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Atomic Polarization in Cavity QED Subject to Spontaneous Emission at Finite Temperature

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Abstract: This work investigates the decoherence dynamics of a fundamental quantum-optical system: a two-level atom dispersively coupled to a quantized field mode. We present a comprehensive solution to the system's master equation, which incorporates not only the coherent dispersive interaction but also the dissipative effects of spontaneous emission into an environment at a finite temperature. By employing advanced superoperator techniques, we develop a method to efficiently unravel the combined unitary and non-unitary dynamics. Specifically, we calculate the time evolution of the atomic polarization, paying particular attention to the case where the field is initially prepared in a coherent state. Our approach rigorously captures the interplay between the field's quantum statistics and the thermal relaxation processes, offering new insights into the decoherence of non-classical states in a dissipative environment.

Keywords: Cavity QED, Spontaneous emission, Atomic polarization.

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1 Introduction

Quantum Electrodynamics (QED) is one of the most precise quantum field theories to describe how light and matter interact. The study of QED allows for very precise calculations of quantum-physical phenomena, whose experimental results coincide with theoretical predictions with great accuracy. In recent years, cavity experiments have also been conducted on a variety of solid-state systems resulting in many interesting applications, of which microlasers, photon bandgap structures and quantum dot structures in cavities [1], nanofiber quantum photonics [2], trapped atoms in cavity QED (CQED), [3, 4].

Probably the most prominent problem in CQED is the interaction between atoms and a quantized electromagnetic field inside a cavity [5,6]. Such interaction produces entanglement [7] which in turn allows the reconstruction of the quantized field in different scenarios [8,9] such as the presence of environments that introduce noise into the system under study [10].

The field measurement usually takes place either by the reconstruction of quasiprobability distribution functions, that even may be done classically [11] or via the characteristic function of the quantized field [8]. Among the variables we can calculate when studying QED systems are entropy, atomic polarization, and population inversion. Reconstruction when different environments affect the quantized field and/or the atom have been shown been shown to work via the recosntruction not of the Wigner function, that only in ideal cases may be measured, but of generalized quasiprobability distribution functions [12,13,14,15,16, 17,18].

Here we are interested in the studying quantum cavity fields in interaction with atoms when they undergo decay even in environments at finite temperature. We will study the quantum cavity subject to spontaneous emission losses, considering a two-level atom interacting with a single-mode electromagnetic field. We consider the case where the transition frequency between the atomic levels and the frequency of the electromagnetic field is very different, known as the dispersive interaction. We solve the master equation associated with the cavity and with the solution we calculate the atomic polarization.

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2 Master equation of a cavity QED subject to spontaneous emission at finite temperature

The master equation of a cavity QED subject to spontaneous emission at finite temperature [19], with a quantized electromagnetic field interacting dispersively with two-level atom [20], is given by (we consider $\hbar = 1$)

$$\begin{aligned}\frac{d\hat{\rho}}{dt} &= -i\left[\hat{H}_{eff},\hat{\rho}\right] \qquad (1)\\ &+ \frac{\gamma(1+\bar{n})}{2}\left(2\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+ - \hat{\rho}\hat{\sigma}_+\hat{\sigma}_- - \hat{\sigma}_+\hat{\sigma}_-\hat{\rho}\right)\\ &+\frac{\gamma\bar{n}}{2}\left(2\hat{\sigma}_+\hat{\rho}\hat{\sigma}_- - \hat{\rho}\hat{\sigma}_-\hat{\sigma}_+ - \hat{\sigma}_-\hat{\sigma}_+\hat{\rho}\right),\end{aligned}$$

where $\hat{H}_{eff} = \chi\hat{n}\hat{\sigma}_z$ is the so-called dispersive Hamiltonian [20] and $\gamma_1 = \frac{\gamma}{2}\left(\bar{n}_k+1\right)$, and $\gamma_2 = \frac{\gamma}{2}\bar{n}_k$ with

$$\bar{n}_k = \frac{1}{e^{\left(\frac{\nu_k}{k_\beta T}\right)}-1}, \qquad (2)$$

the average number of thermal photons and γ the spontaneous emission decay rate. The σ's are the usual Pauli spin matrices, while $\hat{n} = \hat{a}^\dagger\hat{a}$ is the so-called number operator with $\hat{a}$ and $\hat{a}^\dagger$ the annihilation and creation operators, respectively. We may define $\gamma_{12} = \gamma_2 - \gamma_1$ and $\gamma_3 = -\gamma_2 - \gamma_1$, such the above master equation may be rewritten as

$$\begin{aligned}\frac{d\hat{\rho}}{dt} &= i\chi\hat{\rho}\hat{n}\hat{\sigma}_z - i\chi\hat{\sigma}_z\hat{n}\hat{\rho} + 2\gamma_1\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+ + 2\gamma_2\hat{\sigma}_+\hat{\rho}\hat{\sigma}_- \qquad (3)\\ &+ \frac{\gamma_{21}}{2}\left(\hat{\sigma}_z\hat{\rho}+\hat{\rho}\hat{\sigma}_z\right)+\gamma_3\hat{\rho}.\end{aligned}$$

By defining the superoperators

$$\begin{aligned}\hat{R}\hat{\rho} &= i\chi\hat{\rho}\hat{n}\hat{\sigma}_z - i\chi\hat{\sigma}_z\hat{n}\hat{\rho}, \qquad (4)\\ \hat{G}_1\hat{\rho} &= 2\gamma_1\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+,\\ \hat{G}_2\hat{\rho} &= 2\gamma_2\hat{\sigma}_+\hat{\rho}\hat{\sigma}_-,\\ \hat{L}_2\hat{\rho} &= \frac{\gamma_{21}}{2}\left(\hat{\sigma}_z\hat{\rho}+\hat{\rho}\hat{\sigma}_z\right),\\ \hat{S}_1\hat{\rho} &= 2i\chi\left(\hat{n}\hat{\rho}-\hat{\rho}\hat{n}\right),\end{aligned}$$

the above master equation may be rewritten in the more compact form

$$\frac{d\hat{\rho}}{dt} = \hat{R}\hat{\rho}+\hat{G}_1\hat{\rho}+\hat{G}_2\hat{\rho}+\hat{L}_2\hat{\rho}+\gamma_3\hat{\rho}\hat{I}_A. \qquad (5)$$

A formal solution is then possible due to the fact that the relevant superoperators in this equation commute in such a way they close an algebra, in particular, we have that

$$\begin{aligned}\left[\hat{R},\hat{G}_1\right]\hat{\rho} &= \hat{S}_1\hat{G}_1\hat{\rho}, \qquad (6)\\ \left[\hat{R},\hat{G}_2\right]\hat{\rho} &= -\hat{S}_1\hat{G}_2\hat{\rho},\\ \left[\hat{G}_1,\hat{G}_2\right]\hat{\rho} &= -\frac{4\gamma_1\gamma_2}{\gamma_{21}}\hat{L}_2\hat{\rho},\\ \left[\hat{G}_1,\hat{L}_2\right]\hat{\rho} &= 2\gamma_{21}\hat{G}_1\hat{\rho},\\ \left[\hat{G}_2,\hat{L}_2\right]\hat{\rho} &= -2\gamma_{21}\hat{G}_2\hat{\rho},\end{aligned}$$

the commutators between the other superoperators (4) are zero.

3 Solution of the master equation

As just mentioned, a formal solution for Eq. (5) is strightforward

$$\hat{\rho}\left(t\right) = e^{\gamma_3 t}e^{\left(\hat{G}_1+\hat{L}_2+\hat{R}+\hat{G}_2\right)t}\hat{\rho}\left(0\right), \qquad (7)$$

that, because of the fact that the relevant superoperators in the argument of the above exponential commute to give the same superoperators allows us to write an ansatz

$$\hat{\rho}\left(t\right) = e^{f_0(t)}e^{f_1(t)\hat{G}_1}e^{f_2(t)\hat{L}_2}e^{f_3(t)\hat{R}}e^{f_4(t)\hat{G}_2}\hat{\rho}\left(0\right). \qquad (8)$$

In order to show the above factorization, we derive equation (8) with respect to time and obtain

$$\begin{aligned}\frac{d\hat{\rho}\left(t\right)}{dt} &= [\frac{d^{f_0(t)}}{dt}+\frac{df_1\left(t\right)}{dt}\hat{G}_1+\frac{df_2\left(t\right)}{dt}e^{f_1(t)\hat{G}_1}\hat{L}_2e^{-f_1(t)\hat{G}_1}\\ &+\frac{df_3\left(t\right)}{dt}e^{f_1(t)\hat{G}_1}\hat{R}e^{-f_1(t)\hat{G}_1}+\frac{df_4\left(t\right)}{dt}e^{f_1(t)\hat{G}_1} \qquad (9)\\ &\times e^{f_2(t)\hat{L}_2}e^{f_3(t)\hat{R}}\hat{G}_2e^{-f_3(t)\hat{R}}e^{-f_2(t)\hat{L}_2}e^{-f_1(t)\hat{G}_1}]\hat{\rho}\left(t\right).\end{aligned}$$

Using the Hadamard formula $e^{wA}Be^{-wA} = B+w[A,B]+\frac{w^2}{2!}[A,[A,B]]+\ldots$ to obtain the different terms needed in equation (9), namely,

$$\begin{aligned}&e^{f_1(t)\hat{G}_1}\hat{L}_2e^{-f_1(t)\hat{G}_1}\hat{\rho}\left(t\right) = \left[\hat{L}_2+2\gamma_{21}f_1\hat{G}_1\right]\hat{\rho}\left(t\right), \qquad (10)\\ &e^{f_1(t)\hat{G}_1}\hat{R}e^{-f_1(t)\hat{G}_1}\hat{\rho}\left(t\right) = \left[\hat{R}-f_1\hat{S}_1\hat{G}_1\hat{\rho}\right]\hat{\rho}\left(t\right),\\ &e^{f_1(t)\hat{G}_1}e^{f_2(t)\hat{L}_2}e^{f_3(t)\hat{R}}\hat{G}_2e^{-f_3(t)\hat{R}}e^{-f_2(t)\hat{L}_2}e^{-f_1(t)\hat{G}_1}\hat{\rho}\left(t\right)\\ &= e^{2\gamma_{21}f_2(t)-\hat{S}_1f_3(t)}\hat{G}_2\hat{\rho}\left(t\right)-\frac{4\gamma_1\gamma_2}{\gamma_{21}}f_1e^{2\gamma_{21}f_2(t)-\hat{S}_1f_3(t)}\hat{L}_2\hat{\rho}\left(t\right)\\ &-4\gamma_1\gamma_2f_1^2e^{2\gamma_{21}f_2(t)-\hat{S}_1f_3(t)}\hat{G}_1\hat{\rho}\left(t\right).\end{aligned}$$

By substituting these expressions into equation (9) and comparing it with equation (5) we obtain the following set of differential equations

$$\begin{aligned}&\frac{df_0\left(t\right)}{dt} = \gamma_3,\\ &\frac{df_1\left(t\right)}{dt}+2\gamma_{21}f_1\left(t\right)\frac{df_2\left(t\right)}{dt}-f_1\left(t\right)\frac{df_3\left(t\right)}{dt}\hat{S}_1\hat{\rho}\\ &-4\gamma_1\gamma_2f_1^2\left(t\right)e^{2\gamma_{21}f_2(t)-\hat{S}_1f_3(t)}\frac{df_4\left(t\right)}{dt} = 1,\\ &\frac{df_2\left(t\right)}{dt}-\frac{4\gamma_1\gamma_2}{\gamma_{21}}f_1\left(t\right)e^{2\gamma_{21}f_2(t)-\hat{S}_1f_3(t)}\frac{df_4\left(t\right)}{dt} = 1,\\ &\frac{df_3\left(t\right)}{dt} = 1,\\ &e^{2\gamma_{21}f_2(t)-\hat{S}_1f_3(t)}\frac{df_4\left(t\right)}{dt} = 1.\end{aligned}$$

The system of differential equations depends on the superoperator $\hat{S}_1$ but this commutes with the other superoperators defined in (4). Then a simple solution may be found for the different functions

$$
\begin{aligned}
f_0(t) &= \gamma_3 t, \qquad\qquad (11)\\
f_1(t) &= \frac{\left(be^{8a\gamma_1\gamma_2 t}+1\right)}{\left(be^{8a\gamma_1\gamma_2 t}-1\right)}a+\left(\frac{\hat{S}_1-2\gamma_{21}}{8\gamma_1\gamma_2}\right),\\
f_2(t) &= \frac{1}{2\gamma_{21}}\left(\ln\left[\frac{\left(be^{8a\gamma_1\gamma_2 t}-1\right)\left(b-e^{-8a\gamma_1\gamma_2 t}\right)}{(b-1)^2}\right]+\hat{S}_1 t\right),\\
f_3(t) &= t,\\
f_4(t) &= \frac{(b-1)^2}{8ab\gamma_1\gamma_2}\frac{1}{1-be^{8a\gamma_1\gamma_2 t}}+\frac{(b-1)}{8ab\gamma_1\gamma_2}.
\end{aligned}
$$

with

$$a^2=\left(\frac{1}{4\gamma_1\gamma_2}\right)+\left(\frac{\hat{S}_1-2\gamma_{21}}{8\gamma_1\gamma_2}\right)^2, \qquad (12)$$

and

$$b=\frac{\hat{S}_1-2\gamma_{21}-8a\gamma_1\gamma_2}{\hat{S}_1-2\gamma_{21}+8a\gamma_1\gamma_2}, \qquad (13)$$

we succeed in finding the coefficients (functions) needed in equation (8) which can be written more precisely as

$$\hat{\rho}(t)=e^{\gamma_3 t}e^{f_1(t,\hat{S}_1)\hat{G}_1}e^{f_2(t,\hat{S}_1)\hat{L}_2}e^{t\hat{R}}e^{f_4(t,\hat{S}_1)\hat{G}_2}\hat{\rho}(0). \qquad (14)$$

For the calculation of atomic polarization we will consider that the interaction between the field and the atom starts at $t=0$, therefore the density operator $\hat{\rho}(0)=\hat{\rho}_f(0)\hat{\rho}_A(0)$. We set the initial density operator of the field to be that for a coherent state, that is $\hat{\rho}_f(0)=|\alpha\rangle\langle\alpha|$ and that the initial density operator of the atom is $\hat{\rho}_A(0)=|\psi_A(0)\rangle\langle\psi_A(0)|$ with $|\psi_A(0)\rangle=\frac{1}{\sqrt{2}}(|e\rangle+|g\rangle)$, where $|e\rangle$ and $|g\rangle$ are the excited and ground states of the atom.With this, the density operator at $t=0$ becomes

$$\hat{\rho}(0)=\frac{e^{-|\alpha|^2}}{2}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{\alpha^k\alpha^{*l}}{\sqrt{k!l!}}|k\rangle\langle l|\left(\hat{I}+|e\rangle\langle g|+|g\rangle\langle e|\right), \qquad (15)$$

where we express the coherent states in terms of Fock states $|\alpha\rangle=e^{-\frac{|\alpha|^2}{2}}\sum\limits_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$.

The functions (11) include $\hat{S}_1$ and therefore it is necessary to know how $\hat{S}_1$ acts on the Fock states, when applied it is observed that $\hat{S}_1|k\rangle\langle l|=(2i\chi(k-l))|k\rangle\langle l|$ therefore, for any function that depends on $\hat{S}_1$, it holds that $F(\hat{S}_1)|k\rangle\langle l|=F(2i\chi(k-l))|k\rangle\langle l|$.

To evaluate $\hat{\rho}(t)$ considering $\hat{\rho}(0)$ given by (15) , it we will apply each of the exponentials that appear in (8). We begin by defining $\hat{\rho}_1(t)$ as $\hat{\rho}_1(t)=e^{f_4(t)\hat{G}_2}\hat{\rho}(0)$ To calculate it, we develop in series. $e^{f_4(t)\hat{G}_2}$

$$e^{f_4(t)\hat{G}_2}\hat{\rho}(0)=\sum_{s=0}^{\infty}\frac{[f_4(t,2i\chi(k-l))]^s}{s!}\hat{G}_2^s\hat{\rho}(0), \qquad (16)$$

applying $\hat{G}_2^s\hat{\rho}(0)$ shows that $\hat{G}_2^0\hat{\rho}(0)=\hat{\rho}(0),\hat{G}_2\hat{\rho}(0)=2\gamma_2\hat{\sigma}_+\hat{\rho}(0)\hat{\sigma}_-$ and how $\hat{\sigma}_-\hat{\sigma}_-=0$ and $\hat{\sigma}_+\hat{\sigma}_{=}0$ so $\hat{G}_2^2\hat{\rho}(0)=(2\gamma_2)^2\hat{\sigma}_+\hat{\sigma}_+\hat{\rho}(0)\hat{\sigma}_-\hat{\sigma}_-=0$ therefore, we conclude that the terms of the series with a power greater than 1 are zero. This results in the following

$$
\begin{aligned}
\hat{\rho}_1(t) &= \frac{e^{-|\alpha|^2}}{2}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{\alpha^{k+l}}{\sqrt{k!l!}}|k\rangle\langle l| \qquad (17)\\
&\quad [\hat{I}+|e\rangle\langle g|+|g\rangle\langle e|+2\gamma_2 f_4(t,2i\chi(k-l))|e\rangle\langle e|].
\end{aligned}
$$

Now we apply $e^{f_3(t)\hat{R}}$, for this we define $\hat{\rho}_2(t)=e^{f_3(t)\hat{R}}\hat{\rho}_1(t)$ [see equation (4)] such that it is not difficult to prove that $e^{\hat{R}t}\hat{\rho}(0)=e^{-i\chi\hat{n}\hat{\sigma}_z t}\hat{\rho}(0)e^{i\chi\hat{n}\hat{\sigma}_z t}$.

By applying this we obtain

$$
\begin{aligned}
\hat{\rho}_2(t) &= \frac{e^{-|\alpha|^2}}{2}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{\alpha^{k+l}}{\sqrt{k!l!}} \qquad (18)\\
&\quad [e^{-i\chi(k-l)t}|k\rangle\langle l||e\rangle\langle e|+e^{i\chi(k-l)t}|k\rangle\langle l||g\rangle\langle g|\\
&\quad +e^{-i\chi(k+l)t}|k\rangle\langle l||e\rangle\langle g|+e^{i\chi(k+l)t}|k\rangle\langle l||g\rangle\langle e|\\
&\quad +2\gamma_2 f_4(t,2i\chi(k-l))e^{-i\chi(k-l)t}|k\rangle\langle l||e\rangle\langle e|].
\end{aligned}
$$

We apply now $e^{f_2(t,\hat{S}_1)\hat{L}_2}$, for this we define $\hat{\rho}_3(t)$ as $\hat{\rho}_3(t)=e^{f_2(t,\hat{S}_1)\hat{L}_2}\hat{\rho}_2(t)$. Of (4) $\hat{L}_2\hat{\rho}=\frac{\gamma_{21}}{2}(\hat{\sigma}_z\hat{\rho}+\hat{\rho}\hat{\sigma}_z)$ and it's not difficult to prove that $e^{f_2(t,\hat{S}_1)\hat{L}_2}\hat{\rho}(0)=e^{\frac{\gamma_{21}}{2}f_2(t,\hat{S}_1)\hat{\sigma}_z}\hat{\rho}(0)e^{\frac{\gamma_{21}}{2}f_2(t,\hat{S}_1)\hat{\sigma}_z}$ is correct. Applying the above results we obtain

$$
\begin{aligned}
\hat{\rho}_3(t) &= \frac{e^{-|\alpha|^2}}{2}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{\alpha^{k+l}}{\sqrt{k!l!}}\\
&[e^{\gamma_{21}f_2(t,2i\chi(k-l))-i\chi(k-l)t}[1+2\gamma_2 f_4(t,2i\chi(k-l))]|k\rangle\langle l||e\rangle\langle e|\\
&+e^{i\chi(k-l)t-\gamma_{21}f_2(t,2i\chi(k-l))}|k\rangle\langle l||g\rangle\langle g| \qquad (19)\\
&+e^{-i\chi(k+l)t}|k\rangle\langle l||e\rangle\langle g|+e^{i\chi(k+l)t}|k\rangle\langle l||g\rangle\langle e|].
\end{aligned}
$$

Finally we apply $e^{\gamma_3 t}e^{f_1(t,\hat{S}_1)\hat{G}_1}$, with this $\hat{\rho}(t)$ may be written as $\hat{\rho}(t)=e^{\gamma_3 t}e^{f_1(t,\hat{S}_1)\hat{G}_1}\hat{\rho}_3(t)$. To calculate this, we develop in series. $e^{f_1(t,\hat{S}_1)\hat{G}_1}$,

$$e^{f_1(t,\hat{S}_1)\hat{G}_1}\hat{\rho}_3(t)=\sum_{s=0}^{\infty}\frac{f_1^s(t,\hat{S}_1)}{s!}\hat{G}_1^s\hat{\rho}_3(t), \qquad (20)$$

by applying $\hat{G}_1^s\hat{\rho}_3(t)$ and using that $\hat{G}_1^0\hat{\rho}=\hat{\rho}$, $\hat{G}_1\hat{\rho}=2\gamma_1\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+$ and because $\hat{\sigma}_-\hat{\sigma}_-=0$ and $\hat{\sigma}_+\hat{\sigma}_{=}0$ we have that $\hat{G}_1^2\hat{\rho}=(2\gamma_1)^2\hat{\sigma}_-\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+\hat{\sigma}_+=0$. therefore, we conclude that the terms of the series with a power greater than 1 are zero. $\hat{G}_1^2\hat{\rho}=(2\gamma_1)^2\hat{\sigma}_-\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+\hat{\sigma}_+=0$. Finally we obtain

$$
\begin{aligned}
\hat{\rho}(t) &= \frac{e^{\gamma_3 t-|\alpha|^2}}{2}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{\alpha^{k+l}}{\sqrt{k!l!}} \qquad (21)\\
&[e^{\gamma_{21}f_2(t,2i\chi(k-l))-i\chi(k-l)t}[1+2\gamma_2 f_4(t,2i\chi(k-l))]|k\rangle\langle l||e\rangle\langle e|\\
&+e^{-\gamma_{21}f_2(t,2i\chi(k-l))}e^{i\chi(k-l)t}|k\rangle\langle l||g\rangle\langle g|\\
&+e^{-i\chi(k+l)t}|k\rangle\langle l||e\rangle\langle g|+e^{i\chi(k+l)t}|k\rangle\langle l||g\rangle\langle e|\\
&+2\gamma_1 e^{\gamma_{21}f_2(t,2i\chi(k-l))-i\chi(k-l)t}[1+2\gamma_2 f_4(t,2i\chi(k-l))]|k\rangle\langle l||g\rangle\langle g|].
\end{aligned}
$$

4 Calculation of atomic polarization

Once we have calculated the total density matrix we may calculate any expectation value, in particular we can calculate the atomic polarization expectation valueas $\langle\hat{\sigma}_x\rangle$

$$\langle\hat{\sigma}_x\rangle=Tr\{\hat{\sigma}_x\hat{\rho}(t)\}=\begin{matrix}\sum_{m=0}^{\infty}[\langle e|\langle m|\hat{\sigma}_x\hat{\rho}(t)|m\rangle|e\rangle\\ +\langle g|\langle m|\hat{\sigma}_x\hat{\rho}(t)|m\rangle|g\rangle]\end{matrix} \qquad (22)$$

with the atomic polarization given by the operator $\hat{\sigma}_x = |e\rangle\langle g| + |g\rangle\langle e|$. Substituting the solution found for the density matrix in the equation (21). After some algebra, we obtain

$$\langle\hat{\sigma}_x\rangle = e^{\gamma_3 t-|\alpha|^2}\sum_{m=0}^{\infty}\frac{\alpha^{2m}}{m!}\cos(2\chi m t)\,. \tag{23}$$

Using this result, in the following figures (1-4) we show the behavior of atomic polarization in the system under study under the conditions stated in each figure. We see that the periodicity of the decaying revivals depends on the parameters χ, Figs. 1 and 3. that show that for a larger dispersive interaction parameter appear sooner.

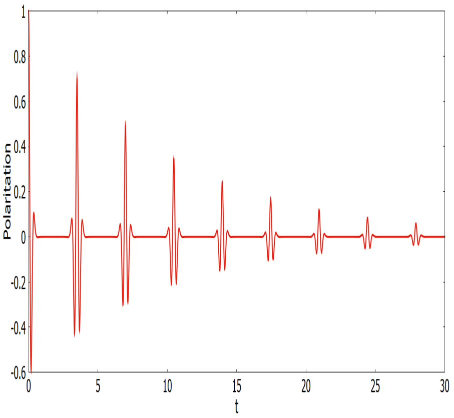

Fig. 1: We plot the polarization $\langle\hat{\sigma}_x\rangle$ as a function of time for the parameters $\gamma_1 = 0.001$, $\gamma_2 = 0.09$, $\alpha = 3$ and $\chi = 0.9$.

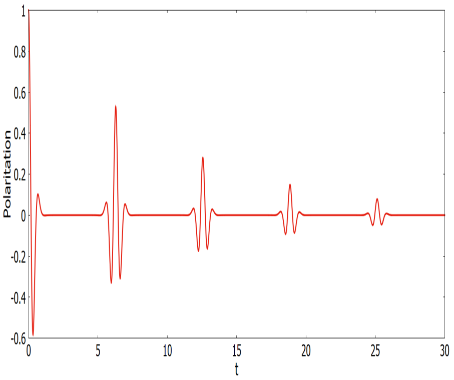

Fig. 2: We plot the polarization $\langle\hat{\sigma}_x\rangle$ as a function of time for the parameters $\gamma_1 = 0.001$, $\gamma_2 = 0.09$, $\alpha = 3$ and $\chi = 0.5$.

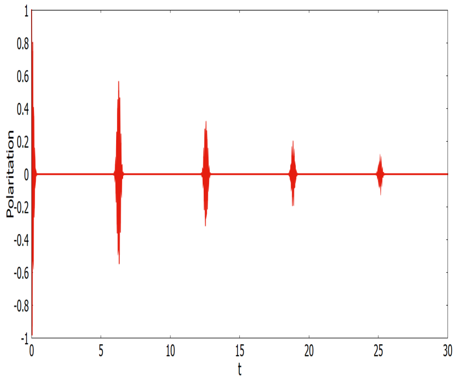

Fig. 3: We plot the polarization $\langle\hat{\sigma}_x\rangle$ as a function of time for the parameters $\gamma_1 = 0.001$, $\gamma_2 = 0.09$, $\alpha = 9$ and $\chi = 0.5$.

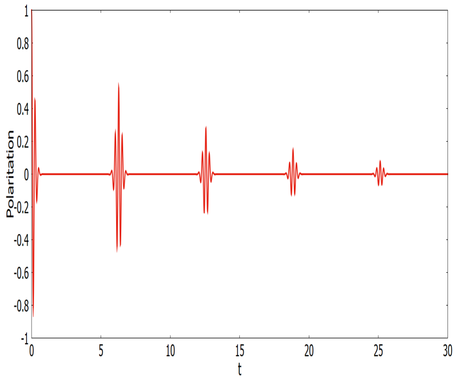

Fig. 4: We plot the polarization $\langle\hat{\sigma}_x\rangle$ as a function of time for the parameters $\gamma_1 = 0.001$, $\gamma_2 = 0.09$, $\alpha = 5$ and $\chi = 0.5$.

5 Conclusions

We have successfully solved the dynamics of a quantum system consisting of a two-level atom dispersively coupled to a quantized electromagnetic field mode inside a cavity. Our model incorporates the inevitable open-system effects of spontaneous emission, which are treated within a finite-temperature reservoir framework. To navigate the complexity of this mixed unitary and dissipative dynamics, we employed advanced superoperator techniques, allowing us to derive an exact solution to the governing master equation.

An analysis of the results, presented in the figures above, reveals two key dependencies governed by the system's primary parameters: the dispersive coupling constant, χ, and the amplitude of the initial coherent field, α.

Influence of dispersive coupling (χ) on collapses and revivals: The coupling constant χ, which quantifies the strength of the effective atom-field interaction, is found to be the primary control parameter for the phenomenon of quantum collapses and revivals.

Prolonged collapses: As χ approaches a specific threshold (near the value 1), we observe a significant lengthening of the initial collapse period. This indicates that the decoherence of the atomic coherence, driven by the quantum fluctuations of the field, is slowed down or takes on a different character as the coupling strengthens.

Accelerated revivals: On the contrary, the time at which the first quantum revival occurs—a purely quantum-mechanical effect where the initial coherence is temporarily restored—decreases as χ approaches this critical value. This suggests that a stronger coupling compresses the timescale of the quantum dynamics, leading to more rapid constructive interference between different number-state components of the field.

Influence of the Initial Field Amplitude, α, on the oscillation frequency: The initial state of the field, prepared as a coherent state with complex amplitude α (where $|\alpha|^2$ represents the mean photon number), dictates the frequency of the oscillatory behavior observed in atomic dynamics.

Our results demonstrate a direct scaling effect: a larger initial field amplitude (higher $|\alpha|$) results in a shorter period for the atomic oscillations. This is consistent with the physics of the dispersive regime, where the atomic frequency shift is proportional to the photon number. A larger α means a broader Poissonian distribution of photon numbers, leading to a wider range of frequency shifts and consequently faster dephasing and higher-frequency beat notes in the signal.

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