Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.18576/jsap/140616

Integrating Composite Models for Enhanced Risk Assessment of the South African Industrial Index (J520)

Sandile C. Shongwe[1,], Zander Greyling[1] and Frans F. Koning[1,2]*

[1] Department of Mathematical Statistics and Actuarial Science, Faculty of Natural and Agricultural Sciences, University of the Free State, Bloemfontein, 9301, South Africa
[2] Fellow of the Actuarial Society of South Africa (FASSA) and Chartered Enterprise Risk Actuary (CERA), South Africa

Received: 2 Sep. 2025, Revised: 12 Oct. 2025, Accepted: 22 Oct. 2025.
Published online: 1 Nov. 2025.

Abstract: In this paper, we examine whether composite models provide superior performance to single-distribution approaches in modeling financial return of the South African Industrial Index (J520). The composite models are developed by piecing together two distributions at a threshold value, where the portion from the left tail to threshold value (captures the frequent but low-to-moderate severity events) is called the 'head distribution' while from the threshold to the right tail (captures the rare but high severity events) is called 'tail distribution'. Thus, using 16 different single distributions, we construct and evaluate a total of 256 composite models, i.e. (16 for the 'head component') × (16 for the 'tail component'). Model performance is assessed by using standard goodness-of-fit criteria and by estimating key risk measures at the 95%, 99%, and 99.5% confidence levels. Loss and gain returns are analyzed separately, with the top 20 models (out of a total of 256) reported in terms of both model fit and risk estimation accuracy. By systematically comparing composite and single-distribution frameworks, the study addresses the practical question of whether greater model flexibility translates into improved risk assessment for real-world financial data. The findings contribute to the growing literature on composite modeling and its applications in actuarial science and risk management.

Keywords: Goodness-of-fit, Heavy-tailed, Industrial index, Tail value-at-risk (TVaR), Value-at-risk (VaR).

1. Introduction

Investors, actuaries and researchers are still seeking models that can accurately capture the characteristics of even the most complex financial data and by association, would be able to predict the outcomes of such data. The financial benefits are clear but should always be modeled with the simplest solution possible. Researchers traditionally rely on single-distribution models, notably the lognormal and exponential distributions, to characterize financial asset returns for their analytical convenience. This foundation underpins the work of Chernobai et al. [1] and Sweeting [2] on traditional single-model distributions. However, Klugman et al. [3] and Eling [4] document that empirical evidence consistently reveals skewness, leptokurtosis and heavy-tailed behavior that these single-model frameworks cannot capture, leading to systematic underestimation of extreme losses and miscalibrated risk measures. Recognizing these shortcomings, researchers have pursued more flexible modeling techniques that preserve interpretability while accommodating complex return dynamics.

A central innovation in this area is the two-component composite model, which partitions the return distribution at a data-driven threshold into a head for frequent, low-severity events and a tail for rare, high-severity outcomes. Li & Liu [5] explain that threshold selection typically balances bias and variance through methods such as the mean excess plot and Kolmogorov-Smirnov minimization, ensuring the segmentation point reflects a clear shift in tail behavior. They also demonstrate that parameters on each side of the cutoff are estimated via maximum likelihood to enforce continuity of the probability density and its derivatives at the joining point. This smooth transition maintains overall distributional integrity and as [5] showed, could enhance the stability of risk measure estimates.

1.1 Literature Review

Table 1 presents a consolidated summary of studies that informs this literature review, detailing each publication's dataset, model type, fitted distributions, fit criteria and risk measures. While other previous work focused on fitting single distributions individually on specific datasets, this study pivots also toward composite models, which combine two or more standard distributions at a calculated threshold(s) to better capture the full spectrum of financial returns behavior.

*Corresponding author e-mail: shongwesc@ufs.ac.za

Table 1: Summative summary of different publications that discuss single and composite distributions fitted on real and simulated financial data

Publication	Dataset	Model Type	No. fitted	Goodness-of-fit	Risk measures
Chikobvu & Jakata [6]	J580 Index	Single	4	AIC,BIC	VaR, TVaR
Maphalla et al. [7]	SA Taxi Claims	Single	6	KS,AD	VaR
Chikobvu & Ndlovu [8]	BTC/USD ZAR/USD	Single	3	AD,AIC,BIC	VaR, TVaR
Marambakuyana & Shongwe [9]	SA Taxi Claims Danish Fire	Single	19	KS,AD,CvM,NLL,AIC,BIC	VaR, TVaR
Shongwe et al. [10]	J580 Index	Single	4	KS,AD,CvM,NLL,AIC,BIC	VaR, TVaR
Jakata & Chikobvu [11]	J520 Index	Single	4	AIC,BIC	VaR, TVaR
Shongwe et al. [12]	J520 Index	Single	22	KS,AD,CvM,NLL,AIC,BIC	VaR, TVaR
Shongwe et al. [13]	J580 Index	Single	19	KS,AD,CvM,NLL,AIC,BIC	VaR, TVaR
Cooray & Ananda [14]	Danish Fire	Composite	1	-	VaR
Scollnik [15]	Danish Fire	Composite	2	KS,AD,CvM	VaR
Scollnik & Sun [16]	Danish Fire	Composite	3	KS,AD,CvM	VaR
Nadarajah & Abu Bakar [17]	Danish Fire	Composite	17	AIC	-
Abu Bakar et al. [18]	Danish Fire Adj. Expenses	Composite	8	NLL,AIC,BIC	VaR, TVaR
Calderín-Ojeda [19]	Danish Fire	Composite	1	-	VaR, TVaR
Calderín-Ojeda & Kwok [20]	Danish Fire	Composite	2	-	VaR, TVaR
Grün & Miljkovic [21]	Danish Fire	Composite	16x16 (256)	KS,AD,AIC,BIC	VaR, TVaR
Marambakuyana & Shongwe [22]	SA Taxi Claims Danish Fire	Composite	16x16 (256)	NLL,AIC,BIC	VaR, TVaR

Acronyms: KS – Kolmogorov Smirnov, AD – Anderson Darling, CvM – Cramer von Mises, NLL – Negative log likelihood, BIC – Bayesian Information Criterion, AIC – Akaike Information Criterion, VaR – Value at Risk, TVaR – Tail VaR, SA – South Africa, J520 – South African Industrial index, J580 – South African Financial index, ZAR – South African Rand, USD – United States Dollar.

Maphalla et al. [7] and Marambakuyana and Shongwe [9] evaluated 6 and 19 distributions respectively on South African taxi claims and Danish fire data. The papers applied maximum likelihood estimation, assessed fit with different tests alongside information criteria and measured the corresponding risk measures. Both studies showed that the GenPareto and Burr families achieved superior tail accuracy for high-severity losses. Chikobvu and Ndlovu [8] split BTC /USD (Bitcoin in United States Dollars) and ZAR/USD (South African Rand per United States Dollars) returns into gains and losses, using Hill, QQ- and PP-plots for diagnostics in tail classification and fitting Weibull, Burr and exponential models to reveal distinct tail behaviors. That work highlights the importance of regime-specific distributional choices in foreign exchange and digital-asset risk analysis.

Chikobvu and Jakata [6], Jakata and Chikobvu [11], Shongwe et al. [10] and Shongwe et al. [13] analysed between 4 and 19 parametric models including Weibull, lognormal and Pareto on daily returns of the South African Financial and Industrial indices, which are denoted as J580 and J520, respectively. They used maximum likelihood for parameter estimation and computed the corresponding information criteria, goodness-of-fit metrics and risk measures. Their results challenged the adequacy of single distribution frameworks for emerging-market equities and thus, here we intend to assess the composite distribution frameworks.

Cooray and Ananda [14] and Scollnik [15] pioneered two-component composite models for Danish fire claims by joining separate body and tail distributions at a data-driven threshold. They matched density continuity at the join point, selected thresholds empirically and used different tests and demonstrated improved risk measures estimation over single-distribution benchmarks. These early composite models showed clear gains in tail capture and risk-measure stability compared to simple parametric frameworks. Nadarajah and Abu Bakar [17], and Abu Bakar et al. [18] introduced richer tail families and refined transition methods in composite models for Danish fire and adjustment-expense data. [17] fitted lognormal bodies with Burr and inverse-family tails; [18] combined Weibull heads with transformed-beta tails. Each study relied on maximum likelihood parameter estimation and information criteria to show that these enhanced composite models capture body and tail behavior more accurately than their predecessors. Additional developments in composite modeling were provided by Scollnik and Sun [16], Calderín-Ojeda [19] and Calderín-Ojeda and Kwok [20], who explored alternative transition mechanisms and tail specifications within two-component composite frameworks. These studies offer further methodological insights into composite model construction and are summarized in Table 1.

Grün and Miljkovic [21] and Marambakuyana and Shongwe [22] used the 16 single distributions listed in Table 2 and constructed (16×16=) 256 two-component (i.e., head and tail combinations) that are shown in Table 3. The latter distributions were fitted to Danish fire and South African taxi-claims datasets. They ranked composite models with information criteria and evaluated risk measures. Composite models such as Weibull-Burr and IPareto-Pareto emerged as consistent top performers. Their comprehensive benchmarking provided clear guidance on composite model selection for complex loss distributions.

Table 2: The probability distribution function (pdf) of 16 single distributions and their parameter(s) information

Distribution	PDF	Parameters	Parameter Description
Burr	$f(x) = \frac{\alpha\gamma(x/\theta)^{\gamma}}{x\left(1+(\frac{x}{\theta})^{\gamma}\right)^{\alpha+1}}, \quad x > 0$	3	Shape = α *and* γ, scale = θ
Exponential	$f(x) = \frac{e^{-x/\theta}}{\theta}, \quad x > 0$	1	Scale = θ
Gamma	$f(x) = \frac{(x/\theta)^{\alpha}e^{-(x/\theta)}}{x\Gamma(\alpha)}, \quad x > 0$	2	Shape = α, scale = θ
Generalized Pareto (GenPareto)	$f(x) = \frac{\Gamma(\alpha+\tau)\theta^{\alpha}x^{\tau-1}}{\Gamma(\alpha)\Gamma(\tau)(x+\theta)^{\alpha+\tau}}, \quad x > 0$	3	Shape = α and τ, scale = θ
Inverse Burr (IBurr)	$f(x) = \frac{\tau\gamma(x/\theta)^{\tau\gamma}}{x\left(1+(\frac{x}{\theta})^{\gamma}\right)^{\tau+1}}, \quad x > 0$	3	Shape = γ *and* τ, scale = θ
Inverse exponential (IExponential)	$f(x) = \frac{\theta e^{-\theta/x}}{x^2}, \quad x > 0$	1	Scale = θ
Inverse gamma (IGamma)	$f(x) = \frac{(\theta/x)^{\alpha}e^{-(\theta/x)}}{x\Gamma(\alpha)}, \quad x > 0$	2	Shape = α, scale = θ
Inverse Gaussian (IGaussian)	$f(x) = \sqrt{\frac{\theta}{2\pi\sigma x^3}}e^{-\frac{\theta(x-\mu)^2}{2\mu^2 x}}, \quad x > 0$	2	Location = μ, scale = θ
Inverse paralogistic (IParalogistic)	$f(x) = \frac{\tau^2(x/\theta)^{\tau^2}}{x[1+(x/\theta)^{\tau}]^{\tau+1}}, \quad x > 0$	2	Shape = τ, scale = θ
Inverse Pareto (IPareto)	$f(x) = \frac{\tau\theta x^{\tau-1}}{(x+\theta)^{\tau+1}}, \quad x > 0$	2	Shape = τ, scale = θ
Inverse Weibull (IWeibull)	$f(x) = \frac{\tau(\theta/x)^{\tau}e^{-(\theta/x)^{\tau}}}{x}, \quad x > 0$	2	Shape = τ, scale = θ
Loglogistic	$f(x) = \frac{\gamma(x/\theta)^{\gamma}}{x[1+(x/\theta)^{\gamma}]^2}, \quad x > 0$	2	Shape = γ, scale = θ
Lognormal	$f(x) = \frac{1}{\sigma x\sqrt{2\pi}}e^{\frac{-(\ln x-\mu)^2}{2\sigma^2}}, \quad x > 0$	2	Location = μ, scale = σ
Paralogistic	$f(x) = \frac{\alpha^2(x/\theta)^{\alpha}}{x[1+(x/\theta)^{\alpha}]^{\alpha+1}}, \quad x > 0$	2	Shape = α, scale = θ
Pareto	$f(x) = \frac{\alpha\theta^{\alpha}}{(x+\theta)^{\alpha+1}}, \quad x > 0$	2	Shape = α, scale = θ
Weibull	$f(x) = \frac{\tau(x/\theta)^{\tau}e^{-(x/\theta)^{\tau}}}{x}, \quad x > 0$	2	Shape = τ, scale = θ

1.2 Research problem

Despite the demonstrated success of composite models in insurance and operational-risk settings, their use in financial-return analysis on the Johannesburg Stock Exchange (JSE) remains uncommon. [11] examines J520 returns using 4 distributions (Burr, exponential, gamma and Weibull) and it is worth mentioning that in their paper, the descriptive statistics are incorrect (correct ones are provided below). In contrast to the 4 distributions, a different study by [12] applies 22 single distributions to the J520 returns. We aim to provide both deeper insight into emerging-market dynamics and practical guidance on the trade-offs between model complexity and interpretability.

Let Y_t be the monthly J520 index for the period June 1995 to January 2018; hence, the corresponding monthly returns (r_t) are computed by

$$r_t = ln\,(Y_t/Y_{t-1}) \;= \begin{cases} Gain, & \text{if } r_t \geq 0 \\ Loss, & \text{if } r_t < 0 \end{cases} \tag{1}$$

since some of the returns are positive (i.e., denote as gains) and others are negative (i.e., denote as losses). The gains and losses are separately analyzed in this study and for ease in analysis, each of the losses (negative returns) is redefined as positive by introducing absolute values thereof: $l_t = |r_t|$ and thus are positive.

A preliminary review of the J520 data highlights the need for more flexible modeling approaches. Unlike [11], who incorrectly analyzes the full dataset descriptives together (see Table 4), we separate the data into 111 negative returns and 160 positive returns. Some descriptive statistics reveal additional complexity: the negative returns have an average of 0.041, skewness of 3.05, and kurtosis of 16.79, indicating a heavy tail, leptokurtic and significant skewness. The positive returns have an average of 0.044, skewness of 0.78, and kurtosis of 3.00. The strong right skew and high peak in the negative returns suggest that standard models may struggle to accurately capture these features. Note that the latter descriptives provide some visualization of how the returns are distributed; however, the manner in which [11] incorrectly computes their descriptives does not reflect separately the negative returns and positive returns data distribution characteristics. Furthermore, the mean-excess plots for both losses and gains in Figure 1 indicate that simple, lighter-tailed distributions might fit the central part of the data but fail to account for the extreme tails. This underscores the need for models capable of better representing such tail behavior, supporting the approach of separating the low-to-moderate events and the extreme ones. The depiction in Figure 1 suggests that no single parametric distribution can adequately model all aspects of the returns data. Therefore, this study aims to explore a composite model approach, testing whether dividing the return data into separate regimes (i.e., head and tail) improves the overall model fit and enhances the accuracy of risk measurements.

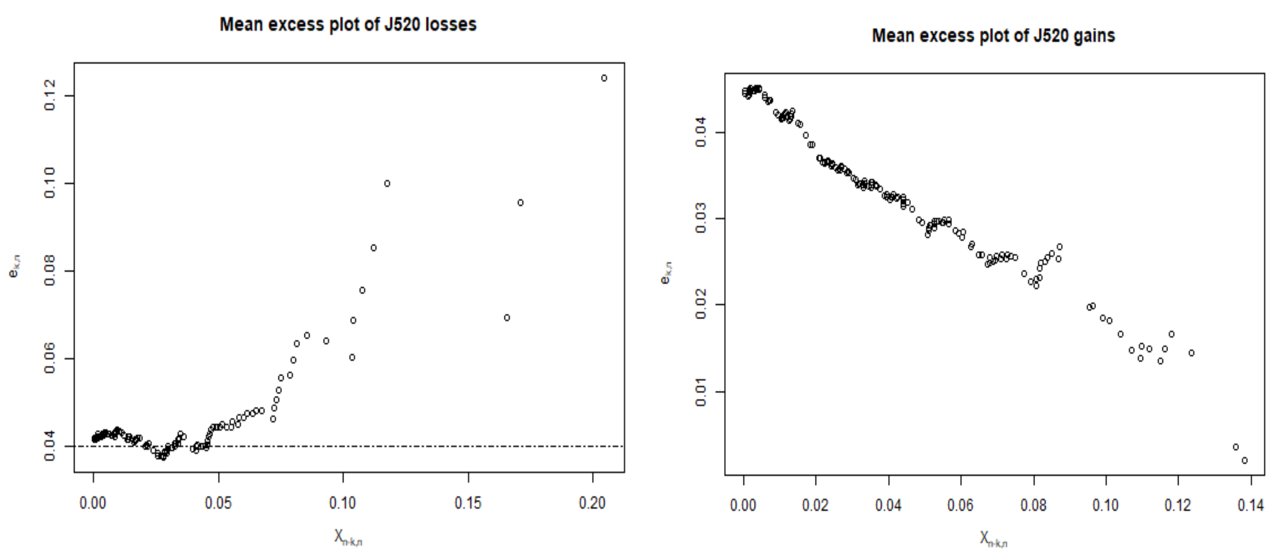
Table 3: List of the 256 composite distributions and their parameter(s) information

No.	Head	Tail	Composite	Parameters	Total
1.	Burr	Burr	Burr-Burr	3+3=6	16
		Exponential	Burr-Exponential	3+1=4	
		Gamma	Burr-Gamma	3+2=5	
		GenPareto	Burr-GenPareto	3+3=6	
		IBurr	Burr-IBurr	3+3=6	
		IExponential	Burr- IExponential	3+1=4	
		IGamma	Burr-IGamma	3+2=5	
		IGaussian	Burr-IGaussian	3+2=5	
		IParalogistic	Burr-IParalogistic	3+2=5	
		IPareto	Burr-IPareto	3+2=5	
		IWeibull	Burr-IWeibull	3+2=5	
		Loglogistic	Burr-Loglogistic	3+2=5	
		Lognormal	Burr-Lognormal	3+2=5	
		Paralogistic	Burr-Paralogistic	3+2=5	
		Pareto	Burr-Pareto	3+2=5	
		Weibull	Burr-Weibull	3+2=5	
2.	Exponential	Burr	Exponential-Burr	1+3=4	16
		Exponential	Exponential-Exponential	1+1=2	
		Gamma	Exponential-Gamma	1+2=3	
		GenPareto	Exponential-GenPareto	1+3=4	
		IBurr	Exponential-IBurr	1+3=4	
		IExponential	Exponential- IExponential	1+1=2	
		IGamma	Exponential-IGamma	1+2=3	
		IGaussian	Exponential-IGaussian	1+2=3	
		IParalogistic	Exponential-IParalogistic	1+2=3	
		IPareto	Exponential-IPareto	1+2=3	
		IWeibull	Exponential-IWeibull	1+2=3	
		Loglogistic	Exponential-Loglogistic	1+2=3	
		Lognormal	Exponential-Lognormal	1+2=3	
		Paralogistic	Exponential-Paralogistic	1+2=3	
		Pareto	Exponential-Pareto	1+2=3	
		Weibull	Exponential-Weibull	1+2=3	
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
16.	Weibull	Burr	Weibull-Burr	2+3=5	16
		Exponential	Weibull—Exponential	2+1=3	
		Gamma	Weibull-Gamma	2+2=4	
		GenPareto	Weibull-GenPareto	2+3=5	
		IBurr	Weibull-Iburr	2+3=5	
		IExponential	Weibull-Iexponential	2+1=3	
		IGamma	Weibull-Igamma	2+2=4	
		IGaussian	Weibull-Igaussian	2+2=4	
		IParalogistic	Weibull-Iparalogistic	2+2=4	
		IPareto	Weibull-Ipareto	2+2=4	
		IWeibull	Weibull-Iweibull	2+2=4	
		Loglogistic	Weibull-Loglogistic	2+2=4	
		Lognormal	Weibull-Lognormal	2+2=4	
		Paralogistic	Weibull-Paralogistic	2+2=4	
		Pareto	Weibull-Pareto	2+2=4	
		Weibull	Weibull-Weibull	2+2=4	
		Overall Number of Composite Models:			**256**

Table 4: Descriptives of the J520 returns

Descriptives	#Combined	Losses	Gains
No. of observations	271	111	160
Minimum	-0.140273	0.00006	0.00025
1st quartile	N/A	0.01051	0.01818
Median/2nd quartile	-0.010478	0.02979	0.03819
Mean/average	-0.009366	0.04123	0.04446
3rd quartile	N/A	0.05426	0.06609
Maximum	0.328471	0.32847	0.14027
Standard deviation	0.057467	0.04605	0.03338
Variance	0.003302	0.00212	0.00111
Coefficient of variation	N/A	1.11707	0.75067
Skewness	1.016932	3.05364	0.77519
Kurtosis	4.420852	16.78625	3.00489
Mean-median ratio	N/A	1.38396	1.16426
Median-mean ratio	N/A	0.72257	0.85892

#Values (which are incorrect) listed in this column are taken from Jakata and Chikobvu [11]



(a) Loss returns (b) Gain returns

Fig. 1: Mean excess plots for the losses and gains returns of the J520 data

1.3 Objective of the paper

The primary objective of this study is to fit the two-component composite models to monthly returns of the J520 index by constructing 256 composite models drawn from 16 candidate distributions. The study then evaluates each model's statistical performance using a suite of goodness-of-fit metrics and risk measures. These composite model results are benchmarked against the top-performing 16 single distributions enabling a detailed assessment of how composite frameworks enhance the modeling of heavy-tail behavior relative to traditional single-distribution approaches. Finally, we pinpoint those composite models that most faithfully reproduce the empirical return characteristics of the J520 and critically examine the trade-off between increased model complexity and interpretability. Through this expanded approach, the study seeks to yield deeper insights into emerging-market return dynamics and guide more robust financial risk-modeling practices.

1.4 Research Questions

This research work intends to provide answers to the following questions regarding the J520's loss and gain returns:

- Which of the possible 256 composite models perform the best in terms of the goodness-of-fit and risk measures?
- Do the more flexible and complex models constitute a better understanding of the underlying heavy-tail nature?

- Are simpler models within the set of 16 single distributions still competitive, or do the more complex models significantly outperform them?

The rest of the paper is structured as follows: In Section 2, the theoretical methodology of goodness-fit metrics and risk measures are discussed. In Section 3, the detailed empirical analysis is conducted, and the corresponding concluding remarks are done in Section 4.

2. Methodology

2.1 Software

All computations, including parameter estimation, goodness-of-fit metrics, and risk measure calculations, were performed using R (version 2025.09.1) with packages such as:

- *Model fitting* & *distributions*: \`fitdistrplus\`, \`actuar\`, \`flexsurv\`, \`gendist\`, \`invgamma\`, \`composite\` (installed from "http://ifas.jku.at/gruen/composite/composite_0.9-3.tar.gz") and \`SMPracticals\`
- *Diagnostics* & *numerics*: \`moments\`, \`pracma\` and \`numDeriv\`
- *Data I/O* & *reporting*: \`readxl\` and \`knitr\`

2.2 Goodness-of-fit

The set of goodness-of-fit metrics are usually called information criterions, and these are the negative log-likelihood (NLL), Akaike information criterion (*AIC*) and Bayesian information criterion (*BIC*). Assume that $l(\theta)$ denote the maximised log-likelihood function of a model, then the *NLL*, *AIC* and *BIC* are defined as Klugman et al. (2019):

$$NLL = -l(\theta), \tag{2}$$

$$AIC = 2NLL + 2p, \tag{3}$$

$$BIC = 2NLL + p \times log(n), \tag{4}$$

respectively, where p is the number of parameters or degrees of freedom and n is the number of observations. The corresponding metrics *NLL*, *AIC*, and *BIC*, incorporate the model parameters in their evaluations, with the *AIC* and *BIC* imposing a greater penalty on models characterized by increased complexity (i.e., a greater number of parameters). A desirable model fit is indicated by lower test scores; therefore, given identical datasets, the distribution exhibiting the lowest value for Equations (2) to (4), separately, is deemed the most appropriate fit.

2.3 Actuarial risk measures

Decision-making regarding risks is very complex and risk measures are essential for actuaries, investors, and financial institutions to make informed decisions about investments and risk management strategies. Two main risk measures are considered, i.e., value-at-risk (VaR) and tail value-at-risk (TVaR). Let $F(\cdot)$ and $F^{-1}(\cdot)$ denote the cdf and inverse cdf of a continuous random variable X, respectively. Then, the VaR of X at a $100p\%$ security level denoted by $VaR_p(X)$, is the $100p\%$ quantile of F such that

$$VaR_i(X) = \begin{cases} F_{i1}^{-1}\big(p(1+\varphi)F_{i1}(\vartheta_1)\big), \; if \; 0 < p \leq \frac{1}{1+\varphi} \\ F_{i2}^{-1}\left((F_{i2}(\vartheta_2) + \frac{1-F_{i2}(\vartheta_2)}{\varphi}\right)(p(1+\varphi)-1)|\vartheta_2), \; if \; \frac{1}{1+\varphi} < p \leq 1, \end{cases} \tag{5}$$

which can be thought of as the lower bound for the capital required to avoid insolvency. Next, TVaR of X at a $100p\%$ security level denoted by $TVaR_p(X)$,

$$ES_i \; or \; TVaR_i(\pi_i) = \begin{cases} \frac{1}{1-p}\left[\frac{\int_{\pi_i}^{\theta} x f_{i1}(\vartheta_1)dx}{F_{i1}(\theta|\vartheta_1)} + \frac{\int_{\theta}^{\infty} x f_{i2}(\vartheta_2)dx}{1-F_{i2}(\theta|\vartheta_2)}\right], \; if \; 0 < p \leq \frac{1}{1+\varphi} \\ \frac{1}{1-p}\frac{1}{(1-F_{i2}(\theta|\vartheta_2))}\left[\int_{\pi_i}^{\infty} x f_{i2}(\vartheta_2)dx\right], \; if \; \frac{1}{1+\varphi} < p \leq 1, \end{cases} \tag{6}$$

which can be thought of as the expected value of total loss, given that it exceeds VaR. In this research work, the fit of the theoretical model is also assessed by comparing the empirical risk estimates to the theoretical risk estimates. The cdf of the empirical distribution is computed by

$$F_n(x) = \frac{1}{n}\#\{i: x_i \leq x\}, \quad (7)$$

where # denotes the number of observations $\leq x$, and n is the total number of observations in the sample. This is done by computing the percentage deviation as follows:

$$\%deviation\ VaR = \frac{(VaR_{Theoretical} - VaR_{Empirical})}{VaR_{Empirical}} \quad (8)$$

$$\%deviation\ TVaR = \frac{(TVaR_{Theoretical} - TVaR_{Empirical})}{TVaR_{Empirical}} \quad (9)$$

where the theoretical implies anyone of the 16 single distributions that are in Table 2 or anyone of the 256 composite distributions that are in Table 3. It is important to note that:

- *Underestimating* the risk measures may result in *under-reserving* - which may lead to insolvency, i.e., not enough capital to cover unexpected losses.
- *Overestimating* the risk measures may result in *over-reserving* - which may negatively affect profitability due to fewer funds available for investment purposes.

3. Analysis

3.1 Goodness-of-fit for losses and gains

3.1.1 Losses

Table 5 presents the goodness-of-fit's NLL, AIC, and BIC metrics (Equations (2) to (4)) that account for the model parameters, calculated for 256 composite models fitted to the J520 loss returns. The top 20 models (sorted here in terms of average ranking (Avg Rank) across each of the three information criteria) were selected for display in Table 5. While all test values warrant consideration, the optimal distribution would ideally demonstrate a strong fit across the majority or all assessments. It is important to evaluate each test value while interpreting its respective strengths and limitations. From Table 5, the most consistent composite models are the loglogistic-head with IParalogistic-tail and the paralogistic-head with IParalogistic-tail as both are indistinguishable across NLL (-245.97), BIC (-473.10), and leading in AIC (-483.93). The IParalogistic-head with IParalogistic-tail ranks third overall, excelling in NLL (-245.97) and BIC (-473.09), its AIC (-483.93) places it on the 3rd rank. The fourth best model, IPareto-head and IParalogistic-tail, performs strongly in AIC (-483.93) but ranks middling in NLL and BIC. Lastly, the IParalogistic-head and loglogistic-tail, along with the IPareto-head and loglogistic-tail, share identical NLL metrics (-245.97) and ranks fifthteenth in BIC (-473.05), with their AIC values ranking in the top 5.

Table 5: Top 20 composite models for loss returns by goodness-of-fit metrics and average ranking

Head	Tail	NLL	Rank NLL	AIC	Rank AIC	BIC	Rank BIC	Avg Rank
Loglogistic	IParalogistic	-245.97	10	-483.93	1	-473.10	11	7.3
Paralogistic	IParalogistic	-245.97	10	-483.93	1	-473.10	11	7.3
IParalogistic	IParalogistic	-245.97	13	-483.93	3	-473.09	13	9.7
IPareto	IParalogistic	-245.97	14	-483.93	4	-473.09	14	10.7
IParalogistic	Loglogistic	-245.95	25	-483.89	5	-473.05	15	15.0
IPareto	Loglogistic	-245.95	25	-483.89	5	-473.05	15	15.0
Loglogistic	Loglogistic	-245.95	25	-483.89	5	-473.05	15	15.0
Paralogistic	Loglogistic	-245.95	25	-483.89	5	-473.05	15	15.0
IParalogistic	Paralogistic	-245.87	37	-483.74	9	-472.91	20	22.0
IPareto	Paralogistic	-245.87	37	-483.74	9	-472.91	20	22.0
Loglogistic	Paralogistic	-245.87	37	-483.74	9	-472.91	20	22.0
Paralogistic	Paralogistic	-245.87	37	-483.74	9	-472.91	20	22.0
Exponential	Loglogistic	-244.36	62	-482.73	17	-474.60	2	27.0
Lognormal	Lognormal	-245.67	43	-483.34	13	-472.50	26	27.3
Exponential	IParalogistic	-244.36	63	-482.72	18	-474.60	3	28.0
IBurr	Exponential	-245.64	44	-483.28	14	-472.44	27	28.3
Exponential	IBurr	-245.64	44	-483.28	14	-472.44	27	28.3
IBurr	IExponential	-245.64	44	-483.28	14	-472.44	27	28.3
IBurr	Pareto	-246.11	2	-482.22	20	-468.67	65	29.0
Loglogistic	IBurr	-245.97	5	-481.95	21	-468.40	67	31.0

Table 6: Worst-performing 5 composite models for loss returns under goodness-of-fit metrics

Head	Tail	NLL	Rank NLL	AIC	Rank AIC	BIC	Rank BIC
IExponential	Pareto	-87.28	252	-168.57	252	-160.44	252
IExponential	IExponential	-80.49	253	-156.99	253	-151.58	253
IExponential	IGamma	-80.49	253	-154.99	254	-146.87	253
IExponential	IWeibull	-80.49	253	-154.99	254	-146.87	253
IExponential	IGaussian	5574.46	256	11154.92	256	11163.05	256

Table 6 shows the 5 worst-performing models among the 256 tested composite models. The IExponential-head paired with either IGamma or IWeibull tails are indistinguishable, tied for 254th overall, with identical NLL (-80.49), BIC (-146.87), and slightly lower AIC (-154.99) values. Finally, the worst model is clearly an outlier value and should not be considered in the analysis, i.e., IExponential-head paired with IGaussian-tail.

We compare the latter top 20 composite models to the 16 single distributions in Table 7 that encompasses the same 16 single distributions used in the creation of the composite models. In comparing top-ranked models, the loglogistic–IParalogistic composite (Table 5) performs poorly compared to the IBurr single (Table 7) on NLL, -245.97 vs. -245.64 (a 0.33 improvement) and BIC, -473.10 vs. -477.15 (a 4.05 decline in BIC), with the single also having a lower AIC due to its simpler form. The second-best composite, paralogistic–IParalogistic, improves on most metrics when compared to the gamma single, with tighter NLL (-245.97 vs. -243.53), stronger AIC and a worse BIC value. For the third comparison, the IParalogistic-IParalogistic composite shows a better fit than the exponential in NLL, -245.97 vs. -242.94 (an 3.03 improvement) and BIC, –473.09 vs. –481.18 (a 8.09 point decline in BIC), with an improved AIC compared to the single model. In each case, the composite model added complexity struggles compared to the single distributions. The consistent BIC advantage across the single distributions suggests that the smaller size of loss returns could hamper the composite model's ability to capture the complexity shown.

Table 7: Ranking and comparative goodness-of-fit metrics for the 16 single distributions fitted on J520 loss returns

Distribution	NLL	Rank NLL	AIC	Rank AIC	BIC	Rank BIC	Avg Rank
IBurr	-245.64	1	-485.28	1	-477.15	5	2.33
Gamma	-243.53	3	-483.07	3	-477.65	2	2.67
Exponential	-242.94	7	-483.89	2	-481.18	1	3.33
Weibull	-243.45	5	-482.9	4	-477.48	3	4.00
GenPareto	-243.63	2	-481.25	6	-473.12	6	4.67
Pareto	-243.40	6	-482.78	5	-477.36	4	5.00
Burr	-243.49	4	-480.98	7	-472.85	7	6.00
Paralogistic	-236.90	8	-469.81	8	-464.39	8	8.00
Loglogistic	-234.30	9	-464.61	9	-459.19	9	9.00
IParalogistic	-231.99	10	-459.99	10	-454.57	10	10.00
IPareto	-229.22	11	-454.43	11	-449.01	11	11.00
Lognormal	-228.34	12	-452.67	12	-447.26	12	12.00
IWeibull	-195.30	13	-386.6	13	-381.18	13	13.00
IGamma	-161.96	14	-319.92	14	-314.5	14	14.00
IGaussian	-155.58	15	-307.16	15	-301.74	15	15.00
IExponential	-80.49	16	-158.98	16	-156.27	16	16.00

Top 1 comparison: the **IBurr** (rank 1 single) vs. the **loglogistic–IParalogistic** composite model (rank 1 composite).

- NLL (–245.97 vs. –245.64) →0.33 improvement
- AIC (–485.28 vs. –483.93) →1.35 decline
- BIC (–473.10 vs. –477.15) →4.05 decline

Top 2 comparison: the **gamma** distribution (rank 2 single) vs. the **paralogistic–IParalogistic** composite model (rank 2 composite).

- NLL (–245.97 vs. –243.53) →2.44 improvement

- AIC (–483.93 vs. –483.07) →0.76 improvement
- BIC (–473.10 vs. –477.65) →4.55 decline

Top 3 comparison: the **exponential** distribution (rank 3 single) vs. the **IParalogistic–IParalogistic** composite model (rank 3 composite).

- NLL (–245.97 vs. –242.94) →3.03 improvement
- AIC (–483.93 vs. –483.89) →0.04 improvement
- BIC (–473.09 vs. –481.18) →8.094 decline

3.1.2 Gains

From Table 8 for gains, the IBurr-head and gamma-tail composite model ranks first across two goodness-of-fit metrics, with leading scores in NLL (–349.83), and AIC (–689.65), with eleventh in BIC (–674.28). The next-best group of models is a cluster of Weibull-tail composite models gamma–Weibull, IParalogistic–Weibull, IPareto-Weibull, paralogistic-Weibull and loglogistic–Weibull. These models are tied equally sixth in NLL with (-347.47), eight in AIC with (-686.95) and fifth in BIC with (-674.65). These results reinforce the inverse Burr-head's versatility, especially when paired with moderately heavy-tailed components.

Table 8: Top 20 composite models for gain returns by goodness-of-fit metrics and average ranking

Head	Tail	NLL	Rank NLL	AIC	Rank AIC	BIC	Rank BIC	Avg Rank
IBurr	Gamma	-349.83	1	-689.65	1	-674.28	11	4.3
Gamma	Weibull	-347.47	6	-686.95	8	-674.65	5	6.3
IParalogistic	Weibull	-347.47	6	-686.95	8	-674.65	5	6.3
IPareto	Weibull	-347.47	6	-686.95	8	-674.65	5	6.3
Loglogistic	Weibull	-347.47	6	-686.95	8	-674.65	5	6.3
Paralogistic	Weibull	-347.47	6	-686.95	8	-674.65	5	6.3
Weibull	Weibull	-347.47	6	-686.95	8	-674.65	5	6.3
IBurr	Paralogistic	-349.62	3	-689.24	2	-673.86	17	7.3
IBurr	IParalogistic	-349.60	4	-689.20	3	-673.83	18	8.3
IBurr	Loglogistic	-349.57	5	-689.15	4	-673.77	19	9.3
Exponential	Weibull	-346.87	39	-687.74	5	-678.52	1	15.0
Weibull	Gamma	-347.14	22	-686.29	14	-673.99	12	16.0
Loglogistic	Gamma	-347.14	23	-686.29	15	-673.99	13	17.0
Paralogistic	Gamma	-347.14	23	-686.29	16	-673.99	13	17.3
IBurr	GenPareto	-349.81	2	-687.62	6	-669.17	52	20.0
Exponential	Gamma	-346.66	51	-687.32	7	-678.10	2	20.0
Gamma	Gamma	-347.14	29	-686.29	17	-673.99	15	20.3
IPareto	Gamma	-347.14	30	-686.29	18	-673.99	16	21.3
Pareto	Weibull	-346.93	36	-685.86	20	-673.56	20	25.3
Exponential	Burr	-346.87	39	-685.74	21	-673.44	21	27.0

Table 9: Worst-performing 5 composite models for gain returns under goodness-of-fit

Head	Tail	NLL	Rank NLL	AIC	Rank AIC	BIC	Rank BIC
IGaussian	IGaussian	-255.51	247	-503.02	250	-490.72	249
IGamma	IExponential	-240.67	252	-475.33	253	-466.11	252
IGamma	IGamma	-240.67	252	-473.33	254	-461.03	253
IExponential	IExponential	-202.93	254	-401.87	255	-395.72	254
IExponential	IGamma	-202.93	254	-399.87	256	-390.64	255

Similar to Table 6, under gain returns, Table 9 provides the worst performing distributions. The IExponential-head and IGamma-tail composite model ranks as the worst performer, with the lowest AIC (–399.87) and tied bottom scores in NLL and BIC. Just before the latter, the IExponential-head and IExponential-tail model has identical NLL and BIC values but slightly improves in AIC. Rounding out the bottom five are three tied models involving IGaussian and IGamma components, all showing weak fit and penalized criteria. These results suggest that overly light-tailed or symmetric pairings may struggle to capture the asymmetry and extremity of gain distributions.

In comparing top-ranked models for gains in Table 8, the IBurr–Gamma composite outperforms the IBurr single (in Table 10) across all goodness-of-fit metrics, with notable improvements in NLL (–349.83 vs. –346.49), AIC, and slight decrease in BIC, confirming a tighter and more generalizable fit. The gamma-Weibull composite similarly surpasses the Weibull single, showing stronger performance across NLL, AIC, and BIC despite added complexity. Third, the IParalogistic-Weibull composite improves on the Burr single in all metrics, reinforcing the value of composite models. These comparisons underscore that while NLL gains are useful, it is the AIC and BIC penalties that validate when added complexity truly enhances model quality.

Table 10: Ranking and comparative goodness-of-fit metrics for the 16 single distributions fitted on J520 gain returns

Distribution	NLL	Rank NLL	AIC	Rank AIC	BIC	Rank BIC	Avg Rank
IBurr	-346.49	1	-686.99	1	-677.76	1	1.00
Weibull	-341.90	2	-679.80	2	-673.65	2	2.00
Burr	-341.90	2	-677.79	3	-668.57	5	3.33
Gamma	-339.91	4	-675.81	4	-669.66	4	4.00
Exponential	-338.09	5	-674.18	5	-671.10	3	4.33
Pareto	-338.09	5	-672.18	6	-666.03	6	5.67
Paralogistic	-327.69	7	-651.38	7	-645.22	7	7.00
Loglogistic	-320.76	8	-637.51	8	-631.36	8	8.00
IParalogistic	-314.17	9	-624.34	9	-618.19	9	9.00
Lognormal	-312.52	10	-621.05	10	-614.9	10	10.00
IPareto	-304.62	11	-605.24	11	-599.10	11	11.00
IWeibull	-267.54	12	-531.08	12	-524.93	12	12.00
GenPareto	-260.68	13	-515.37	13	-506.14	13	13.00
IGaussian	-255.51	14	-505.02	14	-495.8	14	14.00
IGamma	-240.67	15	-477.33	15	-471.18	15	15.00
IExponential	-202.93	16	-403.87	16	-400.80	16	16.00

Top 1 comparison: the **IBurr** (Rank 1 single model) vs. the **IBurr-gamma** (rank 1 composite model).

- NLL (–349.83 vs. –346.49) →3.34 improvement
- AIC (–689.65 vs. –686.99) →2.66 improvement
- BIC (–674.28 vs. –677.76) →3.48 decline

Top 2 comparison: the **Weibull** distribution (rank 2 single) vs. the **gamma–Weibull** composite model (rank 2 composite).

- NLL (–347.47 vs. –341.90) →5.57 improvement
- AIC (–686.95 vs. –679.80) →7.15 improvement
- BIC (–674.65 vs. –673.65) →1.00 improvement

Top 3 comparison: The **Burr** distribution (rank 3 single) vs. the **IBurr–paralogistic** composite model (rank 3 composite).

- NLL (–349.62 vs. –341.90) →7.72 improvement
- AIC (–689.24 vs. –677.79) →11.45 improvement
- BIC (–673.86 vs. –668.57) →5.29 improvement

3.2 Risk measures for losses and gains

3.2.1 Losses

From the full set of 256 composite models, the 50 best-fitting candidates, ranked by their goodness-of-fit performance, were selected for risk-metric estimation. Table 11 presents VaR and TVaR for the J520 loss returns across the top 20 composite models, calculated at 95% (α = 0.05), 99% (α = 0.01), and 99.5% (α = 0.005) confidence levels. The empirical distribution is included as a benchmark "model," and deviations of the theoretical VaR and TVaR estimates from empirical values shown in brackets are computed using Equations (8) and (9). Some composite models closely track the empirical risk metrics, while others exhibit more pronounced deviations. Green as seen in the footer of Table 11 represents values that exceed a 100 % threshold which is improbable for a business to keep more than the total of their funds.

For loss returns, the Weibull-head and Burr-tail composite model leads decisively, ranking first in four out of six VaR and

TVaR metrics and second in the remaining two, yielding an average rank of 1.33, see Table 11. The IBurr-head and Burr-tail model follow closely, consistently placing in the top three across all thresholds. Tied for third are the IBurr-head with Pareto-tail and the loglogistic-head with loglogistic-tail, both showing strong VaR performance and solid TVaR support. The GenPareto-head with paralogistic-tail ranks fifth, maintaining uniformity across all metrics. These results highlight the value of pairing flexible head distributions with heavier-tailed components to better capture extreme loss behavior, especially in risk-sensitive domains.

Table 11: Risk measures for the top 20 composite models on the J520 loss returns and the % deviation from the empirical distribution's risk measure

Head	Tail	$VaR_{0.95}$	$VaR_{0.99}$	$VaR_{0.995}$	$TVaR_{0.95}$	$TVaR_{0.99}$	$TVaR_{0.995}$	Average Rank
Empirical		**0.1099**	**0.2011**	**0.2603**	**0.1831**	**0.2665**	**0.3285**	
Weibull	Burr	0.1298 (18.1%)	0.2061 (2.5%)	0.2396 (-7.9%)	0.1773 (-3.2%)	0.2549 (-4.4%)	0.2888 (-12.1%)	1.33
IBurr	Burr	0.0640 (-41.8%)	0.2121 (5.5%)	0.3053 (17.3%)	0.1648 (-10.0%)	0.3766 (41.3%)	0.5013 (52.6%)	2.67
IBurr	Pareto	0.0457 (-58.4%)	0.1013 (-49.6%)	0.1363 (-47.6%)	0.0828 (-54.8%)	0.1649 (-38.1%)	0.2136 (-35.0%)	3.33
Loglogistic	Loglogistic	0.0972 (-11.6%)	0.1887 (-6.2%)	0.2481 (-4.7%)	0.2754 (50.4%)	0.8688 (226.0%)	1.5239 (363.9%)	3.33
GenPareto	Paralogistic	0.2952 (168.0%)	0.4845 (140.9%)	0.5725 (119.9%)	0.4145 (126.4%)	0.6173 (131.6%)	0.7114 (116.6%)	5.33
IBurr	Loglogistic	19.08 (17264%)	20.8017 (10244%)	21.0306 (7979%)	20.1622 (10911%)	21.0573 (7801.4%)	21.2004 (6353.7%)	6.67
IBurr	GenPareto	19.27 (17432%)	20.9740 (10329.6%)	21.1924 (8041%)	20.3358 (11006%)	21.2068 (7857.5%)	21.3311 (6393.5%)	7.67
IBurr	IBurr	19.31 (17474%)	21.0479 (10366.4%)	21.2686 (8070%)	20.4001 (11041%)	21.2904 (7888.9%)	21.4229 (6421.4%)	8.67
IBurr	Exponential	23.02 (20848%)	40.7098 (20143.6%)	51.4791 (19676%)	35.4093 (19238%)	61.3010 (22902.3%)	77.3219 (23437.9%)	9.67
Exponential	IBurr	23.02 (20848%)	40.7098 (20143.6%)	51.4791 (19676%)	35.4093 (19238%)	61.3010 (22902.3%)	77.3219 (23437.9%)	9.67
IGamma	IBurr	23.02 (20848%)	40.7098 (20143.6%)	51.4791 (19676%)	35.4093 (19238%)	61.3010 (22902.3%)	77.3219 (23437.9%)	9.67
IGaussian	IBurr	23.02 (20848%)	40.7098 (20143.6%)	51.4791 (19676%)	35.4093 (19238%)	61.3010 (22902.3%)	77.3219 (23437.9%)	9.67
IWeibull	IBurr	23.02 (20848%)	40.7098 (20143.6%)	51.4791 (19676%)	35.4093 (19238%)	61.3010 (22902.3%)	77.3219 (23437.9%)	9.67
Pareto	IBurr	23.02 (20848%)	40.7098 (20143.6%)	51.4791 (19676%)	35.4093 (19238%)	61.3010 (22902.3%)	77.3219 (23437.9%)	9.67
Lognormal	IBurr	23.02 (20848%)	40.7098 (20143.6%)	51.4791 (19676%)	35.4093 (19238%)	61.3010 (22902.3%)	77.3219 (23437.9%)	10.67
IBurr	IExponential	23.022(20848.9%)	40.7098 (20143.6%)	51.48 (19676%)	35.41 (19238.8%)	61.3010 (22902.3%)	77.3219 (23437.9%)	11.83
IBurr	Paralogistic	24.70 (22375.5%)	45.0646 (22309.1%)	57.76 (22088%)	39.14 (21276.4%)	69.8919 (26125.9%)	89.35 (27098%)	17.67
IPareto	Burr	70.93 (64442.4%)	204.06 (101372.8%)	359.37(137958%)	942.32 (514550.2%)	4284.27 (1607506.4%)	8304.45 (2527891%)	19.33
IPareto	IParalogistic	118.5(107721.4%)	299.055 (148609%)	491.33(188653%)	1199.20 (654843.2%)	5317.87 (1995349.2%)	10261.8 (3123738%)	20.83
IParalogistic	Burr	102.9 (93508.9%)	544.971 (270895%)	1221.68 (469236%)	26643.14 (14551041.5%)	132396.9 (49679787.4%)	263994.6 (80363553%)	22

The green highlighted cells denote the ones where at least one of the risk measures at 1-α level is greater or equal to 1, this is an impractical scenario in real life.

Table 12: Risk measures for the 16 standard distributions fitted on the J520 *loss* returns and the % deviation from the empirical distribution's risk measure

Distribution	$VaR_{0.95}$	$VaR_{0.99}$	$VaR_{0.995}$	$TVaR_{0.95}$	$TVaR_{0.99}$	$TVaR_{0.995}$
Empirical	**0.1099**	**0.2011**	**0.2603**	**0.1831**	**0.2665**	**0.3285**
Exponential	0.1235 (12.4%)	0.1899 (-5.6%)	0.2184 (-16.1%)	0.1647 (-10.1%)	0.2311 (-13.3%)	0.2596 (-21.0%)
Gamma	0.1291 (17.5%)	0.2023 (0.6%)	0.2340 (-10.1%)	0.1746 (-4.6%)	0.2481 (-6.9%)	0.2798 (-14.8%)
Weibull	0.1298 (18.1%)	0.2061 (2.5%)	0.2397 (-7.9%)	0.1774 (-3.1%)	0.2549 (-4.4%)	0.2889 (-12.1%)
Pareto	0.1281 (16.6%)	0.2102 (4.5%)	0.2489 (-4.4%)	0.1800 (-1.7%)	0.2690 (0.9%)	0.3109 (-5.4%)
IBurr	0.1164 (5.9%)	0.2058 (2.3%)	0.2602 (-0.04%)	0.1790 (-2.2%)	0.3098 (16.2%)	0.3907 (18.9%)
GenPareto	0.1301 (18.4%)	0.2100 (4.4%)	0.2462 (-5.4%)	0.1802 (-1.6%)	0.2638 (-1%)	0.3017 (-8.2%)
Burr	0.1299 (18.2%)	0.2102 (4.5%)	0.2467 (-5.4%)	0.1802 (-1.6%)	0.2645 (-0.8%)	0.3027 (-7.9%)
Paralogistic	0.1792 (63.1%)	0.4857 (141.5%)	0.7346 (182.2%)	0.4536 (147.7%)	1.1826 (343.8%)	1.7775 (441.1%)
Loglogistic	0.2356 (114.4%)	0.8236 (309.6%)	1.3985 (437.3%)	1.0058 (449.3%)	3.428 (1186.6%)	5.804 (1666.8%)
IParalogistic	0.3042 (176.8%)	1.2695 (531.3%)	2.3248 (793.1%)	2.371 (1195.3%)	9.609 (3505.6%)	17.53 (5237.3%)
IPareto	0.4463 (306.1%)	2.3190 (1053.6%)	4.661 (1690.9%)	∞	∞	∞
Lognormal	0.2333 (112.3%)	0.6297 (213.1%)	0.9058 (248%)	0.5224 (185.3%)	1.1808 (343.1%)	1.6161 (392%)
IWeibull	3.315 (2916%)	82.881 (41114%)	327.43(125692%)	Divergent	Divergent	Divergent
IGamma	13.11 (11838%)	1989.3 (989094%)	17294 (7×10^{6}%)	Divergent	Divergent	Divergent
IGaussian	0.1749 (59.1%)	0.6960 (246.1%)	1.0501 (303.4%)	0.5237(186%)	1.2835 (381.6%)	1.7229 (424.5%)
IExponential	0.0499 (-54.6%)	0.2548 (26.7%)	0.5109 (-42%)	∞	∞	∞

For loss returns, the IBurr single distribution stands out as the closest match to the empirical benchmark, with minimal percent deviations across VaR thresholds-just 5.9% at $VaR_{0.95}$, 2.3% at $VaR_{0.99}$, and a near-perfect –0.04% at $VaR_{0.995}$. While its TVaR values rise modestly at higher quantiles (16.2% and 18.9%), they remain far more restrained than the extreme divergences seen in heavier-tailed or inverse forms. In contrast, the best composite model: Weibull-head and Burr-tail, shows slightly larger deviations, such as 18.1% at $VaR_{0.95}$ and –7.9% at $VaR_{0.995}$, suggesting that even well-performing composites may not align as tightly with empirical data and particularly the extreme tail values.

Top 1 comparison: **IBurr** (rank 1 single) vs. **Weibull–Burr** (rank 1 composite)

- VaR(0.95): 0.1164 vs. 0.1298 → inverse Burr closer (+5.9% vs. +18.1% deviation)
- VaR(0.99): 0.2058 vs. 0.2061 → inverse Burr closer (+2.3% vs. +2.5% deviation)
- VaR(0.995): 0.2602 vs. 0.2396 → inverse Burr closer (–0.04% vs. –7.9% deviation)
- TVaR(0.95): 0.1790 vs. 0.1773 → inverse Burr closer (–2.2% vs. –3.2% deviation)
- TVaR(0.99): 0.3098 vs. 0.2549 → Weibull–Burr closer (+16.2% vs. –4.4% deviation)

- TVaR(0.995): 0.3907 vs. 0.2888 → Weibull–Burr closer (+18.9% vs. –12.1% deviation).

Top 2 comparison: **Weibull** (rank 2 single) vs. **IBurr–Burr** (rank 2 composite)

- VaR(0.95): 0.1298 vs. 0.0640 → Weibull closer (+18.1% vs. –41.8%)
- VaR(0.99): 0.2061 vs. 0.2121 → Weibull closer (+2.5% vs. +5.5%)
- VaR(0.995): 0.2397 vs. 0.3053 → Weibull closer (–7.9% vs. +17.3%)
- TVaR(0.95): 0.1774 vs. 0.1648 → Weibull closer (–3.1% vs. –10.0%)
- TVaR(0.99): 0.2549 vs. 0.3766 → Weibull closer (–4.4% vs. +41.3%)
- TVaR(0.995): 0.2889 vs. 0.5013 → Weibull closer (–12.1% vs. +52.6%)

Top 3 comparison: **Burr** (rank 3 single) vs. **IBurr–Pareto** (rank 3 composite)

- VaR(0.95): 0.1299 vs. 0.0457 → Burr closer (+18.2% vs. –58.4%)
- VaR(0.99): 0.2102 vs. 0.1013 → Burr closer (+4.5% vs. –49.6%)
- VaR(0.995): 0.2467 vs. 0.1363 → Burr closer (–5.4% vs. –47.6%)
- TVaR(0.95): 0.1802 vs. 0.0828 → Burr closer (–1.6% vs. –54.8%)
- TVaR(0.99): 0.2645 vs. 0.1649 → Burr closer (–0.8% vs. –38.1%)
- TVaR(0.995): 0.3027 vs. 0.2136 → Burr closer (–7.9% vs. –35.0%)

3.2.2 Gains

Table 13 presents the VaR and TVaR metrics for the J520 gain returns calculated in the same manner as Table 11 with all risk measure tables under the same analogue. Green represents values that exceed a 100 % threshold which is improbable for a business to keep more than the total of their funds, as explained for losses. For gain returns, the loglogistic-head and Weibull-tail composite model is the clear leader, ranking first across all VaR and TVaR thresholds with an average rank of 1.00. The IBurr-head with loglogistic-tail follows closely, consistently placing second in VaR and top three in TVaR metrics. Third is the IBurr-head with generalized Pareto-tail, showing strong tail performance and an average rank of 3.67. The IBurr-head with Gamma-tail ranks fourth, excelling in TVaR$_{0.995}$ (0.3597). Rounding out the top five is the exponential-head with Burr-tail, which maintains consistent mid-tier rankings across all measures. These results suggest that pairing flexible head distributions with heavier-tailed components can yield superior performance in capturing extreme gains.

Table 13: Risk measures for the Top 20 composite models on the J520 gains returns and the % deviation from the empirical distribution's risk measure

Head	Tail	VaR$_{0.95}$	VaR$_{0.99}$	VaR$_{0.995}$	TVaR$_{0.95}$	TVaR$_{0.99}$	TVaR$_{0.995}$	Avg Rank
Empirical		**0.1098**	**0.1368**	**0.1387**	**0.1249**	**0.1393**	**0.1403**	
Loglogistic	Weibull	0.0949 (-13.6%)	0.1292 (-5.6%)	0.1420 (2.4%)	0.1224 (-2.0%)	0.1791 (28.6%)	0.2233 (59.1%)	1
IBurr	Loglogistic	0.1346 (22.6%)	0.2159 (57.8%)	0.2638 (90.2%)	0.1896 (51.8%)	0.3027 (117.3%)	0.3690 (163.0%)	2.33
IBurr	GenPareto	0.1348 (22.7%)	0.2212 (61.7%)	0.2712 (95.5%)	0.1938 (55.2%)	0.3123 (124.2%)	0.3821 (172.4%)	3.67
IBurr	Gamma	0.1357 (23.6%)	0.2258 (65.1%)	0.2813 (102.8%)	0.1937 (55.1%)	0.3045 (118.6%)	0.3597 (156.4%)	3.83
Exponential	Burr	0.2263 (106.1%)	0.3625 (165.0%)	0.4212 (203.7%)	0.3109 (148.9%)	0.4472 (221.0%)	0.5058 (260.5%)	6.67
Loglogistic	Burr	0.0000 (-100.0%)	0.0002 (-99.9%)	0.0005 (-99.7%)	0.1726 (38.2%)	0.8628 (519.4%)	1.7254 (1129.8%)	7
Exponential	Weibull	0.2263 (106.1%)	0.3625 (165.0%)	0.4212 (203.7%)	0.3109 (148.9%)	0.4472 (221.0%)	0.5059 (260.5%)	7.67
IBurr	IParalogistic	0.1351 (23.0%)	0.2288 (67.2%)	0.2950 (112.7%)	0.5634 (351.1%)	1.0243 (635.3%)	1.2386 (782.8%)	8.33
Exponential	Gamma	0.2302 (109.6%)	0.3666 (168.0%)	0.4253 (206.7%)	0.3149 (152.1%)	0.4514 (224.0%)	0.5101 (263.6%)	9
IBurr	Paralogistic	0.1371 (24.9%)	0.2392 (74.9%)	0.3206 (131.2%)	0.4570 (265.9%)	1.5114 (985.0%)	2.9048 (1970.4%)	9.33
Exponential	GenPareto	0.2301 (109.6%)	0.3666 (168.0%)	0.4253 (206.7%)	0.3149 (152.1%)	0.4514 (224.0%)	0.5101 (263.6%)	9.33
Pareto	Gamma	0.2302 (109.6%)	0.3666 (168.0%)	0.4253 (206.7%)	0.3149 (152.1%)	0.4514 (224.0%)	0.5101 (263.6%)	10.67
IBurr	Burr	0.5371 (389.1%)	0.7935 (480.1%)	0.9212 (564.2%)	0.7050 (464.5%)	1.0063 (622.4%)	1.16 (729.5%)	12.17
IBurr	Weibull	2.9033 (2544%)	4.69 (3330%)	5.66 (3980%)	4.10 (3185.2%)	6.39 (4490.2%)	7.68 (5374.4%)	14
IPareto	IBurr	12.29 (11090%)	16.75 (12143%)	18.96 (13566%)	15.23 (12094%)	20.58 (14673.3%)	23.45 (16614.8%)	16
Paralogistic	IBurr	12.50 (11288%)	17.15 (12435%)	19.46 (13930%)	15.52 (12327%)	20.92 (14920.3%)	23.69 (16789.0%)	17
IBurr	Exponential	12.67 (11440%)	17.35 (12585%)	19.69 (14093%)	15.71 (12481%)	21.16 (15093.3%)	23.96 (16979.8%)	18
Exponential	IBurr	12.67 (11440%)	17.35 (12585%)	19.69 (14093%)	15.71 (12481%)	21.16 (15093.3%)	23.96 (16979.8%)	18
IPareto	Paralogistic	3.36 (2962%)	17.89 (12979%)	36.06 (25895%)	109.82 (87826%)	520.95 (373878%)	1016.99 (724771.0%)	18.67
IParalogistic	Paralogistic	4.72 (4196%)	28.02 (20382%)	59.16 (42555%)	423.81 (339222%)	2077.28 (1491130%)	4114.63 (2932639.8%)	20

Table 14: Risk measures for the 16 standard distributions fitted on the J520 *gains* returns and the % deviation from the empirical distribution's risk measure

Distribution	$VaR_{0.95}$	$VaR_{0.99}$	$VaR_{0.995}$	$TVaR_{0.95}$	$TVaR_{0.99}$	$TVaR_{0.995}$
Empirical	**0.1098**	**0.1368**	**0.1387**	**0.1249**	**0.1393**	**0.1403**
IBurr	0.1067 (-2.8%)	0.1461 (6.8%)	0.1657 (19.5%)	0.1323 (5.9%)	0.1781 (27.9%)	0.2017 (43.8%)
Weibull	0.1168 (6.4%)	0.1669 (22%)	0.1875 (35.2%)	0.1478 (18.3%)	0.1960 (40.7%)	0.2160 (54%)
Exponential	0.1332 (21.3%)	0.2048 (49.7%)	0.2356 (69.9%)	0.1777 (42.3%)	0.2492 (78.9%)	0.2800 (99.6%)
Gamma	0.1245 (13.4%)	0.1859 (35.9%)	0.2122 (53%)	0.1626 (30.2%)	0.2237 (60.6%)	0.2498 (78%)
Burr	0.1168 (6.4%)	0.1669 (22.0%)	0.1875 (35.2%)	0.1478 (18.3%)	0.1960 (40.7%)	0.2160 (54%)
Pareto	0.1332 (21.3%)	0.2048 (49.7%)	0.2356 (69.9%)	0.1777 (42.3%)	0.2492 (78.9%)	0.2801 (99.6%)
Paralogistic	0.1571 (43.1%)	0.3311 (142%)	0.4491 (223.8%)	0.2870 (129.8%)	0.5830 (318.5%)	0.7855(459.9%)
Loglogistic	0.2170 (97.6%)	0.6207 (353.7%)	0.9682 (598.1%)	0.6119 (389.9%)	1.7168 (1132.4%)	2.6715(1804%)
IParalogistic	0.3018 (174.9%)	1.0905 (697.1%)	1.88 (1255.2%)	1.4152 (1033.1%)	4.9943 (3485%)	8.5835 (6018%)
Lognormal	0.2106 (91.8%)	0.4861 (255.3%)	0.6603 (376.1%)	0.4018 (221.7%)	0.8074 (479.6%)	1.0548 (651.8%)
IPareto	0.5461 (397.4%)	2.82 (1966.8%)	5.68 (3994.4%)	∞	∞	∞
IWeibull	1.702 (1450.2%)	23.68 (17212%)	72.85 (52426%)	Divergent	Divergent	Divergent
GenPareto	1.0363 (843.8%)	14.61 (10582%)	45.61 (32785%)	Divergent	Divergent	Divergent
IGaussian	0.1893 (72.4%)	0.4828 (252.9%)	0.6451 (365.1%)	0.3775 (202.2%)	0.7363 (428.6%)	0.9194 (555.3%)
IGamma	2.197 (1901.1%)	59.76 (43585%)	247.78 (178542%)	Divergent	Divergent	Divergent
IExponential	0.1472 (34.1%)	0.7512 (449.1%)	1.51 (985.9%)	∞	∞	∞

For gain returns in Table 14, the IBurr distribution offers the closest overall alignment to the empirical benchmark among the 16 standard distributions. It shows minimal deviation at $VaR_{0.95}$ (–2.8%) and moderate increases at $VaR_{0.99}$ (6.8%) and $VaR_{0.995}$ (19.5%), with TVaR deviations ranging from 5.9% to 43.8%. While not perfect, these figures are significantly tighter than those of other candidates. For example, the Weibull and Burr distributions show over 50% deviation at $TVaR_{0.995}$, and the exponential, gamma, and Pareto distributions exceed 99% deviation in the same metric. More extreme models like IGamma, IWeibull, and GenPareto diverge catastrophically, with TVaR values either undefined or thousands of percent above empirical levels. It challenges the notion that composite models are an irreputable improvement over single models. The single models' improvements in risk assessment challenges the immediate notion that additional complexity improves estimation capacity.

Top 1 comparison: **IBurr** (rank 1 distribution) vs. **loglogistic–Weibull** (rank 1 composite)

- VaR(0.95): 0.1067 vs. 0.0949 → loglogistic–Weibull closer (–13.6% vs. –2.8%)
- VaR(0.99): 0.1461 vs. 0.1292 → loglogistic–Weibull closer (–5.6% vs. +6.8%)
- VaR(0.995): 0.1657 vs. 0.1420 → loglogistic–Weibull closer (+2.4% vs. +19.5%)
- TVaR(0.95): 0.1323 vs. 0.1224 → loglogistic–Weibull closer (–2.0% vs. +5.9%)
- TVaR(0.99): 0.1781 vs. 0.1791 → inverse Burr closer (+27.9% vs. +28.6%)
- TVaR(0.995): 0.2017 vs. 0.2233 → inverse Burr closer (+43.8% vs. +59.1%)

Top 2 comparison: **Weibull** (rank 2 distribution) vs. **inverse Burr–loglogistic** (rank 2 composite)

- VaR(0.95): 0.1168 vs. 0.1346 → Weibull closer (+6.4% vs. +22.6%)
- VaR(0.99): 0.1669 vs. 0.2159 → Weibull closer (+22.0% vs. +57.8%)
- VaR(0.995): 0.1875 vs. 0.2638 → Weibull closer (+35.2% vs. +90.2%)
- TVaR(0.95): 0.1478 vs. 0.1896 → Weibull closer (+18.3% vs. +51.8%)
- TVaR(0.99): 0.1960 vs. 0.3027 → Weibull closer (+40.7% vs. +117.3%)
- TVaR(0.995): 0.2160 vs. 0.3690 → Weibull closer (+54.0% vs. +163.0%)

Top 3 comparison: **Burr** (rank 3 distribution) vs. **IBurr–GenPareto** (rank 3 composite)

- VaR(0.95): 0.1168 vs. 0.1348 → Burr closer (+6.4% vs. +22.7%)
- VaR(0.99): 0.1669 vs. 0.2212 → Burr closer (+22.0% vs. +61.7%)
- VaR(0.995): 0.1875 vs. 0.2712 → Burr closer (+35.2% vs. +95.5%)
- TVaR(0.95): 0.1478 vs. 0.1938 → Burr closer (+18.3% vs. +55.2%)
- TVaR(0.99): 0.1960 vs. 0.3123 → Burr closer (+40.7% vs. +124.2%)

- TVaR(0.995): 0.2160 vs. 0.3821 → Burr closer (+54.0% vs. +172.4%)

3.3 Discussion

To mathematically evaluate the performances, we gave each distributions a rank, ordered from first for best-performance to two-hundred and fifty-sixth for worst-performance. The average ranking was calculated by averaging each distributions' ranking across all metrics; thus, it provided a general sense of how well the model performed across different tests. See Tables A1 and A2 in the Appendix for more information.

This study highlights the IBurr distribution's consistent effectiveness as a head component, especially in gain-side modeling, due to its ability to capture lower value behaviors while adapting to various tail characteristics, making it a versatile candidate for more comprehensive risk modeling. It also reveals asymmetry in optimal head-tail pairings: gains are best modeled with a loglogistic-head and Weibull-tail, whereas losses favor a loglogistic-head with a loglogistic-tail, indicating fundamental differences in their distributional characteristics. Tail selection critically impacts model performance, particularly for TVaR, with some models showing strong VaR rankings but weaker TVaR results, emphasizing the importance of evaluating both metrics simultaneously. Additionally, models outside the top tier may produce unstable or extreme risk estimates due to volatile tail behavior or estimation issues, underscoring the need for thorough diagnostics and cautious interpretation.

This pattern reveals a broader insight: single distributions tend to offer closer alignment to empirical risk measures, especially in the central quantiles. Their simplicity may limit tail flexibility, but it also reduces the risk of overfitting and parameter inflation. Composite models, while powerful in capturing tail behavior and structural nuance, often trade off empirical proximity for theoretical richness. This suggests a pragmatic modeling strategy of use single distributions for calibration and benchmarking, and reserve composites for stress testing, tail risk analysis, or when empirical alignment is less critical than structural fidelity.

4. Conclusion

This study advances the field of financial index modeling and extreme value analysis by demonstrating the efficacy of composite models applied to the JSE's highest-capitalization index. Building on prior work that fits a limited number of standard distributions to financial returns, our analysis expands the scope of 22 standard distributions to 256 composite models, applied separately to gain and loss returns. Comparative analysis shows that composite models consistently outperform their single-distribution counterparts on goodness-of-fit metrics, particularly that of NLL, but is similar in BIC which more heavily penalizes complexity. In tail-risk assessment, the IBurr single distribution often rivals composite models with greater empirical fit. This duality underscores that while added complexity can capture nuanced distributional behavior, parsimony remains a powerful tool for robust risk estimation.

A key contribution is in the identification of asymmetric head-tail components between gains and losses. Gains are best modeled using a loglogistic head with a Weibull tail, while losses favor a loglogistic head paired with a loglogistic tail. These findings underscore the importance of accommodating divergences in financial return behavior, rather than imposing symmetry across components. While the gains are noticeable but small, they are statistically and practically significant, especially in high-stakes contexts like risk modeling, where tail behavior can drive outcomes. Tail selection emerges as a decisive factor in risk estimation, especially under TVaR scrutiny. While several models perform well under VaR, their instability under TVaR, particularly models such as the loglogistic-Burr, highlights the need for dual methods of assessment. Some composite models exhibit volatile or inflated risk measures, emphasizing the importance of stable real-world applications. From a portfolio management perspective, we recommend:

- Gain-side modeling: prioritize composites with loglogistic or IBurr heads and less complex tails (Weibull, loglogistic, gamma) for balanced goodness-of-fit and tail control.
- Loss-side modeling: focus on loglogistic-loglogistic or Weibull-Burr composite models to achieve tight alignment with empirical risk benchmarks.

If the composite models are stable under risk estimation standards and demonstrate a statistically good fit under the majority of goodness-of-fit metrics, the models offer meaningful improvements and can enhance portfolio performance when applied thoughtfully. The gain returns offer the greatest information gain from composite modeling, enabling investors to more accurately predict market upticks, while benefitting from natural monitoring and adjustability improvements. Investors can also gain additional insights by extending the historical data range used in estimation. A larger dataset allows composite models to capture tail behavior more reliably, thereby improving the stability and accuracy of extreme-risk estimates.

Given the added flexibility of composite models, investors gain a more powerful framework for stress-testing future market conditions and identifying early signs of structural shifts. By capturing asymmetrical and extreme-tail behavior more accurately, these models support better-timed shorting and opportunistic investing strategies, enabling investors to react more effectively to both downturn risks and upside momentum. Future research should explore mixture models, which improve transitions between regimes, by adjusting the threshold per model. Given the J520's limited sample size, two-component models are currently more feasible than three-component composite models. Exploring flexible new distributions such as arctan-Weibull, Z-Weibull, and HTBPT-Weibull, or even exponentiated composite models, could enhance fit and risk estimation. These directions advance financial modeling by improving adaptability, predictive accuracy, and decision-making under uncertainty. This paper focuses on choosing a specific best distribution out of the possible 256 composite distributions as the best fitting; for the future, the recommend that the reader consult [24] to [26] for a different approach based on grid maps and model averaging.

Appendix

In this section, we provide additional information that was omitted in the main portions of the paper. Tables A1 and A2 provides the overall average rankings of all the composite models that had valid risk measures and information criteria based on J520's loss and gain returns, respectively.

Table A1: The average rankings for each fitted composite model (where all metrics are calculated) for the J520 loss returns

Head	Tail	$\text{VaR}_{0.95}$ Rank	$\text{VaR}_{0.99}$ Rank	$\text{VaR}_{0.995}$ Rank	$\text{TVaR}_{0.95}$ Rank	$\text{TVaR}_{0.99}$ Rank	$\text{TVaR}_{0.995}$ Rank	Rank NLL	Rank AIC	Rank BIC	AVG
Loglogistic	Loglogistic	1	3	1	3	5	5	25	5	15	7.00
IBurr	Pareto	4	4	4	4	2	2	2	20	65	11.89
Weibull	Burr	2	1	2	1	1	1	20	30	76	14.89
Exponential	IBurr	9	9	9	9	9	9	44	14	27	15.44
IBurr	Exponential	9	9	9	9	9	9	44	14	27	15.44
IBurr	IExponential	9	9	9	15	9	16	44	14	27	16.89
IPareto	IParalogistic	22	21	21	19	19	19	14	4	14	17.00
Loglogistic	IParalogistic	21	20	19	24	24	24	10	1	11	17.11
IParalogistic	IParalogistic	23	23	24	23	23	23	13	3	13	18.67
IBurr	Loglogistic	6	6	6	6	6	6	36	39	84	21.67
GenPareto	Paralogistic	5	5	5	5	4	4	37	47	99	23.44
Loglogistic	IBurr	25	24	22	22	22	22	5	21	67	25.56
IBurr	Burr	3	2	3	2	3	3	1	70	144	25.67
IPareto	IBurr	26	25	25	20	20	20	7	23	69	26.11
IPareto	Burr	19	19	20	18	18	18	22	32	78	27.11
Loglogistic	Burr	18	18	18	26	25	25	18	28	74	27.78
IParalogistic	Burr	20	22	23	21	21	21	21	31	77	28.56
IBurr	GenPareto	7	7	7	7	7	7	3	71	145	29.00
IBurr	IBurr	8	8	8	8	8	8	4	72	146	30.00
IGamma	IBurr	9	9	9	9	9	9	44	52	125	30.56
IGaussian	IBurr	9	9	9	9	9	9	44	52	125	30.56
IWeibull	IBurr	9	9	9	9	9	9	44	52	125	30.56
Pareto	IBurr	9	9	9	9	9	9	44	52	125	30.56
Lognormal	IBurr	9	9	9	15	9	9	44	52	125	31.22
IBurr	Paralogistic	17	17	17	17	17	17	37	47	99	31.67

Table A2: The average rankings for each fitted composite model (where all metrics are calculated) for the J520 gain returns

Head	Tail	$\text{VaR}_{0.95}$ Rank	$\text{VaR}_{0.99}$ Rank	$\text{VaR}_{0.995}$ Rank	$\text{TVaR}_{0.95}$ Rank	$\text{TVaR}_{0.99}$ Rank	$\text{TVaR}_{0.995}$ Rank	Rank NLL	Rank AIC	Rank BIC	AVG
Loglogistic	Weibull	1	1	1	1	1	1	6	8	5	2.8
IBurr	Gamma	5	4	5	4	3	2	1	1	11	4.0
IBurr	Loglogistic	2	2	2	3	2	3	5	4	19	4.7
IBurr	IParalogistic	4	5	6	12	12	11	4	3	18	8.3
IBurr	Paralogistic	6	6	7	11	13	13	3	2	17	8.7
IBurr	GenPareto	3	3	3	5	4	4	2	6	52	9.1
Exponential	Weibull	9	9	9	7	6	6	39	5	1	10.1
Exponential	Gamma	11	10	10	9	7	7	51	7	2	12.7
Exponential	Burr	8	8	8	6	5	5	39	21	21	13.4
Loglogistic	Burr	7	7	4	2	10	12	6	36	45	14.3
Exponential	GenPareto	10	11	11	8	8	8	51	25	24	17.3
Pareto	Gamma	12	12	12	10	9	9	50	24	23	17.9
IBurr	Weibull	14	14	14	14	14	14	6	36	45	19.0
Loglogistic	Gamma	32	28	28	21	20	19	23	15	13	22.1
IPareto	Gamma	25	25	25	23	23	23	30	18	16	23.1
Gamma	Gamma	33	29	29	22	21	20	29	17	15	23.9
IPareto	Paralogistic	15	19	19	19	19	21	54	26	25	24.1
IBurr	Exponential	23	17	17	17	17	17	58	31	29	25.1
Exponential	IBurr	23	17	17	17	17	17	58	31	29	25.1
IParalogistic	Paralogistic	16	20	20	20	22	22	55	27	26	25.3
IBurr	Burr	13	13	13	13	11	10	6	78	88	27.2
Loglogistic	Paralogistic	17	21	21	25	25	25	56	28	27	27.2
Paralogistic	IBurr	22	16	16	16	16	16	35	49	59	27.2
Paralogistic	Gamma	35	34	33	33	32	32	23	16	13	27.9
IPareto	IBurr	21	15	15	15	15	15	41	55	62	28.2

Data Availability Statement: All codes used to run these models are available on request. All the data are publicly available on this link:

https://figshare.com/articles/dataset/Modelling_Extremes_of_the_Johannesburg_Stock_Exchange_Industrial_Index_J520_using_the_Generalised_Pareto_Distribution_/6935810/1

Conflicts of Interest Statement

The authors certify that they have NO affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript.

References

[1] Chernobai A.S., Rachev S.T. & Fabozzi F.J. (2007). *Operational Risk: A Guide to Basel II Capital Requirements, Models, and Analysis*. John Wiley & Sons: New Jersey. https://doi.org/10.1002/9781119201922.

[2] Sweeting P. (2017). *Financial Enterprise Risk Management (2nd ed)*. Cambridge University Press: Cambridge. ISBN: 9781107184619. www.cambridge.org/9780521111645.

[3] Klugman S.A., Panjer H.H. & Willmot G.E. (2019). Loss Models: From Data to Decisions (5th ed.). Wiley, (2019). ISBN: 978-1-119-52378-9.

[4] Eling M. (2014). Fitting asset returns to skewed distributions: Are the skew-normal and skew-student good models? *Insurance: Mathematics and Economics*, 59, 45-56. https://doi.org/10.1016/j.insmatheco.2014.08.004.

[5] Li J. & Liu J. (2023). Claims modelling with three-component composite models. *Risks*, 11(11), 196. https://doi.org/10.3390/risks11110196.

[6] Chikobvu D. & Jakata O. (2023). Fitting statistical parent distributions to quantify financial risk in the South African Financial Index (J580). *Journal of Statistics Applications & Probability*, 12(3), 1231-1245. http://doi.org/10.18576/jsap/120329.

[7] Maphalla R., Mokhoabane M., Ndou M. & Shongwe S.C. (2023). Quantifying risk using loss distributions. In A. A. Jaoude (Ed.), *Applied Probability Theory - New Perspectives, Recent Advances and Trends*, pp.139-161. London, United Kingdom: IntechOpen. http://doi.org/10.5772/intechopen.108856.

[8] Chikobvu D. & Ndlovu T. (2024). Statistical Analysis of the bitcoin and South African Rand exchange rates risks when the tails are somewhat heavy. *Journal of Statistics Application & Probability*, 13(5), 1411-1429. https://doi.org/10.18576/jsap/130501.

[9] Marambakuyana W.A. & Shongwe S.C. (2024). Quantifying risk of insurance claims data using various loss distributions. *Journal of Statistical Applications & Probability*, 13(3), 1031-1044. https://doi.org/10.18576/jsap/130315.

[10] Shongwe S.C., Paulse N.A. & Malumane L.E. (2024). Risk quantification using skewed distributions: An application to the South African Financial Index (J580). *ITM Web of Conferences*, 67, 01009. https://doi.org/10.1051/itmconf/20246701009.

[11] Jakata O. & Chikobvu D. (2025). Forecasting financial risk using statistical parent distributions in the South African industrial index (J520) returns. *Thailand Statistician*, 23(3), 598–614. http://statassoc.or.th.

[12] Shongwe S.C., Greyling Z., Zwane T. & Gcanga S. (2025). Analyzing the South African Industrial Index Data using Numerous Standard Statistical Distributions. *Journal of Statistics Applications & Probability*, 14(1), 27-44. https://dx.doi.org/10.18576/jsap/140103.

[13] Shongwe S.C., Malumane L.E., Paulse N.A., Mazondo G. & Sifumba L. (2025). Fitting statistical parent distributions to quantify financial risk in the South African Financial Index (J580): An extended analysis. *Journal of Statistics Applications & Probability*, 14(3), 369-382. https://dx.doi.org/10.18576/jsap/140304.

[14] Cooray K. & Ananda M.M. (2005). Modeling actuarial data with a composite lognormal-Pareto model. *Scandinavian Actuarial Journal*, 2005(5), 321-334. https://doi.org/10.1080/03461230510009763.

[15] Scollnik D.P.M. (2007). On composite lognormal-Pareto models. *Scandinavian Actuarial Journal*, 2007(1), 20-33.

https://doi.org/10.1080/03461230601110447.

[16] Scollnik D.P.M. & Sun C. (2012). Modeling with Weibull-Pareto models. *North American Actuarial Journal*, 16(2), 260-275. https://doi.org/10.1080/10920277.2012.10590640.

[17] Nadarajah S. & Abu Bakar S.A. (2014). New composite models for the Danish fire insurance data. *Scandinavian Actuarial Journal*, 2014(2), 180-187. https://doi.org/10.1080/03461238.2012.695748.

[18] Abu Bakar S.A., Hamzah N.A., Maghsoudi M. & Nadarajah S. (2015). Modeling loss data using composite models. *Insurance: Mathematics and Economics*, 61, 146-154. https://doi.org/10.1016/j.insmatheco.2014.08.008.

[19] Calderín-Ojeda E. (2015). On the composite Weibull–Burr model to describe claim data. *Communications in Statistics: Case Studies, Data Analysis and Applications*, 1(1), 59-69. https://doi.org/10.1080/23737484.2015.1066661.

[20] Calderín-Ojeda E. & Kwok C.F. (2016). Modeling claims data with composite Stoppa models. *Scandinavian Actuarial Journal*, 2016(9), 1-20. https://doi.org/10.1080/03461238.2015.1034763.

[21] Grün B. & Miljkovic T. (2019). Extending composite loss models using a general framework of advanced computational tools. *Scandinavian Actuarial Journal*, 2019(8), 642-660. https://doi.org/10.1080/03461238.2019.1596151.

[22] Marambakuyana W.A. & Shongwe S.C. (2024). Composite and mixture distributions for heavy-tailed data – An application to insurance claims. *Mathematics*, 12(2), 335. https://doi.org/10.3390/math12020335.

[23] FigShare (2024). The South African Industrial Index (J520) data: 06/1995 to 01/2018. https://figshare.com/articles/dataset/Modelling_Extremes_of_the_Johannesburg_Stock_Exchange_Industrial_Index_J520_using_the_Generalised_Pareto_Distribution_/6935810/1. Date Accessed: 24 May 2024.

[24] Miljkovic T. & Grün B. (2021). Using Model Averaging to Determine Suitable Risk Measure Estimates. *North American Actuarial Journal*, 25(4), 562–579. https://doi.org/10.1080/10920277.2021.1911668.

[25] Blostein M. & Miljkovic T. (2019). On modeling left-truncated loss data using mixtures of distributions. *Insurance: Mathematics and Economics*, 85, 35–46. https://doi.org/10.1016/j.insmatheco.2018.12.001.

[26] Mothibe L.B. & Shongwe S.C. (2026). Model Averaging and Grid Maps for Modeling Heavy-Tailed Insurance Data. *Risks*, 14(1), 11. https://doi.org/10.3390/risks14010011