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Conformable NE Transform: Theories, Methods and Applications

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Abstract: In this study, we extend the NE transform formula to applicable fractional orders and examine several fascinating and fundamental properties of the transform. Additionally, we offer a comprehensive analytical solution for both linear and nonlinear fractional differential equations. We investigate the fractional Newell-Whitehead-Segel equation, a significant amplitude equation in physics, employing the conformable NE decomposition method. Furthermore, the method employed to derive approximate and analytical solutions for linear-nonlinear fractional partial differential equations integrates the Adomian decomposition method with the conformable NE transform. Ultimately, the results indicate that our proposed approach is effective and suitable for all situations concerning conformable fractional differential equations. This research supports SDG 4: Quality Education by advancing analytical methods in fractional calculus and mathematical modeling.

Keywords: NE transform; Conformable derivative; Newell-Whitehead-Segel equation; Adomian decomposition method.

1 Introduction

Modern definitions of fractional integrals and fractional derivatives for real-valued functions are crucial within the scope of fractional calculus [1,2]. These definitions are used to set up several models concerning industrial growth, human disease, and economic growth. All of these new definitions have made the examination of this topic more straightforward and efficient [3]. Fractional derivatives have been defined in a variety of ways, including by Riemann-Liouville, Caputo, Hadamard, Grunwald, Letnikov, and Riesz. Several authors introduced new definitions of the fractional derivative, known as the conformable fractional derivative [4,5], which is easier to find than earlier definitions. Conformable fractional descriptions have recently acquired popularity due to their natural formulation and ease. Its uses are fast developing in the remodeling of different dynamical models and an emerging variety of approaches with this definition [6]-[9]. The conformable fractional derivative of order $\alpha \in (0,1]$ of x at t is described by

$$D^{\alpha}x(t) = lim_{\varepsilon\to 0}\frac{x(t+\varepsilon t^{1-\alpha})-x(t)}{\varepsilon}, \tag{1}$$

for all $t>0$. If x is α-differentiable in some interval $(0,\alpha)$ with $\alpha>0$, then we have

$$D^{\alpha}x(0) = lim_{t\to 0^{+}}D^{\alpha}x(t), \tag{2}$$

whenever the boundary of the right side is presented. We note that if x is differentiable, then

$$D^{\alpha}x(t) = t^{1-\alpha}x'(t). \tag{3}$$

Certainly, this is not applicable for $t=0$, and it would be beneficial to address equations and solutions that involve singularities [10].

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Integral transforms frequently employ an exponential function for integration from 0 to ∞. This enables the application of the condition that either e^{-st} or $e^{-t/u}$ approaches 0 as t tends toward infinity. Watugala [11] in 1993 invented the Sumudu transform, which is a modified Laplace transform and Elzaki et al. [12] in 2012 to deal with initial value issues in engineering applications [13]. The quantitative weighted generalization of the Jafari transform has been examined by Serdal et al. [14]. Benattia et al. [15] has investigated Shehu conformable fractional transform, theories and applications. New results on the conformable Sumudu transform theories and applications have been examined by Zhour et al [4].

In this study, we enlarged the concept of the NE transform into a fractional order and derived a set of significant rules and features for this extension. After, we applied this transformation to derive analytical solutions for fractional models. The constant M in a given function in A must have a value that is positive. The integral equation creates a new integral transform, denoted by the operator G(.) [16]:

$$G(s,u) = N[g(t)] = \frac{1}{su}\int_0^{+\infty} e^{\frac{-st}{u}} g(t)dt, \quad s>0, \quad u>0.$$

If the function $g(t)$ is piecewise continuous over the interval $[0,\infty]$. It is called the function $G(s,u)$'s inverse integral transformation.

$$N^{-1}[G(s,u)] = \frac{1}{2\Pi i}\int_{c-\infty}^{c+\infty} e^{\frac{st}{u}} G(s)sds = g(t).$$

2 Conformable NE transform

This part offers a collection of essential fundamental principles and features for the conformable fractional NE transform.

Definition 1. *Allow $g:[0,\infty)\to R$ to be given a function with $0<\alpha\le 1$. The conformable NE transform of g is defined as follows:*

$$N_\alpha[g(t)] = G_\alpha(s,u) = \frac{1}{su}\int_0^{+\infty} e^{\frac{-st^\alpha}{\alpha u}} g(t)t^{\alpha-1}dt, \tag{4}$$

given that the integral exists.

Theorem 1. *Allow $g:[0,\infty)\to R$ to be given a function with $0<\alpha\le 1$. Then*

$$N_\alpha[D^\alpha g(t)] = \frac{sG_\alpha(s,u)}{u} - \frac{g(0)}{su}. \tag{5}$$

Proof. Utilizing Definition (1) and integration by parts, we obtain

$$\begin{aligned}
N_\alpha[D^\alpha g(t)] = &= \frac{1}{su}\int_0^{+\infty} e^{\frac{-st^\alpha}{\alpha u}} D^\alpha g(t)t^{\alpha-1}dt = \frac{1}{su}\int_0^{+\infty} e^{\frac{-st^\alpha}{\alpha u}} t^{1-\alpha}g'(t)t^{\alpha-1}dt \\
&= \frac{1}{su}\int_0^{+\infty} e^{\frac{-st^\alpha}{\alpha u}} g'(t)dt = \frac{1}{su}\left[e^{\frac{-st^\alpha}{\alpha u}} g(t)\right]_0^\infty - \frac{1}{su}\int_0^{+\infty} g(t)\frac{s}{u}e^{\frac{-st^\alpha}{\alpha u}}dt \\
&= \frac{1}{su}\lim_{k\leftarrow\infty}\left[e^{\frac{-st^\alpha}{\alpha u}} g(t)\right]_0^k + \frac{1}{su}\int_0^{+\infty}\left(\frac{s}{u}e^{\frac{-st^\alpha}{\alpha u}}\right)t^{\alpha-1}g(t)dt \\
&= \frac{sG_\alpha(s,u)}{u} - \frac{g(0)}{su},
\end{aligned}$$

this completes the proof.

Theorem 2. *Allow $g:[0,\infty)\to R$ to be given a function with $1\le\alpha<2$. Then*

$$N_\alpha[D^\alpha g(t)] = \frac{s^2G_\alpha(s,u)}{u^2} - \frac{D^\alpha g(0)}{su} - \frac{g(0)}{u^2}. \tag{6}$$

Proof. The proof is similar to the proof of Theorem (1). Therefore, we omitted the proof.

Theorem 3. *Allow $g:[0,\infty)\to R$ to be given a function with $0<\alpha\le 1$. Thus*

$$G_\alpha(s,u) = N\left(g(\alpha t)^{\frac{1}{\alpha}}\right). \tag{7}$$

Proof. Utilizing the Definition (1) and allowing $k = \frac{t^\alpha}{\alpha}$, we obtain:

$$G_\alpha(s,u) = \frac{1}{su}\int_0^{+\infty} e^{\frac{-st^\alpha}{\alpha u}} g(t)t^{\alpha-1}dt = \frac{1}{su}\int_0^{+\infty} e^{\frac{-sk}{u}} g(\alpha k)^{\frac{1}{\alpha}} dk = N\left(g(\alpha k)^{\frac{1}{\alpha}}\right),$$

this completes the proof.

Theorem 4. *Allow* $b, \in R$ *with* $0 < \alpha \leq 1$. *Then*

$$i.\ N_\alpha[b](s,u) = \frac{b}{s^2},$$

$$ii.\ N_\alpha[t^b](s,u) = \alpha^{\frac{b}{\alpha}} \frac{\Gamma(1+\frac{b}{\alpha})}{u^{\frac{-b}{\alpha}} s^{2+\frac{b}{\alpha}}},$$

$$iii.\ N_\alpha\left[e^{b\frac{t^\alpha}{\alpha}}\right](s,u) = \frac{1}{s(s-bu)},$$

$$iv.\ N_\alpha\left[\sin(b\frac{t^\alpha}{\alpha})\right](s,u) = \frac{bu}{s(s^2+b^2u^2)},$$

$$v.\ N_\alpha\left[\cos(b\frac{t^\alpha}{\alpha})\right](s,u) = \frac{1}{s^2+b^2u^2},$$

$$vi.\ N_\alpha\left[\sinh(b\frac{t^\alpha}{\alpha})\right](s,u) = \frac{bu}{s(s^2-b^2u^2)},$$

$$vii.\ N_\alpha\left[\cosh(b\frac{t^\alpha}{\alpha})\right](s,u) = \frac{1}{s^2-b^2u^2}.$$

Proof. When utilizing Definition (1), all of them can be obtained easily. For example;

$$i.\ N_\alpha[b](s,u) = \frac{1}{su}\int_0^{+\infty} e^{\frac{-st^\alpha}{\alpha u}} bt^{\alpha-1}dt = \frac{b}{su}\int_0^{+\infty} e^{\frac{-sv}{u}} dv$$
$$= \frac{b}{s^2}.$$

$$iii.\ N_\alpha\left[e^{b\frac{t^\alpha}{\alpha}}\right](s,u) = \frac{1}{su}\int_0^{+\infty} e^{\frac{-st^\alpha}{\alpha u}} e^{b\frac{t^\alpha}{\alpha}} t^{\alpha-1}dt = \frac{1}{su}\int_0^{+\infty} e^{b-\frac{s}{u}\frac{t^\alpha}{\alpha}} t^{\alpha-1}dt$$
$$= \frac{1}{s(s-bu)}.$$

$$v.\ N_\alpha\left[\cos(b\frac{t^\alpha}{\alpha})\right](s,u) = N_\alpha\left[\cos(b\frac{((\alpha t)^{\frac{1}{\alpha}})^\alpha}{\alpha})\right](s,u) = N[\cos(bt)](s,u)$$
$$= \frac{1}{s^2+b^2u^2}.$$

Theorem 5. *Allow* $b, k_1, k_2 \in R$ *with* $0 < \alpha \leq 1$. *Then*

$$i.\ N_\alpha[k_1 g(t) + k_2 h(t)] = k_1 G_\alpha(s,u) + k_2 H_\alpha(s,u),$$

$$ii.\ N_\alpha\left[e^{b\frac{t^\alpha}{\alpha}}\right] = G_\alpha(s(s+bu)),$$

$$iii.\ N_\alpha[I^\alpha g(t)] = \frac{u}{s} G_\alpha(s,u),$$

$$iv.\ N_\alpha\left[\frac{t^{n\alpha}}{\alpha^n} g(t)\right] = \frac{u^n}{s^n}\frac{d^n}{du^n} u^n G_\alpha(s,u),$$

$$v.\ N_\alpha[(g*h)(t)] = usG_\alpha(s,u)H_\alpha(s,u).$$

Proof.
i. Implementing Definition (1), we get:

$$N_\alpha[k_1g(t)+k_2h(t)] = \frac{1}{su}\int_0^{+\infty} e^{\frac{-st^\alpha}{\alpha u}}(k_1g(t)+k_2h(t))t^{\alpha-1}dt$$
$$= k_1\frac{1}{su}\int_0^{+\infty} e^{\frac{-st^\alpha}{\alpha u}}g(t)t^{\alpha-1}dt + k_2\frac{1}{su}\int_0^{+\infty} e^{\frac{-st^\alpha}{\alpha u}}h(t)t^{\alpha-1}dt$$
$$= k_1G_\alpha(s,u)+k_2H_\alpha(s,u).$$

ii. Utilizing Theorem (3), gives:

$$N_\alpha\left[e^{-b\frac{t^\alpha}{\alpha}}\right] = N\left(e^{\frac{-b}{\alpha}(\alpha t)^{\alpha\frac{1}{\alpha}}}\right) = N\left(e^{-bt}g(\alpha t)^{\frac{1}{\alpha}}\right)$$
$$= N\left(g(\alpha t)^{\frac{1}{\alpha}}\right)|_{s\to s(s+bu)} = G_\alpha(s(s+bu)).$$

iii. When we apply Theorem (2), we obtain:

$$N_\alpha[D^\alpha I^\alpha g(t)] = \frac{s}{u}N_\alpha[I^\alpha g(t)] - \frac{I^\alpha g(0)}{su}.$$

$I^\alpha g(0)=0$, then we acquire:

$$G_\alpha(s,u) = \frac{s}{u}N_\alpha[I^\alpha g(t)],$$

and

$$N_\alpha[I^\alpha g(t)] = \frac{u}{s}G_\alpha(s,u).$$

iv. with Theorem (4), we write:

$$N_\alpha\left[\frac{t^{n\alpha}}{\alpha^n}g(t)\right] = N\left[\frac{(\alpha t)^{n\alpha\frac{1}{\alpha}}}{\alpha^n}g(\alpha t)^{\frac{1}{\alpha}}\right] = N[t^n g(\alpha t)^{\frac{1}{\alpha}}] = \frac{u^n}{s^n}\frac{d^n}{du^n}u^nG_\alpha(s,u).$$

v. Utilizing Theorem (4), yields:

$$N_\alpha[(g*h)(t)] = N\left[(g*h)(\alpha t)^{\frac{1}{\alpha}}\right] = usN\left[g(\alpha t)^{\frac{1}{\alpha}}\right]N\left[h(\alpha t)^{\frac{1}{\alpha}}\right] = usG_\alpha(s,u)H_\alpha(s,u).$$

3 Conformable NE decomposition method

There are other Adomian decomposition method (ADM) modifications and hybrid forms in the literature [17,18,19]. One of the hybrid types of ADM is the conformable NE decomposition method (CNDM). The NE transform is applied to linear-nonlinear fractional partial differential equations (PDEs) in CNDM by combining it with ADM in a conformable way. Let's now present the CNDM algorithm. We examine the following fractional PDEs in general operator form to illustrate the fundamental concept of CNDM.

$$D_t^\alpha g(x,t)+D_x^n g(x,t)+R(g(x,t))+M(g(x,t)) = h(x,t), \quad t>0, \quad x>0, \quad 0<\alpha\le 1, \tag{8}$$

where, D_t^α represents the linear derivative operator of conformable order α in t, R consists of other linear terms with lesser derivatives, M denotes the nonlinear term, and $h(x,t)$ indicates the nonhomogeneous part. The highest order classical linear derivative operator in x is D_x^n. Utilizing the conformable NE transform N_α concerning t on both sides of Eq. (8) leads to the outcome:

$$N_\alpha[D_t^\alpha g]+N_\alpha[D_x^n g]+N_\alpha[R(g)+M(g)] = N_\alpha[h(x,t)]. \tag{9}$$

Eq. (9), derived from the differential property of the conformable NE transform, becomes

$$\frac{s}{u}N_\alpha[g] - \frac{g(x,0)}{su} + N_\alpha[D_x^n g] + N_\alpha[R(g)) + M(g)] = N_\alpha[h(x,t)]. \tag{10}$$

When we simplify Eq. (10):

$$N_\alpha[g] = \frac{g(x,0)}{s^2} + \frac{u}{s}\{N_\alpha[h(x,t)] - N_\alpha[D_x^n g] - N_\alpha[R(g) + M(g)]\}. \tag{11}$$

Applying the conformable inverse NE transform to this equation, yields:

$$g(x,t) = N_\alpha^{-1}\left\{\frac{g(x,0)}{s^2}\right\} + N_\alpha^{-1}\left\{\frac{u}{s}\{N_\alpha[h(x,t)] - N_\alpha[D_x^n g] - N_\alpha[R(g) + M(g)]\}\right\}. \tag{12}$$

The nonlinear term $M(g(x,t))$ of Eq. (8) and the Adomian polynomials A_k, which depend on $g_0, g_1, g_2, ,\ldots, g_k$, are presented as follows [20] in accordance with the ADM, along with the solution $g(x,t)$ with its convergency.

$$g(x,t) = \sum_{k=0}^{\infty} g_k(x,t), \tag{13}$$

$$N(g(x,t)) = \sum_{k=0}^{\infty} A_k. \tag{14}$$

Then, we have

$$\sum_{k=0}^{\infty} g_k = N_\alpha^{-1}\left\{\frac{g(x,0)}{s^2} + \frac{u}{s}\{N_\alpha[h(x,t)]\}\right\} - N_\alpha^{-1}\left\{\frac{u}{s}\left\{N_\alpha[D_x^n \sum_{k=0}^{\infty} g_k]\right\}\right\}$$
$$-N_\alpha^{-1}\left\{\frac{u}{s}\left\{N_\alpha[R(\sum_{k=0}^{\infty} g_k)) + \sum_{k=0}^{\infty} A_k]\right\}\right\}.$$

When both sides of the above equation match, the iterative method that follows is produced.

$$g_0 = N_\alpha^{-1}\left\{\frac{g(x,0)}{s^2} + \frac{u}{s}\{N_\alpha[h(x,t)]\}\right\}, \tag{15}$$

$$g_{k+1} = -N_\alpha^{-1}\left\{\frac{u}{s}N_\alpha\left[D_x^n \sum_{k=0}^{\infty} g_k\right] - \frac{u}{s}N_\alpha\left[R\left(\sum_{k=0}^{\infty} g_k\right) + \sum_{k=0}^{\infty} A_k\right]\right\}. \tag{16}$$

Let's examine a few instances to demonstrate the advantages of this method and the NE transform.

4 Applications

This section provides instances of using the conformable fractional NE transform and CNDM to address both linear, nonlinear ordinary, and conformable PDEs.

Example 1. We consider

$$D_t^\alpha g(t) = g(t) + 1, \qquad 0 < \alpha \leq 1. \tag{17}$$

According to the initial constraint $g(0) = 0$, we apply the conformable fractional NE transform to the above expression, we obtain:

$$N_\alpha[D_t^\alpha g(t)] = N_\alpha(g(t)) + N_\alpha(1),$$

$$\frac{s}{u}G_\alpha(s,u) - \frac{1}{su}g(0) = G_\alpha(s,u) + \frac{1}{s^2},$$

$$G_\alpha(s,u) = \frac{u}{s^2(s-u)}.$$

Implementing N_α^{-1} to the last equation, gives:

$$g(t) = N_\alpha^{-1}\left[\frac{1}{s(s-u)} - \frac{1}{s^2}\right] = \exp\left(\frac{t^\alpha}{\alpha}\right) - 1.$$

Example 2. Let us consider the Riccati differential equation as [21]:

$$D_t^\alpha g(t) = -g^2(t) + 1, \qquad t \geq 0, \qquad 0 < \alpha \leq 1, \tag{18}$$

with the initial condition $g(0) = 0$. When we implement the conformable fractional NE transform to Eq. (18), we have:

$$N_\alpha\left[D_t^\alpha g(t)\right] = N_\alpha(g^2(t)) + N_\alpha(1),$$
$$\frac{s}{u}G_\alpha(s,u) - \frac{1}{su}g(0) = N_\alpha(g^2(t)) + \frac{1}{s^2},$$
$$G_\alpha(s,u) = -\frac{u}{s}N_\alpha(g^2(t)) + \frac{u}{s^3}.$$

Using N_α^{-1}:

$$g(t) = -N_\alpha^{-1}\left[\frac{u}{s}N_\alpha(g^2(t))(s,u)\right] + N_\alpha^{-1}\left[\frac{u}{s^3}\right] = -N_\alpha^{-1}\left[\frac{u}{s}N_\alpha(g^2(t))(s,u)\right] + \frac{t^\alpha}{\alpha}. \tag{19}$$

As $g^2(t)$ is a nonlinear equation, using the Adomian decomposition approach, we write

$$g(t) = z(t) = \sum_{i=0}^{\infty} z_i(t), \qquad g^2(t) = \sum_{i=0}^{\infty} A_i,$$

where A_i are Adomian polynomials, and some of them are as follows:

$$A_0 = g_0^2, \quad A_1 = 2g_1g_0, \quad A_2 = 2g_2g_0 + g_1^2, \cdots.$$

So, the general solution is obtained as:

$$z_0 = \frac{t^\alpha}{\alpha},$$
$$z_{i+1} = -N_\alpha^{-1}\left[\frac{u}{s}N_\alpha(A_i)\right] \qquad i \geq 0,$$

for $i = 0$, we get:

$$z_1 = -N_\alpha^{-1}\left[\frac{u}{s}N_\alpha(A_0)\right] = -N_\alpha^{-1}\left[\frac{u}{s}N_\alpha\left(\frac{t^{2\alpha}}{\alpha^2}\right)\right]$$
$$= -2N_\alpha^{-1}\left[\frac{u^3}{s^5}\right] = -\frac{1}{3}\frac{t^{3\alpha}}{\alpha^3}$$

For $i = 1$, we have:

$$z_2 = -N_\alpha^{-1}\left[\frac{u}{s}N_\alpha(A_1)\right] = -N_\alpha^{-1}\left[\frac{u}{s}N_\alpha\left(\frac{-2}{3}\frac{t^{4\alpha}}{\alpha^4}\right)\right]$$
$$= \frac{2}{3}N_\alpha^{-1}\left[\frac{24u^5}{s^7}\right] = \frac{2}{15}\frac{t^{5\alpha}}{\alpha^5}.$$

for $i = 2$, we acquire:

$$z_3 = -N_\alpha^{-1}\left[\frac{u}{s}N_\alpha(A_2)\right] = -N_\alpha^{-1}\left[\frac{u}{s}N_\alpha(\frac{17}{45}\frac{t^{6\alpha}}{\alpha^6})\right]$$
$$= -\frac{17}{45}N_\alpha^{-1}\left[\frac{720u^7}{s^9}\right] = -\frac{17}{315}\frac{t^{7\alpha}}{\alpha^7}.$$

Thus, the approximate solution is found as follows:

$$z(t) = \frac{t^\alpha}{\alpha} - \frac{1}{3}\frac{t^{3\alpha}}{\alpha^3} + \frac{2}{15}\frac{t^{5\alpha}}{\alpha^5} - \frac{17}{315}\frac{t^{7\alpha}}{\alpha^7} + \cdots,$$

and

$$z(t) = tanh\left(\frac{t^{\alpha}}{\alpha}\right) = \frac{\exp[2\frac{t^{\alpha}}{\alpha}] - 1}{\exp[2\frac{t^{\alpha}}{\alpha}] + 1}.$$

For $\alpha = 1$, the solution of Eq. (18) is achieved as:

$$g(t) = \frac{\exp(2t) - 1}{\exp(2t) + 1}.$$

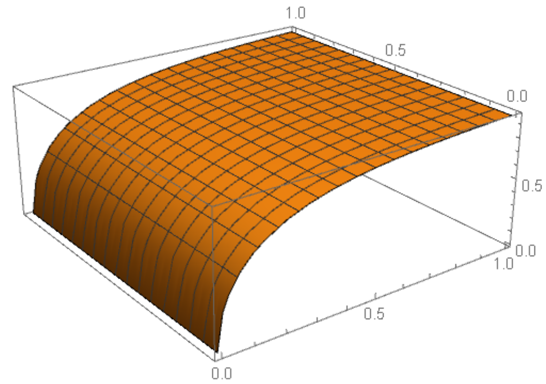

Fig. 1: Numerical simulation for $\alpha = 0.5$ for Example (2).

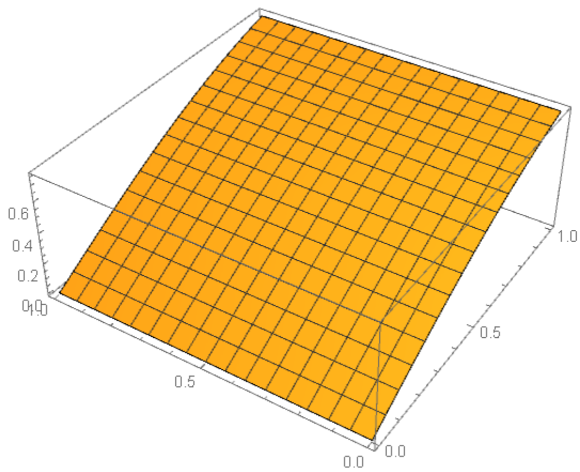

Fig. 2: Numerical simulation for $\alpha = 1$ for Example (2).

Example 3. We take into consideration the following equation;

$$D_t^{2\alpha} g(t) - 4g(t) = 0, \quad g(0) = 1, \quad D^{\alpha} g(0) = 0, \qquad 1 \le \alpha < 2, \tag{20}$$

Implementing the conformable fractional NE transform to Eq. (20), gives:

$$\frac{s^2}{u^2} G_{\alpha}(s,u) - \frac{1}{u^2} g(0) - 4G_{\alpha}(s,u) = 0,$$

$$G_{\alpha}(s,u) = \frac{1}{s^2 - 4u^2}.$$

When we apply the N_{α}^{-1} of both sides of the last equation, the exact solution can be found as follows:

$$g(t) = N_{\alpha}^{-1}\left[\frac{1}{s^2 - 4u^2}\right] = \cosh\left[\frac{2t^{\alpha}}{\alpha}\right].$$

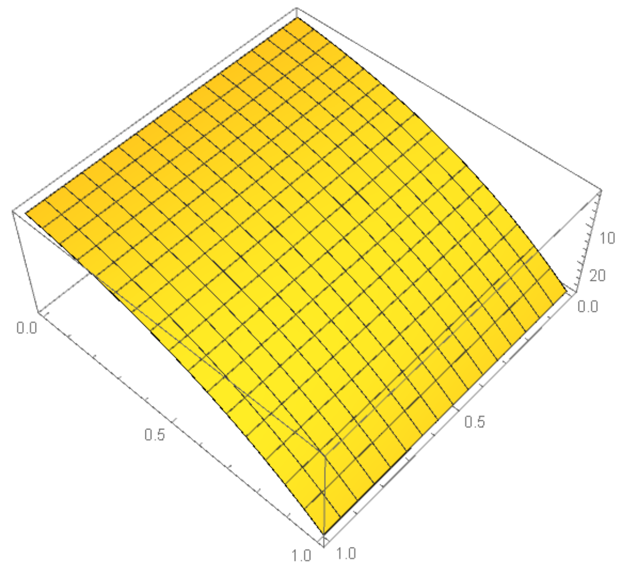

Fig. 3: Simulation for $\alpha = 0.5$ for Example (3).

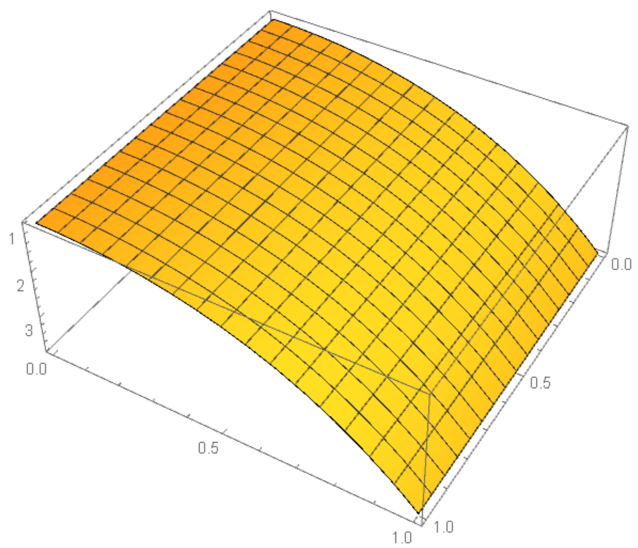

Fig. 4: Simulation for $\alpha = 1$ for Example (3).

Example 4. We take into consideration

$$D_t^{2\alpha} g(t) - g(t) = \sin\left(\frac{2t^\alpha}{\alpha}\right), \quad g(0) = 2, \quad D^\alpha g(0) = 0, \qquad 1 \le \alpha < 2. \tag{21}$$

If we use the conformable fractional NE transform, we will get:

$$\frac{s^2}{u^2} G_\alpha(s,u) - \frac{2}{u^2} - G_\alpha(s,u) = \frac{2u}{s(s^2+4u^2)},$$

$$G_\alpha(s,u) = \frac{2u^3}{s(s^2+4u^2)(s^2+u^2)} + \frac{2}{s^2-u^2}.$$

Implementing the N_α^{-1} of both sides of the last equation, yields:

$$\begin{aligned} g(t) &= N_\alpha^{-1}\left[\frac{-2u}{5s(s^2+4u^2)} + \frac{2u}{5s(s^2-u^2)} + \frac{2}{s^2-u^2}\right] \\ &= \frac{-1}{5}\sin\left(\frac{2t^\alpha}{\alpha}\right) + \frac{2}{5}\sinh\left(\frac{t^\alpha}{\alpha}\right) + 2\cosh\left[\frac{t^\alpha}{\alpha}\right]. \end{aligned}$$

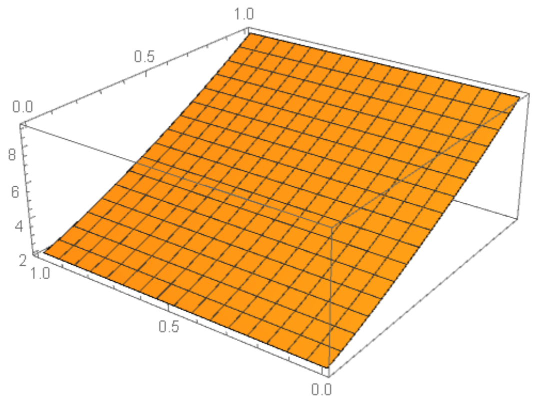

Fig. 5: Simulation for $\alpha = 0.5$ for Example (4).

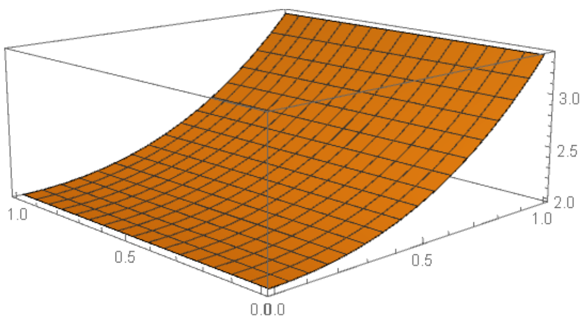

Fig. 6: Simulation for $\alpha = 1$ for Example (4).

Example 5. We examine the fractional NWS equation that is linearly conformable as [22]:

$$D_t^\alpha g(x,t) = D_x^2 g(x,t) - 2g(x,t), \quad g(x,0) = \exp(x), \qquad 0 \le \alpha \le 1. \tag{22}$$

In relation to t, when the conformable fractional NE transform is implemented to the above equation and using the initial condition, it becomes:

$$\frac{s}{u} N_\alpha [g(x,t)] - \frac{g(x,0)}{su} = N_\alpha \left[D_x^2 g(x,t)\right] - 2N_\alpha [g(x,t)],$$
$$N_\alpha[g] = \frac{\exp(x)}{s(s+2u)} + \frac{u}{s+2u} N_\alpha \left[D_x^2 g(x,t)\right].$$

Applying the N_α^{-1} to this equation, gives:

$$g = \exp(x)\exp\left(\frac{-2t^\alpha}{\alpha}\right) + N_\alpha^{-1}\left[\frac{u}{s+2u} N_\alpha \left[D_x^2 g\right]\right].$$

Thus, we write

$$\sum_{k=0}^{\infty} g_k = \exp(x)\exp\left(\frac{-2t^\alpha}{\alpha}\right) + N_\alpha^{-1}\left[\frac{u}{s+2u} N_\alpha \left[D_x^2 \sum_{k=0}^{\infty} g_k\right]\right].$$

When we use the iterative technique, we have

$$g_0 = \exp\left(x - \frac{2t^\alpha}{\alpha}\right),$$

$$g_{i+1} = N_\alpha^{-1}\left[\frac{u}{s+2u} N_\alpha \left[D_x^2 g_k\right]\right] \qquad i \ge 0$$

and

$$g_1 = \frac{t^\alpha}{\alpha}\exp\left(x - \frac{2t^\alpha}{\alpha}\right), \quad g_2 = \frac{t^{2\alpha}}{\alpha^2 2!}\exp\left(x - \frac{2t^\alpha}{\alpha}\right), \quad \cdots,$$

So, the approximate solution is obtained as follows:

$$g(x,t) = \exp\left(x - \frac{2t^\alpha}{\alpha}\right) + \frac{t^\alpha}{\alpha}\exp\left(x - \frac{2t^\alpha}{\alpha}\right) + \frac{t^{2\alpha}}{\alpha^2 2!}\exp\left(x - \frac{2t^\alpha}{\alpha}\right) + \frac{t^{3\alpha}}{\alpha^3 3!}\exp\left(x - \frac{2t^\alpha}{\alpha}\right) + \cdots.$$

and

$$g(x,t) = \exp^{x - \frac{t^\alpha}{\alpha}}.$$

Example 6. We take note of the following nonlinear conformable fractional NWS equation as [22]:

$$D_t^\alpha g(x,t) = 5D_x^{2r} g(x,t) + 2g(x,t) + g^2(x,t), \quad g(x,0) = \tau, \qquad 0 \le \alpha \le 1, \tag{23}$$

where τ is a constant. If we apply the conformable fractional NE transform with respect to t is implemented the equation and using the initial condition, it becomes:

$$\frac{s}{u} N_\alpha [g(x,t)] - \frac{g(x,0)}{su} = 5N_\alpha \left[D_x^2 g(x,t)\right] + 2N_\alpha [g(x,t)] + N_\alpha \left[g^2(x,t)\right],$$

$$N_\alpha[g] = \frac{\tau}{s(s-2u)} + \frac{5u}{s-2u} N_\alpha \left[D_x^2 g(x,t)\right] + \frac{u}{s-2u} N_\alpha \left[g^2(x,t)\right].$$

When the N_α^{-1} is applied to the above equation, we get:

$$g = \tau\exp\left(\frac{2t^\alpha}{\alpha}\right) + N_\alpha^{-1}\left[\frac{5u}{s-2u} N_\alpha \left[D_x^2 g\right]\right] + N_\alpha^{-1}\left[\frac{u}{s-2u} N_\alpha \left[g^2\right]\right].$$

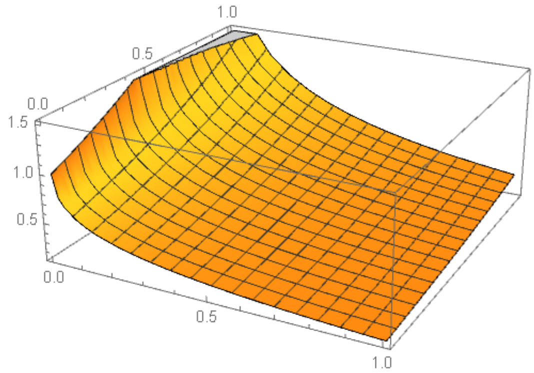

Fig. 7: Numerical simulation for $\alpha = 0.5$ for Example (5).

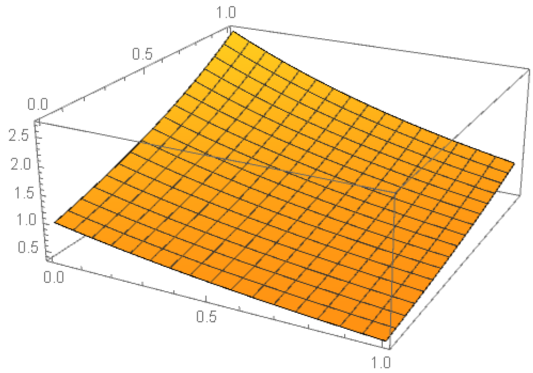

Fig. 8: Numerical simulation for $\alpha = 1$ for Example (5).

So, we have

$$\sum_{k=0}^{\infty} g_k = \tau \exp(\frac{2t^\alpha}{\alpha}) + N_\alpha^{-1}\left[\frac{5u}{s-2u} N_\alpha\left[D_x^2 g_k\right]\right] + N_\alpha^{-1}\left[\frac{u}{s-2u} N_\alpha\left[\sum_{k=0}^{\infty} g_k\right]\right].$$

We give a few components of Adomian polynomials as follows:

$$A_0 = g_0^2, \quad A_1 = 2g_1 g_0, \quad A_2 = 2g_2 g_0 + g_1^2, \cdots.$$

Using the iterative technique gives

$$g_0 = \tau \exp\left(\frac{2t^\alpha}{\alpha}\right),$$

$$g_{i+1} = N_\alpha^{-1}\left[\frac{5u}{s-2u} N_\alpha\left[D_x^2 g_k\right]\right] + N_\alpha^{-1}\left[\frac{u}{s-2u} N_\alpha\left[\sum_{k=0}^{\infty} g_k\right]\right], \qquad i \geq 0,$$

and

$$g_1 = \frac{1}{2}\tau^2 \exp\left(\frac{2t^\alpha}{\alpha}\right)\left(\exp\left(\frac{2t^\alpha}{\alpha}\right) - 1\right), \quad g_2 = \left(\frac{1}{2}\right)^2 \tau^3 \exp\left(\frac{2t^\alpha}{\alpha}\right)\left(\exp\left(\frac{2t^\alpha}{\alpha}\right) - 1\right)^2,$$

$$g_3 = \left(\frac{1}{2}\right)^3 \tau^4 \exp\left(\frac{2t^\alpha}{\alpha}\right)\left(\exp\left(\frac{2t^\alpha}{\alpha}\right) - 1\right)^3, \cdots.$$

Thus, the approximate solution is obtained as:

$$g(x,t) = \tau \exp\left(\frac{2t^\alpha}{\alpha}\right) + \frac{1}{2}\tau^2 \exp\left(\frac{2t^\alpha}{\alpha}\right)\left(\exp\left(\frac{2t^\alpha}{\alpha}\right)-1\right) + \left(\frac{1}{2}\right)^2 \tau^3 \exp\left(\frac{2t^\alpha}{\alpha}\right)\left(\exp\left(\frac{2t^\alpha}{\alpha}\right)-1\right)^2 + \left(\frac{1}{2}\right)^3 \tau^4 \exp\left(\frac{2t^\alpha}{\alpha}\right)\left(\exp\left(\frac{2t^\alpha}{\alpha}\right)-1\right)^3 + \ldots$$

and

$$g(x,t) = \frac{2\tau \exp\left(\frac{2t^\alpha}{\alpha}\right)}{2+\tau\left(1-\exp\left(\frac{2t^\alpha}{\alpha}\right)\right)}.$$

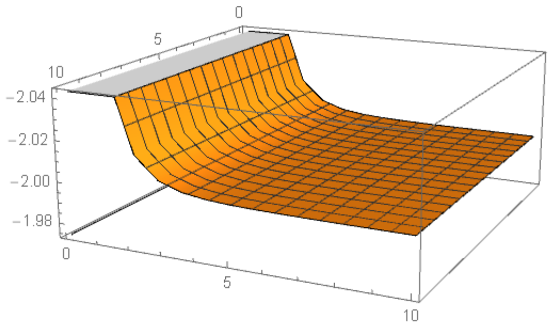

Fig. 9: Numerical simulation for $\alpha = 0.5$ and $\tau = 1$ for Example (6).

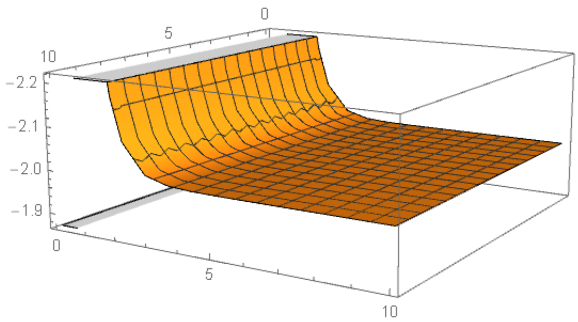

Fig. 10: Numerical simulation for $\alpha = 1$ and $\tau = 1$ for Example (6).

The Conformable NE Transform framework offers a powerful way to introduce fractional-order and nonlocal dynamics into cavity-QED and atom–field interaction models, with direct applications to quantum information tasks such as robust entanglement generation, phase-based encoding and decoherence control. By recasting two-mode and multiphoton Jaynes–Cummings variants in a conformable NE setting one can derive modified evolution operators and analytical expressions for Pancharatnam phase, field entropy and quasiprobability distributions that explicitly reveal the influence of fractional dynamics and Kerr-type nonlinearities on coherence and phase stability [23,27,31]. Those modified phase and entropy behaviors can be exploited for phase-based quantum gates and metrology (through enhanced or tunable Pancharatnam phase sensitivity) and for designing strategies to delay entanglement sudden death or create long-lived entangled states under dissipative conditions [28,30]. Moreover, conformable NE methods can yield compact formulas for entropy squeezing and entropic uncertainty measures, facilitating optimization of squeezing resources and measurement protocols in the presence of Stark shifts, second-harmonic processes or arbitrary nonlinear media [24,25, 26, 29]. Overall, applying the Conformable NE Transform to these models connects microscopic nonlinear and fractional parameters to operational quantum-information metrics (entanglement, entropy squeezing, phase fidelity), enabling targeted design of gates, sensors and decoherence-mitigation schemes in realistic cavity QED platforms [32].

5 Conclusion

The conformable NE transform was devised to provide a universal analytical solution for both linear and nonlinear conformable fractional differential equations. The proposed method appears promising within the framework of fractional calculus theory and can be applied to various linear and nonlinear situations. For the first time in here, the fractional Newell-Whitehead-Segel equation is solved using CNDM. It is evident from the applications that even the approximate three-step solutions to the nonlinear problems provide highly accurate results. Furthermore, this approach provides us with the approximate analytical solutions if an infinite number of terms are taken. This demonstrates that CNDM is a simple and efficient mathematical tool for obtaining the approximate analytical solutions of the given class of linear-nonlinear fractional PDEs. Moreover, CNDM is a potential approach for resolving additional nonlinear fractional PDEs, and it will serve as a guide for researchers studying approximate analytical solutions of fractional PDEs' problems. Graphs demonstrate that changes in the fractional-order of the derivative result in different solution characteristics, especially during convergence to the exact solution.

Availability of data and materials

All data that was used is included in the research.

Competing interests

On behalf of all authors, the corresponding author states that there is no conflict of interest.

Authors' contributions

All authors have contributed, read, and approved the manuscript.

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Ethics approval and consent to participate

Not applicable.

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