform univariate models in climatic forecasting applications. Instead, the choice between modelling approaches should be guided by empirical characteristics of the data, particularly the strength and stability of cross-variable dependencies. As emphasized by Ogallo et al. (2020) and Youngman (2020), model selection in climate forecasting must be grounded in empirical evidence rather than theoretical preference.

From a practical perspective, the study also demonstrates the value of applying a consistent preprocessing framework when comparing alternative forecasting models. By applying the same Box–Cox transformation to all climatic variables prior to model estimation, the analysis ensured that differences in forecasting performance were not driven by scale or distributional inconsistencies across the series. This approach supports the methodological argument advanced by Guerrero (1993), who emphasized the importance of standardized transformations when comparing models with different structural assumptions.

Overall, the empirical findings provide clear evidence that variance-stabilized univariate ARIMA models can deliver highly accurate forecasts for climatic variables when cross-variable dependencies are limited. While multivariate VAR models offer valuable insights into structural relationships among climatic indicators, their forecasting advantages may be limited when correlations between variables are moderate or unstable. These insights are particularly relevant for climate-driven applications such as building environmental simulation, where reliable climatic forecasts are required to support accurate modelling of building performance, energy demand, and environmental sustainability.

## 5. Conclusion

This study investigated the role of variance-stabilizing transformations and time-series modelling approaches in improving the forecasting of key climatic variables—temperature, rainfall, and humidity—with the broader objective of enhancing climatic inputs used in building environmental simulation. Specifically, the research compared univariate ARIMA models and multivariate VAR models within a unified preprocessing framework based on the Box–Cox transformation.

The empirical results demonstrate that the Box–Cox transformation provides a flexible and effective method for stabilizing variance and improving the distributional properties of climatic time series. By regularizing the statistical characteristics of the data prior to modelling, the transformation facilitates more reliable parameter estimation and improves the diagnostic properties of time-series models. These improvements contribute to the development of forecasting models that are both statistically robust and practically applicable.

The comparative analysis revealed that univariate ARIMA models consistently achieved higher forecasting accuracy than the multivariate VAR model for the climatic variables considered in this study. While the VAR framework successfully captured structural interactions among the variables, the strength of these interactions was insufficient to produce superior forecasting performance relative to the univariate models. This finding underscores the importance of carefully evaluating cross-variable dependencies when selecting forecasting methodologies for climatic data.

An additional insight from the analysis concerns the inherent trade-off between transformation objectives and model efficiency criteria. Maximizing the likelihood function for transformation parameter estimation does not necessarily produce a distribution that perfectly satisfies all statistical assumptions. Nevertheless, the results demonstrate that the Box–Cox transformation remains a valuable preprocessing tool for improving modelling conditions and facilitating the identification of simple and effective forecasting models.

Overall, the study highlights the importance of integrating appropriate preprocessing techniques with well-established time-series models to improve the reliability of climatic forecasting. By enhancing the quality of climate data forecasts, the proposed framework contributes to more accurate environmental inputs for building performance modelling, energy analysis, and sustainable building design.

Future research may extend this framework by exploring additional transformation techniques, nonlinear modelling approaches, or hybrid forecasting methods capable of capturing more complex interactions among climatic variables. Such developments could further strengthen the role of advanced time-series analysis in supporting climate-informed decision-making and improving the predictive capabilities of building environmental simulation systems.

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