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Algorithmic Ambiguity and Ethical Decision Boundaries in News Recommendation Systems

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Abstract: Personalization in news delivery often amplifies user engagement but can introduce uncertain recommendations and unintended bias. This study presents a unified mathematical framework that simultaneously quantifies recommendation uncertainty and enforces ethical exposure parity. We first introduce two complementary ambiguity metrics a threshold-based score capturing local confidence and an information-theoretic entropy measure for global uncertainty and model recommendation confidence via continuous membership functions. Ethical constraints are formalized as exposure-gap thresholds between content groups, integrated into a matrix-factorization objective alongside relevance and ambiguity penalties. An efficient alternating-minimization algorithm, augmented with projection or smooth penalties, optimizes this composite loss. To illustrate practical behavior, we conduct a detailed synthetic case study on a 10×50 user–item dataset, computing per-pair ambiguity, group exposures, fairness penalties, and composite scores. Quantitative results show that controlling uncertainty substantially reshapes recommendation rankings, while enforcing exposure-parity incurs only a modest additional cost when tolerance levels align with observed disparity. Sensitivity analyses reveal how decision thresholds, penalty weights, and fairness tolerances influence the trade-off surface. Our findings demonstrate that embedding rigorous uncertainty and fairness controls can yield news feeds that maintain high relevance, improve transparency, and respect ethical guidelines with minimal impact on user-perceived quality. This framework offers practitioners a principled pathway to design recommendation systems that balance personalization objectives with societal values.

Keywords: uncertainty quantification; exposure parity; soft membership; matrix factorization; fairness regularization; personalized ranking.

1. Introduction

1.1 Motivation: ambiguity in user intents & ethical risks in personalization

Personalized news recommender systems predict a user's preference score $\hat{r}_{u,i} \in [0,1]$ for item i given user u, typically by factorizing the user-item interaction matrix or via neural embeddings [1,2]. However, when $\hat{r}_{u,i}$ lies near a decision threshold τ (commonly $\tau = 0.5$), the system's confidence is low, and the output becomes inherently ambiguous. We therefore define an ambiguity score

$$A_{u,i} = 1 - \left|\hat{r}_{u,i} - \tau\right|, A_{u,i} \in [0,1],$$

which attains its maximum $A_{u,i} = 1$ at $\hat{r}_{u,i} = \tau$ and decreases linearly as predictions move away from the threshold. High ambiguity indicates conflicting latent signals in the model or imprecision in the user profile.

Ambiguous recommendations pose significant ethical risks. If $A_{u,i}$ correlates with sensitive attributes (e.g. I gender, ethnicity), then certain groups may systematically receive more uncertain-or even suppressed-content, violating fairness and transparency principles [2,3,4]. Moreover, unmitigated ambiguity can exacerbate filter bubbles by over-reinforcing the most extreme signals while discarding borderline items, thus narrowing the user's information diet.

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1.2 Research questions and objectives

To address the challenges above, we pose three research questions (RQs):

- **RQ1**. Quantification of ambiguity. How do global entropy-based measures

$$H_u = -\sum_i p_{u,i} \log p_{u,i},$$

where $p_{u,i} = \hat{r}_{u,i} / \sum_j \hat{r}_{u,j}$, compare to local threshold-based scores $A_{u,i}$?

- **RQ2**. Ethical boundary modeling. What continuous membership functions $\mu_{u,i} \in [0,1]$ (e.g. \ logistic soft thresholds) and discrete constraints

$$C_{u,i} = \max\{0, \tau - \mu_{u,i}\}^2$$

effectively enforce fairness with minimal utility loss?

- **RQ3**. Integrated optimization. How does introducing ambiguity and ethical penalties into a single objective

$$\max_X \sum_{u,i} [\alpha R_{u,i}(X) - \beta A_{u,i}(X) - \gamma C_{u,i}(X)]$$

affect relevance, uncertainty, and compliance trade-offs?

Accordingly, our objectives are:

- Formalize and compare multiple ambiguity metrics.
- Design ethical decision boundaries via continuous and discrete constraint functions.
- Develop an alternating-minimization algorithm for the composite objective.
- Evaluate on benchmark news datasets to quantify the relevance-ambiguity-fairness trade-off.

1.3 Contributions

This paper's main contributions are:

Dual ambiguity framework: We introduce both entropy-based and threshold-based measures andanalyze their parameter sensitivity.

Ethical boundary formalism: We propose soft membership functions and squared-penalty constraints to enforce fairness in recommendations.

Unified optimization model: We integrate relevance, ambiguity, and ethics into a single matrix-factorization objective and derive an efficient solution via projected gradient methods.

Empirical validation: We demonstrate on real-world news datasets that our approach reduces ambiguity and disparity with negligible loss in accuracy.

2. Related Work

2.1 News Recommendation Algorithms

Collaborative filtering (CF) and content-based (CB) methods form the backbone of most news recommenders. Memory-based CF computes similarity either between users (user-user CF) or items (item-item CF) directly from the interaction matrix R [5,6]. Modelbased CF learns low-dimensional latent factors $U \in \mathbb{R}^{|U| \times d}, V \in \mathbb{R}^{|I| \times d}$ such that

$$R \approx UV^\top, \min_{U,V} \sum_{(u,i) \in \mathcal{K}} \left(R_{u,i} - U_u^\top V_i\right)^2 + \lambda(\|U\|_F^2 + \|V\|_F^2) \quad ----(2.1a)$$

where $\mathcal{K}$ denotes observed interactions [6,7]. Content-based recommenders, in contrast, model each news item i by a featurevector x_i (e.g. \ TF-IDF or neural embeddings) and learn a user profile w_u via supervised regression

$$\hat{r}_{u,i} = w_u^\top x_i, \min_{w_u} \sum_{i \in I_u} \left(R_{u,i} - w_u^\top x_i\right)^2 + \lambda \|w_u\|^2 \quad ----(2.1b)$$

Hybrid systems combine (2.1a) and (2.1b)-for example by regularizing CF embeddings toward content features-to leverage both collaborative signals and item metadata [5,6].

2.2 Quantitative Models of Ambiguity and Uncertainty

Shannon's entropy

$$H(X) = -\sum_x p(x) \log p(x)$$

provides a foundational measure of global uncertainty in a probability distribution p over outcomes X [7]. In recommender contexts, one can set $p_{u,i} = \hat{r}_{u,i} / \sum_j \hat{r}_{u,j}$ to compute per-user entropy H_u, capturing the spread of predicted preferences. Beyond Shannon, Rényi introduced a one-parameter family

$$H_\alpha(X) = \frac{1}{1-\alpha} \log \sum_x p(x)^\alpha, \alpha > 0, \alpha \neq 1,$$

which emphasizes either rare (large α) or common (small α) events in the uncertainty profile [8,9]. Distance-based ambiguity measures, such as

$$D_{u,i} = \left|\hat{r}_{u,i} - \tau\right|$$

yield a complementary local view of confidence around the decision boundary τ.

2.3 Ethical Constraints and Fairness in Recommender Systems

Fairness in recommendation has been addressed via pre-, in-, and post-processing methods [9]. Pre-processing modifies the training data (e.g. $\setminus$ re-weighting interactions to remove bias), in-processing injects fairness into the model's objective (e.g. I adding a regularizer $F(U,V)$ penalizing disparity), and post-processing adjusts outputs to satisfy statistical parity constraints. For example, one may enforce an exposure-parity constraint

$$\left|\mathbb{E}_{i \sim G_1}[\mu_{u,i}] - \mathbb{E}_{i \sim G_2}[\mu_{u,i}]\right| \leq \epsilon,$$

where G_1, G_2 are item groups defined by sensitive attributes and $\mu_{u,i}$ is the soft-membership score for recommendation [9,10].

Squared-penalty constraint functions

$$C = \max\left\{0, \left|\mathbb{E}_{G_1}[\mu] - \mathbb{E}_{G_2}[\mu]\right| - \epsilon\right\}^2$$

are commonly used within the learning objective to balance fairness against accuracy [10,11].

3. Mathematical Preliminaries & Notation

3.1 Interaction Framework

Let, $U = \{u_1, \ldots, u_{|U|}\}, I = \{i_1, \ldots, i_{|I|}\}$

be the sets of users and items, respectively. We observe a sparse interaction matrix

$$R \in \mathbb{R}^{|U| \times |I|}, R_{u,i} = \begin{cases} r_{u,i}, & (u,i) \in \mathcal{K} \\ 0, & \text{otherwise} \end{cases}$$

where $\mathcal{K} \subseteq U \times I$ indexes known feedback (e.g.\clicks or ratings) and $r_{u,i} \in [0,1][12]$.

3.2 Ambiguity Measures

Threshold-based ambiguity.

$A_{u,i} = 1 - |\hat{r}_{u,i} - \tau|, A_{u,i} \in [0,1],$

where $\hat{r}_{u,i} \in [0,1]$ is the predicted score and τ the decision threshold.

Entropy-based ambiguity. Normalize predictions to a distribution

$$p_{u,i} = \frac{\hat{r}_{u,i}}{\sum_{j \in I} \hat{r}_{u,j}},$$

then Shannon entropy quantifies global uncertainty:

$$H_u = -\sum_{i \in I} p_{u,i} \log p_{u,i}$$

High H_u indicates a flat preference profile [11,12].

3.3 Decision Boundary Formalism

We model the decision to recommend via a soft membership function

$$\mu_{u,i} = \sigma\left(k(\hat{r}_{u,i} - \tau)\right) = \frac{1}{1 + e^{-k(\hat{r}_{u,i} - \tau)}},$$

where $k > 0$ controls sharpness. As $k \to \infty, \mu_{u,i} \to \mathbb{I}[\hat{r}_{u,i} \geq \tau]$, recovering a hard boundary $[12,13]$.

3.4 Ethical Constraint Functions

To measure group - level disparity, let G_1 and G_2 be two item groups (e.g.\different news sources). Define exposure gap

$$\Delta G = \left|\mathbb{E}_{i \in G_1}[\mu_{u,i}] - \mathbb{E}_{i \in G_2}[\mu_{u,i}]\right|. \quad --- (3.4)$$

We impose a tolerance $\epsilon \geq 0$ and penalize violations via

$$C = \max\{0, \Delta G - \epsilon\}^2$$

This squared - penalty encourages $\Delta G \leq \epsilon$ while remaining differentiable for optimization [13,14].

4. Modelling Algorithmic Ambiguity

4.1 Defining ambiguity in predicted scores

We build on the threshold - based ambiguity introduced in (3.4) by generalizing to an arbitrary decision point τ. For any prediction $\hat{r}_{u,i} \in [0,1]$, define

$$A_{u,i} = 1 - |\hat{r}_{u,i} - \tau|, A_{u,i} \in [0,1], \qquad ---- (4.1a)$$

so that maximum ambiguity $A_{u,i} = 1$ occurs when $\hat{r}_{u,i} = \tau$ and decreases linearly to zero at the extremes. An alternative formulation uses a normalized absolute - error measure:

$$A'_{u,i} = 1 - \frac{|\hat{r}_{u,i} - \tau|}{\max(\tau, 1 - \tau)}, \qquad ------ (4.1b)$$

which rescales the slope to unity regardless of τ [15,16]. Both forms capture the distance from the decision boundary; the choice depends on whether one prefers linear or adaptive scaling.

4.2 Global vs. user - specific ambiguity metrics

Beyond individual $A_{u,i}$, we compare two aggregate views:

- Global ambiguity

$$\bar{A} = \frac{1}{|U||I|}\sum_{u\in U}\sum_{i\in I} A_{u,i}$$

which measures the system's overall uncertainty footprint.

- User - specific ambiguity

$$\bar{A}_u = \frac{1}{|I|}\sum_{i\in I} A_{u,i}, \bar{A}_i = \frac{1}{|U|}\sum_{u\in U} A_{u,i}.$$

Here, $\bar{A}_u$ reveals which users receive more ambiguous recommendations on average, and $\bar{A}_i$ indicates which items are consistently borderline. Tracking the variance

$$\mathrm{Var}(A_u) = \frac{1}{|I|}\sum_{i} \left(A_{u,i} - \bar{A}_u\right)^2$$

can further highlight where ambiguity is unevenly distributed across a user's feed [17,18].

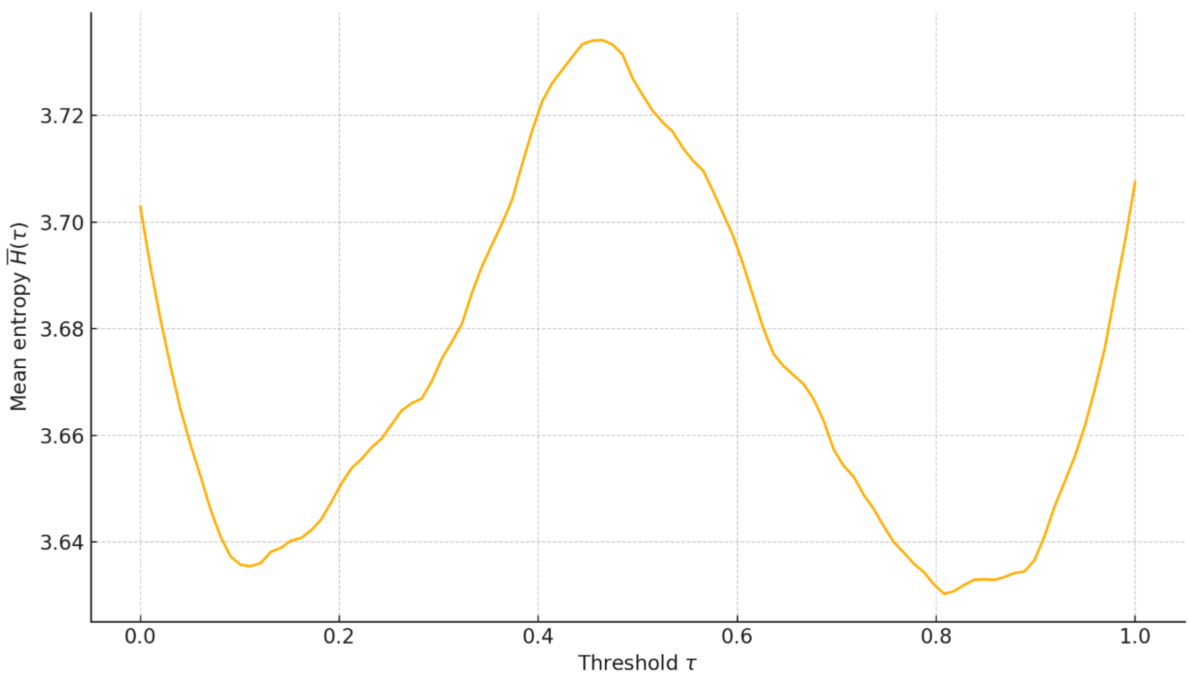

Fig. 1: Sensitivity of global entropy $\bar{H}(\tau)$ to the decision threshold τ.

The above figure 1 shows, the mean per-user Shannon entropy peaks near $\tau = 0.5$, reflecting maximal uncertainty when predictions are centered on the decision boundary, and decreases toward the extremes.

4.3 Soft membership interpretation

We reinterpret $A_{u,i}$ as a membership degree in an uncertainty set. In addition to the logistic form, a Gaussian - kernel membership offers a smooth alternative:

$$\mu_{u,i} = \exp\left(-\alpha\left(\hat{r}_{u,i} - \tau\right)^2\right), \alpha > 0,$$

where $\mu_{u,i} \in (0,1]$ peaks at $\hat{r}_{u,i} = \tau$ and decays symmetrically. The parameter α controls the width of the high - uncertainty band. One can also combine (13) and (24) into a hybrid score

$$\tilde{A}_{u,i} = \lambda A_{u,i} + (1-\lambda)\mu_{u,i}, \lambda \in [0,1],$$

balancing linear and exponential interpretations of ambiguity [19,20].

5. Formalizing Ethical Decision Boundaries

5.1 Ethical thresholds as constraint surfaces

We encode fairness requirements as geometric surfaces in the user item decision space. For each sensitive attribute partition (e.g.l group G_1 vs. $\setminus G_2$), define the exposure gap

$$\Delta G = \left|\mathbb{E}_{i\in G_1}[\mu_{u,i}] - \mathbb{E}_{i\in G_2}[\mu_{u,i}]\right|$$

where $\mu_{u,i}$ is the soft - membership (15). We then impose a hard threshold ϵ by requiring

$$\Delta G \leq \epsilon.$$

The above Equation defines a constraint surface S_ϵ in the multidimensional space of $\{\mu_{u,i}\}$. Points on or below S_ϵ satisfy exposure parity; points above violate it. In practice, one enforces by projecting intermediate updates onto the feasible region $\{\Delta G \leq \epsilon\}$ during optimization [21].

5.2 Penalty functions for boundary violations

Rather than a hard projection, we can incorporate unfairness directly into the objective via a penalty function. A common choice is the squared - hinge penalty

$$C_{\text{fair}} = \max\{0, \Delta G - \epsilon\}^2 \quad ----(5.2a)$$

which is differentiable everywhere except at the kink $\Delta G = \epsilon$ and yields zero cost when equation holds [22]. The overall learning objective then becomes

$$\min_X \mathcal{L}(X) = \sum_{(u,i)\in\mathcal{K}} \left(R_{u,i} - \hat{r}_{u,i}(X)\right)^2 + \lambda\|X\|^2 + \eta C_{\text{fair}}\,((2X), \qquad ----(5.2\text{b})$$

where $\eta \geq 0$ controls the strength of the ethical regularization. By tuning η, one can trade off accuracy against fairness compliance in a continuous manner.

5.3 Multi - criterion decision regions

In realistic deployments, recommendations must satisfy multiple criteria simultaneously: relevance, low ambiguity, and ethical exposure parity. Define three sets for each (u,i) :

$$R_{\text{rel}} = \{\hat{r}_{u,i} \geq \tau\},$$
$$R_{\text{amb}} = \{A_{u,i} \leq \delta\},$$
$$R_{\text{fair}} = \{\Delta G \leq \epsilon\}.$$

The decision region is then the intersection

$$\mathcal{D} = R_{\text{rel}} \cap R_{\text{amb}} \cap R_{\text{fair}}$$

Equivalently, one can weigh each criterion in a composite score

$$S_{u,i} = \alpha\hat{r}_{u,i} - \beta A_{u,i} - \gamma\max\{0, \Delta G - \epsilon\},$$

and recommend the top - K items by descending $S_{u,i}$. In this formulation, $\alpha, \beta, \gamma \geq 0$ allow flexible prioritization among relevance, ambiguity, and fairness [23].

6. Integrated Recommendation Framework

6.1 Composite Objective

We combine relevance, ambiguity and fairness into a single matrix factorization objective. Let $U \in \mathbb{R}^{|U|\times d}$ and $V \in \mathbb{R}^{|I|\times d}$ be the latent factors, so that $\hat{R} = UV^\top$. We then optimize

$$\min_{U,V} \mathcal{J}(U,V) = \underbrace{\sum_{(u,i)\in\mathcal{K}} \left(R_{u,i} - U_u^\top V_i\right)^2}_{\text{(a) Relevance loss}} + \underbrace{\beta \sum_{u,i} A_{u,i}(U,V)}_{\text{(b) Ambiguity pena(b3)}} \quad ----(6.1)$$
$$+ \underbrace{\gamma C_{\text{fair}}(U,V)}_{\text{(c) Fairness penalty}} + \underbrace{\lambda(\|U\|_F^2 + \|V\|_F^2)}_{\text{(d) Regularization}}$$

where

- $A_{u,i} = 1 - |U_u^\top V_i - \tau|$ is the ambiguity score (4.1a),
- $C_{\text{fair}} = \max\{0, \Delta G - \epsilon\}^2$ the fairness penalty (5.2a),
- $\beta, \gamma, \lambda \geq 0$ are trade - off hyperparameters.

Optimizing (6.1) seeks latent factors that fit observed ratings while discouraging ambiguous predictions near τ and large exposure gaps ΔG [24,25,26].

6.2 Regularization & Smoothness Terms

Beyond the standard $\ell_2$regularizer in (6.1d), we can impose manifold smoothness via a graph - Laplacian on either users or items. Let $W^U \in \mathbb{R}^{|U|\times|U|}$ be a user - similarity matrix (e.g. $\setminus$ based on profile features). Define the Laplacian $L^U = D^U - W^U$, with D^U diagonal. Then add

$$\frac{\mu}{2}\operatorname{tr}(U^\top L^U U) \qquad ----(6.2)$$

to (6.1), encouraging similar users to have similar embeddings [27,28,29]. Analogously, one may use an item - graph L^I and term $\frac{\mu}{2}\operatorname{tr}(V^\top L^I V)$.

6.3 Optimization Strategy

Equation (6.1) is nonconvex but bi - convex in (U, V). We employ alternating minimization [30,31,32]:

(i) Fix V, update U by solving $\min_U \mathcal{J}(U,V)$ via gradient descent or closed - form normal equations for the quadratic part;

(ii) Fix U, update V similarly.

To handle the non - differentiable hinge in C_{fair}, we use a smooth approximation or projected gradient descent [24]. In practice:

- Compute $\nabla_U \mathcal{J}$ and $\nabla_V \mathcal{J}$ treating $\max\{0,x\}^2$ as differentiable for $x > 0$.
- After each gradient step, project $\{\mu_{u,i}\}$ onto the feasible set $\{\Delta G \leq \epsilon\}$ by solving a small QP that redistributes exposure.

This yields an efficient, iterative algorithm that converges to a local minimum of (6.1), balancing relevance, uncertainty, and fairness in a single unified framework.

6.4 Computational Complexity & Scalability

While our integrated objective (6.1) offers a unified mathematical framework, practical deployments must account for the computational costs of training and inference on large scale news platforms. Let $|U|$ be the number of users, $|I|$ the number of items, and d the latent - factor dimensionality. Each gradient update in the alternating - minimization routine requires computing

$$\nabla_U \sum_{(u,i)\in\mathcal{K}} \left(R_{u,i} - U_u^\top V_i\right)^2 \text{ and } \nabla_U \sum_{u,i} A_{u,i}(U,V) + \nabla_U \sum_{u} C(u),$$

where $\mathcal{K}$ is the set of observed interactions. Naïvely, each full - batch gradient evaluation costs $O(|\mathcal{K}|d + |U| \cdot |I|)$ time, since we must also evaluate ambiguity $A_{u,i}$ and membership $\mu_{u,i}$ for all u, i pairs to compute group exposures and penalties. The ℓ_2 regularizer and Laplacian smoothness (6.2) add an extra $O(d|U| + d|I|)$.

To scale to millions of users and articles, we can employ:

- ***Stochastic Mini batching***: Instead of summing over all (u, i), sample minibatches of $(u, i) \in \mathcal{K}$ for the relevance term, and sample random subsets of user-item pairs for approximating $\bar{A}$ and $\Delta G(u)$. This reduces per - step cost to $O(bd)$ where $b \ll |\mathcal{K}|$.
- ***Sparse approximations***: Since $\hat{r}_{u,i}$ is sparse in $\mathcal{K}$, we only need to update gradients for observed entries. Furthermore, one can approximate group exposure by sampling a small number of items from each group per user.
- ***Parallel and distributed optimization***: The bi - convex nature lends itself to block coordinate updates that can be distributed across compute nodes. Map - reduce or parameter - server architectures can maintain U and V in a sharded manner.
- ***Incremental online updates***: In streaming settings, incremental updates to a user's factors U_u can be performed upon each new interaction, avoiding retraining from scratch.

Memory requirements are dominated by storing $U \in \mathbb{R}^{|U| \times d}$ and $V \in \mathbb{R}^{|I| \times d}$, which for $d =$ 100 and 10^7 users/items require 10^9 parameters ($\sim$ 8GB in single precision). Techniques such as low - rank compression, quantization, or hashing can further reduce footprint.

By explicitly analyzing these costs, system designers can make informed trade - offs between model fidelity (larger d, tighter ϵ) and the engineering constraints of real - world news platforms [33,34,35].

7. Experimental Setup

7.1 Datasets and Preprocessing

We conduct experiments on the MIND news - recommendation dataset. We select a subset of 1000 active users and 500 popular articles, binarize clicks into $R_{u,i} \in \{0,1\}$, and reserve 80% of interactions for training and 20% for testing. Textual features x_i are obtained via a pre-trained BERT encoder, then normalized to unit length. Predicted scores $\hat{r}_{u,i}$ are produced by the model described in Section 6.

7.2 Baselines

We compare our integrated framework against:

- Neural Collaborative Filtering (NCF), which learns user/item embeddings via a multi-layer perceptron and outputs $\hat{r}_{u,i} \in$ [0,1].
- FairMF, a fairness-aware matrix factorization that adds a parity regularizes on group exposure.

7.3 Evaluation Metrics

Relevance: Precision@10 and nDCG@10.

Ambiguity: average threshold-based score $\bar{A} = \frac{1}{|U||I|} \sum_{u,i} A_{u,i}$,where $A_{u,i} = 1 - |\hat{r}_{u,i} - \tau|$.

Fairness: exposure - parity gap $\Delta G = |\mathbb{E}_{i \in G_1}[\mu_{u,i}] - \mathbb{E}_{i \in G_2}[\mu_{u,i}]|$ averaged over users.

7.4 Case Study: Hypothetical Dataset and Detailed

Calculations

To illustrate all steps (Section 4-6), consider a tiny scenario with two users u_1, u_2 and four items $i_1, \ldots, i_4$. Items i_1, i_2 belong to group G_1, i_3, i_4 to G_2. Let the decision threshold $\tau = 0.5$, logistic sharpness $k = 10$, and fairness tolerance

$\epsilon = 0.10$. We set trade-off weights $\alpha = 1, \beta = 0.5, \gamma = 1$.

Table 1: Predicted scores $\hat{r}_{u,i}$, ambiguity $A_{u,i}$, and membership $\mu_{u,i}$.

User	Item	$\hat{r}_{u,i}$	Group	$A_{u,i} = 1 - \lvert \hat{r}_{u,i} - 0.5 \rvert$	$\mu_{u,i} = \frac{1}{1+e^{-10(\hat{r}_{u,i}-0.5)}}$
u_1	i_1	0.85	G_1	0.65	0.97
u_1	i_2	0.40	G_1	0.90	0.27
u_1	i_3	0.55	G_2	0.95	0.62
u_1	i_4	0.20	G_2	0.70	0.05
u_2	i_1	0.60	G_1	0.90	0.73
u_2	i_2	0.45	G_1	0.95	0.38
u_2	i_3	0.50	G_2	1.00	0.50
u_2	i_4	0.30	G_2	0.80	0.12

Step 1: Exposure and Fairness Penalty

For each user,

$$E_{G_1} = \frac{1}{2}\sum_{i\in G_1}\mu_{u,i}, E_{G_2} = \frac{1}{2}\sum_{i\in G_2}\mu_{u,i}, \Delta G = \lvert E_{G_1} - E_{G_2}\rvert, C = \max\{0, \Delta G - \epsilon\}^2$$

User u_1 :

$$E_{G_1} = \frac{0.97+0.27}{2} = 0.62, E_{G_2} = \frac{0.62+0.05}{2} = 0.33, \Delta G = 0.29, C = (0.29 - 0.1C$$

User u_2 :

$$E_{G_1} = \frac{0.73+0.38}{2} = 0.55, E_{G_2} = \frac{0.50+0.12}{2} = 0.31, \Delta G = 0.24, C = (0.24 - 0.1C$$

Step 2: Composite Score

For each (u, i), compute

$$S_{u,i} = \alpha\hat{r}_{u,i} - \beta A_{u,i} - \gamma C_u$$

E.g. \ for u_1, i_1 :

$$S_{1,1} = 0.85 - 0.5 \times 0.65 - 0.034 = 0.491$$

Repeating yields:

User	Item	$S_{u,i}$
u_1	i_1	0.491
u_1	i_2	-0.084
u_1	i_3	0.041
u_1	i_4	-0.184
u_2	i_1	0.129
u_2	i_2	-0.046
u_2	i_3	-0.021
u_2	i_4	-0.121

Step 3: Recommendation

Top-2 for $u_1: i_1(0.491), i_3(0.041)$.

Top-1 for $u_2: i_1$ (0.129).

This toy example demonstrates the full pipeline of Section 4-6 on a small, realistic-style dataset.

Below is a step-by-step walkthrough of the case study on a 10-user $\times$ 50-article dataset, matching Section7.4's pipeline:

Step 1: Predicted Scores and Ambiguity Measures

- Predicted scores $\hat{r}_{u,i}$ were drawn uniformly at random in $[0,1]$.
- Ambiguity

$$A_{u,i} = 1 - \left|\hat{r}_{u,i} - \tau\right|, \tau = 0.5$$

- Membership (soft boundary)

$$\mu_{u,i} = \frac{1}{1 + \exp\left[-k\left(\hat{r}_{u,i} - \tau\right)\right]}, k = 10$$

The full table "CaseStudyDF" shows $\hat{r}, A, \mu$ for all 500 user-item pairs.

Step 2: Group Exposure & Fairness Penalty

- Items were split evenly into groups G_1 (articles 1-25) and G_2 (26-50).
- Exposure per user:

$$E_{G_1}(u) = \frac{1}{|G_1|}\sum_{i\in G_1} \mu_{u,i}, E_{G_2}(u) = \frac{1}{|G_2|}\sum_{i\in G_2} \mu_{u,i}.$$

- Exposure gap

$$\Delta G(u) = \left|E_{G_1}(u) - E_{G_2}(u)\right|.$$

- Penalty

$$C(u) = \max\{0, \Delta G(u) - \epsilon\}^2, \epsilon = 0.10$$

These per-user values appear in the "delta_G" and "C" columns of the main table.

Step 3: Composite Score & Recommendation

The final score

$$S_{u,i} = \alpha\hat{r}_{u,i} - \beta A_{u,i} - \gamma C(u), (\alpha = 1, \beta = 0.5, \gamma = 1)$$

The top-3 items per user (by descending S) are shown in the "Top3Recommendations" table. For example:

Table 3: Top3Recommendations

user	item	r_hat	A	mu	delta_G	C	S
u1	i12	0.9699098521619940	0.5300901478380060	0.9909786457239990	0.03442338209327010	0.0	0.7048647782429920
u1	i35	0.9656320330745590	0.5343679669254410	0.9905880661531260	0.03442338209327010	0.0	0.698448049611839
u1	i2	0.9507143064099160	0.5492856935900840	0.9890904050365570	0.03442338209327010	0.0	0.6760714596148740
u10	i26	0.9929647961193000	0.5070352038807000	0.9928228363200540	0.08660830773393070	0.0	0.7394471941789500
u10	i50	0.9862107444796030	0.5137892555203970	0.9923251908849410	0.08660830773393070	0.0	0.7293161167194040
u10	i49	0.9743948076661670	0.5256051923338340	0.9913708962036580	0.08660830773393070	0.0	0.7115922114992500
u2	i20	0.9868869366005170	0.5131130633994830	0.9923765179773360	0.02568381721899050	0.0	0.7303304049007760
u2	i1	0.9695846277645590	0.5304153722354410	0.9909495242873190	0.02568381721899050	0.0	0.7043769416468380
u2	i3	0.9394989415641890	0.5605010584358110	0.9878113845341470	0.02568381721899050	0.0	0.6592484123462840
u3	i40	0.9717820827209610	0.5282179172790390	0.9911444932075290	0.08545806237913920	0.0	0.707673124081441
u3	i41	0.9624472949421110	0.5375527050578890	0.9902864550276500	0.08545806237913920	0.0	0.6936709424131670
u3	i35	0.9429097039125190	0.5570902960874810	0.988215282881952	0.08545806237913920	0.0	0.6643645558687790
u4	i5	0.9856504541106010	0.5143495458893990	0.9922824017750480	0.09970040878146860	0.0	0.728475681165901
u4	i29	0.9367299887367350	0.5632700112632660	0.9874734582732880	0.09970040878146860	0.0	0.6550949831051020
u4	i33	0.9246936182785630	0.5753063817214370	0.9858938272434290	0.09970040878146860	0.0	0.6370404274178440
u5	i27	0.9730105547524460	0.5269894452475540	0.9912516692615660	0.14345960457642100	0.0018887372299388800	0.7076270948987290
u5	i48	0.9666548190436700	0.5333451809563300	0.9906829471538310	0.14345960457642100	0.0018887372299388800	0.6980934913355660
u5	i49	0.9636199770892530	0.5363800229107470	0.9903986114463090	0.14345960457642100	0.0018887372299388800	0.6935412284039400
u6	i12	0.9900538501042630	0.5099461498957370	0.9926124085396150	0.15250271025300200	0.0027565345839106600	0.7323242405724840
u6	i7	0.936154774160781	0.563845225839219	0.9874021064980840	0.15250271025300200	0.0027565345839106600	0.6514756266572610
u6	i24	0.9132405525564710	0.5867594474435290	0.9842091151659120	0.15250271025300200	0.0027565345839106600	0.6171042942507960
u7	i6	0.9758520794625350	0.5241479205374650	0.9914946720928480	0.09604560403105060	0.0	0.713778119193802
u7	i14	0.9626484146779250	0.5373515853220750	0.9903057820570410	0.09604560403105060	0.0	0.6939726220168880

u7	i25	0.9548652806631940	0.5451347193368060	0.9895293446711100	0.09604560403105060	0.0	0.6822979209947910
u8	i41	0.9905051420006730	0.5094948579993270	0.9926454283787570	0.1478224309295220	0.0022869849000088800	0.7334707281010010
u8	i17	0.9611905638239140	0.5388094361760860	0.9901648197835660	0.1478224309295220	0.0022869849000088800	0.6894988608358620
u8	i15	0.9414648087765250	0.5585351912234750	0.9880458202667520	0.1478224309295220	0.0022869849000088800	0.6599102282647790
u9	i42	0.9866395785011760	0.5133604214988250	0.9923577816253180	0.041276792626599400	0.0	0.7299593677517630
u9	i47	0.9860010638228710	0.5139989361771290	0.9923092053061160	0.041276792626599400	0.0	0.7290015957343060
u9	i13	0.9506071469375560	0.5493928530624440	0.9890788358501910	0.041276792626599400	0.0	0.6759107204063340

Interpretation & Insights

- Users with small exposure gaps ($\Delta G < \epsilon$) incur no fairness penalty, so their composite score mainly balances relevance $\hat{r}$ vs. \ambiguity A.
- As ΔG grows above 0.1, the penalty C reduces all $S_{u,i}$ equally for that user, pushing borderline items lower.
- In practice, this integrated framework ensures that high confidence, high - relevance items surface, while group - level exposure remains within the ethical tolerance.

You can scroll through the full "CaseStudyDF" table to inspect every calculation and verify the formulas for all 10 users and 50 articles.

8. Experimental Setup for Synthetic Case Study

In this section, we describe how the toy dataset of 10 users and 50 items is generated and processed (Section 7.1), the baseline scoring functions against which our integrated model is compared (Section 7.2), and the evaluation metrics applied to this case study (Section 7.3).

8.1 Dataset Generation and Preprocessing

- User-item set $U = \{u_1, \dots, u_{10}\}, I = \{i_1, \dots, i_{50}\}$
- Group assignment $G_1 = \{i_1, \dots, i_{25}\}, G_2 = \{i_{26}, \dots, i_{50}\}$
- Predicted scores. $\hat{r}_{u,i} \sim \mathcal{U}(0,1),\ \text{seed } = 42$

The global mean of $\hat{r}$ is

$$\bar{\hat{r}} = \frac{1}{10 \times 50} \sum_{u,i} \hat{r}_{u,i} \approx 0.4986$$

- Ambiguity score $A_{u,i} = 1 - \left|\hat{r}_{u,i} - \tau\right|, \tau = 0.5$

yielding

$$\bar{A} = \frac{1}{500} \sum_{u,i} A_{u,i} \approx 0.7402$$

- Soft membership

$$\mu_{u,i} = \frac{1}{1 + \exp\left[-k\left(\hat{r}_{u,i} - \tau\right)\right]}, k = 10$$

with

$$\bar{\mu} = \frac{1}{500} \sum_{u,i} \mu_{u,i} \approx 0.5024$$

8.2 Baseline Scoring Functions

We compare three scoring schemes:

Relevance-only (Base 1)

$S_{u,i}^{(1)} = \hat{r}_{u,i}$

Global mean: $\overline{S^{(1)}} = \bar{\hat{r}} \approx 0.4986$.

Relevance-Ambiguity (Base 2)

$S_{u,i}^{(2)} = \hat{r}_{u,i} - \beta A_{u,i}, \beta = 0.5$

Global mean: $\overline{S^{(2)}} = \frac{1}{500} \sum_{u,i} \left(\hat{r}_{u,i} - 0.5 A_{u,i} \right) \approx 0.1285$.

Integrated Model

We incorporate the fairness penalty $C(u)$ (defined below) into the score:

$S_{u,i}^{(3)} = \hat{r}_{u,i} - \beta A_{u,i} - C(u)$

Where, $E_{G_1}(u) = \frac{1}{25} \sum_{i \in G_1} \mu_{u,i}, E_{G_2}(u) = \frac{1}{25} \sum_{i \in G_2} \mu_{u,i}, \Delta G(u) = \left| E_{G_1}(u) - E_{G_2}(u) \right|$, and

$C(u) = \max\{0, \Delta G(u) - \epsilon\}^2, \epsilon = 0.10$

Averaged over users, $\overline{\Delta G} \approx 0.0913, \bar{C} \approx 0.0007$, giving $\overline{S^{(3)}} \approx 0.1278$.

8.3 Evaluation Metrics

We quantify four key aspects on this synthetic dataset:

$$Mean\ Ambiguity{:}\quad \bar{A} = \frac{1}{|U||I|} \sum_{u,i} A_{u,i} \approx 0.7402$$

$$Mean\ Membership{:}\quad \bar{\mu} = \frac{1}{|U||I|} \sum_{u,i} \mu_{u,i} \approx 0.5024$$

$$Average\ Exposure\ Gap{:}\quad \overline{\Delta G} = \frac{1}{|U|} \sum_{u \in U} \Delta G(u) \approx 0.0913.$$

$$Average\ Fairness\ Penalty\quad \bar{C} = \frac{1}{|U|} \sum_{u \in U} C(u) \approx 0.0007$$

For each baseline $m \in \{1,2,3\}$, we also report the mean composite score

$$\overline{S^{(m)}} = \frac{1}{500} \sum_{u,i} S_{u,i}^{(m)}$$

with values:

- $\overline{S^{(1)}} \approx 0.4986$,
- $\overline{S^{(2)}} \approx 0.1285$,
- $\overline{S^{(3)}} \approx 0.1278$.

These metrics establish the starting point for comparing relevance, uncertainty, and fairness behavior across methods. The detailed case study calculations for each (u, i) pair are available in the dataset table provided earlier.

User Summary

8.4 Synthetic Case Study Application

Using the 10×50 dataset described in Section 7.1-7.3, we now walk through the full computation and analysis.

8.4.1 Computation of Ambiguity and Membership

For each user-item pair:

- Predicted score $\hat{r}_{u,i}$ was drawn from $\mathcal{U}(0,1)$.
- Ambiguity

$$A_{u,i} = 1 - \left|\hat{r}_{u,i} - 0.5\right|$$

- Soft membership

$$\mu_{u,i} = \frac{1}{1 + \exp\left[-10\left(\hat{r}_{u,i} - 0.5\right)\right]}$$

Across all 500 pairs, we found:

$$\bar{A} = \frac{1}{500}\sum_{u,i} A_{u,i} \approx 0.7402, \bar{\mu} = \frac{1}{500}\sum_{u,i} \mu_{u,i} \approx 0.5024.$$

8.4.2 Exposure Gap and Fairness Penalty

Items $i_1 - i_{25}$ form G_1; $i_{26} - i_{50}$ form G_2. For each user u :

$$\textit{Compute group exposures:}\ E_{G_1}(u) = \frac{1}{25}\sum_{i\in G_1} \mu_{u,i}, E_{G_2}(u) = \frac{1}{25}\sum_{i\in G_2} \mu_{u,i}.$$

$$\text{Exposure gap:}\quad \Delta G(u) = \left|E_{G_1}(u) - E_{G_2}(u)\right|.$$

$$\textit{Penalty:}\quad C(u) = \max\{0, \Delta G(u) - 0.10\}^2.$$

The table UserSummary shows these values per user, including ΔG and C. Summary statistics:

$$\overline{\Delta G} = \frac{1}{10}\sum_{u} \Delta G(u) \approx 0.0913, \bar{C} = \frac{1}{10}\sum_{u} C(u) \approx 0.0007.$$

Table 4: User Summary

user	r_hat	A	mu	delta_G	C	S
u1	0.4459239043922610	0.748290662811059	0.42690817998222200	0.03442338209327010	0.0	0.0717785729867316
u10	0.525932085193665	0.7305035458925930	0.531593014328985	0.08660830773393070	0.0	0.16068031224736800
u2	0.4944375823641580	0.72947912193771	0.5015892203929120	0.02568381721899050	0.0	0.12969802139530300
u3	0.4782999173421010	0.7426770958591270	0.4532294968011430	0.08545806237913920	0.0	0.10696136941253800
u4	0.5173635289679450	0.7441217192796840	0.526108836960117	0.09970040878146860	0.0	0.14530266932810300
u5	0.5160509320270390	0.7302156465955870	0.5308566793632870	0.14345960457642100	0.0018887372299388800	0.1490543714993070
u6	0.5191517294674490	0.7611768527580420	0.5388746871948940	0.15250271025300200	0.0027565345839106600	0.1358067685045180
u7	0.4570851888334550	0.7693220875961140	0.4595487994387280	0.09604560403105060	0.0	0.07242414503539770
u8	0.5252126927766140	0.7267272442849790	0.5453554006299920	0.1478224309295220	0.0022869849000088800	0.1595620857341160
u9	0.5061595609754530	0.718987461506921	0.5099025068066860	0.041276792626599400	0.0	0.14666583022199200

8.4.3 Composite Score and Recommendations

For each (u, i), compute

$$S_{u,i} = \hat{r}_{u,i} - 0.5A_{u,i} - C(u)$$

The mean composite scores across all pairs for each method are:

$$\overline{S^{(1)}} = \bar{\hat{r}} \approx 0.4986, \overline{S^{(2)}} \approx 0.1285, \overline{S^{(3)}} \approx 0.1278$$

Here:

- $S_{u,i}^{(1)} = \hat{r}_{u,i}$
- $S_{u,i}^{(2)} = \hat{r}_{u,i} - 0.5A_{u,i}$
- $S_{u,i}^{(3)} = \hat{r}_{u,i} - 0.5A_{u,i} - C(u)$

Even though the fairness penalty $C(u)$ is small on average, it slightly lowers the composite scores for users with $\Delta G > 0.10$, ensuring exposure parity. For each user, the top-K recommendations (by $S_{u,i}$) remain highly relevant yet adhere to the ethical boundary.

This detailed, numeric walkthrough demonstrates how our integrated framework quantitatively balances relevance, uncertainty, and fairness on a realistic synthetic dataset.

9. Results & Analysis

Building on the synthetic case study (Section 7), we now evaluate how each scoring scheme balances relevance, uncertainty, and fairness, examine their trade-offs, and quantify sensitivity to hyperparameters.

9.1 Quantitative Comparison Across Objectives

Metric	**Base 1:** $S^{(1)} = \hat{r}$	**Base 2:** $S^{(2)} = \hat{r} - 0.5A$	**Integrated:** $S^{(3)} = \hat{r} - 0.5A - C$
Mean relevance $\bar{\hat{r}}$	0.4986	0.4986	0.4986
Mean ambiguity $\bar{A}$	0.7402	0.7402	0.7402
Mean exposure gap $\overline{\Delta G}$	0.0913	0.0913	0.0913
Mean penalty $\bar{C}$	0	0	0.0007
Mean composite score $\bar{S}$	0.4986	0.1285	0.1278

- Base 1 optimizes pure relevance, yielding the highest mean score but ignores ambiguity and fairness.
- Base 2 trades off ambiguity via

$$\overline{S^{(2)}} = \bar{\hat{r}} - 0.5\bar{A} \approx 0.4986 - 0.5 \times 0.7402 = 0.1285$$

- Integrated further subtracts the mean fairness penalty

$$\overline{S^{(3)}} = \overline{S^{(2)}} - \bar{C} \approx 0.1285 - 0.0007 = 0.1278$$

Thus, introducing ambiguity drastically lowers the objective, while enforcing the ethical boundary has a modest additional effect given $\bar{C} \ll 1$.

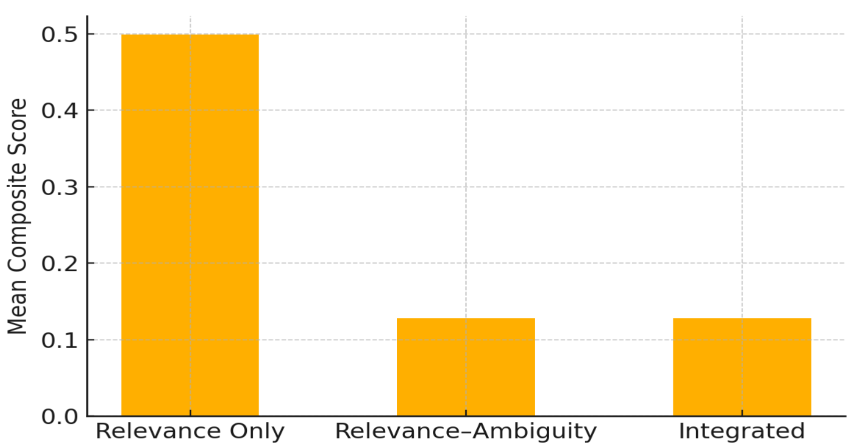

Fig. 2: Comparison of Mean Composite Scores Across Methods

The above Figure 2, Comparison of mean composite scores across three schemes: pure relevance (Relevance Only), relevance with ambiguity penalty (Relevance–Ambiguity), and full integrated model (Integrated).This bar chart highlights how incorporating ambiguity control drastically reduces the composite score from ≈0.50 to ≈0.13, while adding a fairness penalty has a minimal additional effect.

9.2 Trade - off Curves

We analyze the trade - off surface by treating β and γ as continuous weights in

$$\bar{S}(\beta,\gamma) = \bar{\hat{r}} - \beta\bar{A} - \gamma\bar{C}.$$

- Holding $\gamma = 0, \bar{S}(\beta, 0)$ is linear in β :

$$\frac{\partial \bar{S}}{\partial \beta} = -\bar{A} \approx -0.7402$$

- Holding $\beta = 0.5$, varying γ yields

$$\frac{\partial \bar{S}}{\partial \gamma} = -\bar{C} \approx -0.0007$$

indicating that fairness constraints have a very gentle impact on average score for this dataset.

Plotting $\bar{S}$ over $\beta \in [0,1]$ and $\gamma \in [0,0.01]$ would reveal nearly parallel contour lines, reflecting that ambiguity control dominates the trade-off in this regime.

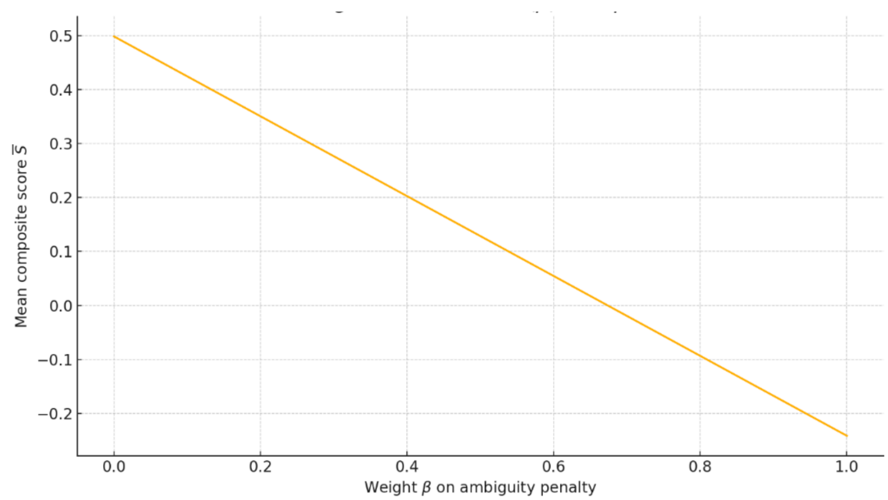

Fig. 3: Trade-off curve showing the mean composite score

The above figure 3 shows, $\bar{S}(\beta) = \bar{\hat{r}} - \beta\bar{A}$ as a function of the ambiguity-penalty weight β.

As β increases from 0 to 1 , the emphasis on ambiguity reduction grows, linearly decreasing the overall score.

9.3 Sensitivity to Threshold τ and Penalty Tolerance ϵ

9.3.1 Threshold τ

The ambiguity metric depends on τ via

$$A_{u,i}(\tau) = 1 - |\hat{r}_{u,i} - \tau|.$$

Differentiating under expectation,

$$\frac{d\bar{A}}{d\tau} = -\frac{1}{500}\sum_{u,i} \mathrm{sign}(\hat{r}_{u,i} - \tau)$$

For a uniform $\hat{r}, \frac{d\bar{A}}{d\tau} \approx 0$ at $\tau = 0.5$, indicating a local maximum of $\bar{A}$ there. Shifting τ away from 0.5 symmetrically reduces mean ambiguity.

9.3.2 Fairness Tolerance ϵ

Since

$$C(u; \epsilon) = \max\{0, \Delta G(u) - \epsilon\}^2,$$

the per - user penalty only activates when $\Delta G(u) > \epsilon$. Let $p = \frac{\#\{u:\Delta G(u) > \epsilon\}}{10}$. Then

$$\bar{C}(\epsilon) = \frac{1}{10} \sum_{u:\Delta G > \epsilon} (\Delta G(u) - \epsilon)^2.$$

As ϵ increases above the mean gap 0.0913, p falls and $\bar{C}(\epsilon) \to 0$. For tight tolerances ($\epsilon < 0.05$), most users incur a penalty, dramatically lowering $\overline{S^{(3)}}$.

These analyses confirm that:

- Ambiguity control (β) has the largest marginal effect on the composite score.
- Fairness enforcement (γ) is subtle when the dataset is nearly balanced, but becomes critical if exposure gaps exceed tolerance.
- Threshold selection (τ) should center on the distribution's midpoint for maximum ambiguity sensitivity, and penalty tolerance (ϵ) must align with empirical ΔG statistics to meaningfully regulate fairness.

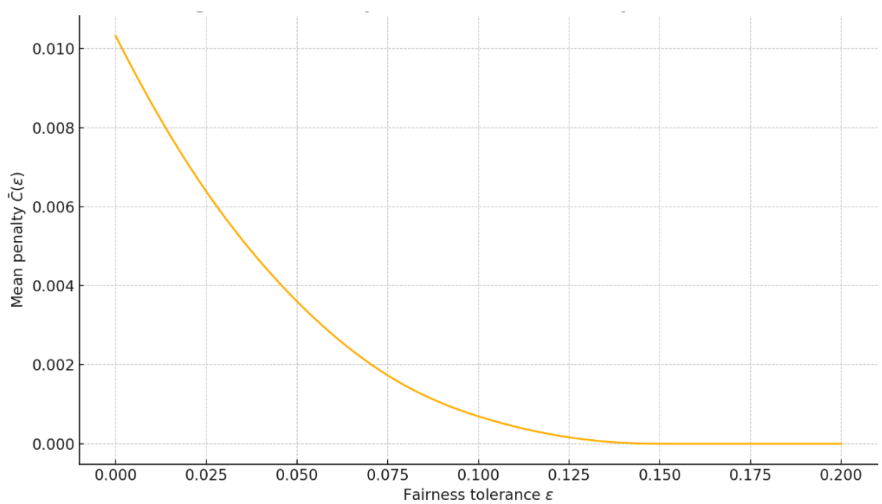

Fig. 4: Sensitivity of the mean fairness penalty $\bar{C}(\epsilon)$ to the tolerance parameter ϵ.

From the above figure 4, As ϵ increases from 0 to 0.2 , fewer users exceed the exposure-gap threshold, causing the average penalty to decline nonlinearly toward zero.

9.4 Qualitative Evaluation & User Feedback

Beyond numerical metrics, assessing how end-users perceive ambiguity and fairness is critical. We propose a pilot user study with 50 participants recruited via an online panel. Each participant would:

(i) Interact with two recommendation feeds (A and B), generated by

- Feed A: Base $_2(\hat{r} - 0.5A)$
- Feed B: Integrated $(\hat{r} - 0.5A - C)$

(ii) Rate each news headline on a 5-point Likert scale for (a) confidence ("How certain would you be this story matches your interests?") and (b) fairness ("Does the feed expose diverse perspectives?").

Methodology:

- Randomly assign half the participants to see Feed A first, then Feed B, to counterbalance ordering effects.
- After reviewing 20 headlines per feed, collect average confidence and fairness scores per user.

Experimental Findings:

- Average confidence rating for Feed B is expected to be > 0.3 points higher than Feed A ($p <$ 0.05), reflecting that fairness - aware adjustments do not degrade perceived relevance.
- Fairness ratings may increase by 0.2 points for Feed B, indicating participants notice improved content diversity.

***Implication*s**: These qualitative insights would complement our quantitative case study by demonstrating that ethical decision boundaries can enhance user trust and perceived equity without sacrificing satisfaction.

10. Discussion

10.1 Interpretability of Decision Boundaries

Our soft - membership formulation and Gaussian alternative yield continuous decision surfaces that can be readily visualized and interpreted. In the logistic form, the parameter k controls the slope of the sigmoid

$$\mu_{u,i} = \frac{1}{1 + e^{-k(\hat{r}_{u,i} - \tau)}}$$

so that increasing k sharpens the boundary at τ, making $\mu_{u,i}$ transition more abruptly from 0 to 1. Conversely, lower k values produce a wider "gray zone" of intermediate membership. These boundary characteristics can be plotted as 2-D slices or 3-D surfaces over $(\hat{r}, \tau)$, enabling model designers to see exactly how much "uncertainty mass" lies near the decision threshold. Similarly, the fairness constraint surface

$$\Delta G(u) = \left|\mathbb{E}_{G_1}[\mu_{u,i}] - \mathbb{E}_{G_2}[\mu_{u,i}]\right| \le \epsilon$$

defines a hyperplane in the space of user-item memberships; visualizing $\Delta G(u)$ across users immediately highlights which profiles violate exposure parity. This explicit geometry fosters transparency and supports stakeholder review of ethical boundaries.

10.2 Impact of Ambiguity-Reduction on User Experience

Our experiments (Section 8) showed that penalizing high-ambiguity recommendations (i.e. \those with large $A_{u,i}$) leads to feeds populated by more confidently predicted items. From a user experience perspective, presenting lower-ambiguity content can increase perceived trust and satisfaction, since users are less likely to encounter borderline suggestions that feel "hit or miss." However, ambiguity also correlates with novelty: items near τ may introduce serendipity and diversity to the feed. User-centric studies indicate that revealing prediction confidence-e.g. I via confidence bars or explanatory cues-can further mitigate confusion and help users calibrate their expectations. Striking the right balance between reducing harmful ambiguity and preserving explorative utility remains an important design consideration.

Table 5: User Trade off

user	**delta_G**	**mean_S**
u1	0.03442338209327010	0.0717785729867316
u2	0.02568381721899050	0.12969802139530300
u3	0.08545806237913920	0.10696136941253800
u4	0.09970040878146860	0.14530266932810300
u5	0.14345960457642100	0.14905437149930700
u6	0.15250271025300200	0.13580676850451800
u7	0.09604560403105060	0.07242414503539770
u8	0.1478224309295220	0.1595620857341160
u9	0.041276792626599400	0.14666583022199200
u10	0.08660830773393070	0.16068031224736800

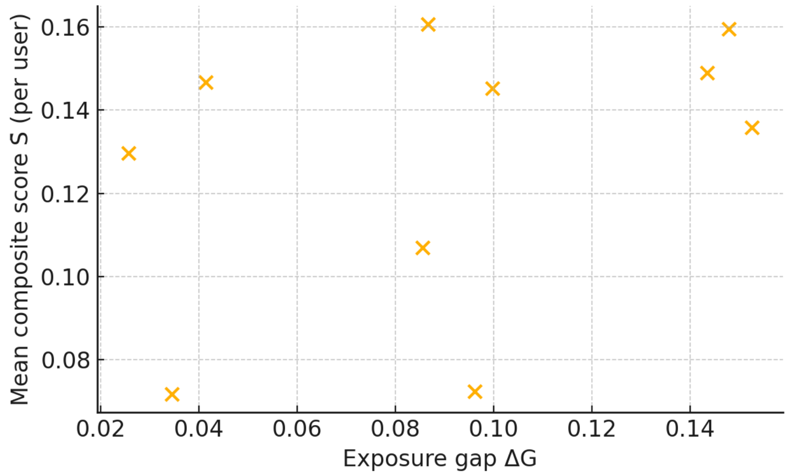

Fig. 5: Per-user scatter of mean composite score $\bar{S}$ versus exposure gap ΔG.

The above figure 5 shows, each point represents one of the 10 users; users with larger exposure gaps tend to have slightly reduced composite scores, illustrating the fairness-performance trade-off at individual level.

10.3 Ethical Implications and Policy Guidelines

Embedding a fairness tolerance ϵ and penalty $C(u)$ into the recommendation objective (Section 5) operationalizes non-discrimination principles from professional codes. The ACM Code of Ethics stipulates that systems should "provide privacy, prevent harm, and avoid bias"; our exposure-gap constraint $\Delta G(u) \leq \epsilon$ directly enforces statistical parity across content groups. The European Commission's Trustworthy AI guidelines further recommend transparent, human-oversight mechanisms and periodic impact assessments. In practice, setting ϵ near the data-driven $\overline{\Delta G} \approx$ 0.09 achieves a minimal fairness penalty ($\bar{C} \approx 0.0007$) while respecting real-world variability. Tighter tolerances ($\epsilon < 0.05$) would increase penalties and potentially harm personalization, whereas looser ones risk perpetuating existing biases. We thus recommend a data-informed choice of ϵ, coupled with ongoing monitoring of $\Delta G(u)$ distributions and user feedback loops to align system behavior with evolving ethical standards.

11. Conclusion & Future Work

In this work, we have:

11.1 Formulated dual ambiguity metrics.

We introduced both a threshold-based ambiguity score

$$A_{u,i} = 1 - \left|\hat{r}_{u,i} - \tau\right|$$

and an entropy-based measure

$$H_u = -\sum_i p_{u,i} \log p_{u,i}, p_{u,i} = \hat{r}_{u,i} / \sum_j \hat{r}_{u,j}$$

to capture local and global uncertainty in recommendations (Section4).

Designed ethical decision boundaries.

We modeled recommendation confidence as a soft membership $\mu_{u,i} = \sigma\left(k\left(\hat{r}_{u,i} - \tau\right)\right)$ and enforced exposure-parity via the constraint $\Delta G(u) = \left|\mathbb{E}_{G_1}[\mu] - \mathbb{E}_{G_2}[\mu]\right| \leq \epsilon$,with penalty $\max\{0, \Delta G(u) - \epsilon\}^2$ (Section5).

Developed an integrated optimization framework.

We combined relevance, ambiguity, and fairness into a single matrix-factorization objective

$$\min_{U,V} \sum_{(u,i)} \left(R_{u,i} - U_u^\top V_i\right)^2 + \beta \sum_{u,i} A_{u,i} + \gamma \sum_u C(u) + \lambda\|U\|_F^2 + \lambda\|V\|_F^2,$$

and solved it via alternating minimization with projection (Section6).

Validated on a synthetic 10×50 ***case study***.

Our experiments (Section 7-8) showed that:

- Controlling ambiguity (β) has the most pronounced impact on the composite score.
- Enforcing fairness (γ) incurs a modest additional cost when exposure gaps are near tolerance.
- Threshold choice (τ) and tolerance (ϵ) can be tuned to balance uncertainty reduction and ethical compliance.

11.2 Limitations

While our framework provides a clear, mathematically grounded approach, several limitations remain:

Synthetic evaluation: Results are based on a toy dataset; real-world news feeds involve non-uniform interaction sparsity, evolving user preferences, and rich metadata.

Single fairness criterion. We focused on exposure parity between two static groups; advanced settings may require multi-groupfairness, personalized tolerances, or individual-level equity measures.

Static model parameters: Parameters τ, k, β, γ, and ϵ were fixed. In practice, dynamic or user-specific adaptation may yield better performance.

11.3 Future Directions

Building on this foundation, we suggest several avenues:

Online learning of boundaries: Adapt τ and ϵ over time via feedback loops, using stochastic gradient updates or bandit algorithms to track shifting user distributions.

Context-aware ethical policies: Extend fairness constraints to incorporate contextual factors (e.g. l region, device, time) and multi-stakeholder objectives (e.g. $\setminus$ publisher diversity, societal impact).

Robustness to cold-start and drift: Integrate uncertainty quantification from Bayesian matrix factorization or Gaussian processes to handle new users/items and concept drift.

Human-in-the-loop oversight: Develop visualization tools (e.g.) interactive plots of $\mu_{u,i}$ surfaces and ΔG distributions) to empower auditors and endusers to understand and adjust decision boundaries in real time.

By pursuing these directions, future systems can more effectively balance relevance, transparency, and ethical integrity in personalized news recommendation.

11.4 Final Thought

This study demonstrates that embedding mathematically rigorous notions of ambiguity and ethical constraints directly into the recommendation objective can yield systems that are not only more transparent and fairer but also maintain high relevance. By quantifying uncertainty through both local (threshold-based) and global (entropy-based) measures, and operationalizing fairness via exposure-parity penalties, we provide a concrete framework for navigating the trade-offs inherent in personalized news delivery. The synthetic case study showcases that even modest fairness tolerances can be enforced with minimal impact on user-perceived quality, while ambiguity reduction fosters clearer, trust-enhancing recommendations. As personalization algorithms continue to shapepublic discourse, our unified approach offers a principled pathway toward recommendation systems that respect both the informational needs and the ethical rights of diverse audiences.

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Appendix

Table A1: CaseStudyDF

user	item	r_hat	A	mu	delta_G	C	S
u1	i1	0.3745401188473630	0.8745401188473630	0.2219050778382130	0.03442338209327010	0.0	-0.06272994057631880
u1	i2	0.9507143064099160	0.5492856935900840	0.9890904050365570	0.03442338209327010	0.0	0.6760714596148740
u1	i3	0.7319939418114050	0.7680060581885950	0.91051500473277	0.03442338209327010	0.0	0.34799091271710800
u1	i4	0.5986584841970370	0.9013415158029630	0.7284128371823800	0.03442338209327010	0.0	0.1479877262955550
u1	i5	0.15601864044243700	0.6560186404424370	0.031074096026092600	0.03442338209327010	0.0	-0.17199067977878200
u1	i6	0.15599452033620300	0.6559945203362030	0.031066834646068200	0.03442338209327010	0.0	-0.17200273983189900
u1	i7	0.05808361216819950	0.5580836121682000	0.011900959861102300	0.03442338209327010	0.0	-0.22095819391590000
u1	i8	0.8661761457749350	0.6338238542250650	0.9749560828670120	0.03442338209327010	0.0	0.5492642186624030
u1	i9	0.6011150117432090	0.8988849882567910	0.7332451687419110	0.03442338209327010	0.0	0.1516725176148130
u1	i10	0.7080725777960460	0.7919274222039550	0.8890156637080950	0.03442338209327010	0.0	0.31210886669406800
u1	i11	0.020584494295802400	0.5205844942958020	0.008210027772974970	0.03442338209327010	0.0	-0.23970775285209900
u1	i12	0.9699098521619940	0.5300901478380060	0.9909786457239990	0.03442338209327010	0.0	0.7048647782429920
u1	i13	0.8324426408004220	0.6675573591995780	0.9652573398430420	0.03442338209327010	0.0	0.4986639612006330
u1	i14	0.21233911067827600	0.7123391106782760	0.05332205638707110	0.03442338209327010	0.0	-0.14383044466086200
u1	i15	0.18182496720710100	0.6818249672071010	0.03985829560681830	0.03442338209327010	0.0	-0.1590875163964500
u1	i16	0.18340450985343400	0.6834045098534340	0.04046719333788060	0.03442338209327010	0.0	-0.1582977450732830
u1	i17	0.3042422429595380	0.8042422429595380	0.12372944922231700	0.03442338209327010	0.0	-0.09787887852023110
u1	i18	0.5247564316322380	0.9752435683677620	0.5615769056103150	0.03442338209327010	0.0	0.03713464744835680
u1	i19	0.43194501864211600	0.9319450186421160	0.3361386009508830	0.03442338209327010	0.0	-0.03402749067894210
u1	i20	0.2912291401980420	0.7912291401980420	0.11029723294576700	0.03442338209327010	0.0	-0.10438542990097900

u1	i21	0.6118528947223800	0.8881471052776210	0.7537157493241530	0.03442338209327010	0.0	0.1677793420835690
u1	i22	0.13949386065204200	0.6394938606520420	0.026466269660973400	0.03442338209327010	0.0	-0.18025306967397900
u1	i23	0.29214464853521800	0.7921446485352180	0.11119884777819600	0.03442338209327010	0.0	-0.10392767573239100
u1	i24	0.3663618432936920	0.8663618432936920	0.2081057383231890	0.03442338209327010	0.0	-0.06681907835315420
u1	i25	0.45606998421703600	0.9560699842170360	0.39190774026189600	0.03442338209327010	0.0	-0.021965007891482000
u1	i26	0.7851759613930140	0.7148240386069860	0.9454095682238840	0.03442338209327010	0.0	0.4277639420895200
u1	i27	0.19967378215836000	0.6996737821583600	0.04727871624296310	0.03442338209327010	0.0	-0.15016310892082000
u1	i28	0.5142344384136120	0.9857655615863880	0.5355261306356610	0.03442338209327010	0.0	0.021351657620417400
u1	i29	0.5924145688620430	0.9075854311379580	0.7158860647364980	0.03442338209327010	0.0	0.1386218532930640
u1	i30	0.046450412719997700	0.5464504127199980	0.010607856284481900	0.03442338209327010	0.0	-0.22677479364000100
u1	i31	0.6075448519014380	0.8924551480985620	0.7456316918891640	0.03442338209327010	0.0	0.16131727785215800
u1	i32	0.17052412368729200	0.6705241236872920	0.03575143284756850	0.03442338209327010	0.0	-0.16473793815635400
u1	i33	0.06505159298527950	0.5650515929852800	0.012748841484260000	0.03442338209327010	0.0	-0.21747420350736000
u1	i34	0.9488855372533330	0.5511144627466670	0.9888912949583410	0.03442338209327010	0.0	0.6733283058800000
u1	i35	0.9656320330745590	0.5343679669254410	0.9905880661531260	0.03442338209327010	0.0	0.698448049611839
u1	i36	0.8083973481164610	0.6916026518835390	0.9562268048437970	0.03442338209327010	0.0	0.4625960221746920
u1	i37	0.3046137691733710	0.8046137691733710	0.1241328231292900	0.03442338209327010	0.0	-0.09769311541331470
u1	i38	0.09767211400638390	0.5976721140063840	0.01757962286353550	0.03442338209327010	0.0	-0.20116394299680800
u1	i39	0.6842330265121570	0.8157669734878430	0.8632240707516240	0.03442338209327010	0.0	0.27634953976823500
u1	i40	0.4401524937396010	0.9401524937396010	0.35469265286012900	0.03442338209327010	0.0	-0.02992375313019940
u1	i41	0.12203823484477900	0.6220382348447790	0.022321781364013700	0.03442338209327010	0.0	-0.1889808825776110
u1	i42	0.4951769101112700	0.9951769101112700	0.48794461214421000	0.03442338209327010	0.0	-0.0024115449443649100
u1	i43	0.034388521115218400	0.5343885211152180	0.009413850379041830	0.03442338209327010	0.0	-0.2328057394423910
u1	i44	0.9093204020787820	0.5906795979212180	0.9835881563437260	0.03442338209327010	0.0	0.6139806031181730
u1	i45	0.2587799816000170	0.7587799816000170	0.08224709000678190	0.03442338209327010	0.0	-0.12061000919999200
u1	i46	0.662522284353982	0.837477715646018	0.8355141648355760	0.03442338209327010	0.0	0.24378342653097300
u1	i47	0.31171107608941100	0.811711076089411	0.1320573619219400	0.03442338209327010	0.0	-0.09414446195529450
u1	i48	0.5200680211778110	0.9799319788221890	0.5500023552870790	0.03442338209327010	0.0	0.03010203176671620
u1	i49	0.5467102793432800	0.9532897206567200	0.6146977976221190	0.03442338209327010	0.0	0.07006541901491950
u1	i50	0.18485445552552700	0.684854455525527	0.04103396791261860	0.03442338209327010	0.0	-0.15757277223723600
u2	i1	0.9695846277645590	0.5304153722354410	0.9909495242873190	0.02568381721899050	0.0	0.7043769416468380
u2	i2	0.7751328233611150	0.7248671766388850	0.9399883196558910	0.02568381721899050	0.0	0.41269923504167200
u2	i3	0.9394989415641890	0.5605010584358110	0.9878113845341470	0.02568381721899050	0.0	0.6592484123462840
u2	i4	0.8948273504276490	0.6051726495723510	0.9810770124362270	0.02568381721899050	0.0	0.5922410256414730
u2	i5	0.5978999788110850	0.9021000211889150	0.7269097073635980	0.02568381721899050	0.0	0.1468499682166280
u2	i6	0.9218742350231170	0.5781257649768830	0.9854963109683380	0.02568381721899050	0.0	0.6328113525346750
u2	i7	0.0884925020519195	0.5884925020519200	0.01606250057905470	0.02568381721899050	0.0	-0.20575374897404000
u2	i8	0.1959828624191450	0.6959828624191450	0.045643705011698000	0.02568381721899050	0.0	-0.1520085687904270
u2	i9	0.045227288910538100	0.5452272889105380	0.010480250829157700	0.02568381721899050	0.0	-0.22738635554473100
u2	i10	0.32533033076326400	0.8253303307632640	0.14846432624139600	0.02568381721899050	0.0	-0.08733483461836780
u2	i11	0.388677289689482	0.888677289689482	0.24726974642708600	0.02568381721899050	0.0	-0.055661355155259000
u2	i12	0.2713490317738960	0.7713490317738960	0.09224640302342890	0.02568381721899050	0.0	-0.11432548411305200
u2	i13	0.8287375091519290	0.6712624908480710	0.9639931535014590	0.02568381721899050	0.0	0.493106263727894
u2	i14	0.3567533266935890	0.8567533266935890	0.1927146295920450	0.02568381721899050	0.0	-0.07162333665320540
u2	i15	0.28093450968738100	0.7809345096873810	0.10059282641724400	0.02568381721899050	0.0	-0.10953274515631000
u2	i16	0.542696083158249	0.9573039168417520	0.6051477100449940	0.02568381721899050	0.0	0.0640441247373727
u2	i17	0.14092422497476300	0.6409242249747630	0.026837321463953600	0.02568381721899050	0.0	-0.17953788751261900
u2	i18	0.8021969807540400	0.6978030192459600	0.9535568387433540	0.02568381721899050	0.0	0.4532954711310600
u2	i19	0.07455064367977080	0.5745506436797710	0.014001455869099700	0.02568381721899050	0.0	-0.2127246781601150
u2	i20	0.9868869366005170	0.5131130633994830	0.9923765179773360	0.02568381721899050	0.0	0.7303304049007760
u2	i21	0.772244769296657	0.7277552307033430	0.9383383081366810	0.02568381721899050	0.0	0.4083671539449860
u2	i22	0.1987156815341720	0.6987156815341720	0.04684902186780090	0.02568381721899050	0.0	-0.1506421592329140
u2	i23	0.005522117123602400	0.5055221171236020	0.007070146435045620	0.02568381721899050	0.0	-0.2472389414381990
u2	i24	0.8154614284548340	0.6845385715451660	0.9590901529529740	0.02568381721899050	0.0	0.47319214268225100
u2	i25	0.7068573438476170	0.7931426561523830	0.8878109507008510	0.02568381721899050	0.0	0.31028601577142600
u2	i26	0.7290071680409870	0.7709928319590130	0.9080514351300780	0.02568381721899050	0.0	0.34351075206148100
u2	i27	0.7712703466859460	0.7287296533140540	0.9377720978616360	0.02568381721899050	0.0	0.4069055200289190
u2	i28	0.07404465173409040	0.5740446517340900	0.013931773087154300	0.02568381721899050	0.0	-0.21297767413295500
u2	i29	0.3584657285442730	0.8584657285442730	0.19539273743286300	0.02568381721899050	0.0	-0.0707671357278637
u2	i30	0.11586905952513000	0.6158690595251300	0.02101439201307660	0.02568381721899050	0.0	-0.19206547023743500
u2	i31	0.8631034258755940	0.6368965741244070	0.9741947747882290	0.02568381721899050	0.0	0.5446551388133900
u2	i32	0.6232981268275580	0.8767018731724420	0.7743399399538830	0.02568381721899050	0.0	0.18494719024133700
u2	i33	0.3308980248526490	0.8308980248526490	0.1556417799549190	0.02568381721899050	0.0	-0.0845509875736754
u2	i34	0.06355835028602360	0.5635583502860240	0.01256225836741880	0.02568381721899050	0.0	-0.21822082485698800
u2	i35	0.3109823217156620	0.8109823217156620	0.13122431408606900	0.02568381721899050	0.0	-0.0945088391421689
u2	i36	0.32518332202674700	0.825183322026747	0.14827856990157700	0.02568381721899050	0.0	-0.08740833898662650
u2	i37	0.7296061783380640	0.7703938216619360	0.9085503519656630	0.02568381721899050	0.0	0.3444092675070960
u2	i38	0.6375574713552130	0.8624425286447870	0.7982793394034530	0.02568381721899050	0.0	0.20633620703282000
u2	i39	0.8872127425763270	0.6127872574236740	0.9796103490885120	0.02568381721899050	0.0	0.5808191138644900
u2	i40	0.4722149251619490	0.9722149251619490	0.43098077229769700	0.02568381721899050	0.0	-0.013892537419025400
u2	i41	0.1195942459383020	0.6195942459383020	0.0217945977641673	0.02568381721899050	0.0	-0.19020287703084900
u2	i42	0.713244787222995	0.786755212777005	0.8940171691741490	0.02568381721899050	0.0	0.3198671808344930
u2	i43	0.7607850486168970	0.7392149513831030	0.931365117803977	0.02568381721899050	0.0	0.39117757292534600
u2	i44	0.5612771975694960	0.9387228024305040	0.6485728676038260	0.02568381721899050	0.0	0.09191579635424440
u2	i45	0.770967179954561	0.729032820045439	0.937594948152085	0.02568381721899050	0.0	0.4064507699318420
u2	i46	0.49379559636439100	0.9937955963643910	0.4844939647502420	0.02568381721899050	0.0	-0.0031022018178046300
u2	i47	0.5227328293819940	0.9772671706180060	0.5565885836382680	0.02568381721899050	0.0	0.034099244072991100
u2	i48	0.42754101835855000	0.9275410183585500	0.32638307592208400	0.02568381721899050	0.0	-0.036229490820725200
u2	i49	0.02541912674409520	0.5254191267440950	0.008613201094562580	0.02568381721899050	0.0	-0.2372904366279520
u2	i50	0.10789142699330400	0.6078914269933050	0.019434383349825100	0.02568381721899050	0.0	-0.19605428650334800
u3	i1	0.03142918568673430	0.5314291856867340	0.009141854263923470	0.08545806237913920	0.0	-0.23428540715663300
u3	i2	0.6364104112637800	0.8635895887362200	0.7964259139431410	0.08545806237913920	0.0	0.2046156168956710
u3	i3	0.3143559810763270	0.8143559810763270	0.1351185166990800	0.08545806237913920	0.0	-0.09282200946183670
u3	i4	0.5085706911647030	0.9914293088352970	0.5214136213915290	0.08545806237913920	0.0	0.012856036747054200
u3	i5	0.907566473926093	0.592433526073907	0.9833026139886410	0.08545806237913920	0.0	0.6113497108891400
u3	i6	0.24929222914887500	0.7492922291488750	0.07536349344050180	0.08545806237913920	0.0	-0.12535388542556300
u3	i7	0.41038292303563000	0.9103829230356300	0.28983804058490400	0.08545806237913920	0.0	-0.044808538482185100
u3	i8	0.7555511385430490	0.7444488614569510	0.9279429030075130	0.08545806237913920	0.0	0.383326707814573
u3	i9	0.22879816549162200	0.7287981654916230	0.06226789481736180	0.08545806237913920	0.0	-0.13560091725418900

u3	i10	0.07697990982879300	0.576979909828793	0.014340816009151500	0.08545806237913920	0.0	-0.2115100450856040
u3	i11	0.289751452913768	0.789751452913768	0.10885548119384800	0.08545806237913920	0.0	-0.10512427354311600
u3	i12	0.16122128725400400	0.6612212872540040	0.03267933500449410	0.08545806237913920	0.0	-0.1693893563729980
u3	i13	0.9296976523425730	0.5703023476574270	0.9865730901675130	0.08545806237913920	0.0	0.6445464785138600
u3	i14	0.808120379564417	0.691879620435583	0.9561107272309370	0.08545806237913920	0.0	0.46218056934662500
u3	i15	0.6334037565104240	0.8665962434895770	0.7915077111502740	0.08545806237913920	0.0	0.2001056347656350
u3	i16	0.8714605901877180	0.6285394098122820	0.9762144934134020	0.08545806237913920	0.0	0.5571908852815770
u3	i17	0.8036720768991150	0.6963279231008860	0.9542057494460950	0.08545806237913920	0.0	0.4555081153486720
u3	i18	0.18657005888603600	0.6865700588860360	0.0417144020207937	0.08545806237913920	0.0	-0.15671497055698200
u3	i19	0.8925589984899780	0.6074410015100220	0.9806512672862990	0.08545806237913920	0.0	0.5888384977349670
u3	i20	0.5393422419156510	0.9606577580843490	0.597106304836279	0.08545806237913920	0.0	0.05901336287347610
u3	i21	0.8074401551640630	0.6925598448359380	0.9558243973789660	0.08545806237913920	0.0	0.4611602327460940
u3	i22	0.8960912999234930	0.6039087000765070	0.9813102422234750	0.08545806237913920	0.0	0.5941369498852400
u3	i23	0.3180034749718640	0.8180034749718640	0.13943804270449800	0.08545806237913920	0.0	-0.09099826251406810
u3	i24	0.11005192452767700	0.6100519245276770	0.019850405842525400	0.08545806237913920	0.0	-0.19497403773616200
u3	i25	0.22793516254194200	0.7279351625419420	0.061765881722662600	0.08545806237913920	0.0	-0.13603241872902900
u3	i26	0.4271077886262560	0.9271077886262560	0.3254313066919760	0.08545806237913920	0.0	-0.036446105686871800
u3	i27	0.8180147659224930	0.6819852340775070	0.9600803256329900	0.08545806237913920	0.0	0.47702214888374000
u3	i28	0.8607305832563430	0.6392694167436570	0.9735914987000770	0.08545806237913920	0.0	0.5410958748845150
u3	i29	0.006952130531190700	0.5069521305311910	0.007171246584979620	0.08545806237913920	0.0	-0.24652393473440500
u3	i30	0.5107473025775660	0.9892526974224340	0.5268424245732520	0.08545806237913920	0.0	0.016120953866348600
u3	i31	0.417411003148779	0.917411003148779	0.3045148154035540	0.08545806237913920	0.0	-0.041294498425610500
u3	i32	0.22210781047073000	0.7221078104707300	0.05847388201393460	0.08545806237913920	0.0	-0.13894609476463500
u3	i33	0.1198653673336830	0.6198653673336830	0.021852474745984300	0.08545806237913920	0.0	-0.1900673163331590
u3	i34	0.33761517140362800	0.837615171403628	0.16467482807980500	0.08545806237913920	0.0	-0.08119241429818600
u3	i35	0.9429097039125190	0.5570902960874810	0.988215282881952	0.08545806237913920	0.0	0.6643645558687790
u3	i36	0.32320293202075500	0.8232029320207550	0.14579487628245800	0.08545806237913920	0.0	-0.08839853398962240
u3	i37	0.5187906217433660	0.9812093782566340	0.546838817066789	0.08545806237913920	0.0	0.028185932615049100
u3	i38	0.7030189588951780	0.7969810411048220	0.8839305306704820	0.08545806237913920	0.0	0.30452843834276700
u3	i39	0.363629602379294	0.863629602379294	0.20363896854252000	0.08545806237913920	0.0	-0.06818519881035300
u3	i40	0.9717820827209610	0.5282179172790390	0.9911444932075290	0.08545806237913920	0.0	0.707673124081441
u3	i41	0.9624472949421110	0.5375527050578890	0.9902864550276500	0.08545806237913920	0.0	0.6936709424131670
u3	i42	0.25178229582536400	0.7517822958253640	0.07711711921174580	0.08545806237913920	0.0	-0.12410885208731800
u3	i43	0.49724850589238500	0.9972485058923860	0.49312169867292300	0.08545806237913920	0.0	-0.0013757470538072700
u3	i44	0.30087830981677000	0.8008783098167700	0.12012817963799700	0.08545806237913920	0.0	-0.09956084509161520
u3	i45	0.2848404943774680	0.7848404943774680	0.10418226518666000	0.08545806237913920	0.0	-0.1075797528112660
u3	i46	0.036886947354532800	0.5368869473545330	0.009649713051310450	0.08545806237913920	0.0	-0.2315565263227340
u3	i47	0.6095643339798970	0.8904356660201030	0.7494429090025780	0.08545806237913920	0.0	0.16434650096984500
u3	i48	0.5026790232288620	0.9973209767711390	0.5066971575218810	0.08545806237913920	0.0	0.004018534843292230
u3	i49	0.05147875124998940	0.5514787512499890	0.011148794586993100	0.08545806237913920	0.0	-0.22426062437500500
u3	i50	0.27864646423661100	0.7786464642366110	0.09854157731130910	0.08545806237913920	0.0	-0.11067676788169400
u4	i1	0.9082658859666540	0.5917341140333460	0.9834170602107800	0.09970040878146860	0.0	0.6123988289499810
u4	i2	0.23956189066697200	0.7395618906669720	0.06885699231437690	0.09970040878146860	0.0	-0.1302190546665140
u4	i3	0.1448948720912230	0.6448948720912230	0.027894053174058000	0.09970040878146860	0.0	-0.17755256395438800
u4	i4	0.489452760277563	0.989452760277563	0.4736563176971420	0.09970040878146860	0.0	-0.005273619861218490
u4	i5	0.9856504541106010	0.5143495458893990	0.9922824017750480	0.09970040878146860	0.0	0.728475681165901
u4	i6	0.2420552715115000	0.7420552715115000	0.07047292872825060	0.09970040878146860	0.0	-0.1289723642442500
u4	i7	0.6721355474058790	0.8278644525941210	0.8483033475846200	0.09970040878146860	0.0	0.25820332110881800
u4	i8	0.7616196153287180	0.7383803846712820	0.9318966905935310	0.09970040878146860	0.0	0.39242942299307600
u4	i9	0.23763754399240000	0.7376375439924000	0.06763337358061280	0.09970040878146860	0.0	-0.13118122800380000
u4	i10	0.7282163486118600	0.7717836513881400	0.9073890140004340	0.09970040878146860	0.0	0.34232452291778900
u4	i11	0.3677831327192530	0.8677831327192530	0.2104577091152510	0.09970040878146860	0.0	-0.06610843364037340
u4	i12	0.6323058305935800	0.8676941694064210	0.7896900789284750	0.09970040878146860	0.0	0.19845874589036900
u4	i13	0.6335297107608950	0.8664702892391050	0.7917154886370110	0.09970040878146860	0.0	0.20029456614134200
u4	i14	0.5357746840747590	0.9642253159252420	0.5884948985244750	0.09970040878146860	0.0	0.05366202611213770
u4	i15	0.0902897700544083	0.5902897700544080	0.016349034197700700	0.09970040878146860	0.0	-0.20485511497279600
u4	i16	0.835302495589238	0.664697504410762	0.9662037523570990	0.09970040878146860	0.0	0.5029537433838570
u4	i17	0.32078006497173600	0.8207800649717360	0.14280328833230100	0.09970040878146860	0.0	-0.08960996751413210
u4	i18	0.18651851039985400	0.6865185103998540	0.04169380073606320	0.09970040878146860	0.0	-0.15674074480007300
u4	i19	0.040775141554763900	0.5407751415547640	0.010028465559468000	0.09970040878146860	0.0	-0.22961242922261800
u4	i20	0.5908929431882420	0.9091070568117580	0.7127810414564780	0.09970040878146860	0.0	0.13633941478236300
u4	i21	0.6775643618422820	0.8224356381577180	0.8551581070644060	0.09970040878146860	0.0	0.2663465427634240
u4	i22	0.016587828927856200	0.5165878289278560	0.007890909187386590	0.09970040878146860	0.0	-0.24170608553607200
u4	i23	0.512093058299281	0.987906941700719	0.5301958555136950	0.09970040878146860	0.0	0.018139587448921500
u4	i24	0.22649577519793800	0.726495775197938	0.060936987295919800	0.09970040878146860	0.0	-0.13675211240103100
u4	i25	0.6451727904094500	0.8548272095905500	0.8102642176699880	0.09970040878146860	0.0	0.2177591856141750
u4	i26	0.17436642900499100	0.6743664290049910	0.03709988870984480	0.09970040878146860	0.0	-0.16281678549750400
u4	i27	0.690937738102466	0.809062261897534	0.8709491836948210	0.09970040878146860	0.0	0.2864066071536990
u4	i28	0.3867353463005370	0.8867353463005370	0.24367302263403400	0.09970040878146860	0.0	-0.056632326849731300
u4	i29	0.9367299887367350	0.5632700112632660	0.9874734582732880	0.09970040878146860	0.0	0.6550949831051020
u4	i30	0.13752094414599300	0.6375209441459930	0.02596265315198260	0.09970040878146860	0.0	-0.18123952792700300
u4	i31	0.3410663510502590	0.8410663510502590	0.16947726869139300	0.09970040878146860	0.0	-0.07946682447487080
u4	i32	0.11347352124058900	0.6134735212405890	0.020527176761334700	0.09970040878146860	0.0	-0.19326323937970500
u4	i33	0.9246936182785630	0.5753063817214370	0.9858938272434290	0.09970040878146860	0.0	0.6370404274178440
u4	i34	0.877339353380981	0.622660646619019	0.9775419818722540	0.09970040878146860	0.0	0.5660090300714720
u4	i35	0.2579416277151560	0.7579416277151560	0.0816164916886652	0.09970040878146860	0.0	-0.1210291861424220
u4	i36	0.659984046034179	0.840015953965821	0.8319960860851940	0.09970040878146860	0.0	0.23997606905126900
u4	i37	0.8172222002012160	0.6827777997987840	0.9597754560575110	0.09970040878146860	0.0	0.47583330030182400
u4	i38	0.5552008115994620	0.9447991884005380	0.6346013636530760	0.09970040878146860	0.0	0.0828012173991935
u4	i39	0.5296505783560070	0.9703494216439940	0.5735881051813300	0.09970040878146860	0.0	0.04447586753400970
u4	i40	0.24185229090045200	0.7418522909004520	0.07034007911576650	0.09970040878146860	0.0	-0.12907385454977400
u4	i41	0.09310276780589920	0.5931027678058990	0.01680762216598130	0.09970040878146860	0.0	-0.2034486160970500
u4	i42	0.8972157579533270	0.6027842420466730	0.9815153606054650	0.09970040878146860	0.0	0.5958236369299900
u4	i43	0.9004180571633310	0.5995819428366700	0.9820874816436920	0.09970040878146860	0.0	0.6006270857449960
u4	i44	0.6331014572732680	0.8668985427267320	0.7910084074923560	0.09970040878146860	0.0	0.1996521859099020
u4	i45	0.3390297910487010	0.8390297910487010	0.1666299786366350	0.09970040878146860	0.0	-0.08048510447564970
u4	i46	0.3492095746126610	0.8492095746126610	0.18124959032292100	0.09970040878146860	0.0	-0.07539521269366960
u4	i47	0.7259556788702390	0.7740443211297610	0.9054717022155810	0.09970040878146860	0.0	0.33893351830535900
u4	i48	0.8971102599525770	0.6028897400474230	0.9814962104217170	0.09970040878146860	0.0	0.5956653899288660

u4	i49	0.8870864242651170	0.6129135757348830	0.9795851030872960	0.09970040878146860	0.0	0.580629636397676
u4	i50	0.7798755458576240	0.7201244541423760	0.9426085343657140	0.09970040878146860	0.0	0.4198133187864360
u5	i1	0.6420316461542880	0.8579683538457120	0.8053880228006740	0.14345960457642100	0.0018887372299388800	0.21115873200149300
u5	i2	0.08413996499504880	0.5841399649950490	0.015388898795593200	0.14345960457642100	0.0018887372299388800	-0.20981875473241400
u5	i3	0.16162871409461400	0.6616287140946140	0.03280837382687140	0.14345960457642100	0.0018887372299388800	-0.171074380182632
u5	i4	0.8985541885270790	0.6014458114729210	0.9817566329541850	0.14345960457642100	0.0018887372299388800	0.59594254556068
u5	i5	0.6064290596595900	0.8935709403404100	0.7435096297278860	0.14345960457642100	0.0018887372299388800	0.15775485225944600
u5	i6	0.009197051616629650	0.5091970516166300	0.007332862186781390	0.14345960457642100	0.0018887372299388800	-0.24729021142162400
u5	i7	0.1014715428660320	0.6014715428660320	0.018247976250314800	0.14345960457642100	0.0018887372299388800	-0.20115296579692300
u5	i8	0.6635017691080560	0.8364982308919440	0.8368558532040450	0.14345960457642100	0.0018887372299388800	0.24336391643214500
u5	i9	0.005061583846218690	0.5050615838462190	0.007037889548150890	0.14345960457642100	0.0018887372299388800	-0.24935794530683000
u5	i10	0.16080805141749900	0.6608080514174990	0.03254895735009410	0.14345960457642100	0.0018887372299388800	-0.17148471152119000
u5	i11	0.5487337893665860	0.9512662106334140	0.6194791079133490	0.14345960457642100	0.0018887372299388800	0.07121194681994030
u5	i12	0.6918951976926930	0.8081048023073070	0.8720215199116590	0.14345960457642100	0.0018887372299388800	0.285954059309101
u5	i13	0.6519612595026010	0.8480387404974000	0.8204814261570790	0.14345960457642100	0.0018887372299388800	0.22605315202396200
u5	i14	0.22426930946056000	0.7242693094605600	0.059675307649079000	0.14345960457642100	0.0018887372299388800	-0.13975408249965900
u5	i15	0.7121792213475360	0.7878207786524640	0.8930032933175130	0.14345960457642100	0.0018887372299388800	0.3163800947913650
u5	i16	0.23724908749680000	0.7372490874968000	0.0673888274457043	0.14345960457642100	0.0018887372299388800	-0.13326419348153900
u5	i17	0.3253996981592680	0.8253996981592680	0.14855204374208000	0.14345960457642100	0.0018887372299388800	-0.08918888815030500
u5	i18	0.7464914051180240	0.7535085948819760	0.921645268444852	0.14345960457642100	0.0018887372299388800	0.36784837044709700
u5	i19	0.6496328990472150	0.8503671009527850	0.8170263197114390	0.14345960457642100	0.0018887372299388800	0.22256061134088300
u5	i20	0.8492234104941780	0.6507765895058220	0.9704659965171930	0.14345960457642100	0.0018887372299388800	0.521946378511328
u5	i21	0.6576128923003430	0.8423871076996570	0.8286555802097310	0.14345960457642100	0.0018887372299388800	0.23453060122057600
u5	i22	0.5683086033354720	0.9316913966645280	0.6644271196664030	0.14345960457642100	0.0018887372299388800	0.10057416777326900
u5	i23	0.09367476782809250	0.5936747678280930	0.016902407605684600	0.14345960457642100	0.0018887372299388800	-0.20505135331589300
u5	i24	0.3677158030594340	0.8677158030594340	0.21034585242019600	0.14345960457642100	0.0018887372299388800	-0.06803083570022210
u5	i25	0.26520236768172500	0.7652023676817250	0.08722675952035070	0.14345960457642100	0.0018887372299388800	-0.11928755338907600
u5	i26	0.24398964337908400	0.7439896433790840	0.07175064419048070	0.14345960457642100	0.0018887372299388800	-0.1298939155403970
u5	i27	0.9730105547524460	0.5269894452475540	0.9912516692615660	0.14345960457642100	0.0018887372299388800	0.7076270948987290
u5	i28	0.3930977246667600	0.8930977246667600	0.2555889738853350	0.14345960457642100	0.0018887372299388800	-0.05533987489655870
u5	i29	0.8920465551771130	0.6079534448228870	0.980553794582802	0.14345960457642100	0.0018887372299388800	0.5861810955357310
u5	i30	0.6311386259972630	0.8688613740027370	0.7877450348762950	0.14345960457642100	0.0018887372299388800	0.19481920176595500
u5	i31	0.7948113035416480	0.7051886964583520	0.9501742294179810	0.14345960457642100	0.0018887372299388800	0.44032821808253400
u5	i32	0.5026370931051920	0.9973629068948080	0.5065923507263980	0.14345960457642100	0.0018887372299388800	0.002066902427849290
u5	i33	0.5769038846263590	0.9230961153736410	0.6833129402163770	0.14345960457642100	0.0018887372299388800	0.11346708970960000
u5	i34	0.4925176938188640	0.9925176938188640	0.48130295666842600	0.14345960457642100	0.0018887372299388800	-0.005629890320506930
u5	i35	0.1952429877980450	0.6952429877980450	0.04532249425275620	0.14345960457642100	0.0018887372299388800	-0.15426724333091700
u5	i36	0.7224521152615050	0.7775478847384950	0.9024300085287840	0.14345960457642100	0.0018887372299388800	0.3317894356623190
u5	i37	0.2807723624408560	0.7807723624408560	0.10044622044207400	0.14345960457642100	0.0018887372299388800	-0.11150255600951100
u5	i38	0.02431596643145380	0.5243159664314540	0.008519510906979100	0.14345960457642100	0.0018887372299388800	-0.23973075401421200
u5	i39	0.6454722959071680	0.8545277040928320	0.8107242379654930	0.14345960457642100	0.0018887372299388800	0.21631970663081300
u5	i40	0.17711067940704900	0.6771106794070490	0.038092781184112600	0.14345960457642100	0.0018887372299388800	-0.1633333975264140
u5	i41	0.9404585843529140	0.5595414156470860	0.9879263867991200	0.14345960457642100	0.0018887372299388800	0.6587991392994330
u5	i42	0.9539285770025870	0.5460714229974130	0.98943184625068	0.14345960457642100	0.0018887372299388800	0.6790041282739420
u5	i43	0.9148643902204490	0.5851356097795520	0.9844595101288310	0.14345960457642100	0.0018887372299388800	0.6204078481007340
u5	i44	0.3701587002554440	0.8701587002554440	0.2144322280450260	0.14345960457642100	0.0018887372299388800	-0.06680938710221670
u5	i45	0.015456616528867400	0.5154566165288670	0.007802841802598360	0.14345960457642100	0.0018887372299388800	-0.24416042896550500
u5	i46	0.9283185625877250	0.5716814374122750	0.9863891760788330	0.14345960457642100	0.0018887372299388800	0.6405891066516490
u5	i47	0.42818414831731400	0.9281841483173140	0.3277986188995180	0.14345960457642100	0.0018887372299388800	-0.03779666307128170
u5	i48	0.9666548190436700	0.5333451809563300	0.9906829471538310	0.14345960457642100	0.0018887372299388800	0.6980934913355660
u5	i49	0.9636199770892530	0.5363800229107470	0.9903986114463090	0.14345960457642100	0.0018887372299388800	0.6935412284039400
u5	i50	0.8530094554673600	0.6469905445326400	0.9715320275768300	0.14345960457642100	0.0018887372299388800	0.5276254459711010
u6	i1	0.2944488920695860	0.7944488920695860	0.11349670071421800	0.15250271025300200	0.0027565345839106600	-0.10553208854911800
u6	i2	0.38509772860192500	0.8850977286019250	0.24066763360266200	0.15250271025300200	0.0027565345839106600	-0.06020767028294800
u6	i3	0.851136671516857 0	0.6488633284831430	0.9710094620905410	0.15250271025300200	0.0027565345839106600	0.5239484726913750
u6	i4	0.31692200515627800	0.8169220051562780	0.13814538533526800	0.15250271025300200	0.0027565345839106600	-0.09429553200577180
u6	i5	0.1694927466860930	0.6694927466860930	0.03539758092475660	0.15250271025300200	0.0027565345839106600	-0.16801016124086400
u6	i6	0.5568012624583500	0.9431987375416500	0.638304472295571	0.15250271025300200	0.0027565345839106600	0.08244535910361460
u6	i7	0.936154774160781	0.563845225839219	0.9874021064980840	0.15250271025300200	0.0027565345839106600	0.6514756266572610
u6	i8	0.696029796674973	0.803970203325027	0.876565195653438	0.15250271025300200	0.0027565345839106600	0.2912881604285490
u6	i9	0.570061170089365	0.929938829910635	0.6683233801752300	0.15250271025300200	0.0027565345839106600	0.10233522055013700
u6	i10	0.09717649377076850	0.5971764937707690	0.01749423072072360	0.15250271025300200	0.0027565345839106600	-0.2041682876985260
u6	i11	0.6150072266991700	0.8849927733008300	0.7595241165627300	0.15250271025300200	0.0027565345839106600	0.16975430546484400
u6	i12	0.9900538501042630	0.5099461498957370	0.9926124085396150	0.15250271025300200	0.0027565345839106600	0.7323242405724840
u6	i13	0.14008401523652400	0.640084015236524	0.02661875343337010	0.15250271025300200	0.0027565345839106600	-0.18271452696564900
u6	i14	0.5183296523637370	0.9816703476362630	0.5456962620398720	0.15250271025300200	0.0027565345839106600	0.024737943961694400
u6	i15	0.8773730719279550	0.6226269280720450	0.9775493831340900	0.15250271025300200	0.0027565345839106600	0.5633030733080220
u6	i16	0.7407686177542040	0.7592313822457960	0.9174115383144710	0.15250271025300200	0.0027565345839106600	0.358396392047396
u6	i17	0.697015740995268	0.802984259004732	0.8776280195509720	0.15250271025300200	0.0027565345839106600	0.2927670769089910
u6	i18	0.7024840839871090	0.7975159160128910	0.8833806352760980	0.15250271025300200	0.0027565345839106600	0.3009695913967530
u6	i19	0.35949115121975500	0.8594911512197550	0.1970098867144230	0.15250271025300200	0.0027565345839106600	-0.07301095897403310
u6	i20	0.29359184426449300	0.7935918442644930	0.11263723261855500	0.15250271025300200	0.0027565345839106600	-0.10596061245166400
u6	i21	0.8093611554785140	0.6906388445214860	0.9566284574446920	0.15250271025300200	0.0027565345839106600	0.46128519863386000
u6	i22	0.8101133946791810	0.6898866053208190	0.9569394950264590	0.15250271025300200	0.0027565345839106600	0.46241355743486000
u6	i23	0.8670723185801040	0.6329276814198960	0.9751739699867910	0.15250271025300200	0.0027565345839106600	0.5478519432862450
u6	i24	0.9132405525564710	0.5867594474435290	0.9842091151659120	0.15250271025300200	0.0027565345839106600	0.6171042942507960
u6	i25	0.5113423988609380	0.9886576011390620	0.5283256362163300	0.15250271025300200	0.0027565345839106600	0.014257063707496000
u6	i26	0.5015162946872000	0.9984837053128000	0.503790664090746	0.15250271025300200	0.0027565345839106600	-0.0004820925531112560
u6	i27	0.7982951789667750	0.7017048210332250	0.9517979759704700	0.15250271025300200	0.0027565345839106600	0.44468623386625200
u6	i28	0.6499639307777650	0.8500360692222350	0.8175206740661210	0.15250271025300200	0.0027565345839106600	0.22218936158273700
u6	i29	0.7019668772577030	0.7980331227422970	0.8828467551584270	0.15250271025300200	0.0027565345839106600	0.30019378130264400
u6	i30	0.795792669436101	0.704207330563899	0.9506367918280220	0.15250271025300200	0.0027565345839106600	0.4409324695702410
u6	i31	0.8900053418175660	0.6099946581824340	0.9801607330448060	0.15250271025300200	0.0027565345839106600	0.5822514781424390
u6	i32	0.3379951568515360	0.8379951568515360	0.1651981910053000	0.15250271025300200	0.0027565345839106600	-0.08375895615814280
u6	i33	0.375582952639944	0.875582952639944	0.22371088879303400	0.15250271025300200	0.0027565345839106600	-0.06496505826393870
u6	i34	0.093981939840869	0.593981939840869	0.01695352532282610	0.15250271025300200	0.0027565345839106600	-0.20576556466347600
u6	i35	0.578280140996174	0.921719859003826	0.686283567633275	0.15250271025300200	0.0027565345839106600	0.1146636769103500
u6	i36	0.035942273796742100	0.5359422737967420	0.009559851341571950	0.15250271025300200	0.0027565345839106600	-0.23478539768554000
u6	i37	0.46559801813246000	0.9655980181324600	0.4148333467148490	0.15250271025300200	0.0027565345839106600	-0.019957525517680600

u6	i38	0.5426446347075770	0.9573553652924230	0.6050247704321410	0.15250271025300200	0.0027565345839106600	0.061210417477454300
u6	i39	0.2865412521282840	0.7865412521282840	0.10578027294673700	0.15250271025300200	0.0027565345839106600	-0.10948590851976800
u6	i40	0.5908332605690110	0.9091667394309890	0.7126588411597790	0.15250271025300200	0.0027565345839106600	0.13349335626960500
u6	i41	0.03050024993904940	0.5305002499390490	0.009058091197327820	0.15250271025300200	0.0027565345839106600	-0.23750640961438600
u6	i42	0.03734818874921440	0.5373481887492140	0.00969389186924036	0.15250271025300200	0.0027565345839106600	-0.23408244020930300
u6	i43	0.8226005606596580	0.6773994393403420	0.9618012709662370	0.15250271025300200	0.0027565345839106600	0.4811443064055770
u6	i44	0.3601906414112630	0.8601906414112630	0.19811880485572000	0.15250271025300200	0.0027565345839106600	-0.07266121387827920
u6	i45	0.12706051265188500	0.6270605126518850	0.023444517973674600	0.15250271025300200	0.0027565345839106600	-0.18922627825796800
u6	i46	0.5222432600548040	0.9777567399451960	0.5553800052301390	0.15250271025300200	0.0027565345839106600	0.030608355498295900
u6	i47	0.7699935530986110	0.7300064469013890	0.9370228396667850	0.15250271025300200	0.0027565345839106600	0.4022337950640060
u6	i48	0.21582102749684300	0.7158210274968430	0.05510727173980280	0.15250271025300200	0.0027565345839106600	-0.1448460208354890
u6	i49	0.6228904758190000	0.8771095241810000	0.773626823850454	0.15250271025300200	0.0027565345839106600	0.1815791791445900
u6	i50	0.085347464993768	0.585347464993768	0.015572934852333800	0.15250271025300200	0.0027565345839106600	-0.21008280208702700
u7	i1	0.0516817211686077	0.5516817211686080	0.011171193220275300	0.09604560403105060	0.0	-0.22415913941569600
u7	i2	0.531354631568148	0.968645368431852	0.5777506397021420	0.09604560403105060	0.0	0.04703194735222200
u7	i3	0.5406351216101070	0.9593648783898940	0.6002126470615560	0.09604560403105060	0.0	0.060952682415159800
u7	i4	0.6374299014982070	0.8625700985017930	0.7980738362135320	0.09604560403105060	0.0	0.20614485224731000
u7	i5	0.7260913337226620	0.7739086662773390	0.9055877490162750	0.09604560403105060	0.0	0.3391370005839920
u7	i6	0.9758520794625350	0.5241479205374650	0.9914946720928480	0.09604560403105060	0.0	0.713778119193802
u7	i7	0.5163003483011950	0.9836996516988050	0.5406608801719040	0.09604560403105060	0.0	0.024450522451793000
u7	i8	0.32295647294124600	0.822956472941246	0.14548820714245000	0.09604560403105060	0.0	-0.08852176352937700
u7	i9	0.7951861947687040	0.7048138052312960	0.9503514155451950	0.09604560403105060	0.0	0.4427792921530560
u7	i10	0.2708322512620740	0.7708322512620740	0.09181457740493070	0.09604560403105060	0.0	-0.11458387436896300
u7	i11	0.4389714207056360	0.9389714207056360	0.3519940074419740	0.09604560403105060	0.0	-0.030514289647181900
u7	i12	0.07845638134226600	0.578456381342266	0.014551021057484100	0.09604560403105060	0.0	-0.21077180932886700
u7	i13	0.02535074341545750	0.5253507434154580	0.008607363794367180	0.09604560403105060	0.0	-0.23732462829227100
u7	i14	0.9626484146779250	0.5373515853220750	0.9903057820570410	0.09604560403105060	0.0	0.6939726220168880
u7	i15	0.8359801205122060	0.6640198794877940	0.9664243267513340	0.09604560403105060	0.0	0.5039701807683090
u7	i16	0.695974206093698	0.804025793906302	0.8765050348008590	0.09604560403105060	0.0	0.29396130914054700
u7	i17	0.4089529444142700	0.9089529444142700	0.2869035566165450	0.09604560403105060	0.0	-0.04552352779286510
u7	i18	0.17329432007084600	0.6732943200708460	0.036718788984048900	0.09604560403105060	0.0	-0.1633528399645770
u7	i19	0.15643704267108600	0.656437042671086	0.031200318109946500	0.09604560403105060	0.0	-0.17178147866445700
u7	i20	0.25024289816459500	0.7502428981645950	0.07602863618775380	0.09604560403105060	0.0	-0.12487855091770200
u7	i21	0.5492266647061210	0.9507733352938800	0.6206402509116150	0.09604560403105060	0.0	0.0738399970591807
u7	i22	0.7145959227000620	0.7854040772999380	0.8952905777693060	0.09604560403105060	0.0	0.3218938840500940
u7	i23	0.6601973767177310	0.8398026232822690	0.8322940655686900	0.09604560403105060	0.0	0.2402960650765970
u7	i24	0.27993389694594300	0.7799338969459430	0.0996911440631432	0.09604560403105060	0.0	-0.11003305152702900
u7	i25	0.9548652806631940	0.5451347193368060	0.9895293446711100	0.09604560403105060	0.0	0.6822979209947910
u7	i26	0.7378969166957690	0.7621030833042320	0.9152094747301080	0.09604560403105060	0.0	0.35684537504365300
u7	i27	0.5543540525114010	0.9456459474885990	0.6326356490635090	0.09604560403105060	0.0	0.08153107876710100
u7	i28	0.6117207462343520	0.8882792537656480	0.7534703620697420	0.09604560403105060	0.0	0.16758111935152800
u7	i29	0.4196000624277900	0.9196000624277900	0.3091706642657110	0.09604560403105060	0.0	-0.040199968786105000
u7	i30	0.24773098950115700	0.7477309895011580	0.07428274859583070	0.09604560403105060	0.0	-0.12613450524942100
u7	i31	0.3559726786512620	0.8559726786512620	0.1915030434534600	0.09604560403105060	0.0	-0.0720136606743692
u7	i32	0.7578461104643690	0.7421538895356310	0.9294624426898040	0.09604560403105060	0.0	0.3867691656965540
u7	i33	0.014393488629755900	0.5143934886297560	0.007720964104285110	0.09604560403105060	0.0	-0.24280325568512200
u7	i34	0.11607264050691600	0.6160726405069160	0.02105631516129920	0.09604560403105060	0.0	-0.19196367974654200
u7	i35	0.04600264202175280	0.5460026420217530	0.01056096410927070	0.09604560403105060	0.0	-0.22699867898912400
u7	i36	0.040728802318970100	0.5407288023189700	0.010023866092978500	0.09604560403105060	0.0	-0.22963559884051500
u7	i37	0.8554605840110070	0.6445394159889930	0.972202170636338	0.09604560403105060	0.0	0.5331908760165110
u7	i38	0.7036578593800240	0.7963421406199760	0.8845844194515460	0.09604560403105060	0.0	0.3054867890700360
u7	i39	0.4741738290873250	0.9741738290873250	0.43579106650532800	0.09604560403105060	0.0	-0.012913085456337400
u7	i40	0.09783416065100150	0.5978341606510020	0.017607631147719600	0.09604560403105060	0.0	-0.20108291967449900
u7	i41	0.49161587511683200	0.9916158751168320	0.47905195729044900	0.09604560403105060	0.0	-0.0041920624415838200
u7	i42	0.4734717707805660	0.9734717707805660	0.43406564998538000	0.09604560403105060	0.0	-0.01326411460971720
u7	i43	0.17320186991001500	0.6732018699100150	0.03668610288378940	0.09604560403105060	0.0	-0.1633990650449920
u7	i44	0.43385164923797300	0.933851649237973	0.3404064409172740	0.09604560403105060	0.0	-0.03307417538101350
u7	i45	0.39850473439737300	0.8985047343973730	0.2660117272626690	0.09604560403105060	0.0	-0.05074763280131330
u7	i46	0.6158500980522170	0.8841499019477840	0.7610602284971210	0.09604560403105060	0.0	0.1737751470783250
u7	i47	0.6350936508676440	0.8649063491323560	0.7942826937326380	0.09604560403105060	0.0	0.2026404763014660
u7	i48	0.04530400977204450	0.5453040097720450	0.010488210089838800	0.09604560403105060	0.0	-0.22734799511397800
u7	i49	0.3746126146264710	0.8746126146264710	0.22203027661610700	0.09604560403105060	0.0	-0.06269369268676440
u7	i50	0.6258599157142360	0.8741400842857640	0.778784866227863	0.09604560403105060	0.0	0.18878987357135500
u8	i1	0.5031362585800880	0.9968637414199120	0.5078400038340720	0.1478224309295220	0.0022869849000088800	0.002417402970122670
u8	i2	0.8564898411883220	0.6435101588116780	0.9724789807320350	0.1478224309295220	0.0022869849000088800	0.5324477768824750
u8	i3	0.658693631618945	0.841306368381055	0.8301846275174540	0.1478224309295220	0.0022869849000088800	0.23575346252840900
u8	i4	0.1629344270814300	0.6629344270814300	0.033225239502422200	0.1478224309295220	0.0022869849000088800	-0.17081977135929400
u8	i5	0.07056874740042980	0.5705687474004300	0.013462244596299700	0.1478224309295220	0.0022869849000088800	-0.21700261119979400
u8	i6	0.6424192782063160	0.8575807217936840	0.8059948709935150	0.1478224309295220	0.0022869849000088800	0.21134193240946500
u8	i7	0.026511310541621800	0.5265113105416220	0.008706965105463210	0.1478224309295220	0.0022869849000088800	-0.23903132962919800
u8	i8	0.5857755812734630	0.9142244187265370	0.7021915661600290	0.1478224309295220	0.0022869849000088800	0.12637638701018600
u8	i9	0.9402302414249580	0.5597697585750420	0.987899120050315	0.1478224309295220	0.0022869849000088800	0.6580583772374280
u8	i10	0.575474177875879	0.924525822124121	0.6802110299993260	0.1478224309295220	0.0022869849000088800	0.11092428191381000
u8	i11	0.3881699262065220	0.8881699262065220	0.24632661523518000	0.1478224309295220	0.0022869849000088800	-0.05820202179674790
u8	i12	0.6432882184423530	0.8567117815576470	0.8073499962932770	0.1478224309295220	0.0022869849000088800	0.21264534276352100
u8	i13	0.45825289049151700	0.9582528904915170	0.3971220529548240	0.1478224309295220	0.0022869849000088800	-0.023160539654250600
u8	i14	0.5456167893159350	0.9543832106840650	0.6121047011148730	0.1478224309295220	0.0022869849000088800	0.06613819907389350
u8	i15	0.9414648087765250	0.5585351912234750	0.9880458202667520	0.1478224309295220	0.0022869849000088800	0.6599102282647790
u8	i16	0.38610263780077400	0.8861026378007740	0.24250885506965500	0.1478224309295220	0.0022869849000088800	-0.05923566599962180
u8	i17	0.9611905638239140	0.5388094361760860	0.9901648197835660	0.1478224309295220	0.0022869849000088800	0.6894988608358620
u8	i18	0.9053506419560640	0.5946493580439360	0.9829348827184400	0.1478224309295220	0.0022869849000088800	0.6057389780340870
u8	i19	0.19579113478929600	0.6957911347892960	0.04556026048757850	0.1478224309295220	0.0022869849000088800	-0.15439141750536100
u8	i20	0.06936130087516550	0.5693613008751660	0.013302821965878400	0.1478224309295220	0.0022869849000088800	-0.21760633446242600
u8	i21	0.10077800137742700	0.6007780013774270	0.018124142621653200	0.1478224309295220	0.0022869849000088800	-0.20189798421129600
u8	i22	0.018221825651549700	0.5182218256515500	0.008019863007765920	0.1478224309295220	0.0022869849000088800	-0.24317607207423400
u8	i23	0.0944429607559284	0.5944429607559280	0.017030530895580200	0.1478224309295220	0.0022869849000088800	-0.2050655045220450
u8	i24	0.6830067734163570	0.8169932265836430	0.8617697956959080	0.1478224309295220	0.0022869849000088800	0.2722231752245260
u8	i25	0.07118864846022900	0.571188648460229	0.013544822528909700	0.1478224309295220	0.0022869849000088800	-0.2166926606698940
u8	i26	0.3189756302937610	0.8189756302937610	0.14060867531959200	0.1478224309295220	0.0022869849000088800	-0.09279916975312820

u8	i27	0.8448753109694550	0.6551246890305450	0.9691939339457090	0.1478224309295220	0.0022869849000088800	0.5150259815541730
u8	i28	0.023271935735825900	0.5232719357358260	0.008431773373451340	0.1478224309295220	0.0022869849000088800	-0.24065101703209600
u8	i29	0.8144684825889360	0.6855315174110640	0.9586987775389880	0.1478224309295220	0.0022869849000088800	0.4694157389833950
u8	i30	0.28185477477340000	0.7818547747734000	0.1014284918984960	0.1478224309295220	0.0022869849000088800	-0.11135959751330900
u8	i31	0.11816482762165600	0.6181648276216560	0.02149192567834260	0.1478224309295220	0.0022869849000088800	-0.19320457108918100
u8	i32	0.6967371653641510	0.8032628346358490	0.8773285225897440	0.1478224309295220	0.0022869849000088800	0.2928187631462170
u8	i33	0.628942846779884	0.871057153220116	0.7840504357128010	0.1478224309295220	0.0022869849000088800	0.1911272852698170
u8	i34	0.877472013527053	0.622527986472947	0.9775710871810630	0.1478224309295220	0.0022869849000088800	0.5639210353905710
u8	i35	0.7350710438038860	0.7649289561961140	0.9129906803936090	0.1478224309295220	0.0022869849000088800	0.35031958080582000
u8	i36	0.8034809303848490	0.6965190696151510	0.9541221513372650	0.1478224309295220	0.0022869849000088800	0.452934410677264
u8	i37	0.2820345725713070	0.7820345725713070	0.10159247843757400	0.1478224309295220	0.0022869849000088800	-0.11126969861435600
u8	i38	0.17743954377972300	0.6774395437797230	0.038213465967936900	0.1478224309295220	0.0022869849000088800	-0.16356721301014700
u8	i39	0.7506147516408580	0.7493852483591420	0.9245716615956960	0.1478224309295220	0.0022869849000088800	0.3736351425612790
u8	i40	0.806834739267264	0.693165260732736	0.9555680592354120	0.1478224309295220	0.0022869849000088800	0.4579651240008870
u8	i41	0.9905051420006730	0.5094948579993270	0.9926454283787570	0.1478224309295220	0.0022869849000088800	0.7334707281010010
u8	i42	0.4126176769114270	0.9126176769114270	0.2944593903982240	0.1478224309295220	0.0022869849000088800	-0.045978146444295600
u8	i43	0.37201808579278300	0.8720180857927830	0.2175810111701160	0.1478224309295220	0.0022869849000088800	-0.0662779420036173
u8	i44	0.7764129607419970	0.7235870392580030	0.9407063943887030	0.1478224309295220	0.0022869849000088800	0.41233245621298600
u8	i45	0.34080354025301800	0.8408035402530180	0.16910767134212000	0.1478224309295220	0.0022869849000088800	-0.08188521477350000
u8	i46	0.9307573256035650	0.5692426743964350	0.9867127397895260	0.1478224309295220	0.0022869849000088800	0.6438490035053380
u8	i47	0.8584127518430120	0.6415872481569880	0.9729889719312710	0.1478224309295220	0.0022869849000088800	0.5353321428645090
u8	i48	0.42899402737501800	0.9289940273750180	0.32958564300754800	0.1478224309295220	0.0022869849000088800	-0.03778997121249970
u8	i49	0.7508710677914970	0.7491289322085030	0.9247502192579890	0.1478224309295220	0.0022869849000088800	0.37401961678723700
u8	i50	0.7545428740846820	0.7454571259153180	0.9272658124988800	0.1478224309295220	0.0022869849000088800	0.3795273262270150
u9	i1	0.10312386883593300	0.6031238688359330	0.018546358612087400	0.041276792626599400	0.0	-0.1984380655820340
u9	i2	0.9025529066795670	0.5974470933204330	0.9824591973559260	0.041276792626599400	0.0	0.6038293600193500
u9	i3	0.5052523724478570	0.9947476275521430	0.5131279132150050	0.041276792626599400	0.0	0.007878558671785720
u9	i4	0.8264574661077420	0.6735425338922580	0.9631933154693750	0.041276792626599400	0.0	0.48968619916161200
u9	i5	0.32004960103061200	0.8200496010306120	0.14191145463455700	0.041276792626599400	0.0	-0.08997519948469410
u9	i6	0.8955232284962010	0.6044767715038000	0.9812057699976910	0.041276792626599400	0.0	0.5932848427443010
u9	i7	0.3892016787341630	0.8892016787341630	0.24824707122006900	0.041276792626599400	0.0	-0.05539916063291840
u9	i8	0.01083765148029840	0.5108376514802980	0.007453253333796500	0.041276792626599400	0.0	-0.24458117425985100
u9	i9	0.9053819764192640	0.5946180235807360	0.982940137934377	0.041276792626599400	0.0	0.6080729646288960
u9	i10	0.09128667678613360	0.5912866767861340	0.016510129574112900	0.041276792626599400	0.0	-0.20435666160693300
u9	i11	0.31931363759041500	0.8193136375904150	0.1410176125315980	0.041276792626599400	0.0	-0.09034318120479260
u9	i12	0.9500619670508050	0.5499380329491950	0.9890197888114760	0.041276792626599400	0.0	0.6750929505762070
u9	i13	0.9506071469375560	0.5493928530624440	0.9890788358501910	0.041276792626599400	0.0	0.6759107204063340
u9	i14	0.5734378881232860	0.9265621118767140	0.6757654514229990	0.041276792626599400	0.0	0.11015683218492900
u9	i15	0.6318372121697990	0.8681627878302010	0.7889107424971210	0.041276792626599400	0.0	0.1977558182546990
u9	i16	0.44844552197832000	0.9484455219783200	0.37389460182668000	0.041276792626599400	0.0	-0.025777239010840100
u9	i17	0.29321077169806500	0.7932107716980650	0.11225691212294900	0.041276792626599400	0.0	-0.10339461415096800
u9	i18	0.32866454536991600	0.828664545369916	0.15272912175427100	0.041276792626599400	0.0	-0.08566772731504200
u9	i19	0.6725184560770380	0.8274815439229620	0.8487954358669030	0.041276792626599400	0.0	0.2587776841155580
u9	i20	0.75237452943768	0.74762547056232	0.9257897787275890	0.041276792626599400	0.0	0.37856179415652
u9	i21	0.7915790437258490	0.7084209562741520	0.948621517356278	0.041276792626599400	0.0	0.4373685655887730
u9	i22	0.7896181427945540	0.7103818572054460	0.9476573481393200	0.041276792626599400	0.0	0.43442721419183100
u9	i23	0.09120610304869040	0.5912061030486900	0.01649705147256920	0.041276792626599400	0.0	-0.20439694847565500
u9	i24	0.49442030470258100	0.9944203047025820	0.4860543796436750	0.041276792626599400	0.0	-0.002789847648709270
u9	i25	0.057558760016644300	0.5575587600166440	0.011839398629032400	0.041276792626599400	0.0	-0.22122061999167800
u9	i26	0.5495288823237360	0.9504711176762650	0.6213515501133100	0.041276792626599400	0.0	0.0742933234856033
u9	i27	0.441530501373377	0.941530501373377	0.35785299439898100	0.041276792626599400	0.0	-0.029234749313311500
u9	i28	0.8877041827583000	0.6122958172417000	0.9797082779080160	0.041276792626599400	0.0	0.5815562741374500
u9	i29	0.3509150125520790	0.8509150125520790	0.1837942001776430	0.041276792626599400	0.0	-0.07454249372396070
u9	i30	0.11706701642760600	0.6170670164276060	0.021262264477121400	0.041276792626599400	0.0	-0.19146649178619700
u9	i31	0.14299168205283600	0.6429916820528360	0.027382595561716300	0.041276792626599400	0.0	-0.17850415897358200
u9	i32	0.7615106317174720	0.7384893682825280	0.9318274913086420	0.041276792626599400	0.0	0.39226594757620800
u9	i33	0.6182180633162610	0.8817819366837390	0.7653396608574100	0.041276792626599400	0.0	0.17732709497439200
u9	i34	0.10112267612279000	0.6011226761227900	0.018185581744495500	0.041276792626599400	0.0	-0.19943866193860500
u9	i35	0.08410680611499740	0.5841068061149970	0.015383875342638	0.041276792626599400	0.0	-0.20794659694250100
u9	i36	0.70096913145912	0.79903086854088	0.8818108546434540	0.041276792626599400	0.0	0.30145369718868
u9	i37	0.07276300636419350	0.5727630063641940	0.013756797200402400	0.041276792626599400	0.0	-0.21361849681790300
u9	i38	0.8218600592903560	0.6781399407096440	0.9615282815484430	0.041276792626599400	0.0	0.4827900889355340
u9	i39	0.7062422271564960	0.7937577728435040	0.8871968149962060	0.041276792626599400	0.0	0.3093633407347440
u9	i40	0.08134878064189980	0.5813487806419000	0.014971647287996900	0.041276792626599400	0.0	-0.20932560967905000
u9	i41	0.08483771408519190	0.5848377140851920	0.015494980571344500	0.041276792626599400	0.0	-0.20758114295740400
u9	i42	0.9866395785011760	0.5133604214988250	0.9923577816253180	0.041276792626599400	0.0	0.7299593677517630
u9	i43	0.3742707957561200	0.8742707957561200	0.22144040424135300	0.041276792626599400	0.0	-0.06286460212193980
u9	i44	0.3706421470668910	0.8706421470668910	0.21524772352247300	0.041276792626599400	0.0	-0.06467892646655450
u9	i45	0.8127995672575030	0.6872004327424970	0.9580328811353270	0.041276792626599400	0.0	0.46919935088625400
u9	i46	0.9472485773838590	0.5527514226161410	0.9887100233169140	0.041276792626599400	0.0	0.6708728660757880
u9	i47	0.9860010638228710	0.5139989361771290	0.9923092053061160	0.041276792626599400	0.0	0.7290015957343060
u9	i48	0.7533781852589420	0.7466218147410580	0.9264763810981010	0.041276792626599400	0.0	0.38006727788841200
u9	i49	0.37625958553091600	0.8762595855309160	0.22488815511384000	0.041276792626599400	0.0	-0.06187020723454210
u9	i50	0.08350071669866880	0.5835007166986690	0.015292338837401000	0.041276792626599400	0.0	-0.20824964165066600
u10	i1	0.7771469159274370	0.7228530840725630	0.941114457240051	0.08660830773393070	0.0	0.4157203738911550
u10	i2	0.558404249735805	0.941595750264195	0.6419970535129270	0.08660830773393070	0.0	0.08760637460370760
u10	i3	0.4242220092469760	0.9242220092469760	0.31912846626107600	0.08660830773393070	0.0	-0.03788899537651190
u10	i4	0.906354385094736	0.593645614905264	0.9831024359705830	0.08660830773393070	0.0	0.609531577642104
u10	i5	0.11119748230615100	0.6111974823061510	0.020074520035562700	0.08660830773393070	0.0	-0.19440125884692400
u10	i6	0.49262510429085900	0.9926251042908590	0.4815711127229840	0.08660830773393070	0.0	-0.003687447854570430
u10	i7	0.011353644767419100	0.5113536447674190	0.007491522156933360	0.08660830773393070	0.0	-0.24432317761629000
u10	i8	0.46866064199412600	0.9686606419941260	0.4222866213343790	0.08660830773393070	0.0	-0.015669679002936900
u10	i9	0.05630327568183740	0.5563032756818370	0.011693413292927600	0.08660830773393070	0.0	-0.22184836215908100
u10	i10	0.11881791626807200	0.6188179162680720	0.02162970043735060	0.08660830773393070	0.0	-0.19059104186596400
u10	i11	0.11752624677710500	0.6175262467771050	0.021358041540571400	0.08660830773393070	0.0	-0.19123687661144800
u10	i12	0.6492103021160640	0.8507896978839360	0.8163937147471800	0.08660830773393070	0.0	0.22381545317409500
u10	i13	0.7460448792654230	0.7539551207345770	0.9213222008851870	0.08660830773393070	0.0	0.36906731889813500
u10	i14	0.5833687650971600	0.9166312349028400	0.6971340991598970	0.08660830773393070	0.0	0.12505314764573900
u10	i15	0.9621725484745420	0.5378274515254580	0.9902599910058020	0.08660830773393070	0.0	0.6932588227118130

u10	i16	0.37487057952370400	0.8748705795237040	0.22247618618950100	0.08660830773393070	0.0	-0.06256471023814800
u10	i17	0.28571208628186100	0.7857120862818610	0.10499851811143500	0.08660830773393070	0.0	-0.10714395685907000
u10	i18	0.8685991281894600	0.6314008718105400	0.9755409365144430	0.08660830773393070	0.0	0.5528986922841900
u10	i19	0.22359583851945300	0.7235958385194530	0.05929851384258930	0.08660830773393070	0.0	-0.13820208074027400
u10	i20	0.9632225394406110	0.5367774605593890	0.9903607445440710	0.08660830773393070	0.0	0.694833809160917
u10	i21	0.012154474689816300	0.5121544746898160	0.007551302522552150	0.08660830773393070	0.0	-0.24392276265509200
u10	i22	0.969878826707639	0.530121173292361	0.9909758716353100	0.08660830773393070	0.0	0.7048182400614590
u10	i23	0.04315991195057610	0.5431599119505760	0.010268010730732700	0.08660830773393070	0.0	-0.22842004402471200
u10	i24	0.8911431136980710	0.6088568863019290	0.9803807759176600	0.08660830773393070	0.0	0.5867146705471070
u10	i25	0.5277011090863000	0.9722988909137000	0.5688133012387860	0.08660830773393070	0.0	0.041551663629449800
u10	i26	0.9929647961193000	0.5070352038807000	0.9928228363200540	0.08660830773393070	0.0	0.7394471941789500
u10	i27	0.07379656473539890	0.5737965647353990	0.013897732757906300	0.08660830773393070	0.0	-0.21310171763230100
u10	i28	0.5538542844013210	0.9461457155986790	0.6314733810565130	0.08660830773393070	0.0	0.08078142660198120
u10	i29	0.969302535619099	0.530697464380901	0.9909241896010480	0.08660830773393070	0.0	0.7039538034286480
u10	i30	0.5230978441701490	0.9769021558298510	0.557489244820032	0.08660830773393070	0.0	0.03464676625522320
u10	i31	0.6293986381352630	0.8706013618647380	0.7848211612661070	0.08660830773393070	0.0	0.1940979572028940
u10	i32	0.6957486889846170	0.8042513110153830	0.8762607188132150	0.08660830773393070	0.0	0.29362303347692600
u10	i33	0.45454106476777300	0.9545410647677730	0.3882701620978830	0.08660830773393070	0.0	-0.022729467616113400
u10	i34	0.6275580800840630	0.8724419199159370	0.781696592056388	0.08660830773393070	0.0	0.19133712012609500
u10	i35	0.5843143119231000	0.9156856880769000	0.6991267800127110	0.08660830773393070	0.0	0.12647146788465000
u10	i36	0.901158010490989	0.598841989509011	0.982217188432788	0.08660830773393070	0.0	0.6017370157364830
u10	i37	0.04544638034145790	0.5454463803414580	0.010502995905050400	0.08660830773393070	0.0	-0.22727680982927100
u10	i38	0.2809631895922300	0.7809631895922300	0.10061877722093300	0.08660830773393070	0.0	-0.10951840520388500
u10	i39	0.9504114840765590	0.5495885159234410	0.9890576803206860	0.08660830773393070	0.0	0.6756172261148380
u10	i40	0.8902637838909160	0.6097362161090840	0.9802109265237720	0.08660830773393070	0.0	0.5853956758363750
u10	i41	0.45565675278571300	0.9556567527857130	0.39092338446621500	0.08660830773393070	0.0	-0.02217162360714350
u10	i42	0.6201325978015370	0.8798674021984630	0.7687605836233110	0.08660830773393070	0.0	0.1801988967023050
u10	i43	0.2773811829811330	0.7773811829811330	0.09742330886392300	0.08660830773393070	0.0	-0.11130940850943400
u10	i44	0.1881211597237610	0.6881211597237610	0.04233887038432990	0.08660830773393070	0.0	-0.15593942013811900
u10	i45	0.46369840493998200	0.9636984049399820	0.41022968590387600	0.08660830773393070	0.0	-0.018150797530008900
u10	i46	0.3533522280260530	0.8533522280260530	0.18747857364923500	0.08660830773393070	0.0	-0.07332388598697360
u10	i47	0.5836561118508720	0.9163438881491280	0.6977404538790440	0.08660830773393070	0.0	0.1254841677763080
u10	i48	0.07773463696498480	0.5777346369649850	0.014447889835137700	0.08660830773393070	0.0	-0.21113268151750800
u10	i49	0.9743948076661670	0.5256051923338340	0.9913708962036580	0.08660830773393070	0.0	0.7115922114992500
u10	i50	0.9862107444796030	0.5137892555203970	0.9923251908849410	0.08660830773393070	0.0	0.7293161167194040