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An Improved Volatility Forecasting with GMLE-Based MS-GARCH Models: Evidence from BWP/USD, ZAR/USD, and BTC/USD

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Abstract: In this study, the growing interconnectedness of the world's financial markets is recognised, and we seek to investigate the complicated volatility dynamics and downside risk inherent in BWP/USD, ZAR/USD, and BTC/USD exchange rate returns. Aiming to overcome the shortcomings of conventional methods, this work seeks to provide a strong and computationally efficient approach for predicting and modelling volatility, and this approach is two-stage. We first use the GMLE method to estimate parameters within MS-GARCH models. The second part of the research evaluates how well the Student's t, skewed Student's t, and generalised error distribution capture the stylised features of financial returns. To reduce regime path dependency, the Hamilton filter is used; quasi-likelihood ratio tests verify statistically meaningful regime switching. Using AIC and BIC for model selection, the MS-GJR-GARCH model is found to be the best fit for BTC/USD, featuring two regime parameters and a skewed Student t distribution. By contrast, the MS-EGARCH specification is recommended for BWP/USD and ZAR/USD with the same student t distribution. With BTC/USD showing far more negative risk, especially over long forecasting horizons, empirical results show different volatility patterns across all three exchange rates. These findings draw attention to the increased investment risk of Bitcoin. The study finds that a strong foundation for modelling exchange rate volatility is provided by regime-switching models, including generalised maximum likelihood estimation and flexible error distributions. Future studies should look at macroeconomic factors that influence regime changes and expand the investigation to include additional developing market currencies.

Keywords: Computational Finance; Downside Risk; Emerging Market Currencies; Exchange Rate Volatility; Financial Risk Modeling.

1 Introduction

A primary indicator of a developed economy is the government's capacity to sustain price stability amid shocks and currency fluctuations. Exchange rates play a significant role in global economics, influencing international trade, investments, and monetary policies. Traders can enhance their profits by making timely assessments of exchange rate volatility, as indicated by [33]. Volatility, serving as an indicator of uncertainty, is fundamental to numerous contemporary financial theories. Intrinsic fluctuations in exchange rates present substantial obstacles to businesses, investors, and policymakers in their efforts to manage risk and make informed decisions [37, 39]. The volatility of rates poses a significant concern for international businesses, financial institutions, and policymakers; thus, fluctuations in exchange rates have a considerable effect on trade, investments, and economic stability. Managing risk well requires a detailed look at how much exchange rates change, and according to [1], financial analysts and investors worry about the uncertainty of investment returns caused by changing market prices (market risk) and the unpredictability of how businesses perform.

Many strategies have been developed to assess the risk of the exchange rate, and most researchers, such as [41] among others, have employed univariate and multivariate linear models to predict the volatility of the exchange rate. However, these models are inadequate for predicting highly erratic series such as exchange rates. Models like

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autoregressive moving average (ARMA) depend on linear assumptions, which explains why they fail to model exchange rate volatility. Furthermore, [14] asserts that linear models imply a constant connection between variables throughout time, hence disregarding any nonlinear interactions and dynamics in the data. This results in erroneous predictions and biased parameter estimations. Nonlinear models were then developed to accurately capture the complicated relationships and changes in exchange rate movements, which often display sudden shifts or changes in behaviour, thus solving this problem. For example, models such as exponentially smoothed transition autoregression (ESTAR) have been proven to characterise actual exchange rates effectively, tolerating mean-reversion patterns that linear models may ignore [27].

In addition, [8] presented the generalised autoregressive conditional heteroscedasticity (GARCH) model, and [7] has underlined that these models fall short when the data set incorporates structural changes. The GARCH model assumes that the data follows a normal distribution even when volatility changes; as a result, it often expects a stable environment where the parameters stay the same over time. However, in real-world applications, financial markets are characterised by abrupt shifts and varying regimes, which lead to significant deviations from the assumptions of the GARCH model. These changes are caused by economic crises or policy changes, among other things [21], where now the GARCH models do not account for these breaks, which leads to biased estimates and poor forecast accuracy during periods of sudden changes. The assumptions underpinning GARCH models, for instance, become erroneous during times of economic crisis, which results in significant prediction errors. Consequently, researchers have explored alternative approaches, such as regime-switching models and machine learning techniques, to better capture these dynamics and provide more accurate forecasts. For exchange rates and other fast-changing financial data, where returns often have heavy tails and are uneven due to sudden market shifts, this assumption is an issue; as a result, not being able to use normal distributions leads to a lower estimate of the risk of extreme market changes in exchange rates and other financial markets [24, 4].

However, [23] identified the limitations of GARCH models and proposed Markov-switching GARCH (MS-GARCH) models, which effectively capture simultaneous periods of high and low volatility while accommodating asymmetry in both regimes. These models are capable of capturing asymmetries in volatility reactions to both positive and negative shocks as indicated by [38]. For example, these models permit increased volatility after negative returns in contrast to positive returns, illustrating actual market behaviour where adverse news typically results in more significant volatility spikes. While they offer certain advantages in capturing volatility dynamics in financial time series, the MS-GARCH models also present several disadvantages. The estimation of MS-GARCH models is frequently complex because it requires consideration of multiple regimes and path-dependent characteristics, respectively. The exponential increase in potential regime paths over time results in computational challenges that render estimation intractable; thus, [2] pointed out that this complexity can cause longer computation times and difficulties in getting reliable parameter estimates. In general, the MS-GARCH models depend on a limited set of discrete regimes, which may impose constraints; hence, this discretisation fails to present the continuous nature of volatility changes that are evident in empirical data accurately. Consequently, the neglect of significant nuances in volatility behaviour leads to potential misspecification of the model and all parameters within a regime are assumed to be contingent upon the same regime sequence, thereby limiting flexibility. As a result, [13] stated that as this occurs, various parameters may lack their unique regime variables, which results in a less precise depiction of the underlying data dynamics. Finally, although MS-GARCH models can account for structural breaks, they remain dependent on the assumption of stationarity within each regime; hence, violations of these assumptions resulting from extreme market events or extended structural changes undermine the validity and forecasting abilities of the model.

Therefore, this study develops a two-stage procedure to address these problems. We employ the generalised maximum likelihood (GMLE) method for parameter estimation rather than the maximum likelihood estimation (MLE) or Markov chain Monte Carlo (MCMC) methods, and to the best of our knowledge, this is the first study to apply this algorithm to Markov-switching models or financial applications. The generalised maximum likelihood estimation method in this study offers distinct advantages over the MLE method, particularly in terms of computational speed and robustness in complex modelling contexts. Unlike MLE, which typically requires a complete and complex likelihood function, GMLE employs simpler approximations that facilitate calculations without compromising accuracy in the estimates. This streamlined approach enables faster convergence, making it especially suitable for models involving regime switching or high-dimensional parameters, such as those found in Markov-switching models. Additionally, GMLE is usually better at handling situations where the model is not perfectly accurate and provides more reliable estimates when MLE might struggle or fail to reach a solution.

Nonetheless, the impact of computational complexity on model performance is particularly significant in practical applications, where prompt inference and scalability are essential. Significant computing demands restrict the practicality of using MLE approaches in settings that need quick updates, such as financial markets or real-time monitoring systems. In these circumstances, GMLE's efficacy enables more agile modelling while maintaining analytical rigour. In financial forecasting, where prices often change rapidly and suddenly shift patterns, GMLE helps quickly estimate the important parameters needed for making useful recommendations. Bayesian methods, on the other hand, often require taking samples from complicated distributions, which can be very slow and use a lot of computer power, especially when dealing with many variables or long data sets like exchange rates. The GMLE, by contrast, directly optimises a simplified or approximate likelihood function, enabling faster parameter estimation without the need for iterative simulations. Bayesian methods require the specification of prior distributions, which introduces subjectivity and can influence the results, particularly when data is scarce. In contrast, GMLE is a frequentist method and does not rely on prior beliefs, making it more objective in its inference. This trait can be particularly advantageous when prior information is unreliable or unavailable. Nonetheless, in the MS-GARCH models, the regimes are unobserved (latent), making the MLE challenging due to the need to integrate all possible regime paths. But, with the generalised maximum likelihood estimation, we effectively manage this complexity by allowing for a more tractable estimation process, often utilising data augmentation techniques that treat latent states as parameters.

The second contribution lies in the application of the Hamilton Filter to tackle the issue of regime-dependent paths. The Hamilton Filter effectively avoids the necessity of explicitly listing and assessing all potential paths because it is conceptually more straightforward to implement than the Kim filter. This feature presents a notable advantage in the context of managing multiple assets, as is our current situation. The Hamilton filter simplifies the estimation process and allows for direct interpretation of the filtered regime probabilities. According to [29], the ”filtering” process directly addresses the issue of path dependence, which facilitates tractable estimation and inference. The capacity to effectively manage path dependence is crucial for currency and cryptocurrency returns, given the frequent and significant nature of regime shifts. The calculation determines the probability of being in a specific regime at time t, based on the information available up to that time. This function facilitates the assessment of the probability that the BWP/USD, ZAR/BWP, and BTC/USD exchange rates are in a ”high volatility” regime at a given moment. This interpretability is essential for comprehending market dynamics and may guide decisions concerning risk management or trading strategies.

The second phase of the study uses three error distributions to properly represent and capture volatility and extremes. The distributions include (1) the Student’s t-distribution, renowned for its fat tails compared to the normal distribution. This quality makes it especially appropriate for forecasting financial returns, which always show extreme values. The model can handle the existence of severe market fluctuations and helps provide better risk assessments and more precise volatility projections. (2) Skewed distributions, especially the skewed Student’s t-distribution, are used to permit asymmetry in the data, which is crucial in the foreign currency market because negative and positive shocks have varying effects on volatility. Implementing skewed distributions, on the other hand, provides a more accurate picture of market behaviour by capturing events like leverage effects, where negative returns cause bigger future volatility spikes than positive returns. Especially, the skewed Student’s t-distribution adds a skewness factor, letting the distribution be uneven where the tail behaviour is controlled by the degrees of freedom parameter; lower values lead to heavier tails, while larger values bring the distribution closer to normal. This adaptability allows for improved fitting to different datasets. (3) The generalised error distribution (GED), which includes normal and exponential distributions as specific examples, can model different levels of kurtosis and skewness. Its adaptability to various empirical situations enables it to be appropriate for capturing a broad spectrum of volatility patterns seen in financial markets.

Ultimately, we evaluate the downside risk for BTC/USD, ZAR/USD, and BWP/USD using the most effective model developed. Furthermore, different distributions provide differing degrees of resilience to outliers. By comparing them, we can identify the model and distribution that best balance capturing overall trends while also considering unusual results. Furthermore, high-frequency data, such as exchange rates used in this work, often have fat tails, indicating that extreme occurrences occur more frequently than predicted by a normal distribution. By evaluating how various distributions address these tails, we evaluate and mitigate tail risk, which is essential in finance. Variations in distributions may provide disparate parameter estimations about parameter stability and interoperability. Consequently, comparative analysis enables us to assess the stability of these characteristics across various distributions. A stable model with interpretable parameters is more likely to be dependable for forecasting and decision-making. See for instance [19].

1.1 Research Highlights and Key Findings

This study delves into the intricate volatility dynamics of BWP/USD, ZAR/USD, and BTC/USD exchange rate returns, employing a sophisticated two-stage methodological approach built upon Markov-Switching GARCH models. A central methodological innovation lies in the implementation of generalised maximum likelihood estimation for parameter estimation, a technique strategically chosen to mitigate the computational challenges inherent in traditional estimation paradigms. Furthermore, the study leverages the flexibility afforded by alternative error distributional assumptions and the analytical power of the Hamilton filter to augment model accuracy and computational tractability. The salient findings are presented below.

1. Compelling Evidence of Regime Switching: Rigorous quasi-likelihood ratio tests provide compelling statistical evidence against the null hypothesis of a single regime for all three exchange rates (BWP/USD, ZAR/USD, and BTC/USD), thereby justifying the adoption of two-regime MS-GARCH models. Unlike [47], who utilised the standard likelihood ratio test, this study employs the quasi-likelihood ratio test, a more appropriate methodology in

this context due to the well-documented challenges associated with the likelihood ratio test when dealing with nuisance parameters on the boundary of the parameter space, specifically the transition probabilities in regime-switching models.
2. Employing established information criteria (AIC, BIC) and likelihood values, the study identifies the optimal model specification for each exchange rate. The skewed Student's t-distribution in conjunction with the MS(2)-GJR-GARCH(1,1) model emerges as the preferred specification for BTC/USD, whilst the same distribution coupled with the MS(2)-EGARCH(1,1) model is selected for both BWP/USD and ZAR/USD.
3. The study robustly confirms the presence of volatility clustering across all three exchange rates, with the analysis elucidating the distinct contributions of within-regime persistence and regime persistence.
4. Our findings indicated that BTC/USD is identified as the riskiest exchange rate among the three, followed by ZAR/USD, with BWP/USD demonstrating the lowest and most stable level of downside risk.

This study reveals compelling evidence of regime switching in BWP/USD, ZAR/USD, and BTC/USD exchange rates, with BTC/USD exhibiting substantially higher downside risk compared to the other two currencies, which demonstrate greater stability. These findings offer valuable insights for risk management and investment decisions.
The remaining parts of this paper are organised as follows. Section 2 discusses the methodology considered and the results and discussion are contained in Section 3. Lastly, the conclusion of the paper is given in Section 4.

2 Methods

This section of the study presents the methods and procedures followed. The currency data from January 2, 2015, to February 19, 2025, was obtained from Yahoo finance website. Bitcoin is traded daily by traders, leading to 3410 observations. On weekends and public holidays, the trading of the South African rand and Botswana pula is suspended, resulting in 2431 observations. [10] advised that to analyse the data effectively, the missing values of the ZAR / USD exchange rates should be filled with zero (0), since there are no gains or losses for the local currency holder on weekends or public holidays; therefore, BWP / USD is not an exception. Therefore, we analysed the entire data set using the R software environment [40] and presented the data in tables and graphs. The exchange rate returns series for the period t is computed by

$$R_t = \ln P_t - \ln P_{t-1} = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

where P_t and P_{t-1} represent the present and past values of a time series respectively. The study by [16] shows that logarithmic returns are preferred to prices that are mainly non-stationary because they offer a more precise financial risk assessment and are a reliable way to compute risk measures such as standard deviation, semivariance, value at risk, and expected shortfall.

Let ε_t be the shock at time t and F_t be the set of information of all the information over time t. Model (2) involves the joint estimation of the mean equation and conditional variance equation and is given by

$$\begin{aligned} R_t &= \mathbb{E}(R_t|F_{t-1}) + \varepsilon_t, \quad \varepsilon_t \sim N(0,1) \\ R_t &= \mu + \varepsilon_t. \end{aligned} \tag{2}$$

where μ is the conditional mean of R_t given information through time $t-1$, R_t is the return at time t, F_{t-1} denotes the information set available at time $t-1$. The term ε_t is assumed to be a non-constant quantity for time and is given by

$$\varepsilon_t = \sigma_t \alpha_t \tag{3}$$

and,

$$\begin{aligned} \sigma_t &= \sqrt{Var(R_t|F_{t-1})} \\ &= \sqrt{\mathbb{E}\left[(R_t-\mu)^2|F_{t-1}\right]} \\ &= \sqrt{\frac{1}{N-1}\sum_{t=1}^{N}(R_t-\mu)^2} \end{aligned}$$

where σ_t is the volatility that evolves over time and $\alpha_t \sim N(0,1)$ which is independent and identically distributed (i.i.d.) residuals. This study focusses on the distribution of α_t which is modelled using the error distributions discussed in Section (1).

2.1 Volatility Modelling with Regime Switching GARCH Models

This section presents the Markov switching GARCH model, which incorporates various conditional volatility models and conditional distributions. [44, 49] argues that models are selected based on their ability to capture extreme hurdles in stock returns series. Thus, this study adopts the use of the MS-TGARCH, MS-EGARCH, and MS-GJRGARCH models because these three models incorporate asymmetry in the impact of past returns on current volatility. However, they do so in different ways: (1) The MS-TGARCH allows for different responses to positive and negative shocks depending on the regime. This is particularly useful in financial markets, where the impact of shocks may vary during periods of high and low volatility. Similar to TGARCH, MS-TGARCH captures the leverage effect, where negative shocks increase volatility more than positive shocks of the same magnitude. (2) MS-EGARCH captures the asymmetry in volatility (leverage effect) where bad news has a different impact on volatility than good news, with this effect potentially varying across regimes. The exponential form allows for capturing more complex features in the data, such as skewness and kurtosis, and regime-switching enhances this ability by allowing different regimes to have different volatility dynamics. (3) MS-GJRGARCH is capable of adjusting to different market conditions, making it more robust in environments where market behaviour changes abruptly. The ability to switch between regimes helps to better model and forecast volatility during financial crises or periods of market turbulence, providing more accurate risk assessments [11].

2.1.1 Markov-Switching Exponential GARCH Model

The model proposed by [35] addresses asymmetric responses of time-varying variance to shocks while maintaining positive variance. The EGARCH (r,s) specification is expressed as follows:

$$\log \sigma_t^2 = \alpha_0 + \sum_{i=1}^{r} \frac{\alpha_i |\varepsilon_{t-i}| + \gamma_t \varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{s} \beta_j \log\left(\sigma_{t-j}^2\right) \tag{4}$$

Here, γ_t is known as the asymmetric response or leverage parameter. Typically, γ_t is positive, which implies that a negative shock will increase future volatility or uncertainty, while a positive shock will lessen future uncertainty. The model effectively captures heavy tails in returns and exhibits volatility clustering but is not able to model the leverage effect because the conditional variance depends on the magnitude rather than the sign of past values [14]. In addition, there are no parameter restrictions in the model. The EGARCH model guarantees a positive conditional variance regardless of the signs of the estimated parameters, eliminating the need for such restrictions. The conditional variance of an MS(k)-EGARCH(r,s), an extension of the standard EGARCH(r, s) model, is defined as:

$$\log \sigma_t^2(S_t) = \alpha_0(S_t) + \sum_{i=1}^{r} \frac{\alpha_i(S_t) |\varepsilon_{t-i}| + \gamma_t(S_t) \varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{s} \beta_j(S_t) \log\left(\sigma_{i-j}^2\right) \tag{5}$$

In this context, S_t is the latent state component, which takes the value 1 during high regime periods and 0 during low regime periods. It is important to highlight that model (5) allows for the estimation of low- and high-volatility structures in exchange rate returns.

2.1.2 Markov-Switching Threshold GARCH Model

The volatility model introduced by [17] is a notable approach frequently employed to address the leverage effect. The TGARCH (r,s) model of conditional variance is formulated as follows:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{r} (\alpha_t + \gamma_i N_{t-1}) \varepsilon_{t-1}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2 \tag{6}$$

Here, N_{t-1} is defined as:

$$N_{t-1} = \begin{cases} 1, \text{ if } & \varepsilon_{t-1} < 0 \\ 0, \text{ if } & \varepsilon_{t-1} \geq 0 \end{cases} \tag{7}$$

In this context, γ_i denotes the asymmetric response or leverage parameter, and α_t and β_i are non-negative parameters that satisfy conditions similar to those in GARCH models. If $\gamma_i = 0$, the model reduces to a standard GARCH (p,q) process. Positive shocks influence volatility by α_i, while negative shocks affect volatility by $\alpha_i + \gamma_i$. Typically, for $\alpha_i > 0$, negative shocks exert a greater influence on conditional variance than positive shocks. According to [6], the impact of ε_{t-1}^2 on the conditional variance σ_t^2 varies depending on whether ε_{t-1} is above or below the threshold value. This model provides an alternative method to account for the asymmetric effects of positive and negative shocks on volatility. One major advantage of the TGARCH model is its ability to investigate the asymmetric response of volatility, such as the leverage effect, where negative shocks have a more pronounced impact on conditional volatility than positive shocks of the same magnitude. The MS(k)-TGARCH(r,s) model, an extension of the standard TGARCH model, is described by:

$$\sigma_t^2(S_t) = \alpha_0(S_t) + \sum_{i=1}^{r} \varepsilon_{t-1}^2 (\alpha_t(S_t) + \gamma_i N_{t-1}(S_t)) + \sum_{j=1}^{s} \beta_j(S_t)\sigma_{t-j}^2 \tag{8}$$

where S_t is the latent state component, taking the value of 1 for high regime periods and 0 for low regime periods. Equation (8) facilitates the estimation of low- and high-volatility structures in exchange rate returns. This model comprises four key components: the conditional mean, the conditional variance, the regime process, and the conditional distribution.

2.1.3 Markov-Switching GJR GARCH Model

An extension of the GARCH model that considers asymmetry is the GJR-GARCH model, introduced by [17]. The equation for the conditional variance in the GJR-GARCH (r,s) framework is as follows:

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{r} \alpha_i \varepsilon_{t-1}^2 + \sum_{i=1}^{r} \gamma_t \varepsilon_{t-i}^2 N_{t-1}^- + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2 \tag{9}$$

where N_{t-1}^- is an indicator variable that is equal to one if ε_{t-1} is negative, and zero otherwise. A positive γ suggests that negative shocks (bad news) have a greater effect than positive shocks (good news) [9]. When all leverage coefficients are zero, the GJR-GARCH model simplifies to the standard GARCH model. The MS(k)-GJR-GARCH(r,s) model is an expanded version of the standard TGARCH model, and according to [3], it is represented by:

$$\sigma_t^2(S_t) = \alpha_0(S_t) + \sum_{j=1}^{r} \alpha_i(S_t)\varepsilon_{t-1}^2 + \sum_{i=1}^{r} \gamma_t(S_t)\varepsilon_{t-i}^2 N_{t-1}^- + \sum_{j=1}^{s} \beta_j(S_t)\sigma_{t-j}^2 \tag{10}$$

The Generalized Maximum Likelihood Estimation (GMLE) modifies the standard Maximum Likelihood Estimation (MLE) function to accommodate complex models with time-varying parameters [18]. The MS(k)-GARCH type models utilised in this study were fitted using the GMLE under the assumptions of three error distributions: the generalised error distribution (GED), the student t distribution, and the skewed student t distribution. [50] argues that this method is instrumental in analysing non-stationary time series and the GMLE estimators are plausible and worthwhile.

2.2 Distribution assumptions of the error

Table 2 and Figures 1, 2 and 3 demonstrate that the distribution of the residuals fails the normality test. The distribution of residual returns shows evidence of heavy tails and excess kurtosis. To address the excess kurtosis and fat tails in the return series, the Student-t, Generalized Error, and Skewed Student-t distributions are used for the error term in the models considered in this study presented in Subsection 2.1. For details of the Student-t distribution, Skewed student-t distribution and GED see [20] and [15]. These distributions are appropriate to capture excess kurtosis and skewness in residual return series. [30] articulates that these distributions are chosen since they account for heavy-tailedness, skewness, and excess kurtosis in the return series.

2.3 Model selection

The Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC), and Log-likelihood (LL) metrics are used to select the best model, similar to the approach in the study by [36]. The AIC, BIC, and LL assess both the accuracy of the model fit and the number of parameters used. [20] argues that they reward a better fit while penalising an increase in the number of parameters in the return series data , and are defined by equations (11), (12) and (13).

$$AIC = -2\log L + 2k \tag{11}$$

$$BIC = -2\log L + k\log n \tag{12}$$

$$D = 2\{\log(\text{likelihood for alternative model}) - \log(\text{likelood for the null model})\} \tag{13}$$

where the likelihood function is expressed as L and the model's estimated parameters are denoted by k. The model with the smallest AIC and BIC values and largest LL value were selected as optimal. The best-fitting models for BTC/USD, ZAR/USD and BWP/USD are presented in Tables 4, 6, and 5.

2.4 Risk Measures

Risk measures are quantitative tools that predict investment risk and volatility based on historical data. They are used to evaluate and assess potential loss or uncertainty in the value of an asset or a stock due to changes in market conditions [28]. In this section, we discuss the proposed risk measures considered in the study.

2.4.1 Value at Risk

The value at risk (VaR) is the maximum loss over a given time horizon at a given confidence level. Let P_t be the closing index on day t. A h-day VaR on day t is defined by,

$$\mathbb{P}\left(P_{t+h} - P_t < VaR_{t,h,\alpha} \mid F_t\right) = \alpha$$

where F_t is the available information set until time t, h is the forecast horizon and α is the probability value.
In this work, we will compute VaR using the parametric approach which will be based on the logarithmic returns calculated in equation (1). The VaR will be given by,

$$\text{VaR}_{t,h,\alpha} = \left(\mathbb{E}\left[R_{t+h} \mid F_t\right] + q_\alpha^z \sqrt{\text{Var}\left[R_{t+h} \mid F_t\right]}\right) P_t$$

where q_α^z is the quantile α of the standardised distribution of returns, Var[.] is the variance and $\mathbb{E}[.]$ is the expected value. This study will consider the values of α in 95% and 99%, while h is usually one day or 20 days, depending on the intuition of the research.

2.4.2 Expected shortfall

The ES refers to a measure of the expected loss in a portfolio conditioned on the fact that the VaR value has been breached [12]. If we consider a violation in the VaR, the ES expresses the expected value of the loss and is estimated by;

$$\begin{aligned} ES_{h,\alpha} &= \mathbb{E}\left[R_{t+h} < VaR_{t,h,\alpha}\right] \\ &= \frac{1}{\alpha}\int_0^\alpha VaR_{t,h,x}dx \end{aligned}$$

3 Results and discussion

Following the methodology described in Section 2, this section addresses data analysis. We present the findings using tables and graphs, and Figure 1 to Figure 3 show the exchange rate plots. The kernel density plot indicates that the distribution of the exchange rates is leptokurtic even though the data appear non-normal in Figure 1 (c and d), Figure 2(c and d), and Figure 3(c and d). Specifically, seasonality is linked to certain positive and negative trends in Figure 1(a), Figure 2(a), and Figure 3(a). Events such as the COVID-19 pandemic contribute to these clusters of volatility. These currencies also show the most concentrated return losses. These numbers also indicate a potential benefit if one considers conditional heteroscedasticity. We therefore underline two key factors: the reason for weight loss and its erratic character. While the former contends that downturn volatility follows these shocks rather than substantial losses or profits, the latter contends that irregular shocks in the actual business sector have a larger influence on future volatility. Financial market shocks result in unique and significant rewards. However, the research conducted by [32] found outcomes that are comparable to those observed in this study.

Table 1 shows the descriptive statistics of the three (3) exchange rates used in this study. The returns on BTC/USD and BWP/USD portrayed negative skewness, while ZAR/USD gave positive skewness. This finding indicates that all the return series are asymmetric. For BTC/USD and BWP/USD, the negative skewness suggests that the return distributions for these currency pairs have longer left tails, implying that extreme negative returns (losses) are more frequent or severe. In other words, there are more substantial downside risks or shocks in these markets. A positive skewness for ZAR/USD means that the return distribution has a longer right tail, indicating more extreme positive returns (gains). Thus, the ZAR/USD exchange rate shows a tendency for more substantial upward movements or profits. Moreover, the BTC/USD and BWP/USD returns have kurtosis above 3, indicating that their distributions are leptokurtic, meaning they have heavier tails and more pronounced peaks than the normal distribution. This data suggests a higher likelihood of extreme returns (both positive and negative), implying more risk in these markets due to the potential for large and infrequent movements. However, the ZAR/USD returns have a kurtosis value of less than 3, meaning that the ZAR/USD return distribution is platykurtic, indicating lighter tails and fewer extreme values compared to the normal distribution. In this case, the distribution is flatter and suggests that the ZAR/USD market experiences fewer large unexpected moves (outliers), with returns clustering more closely around the mean.

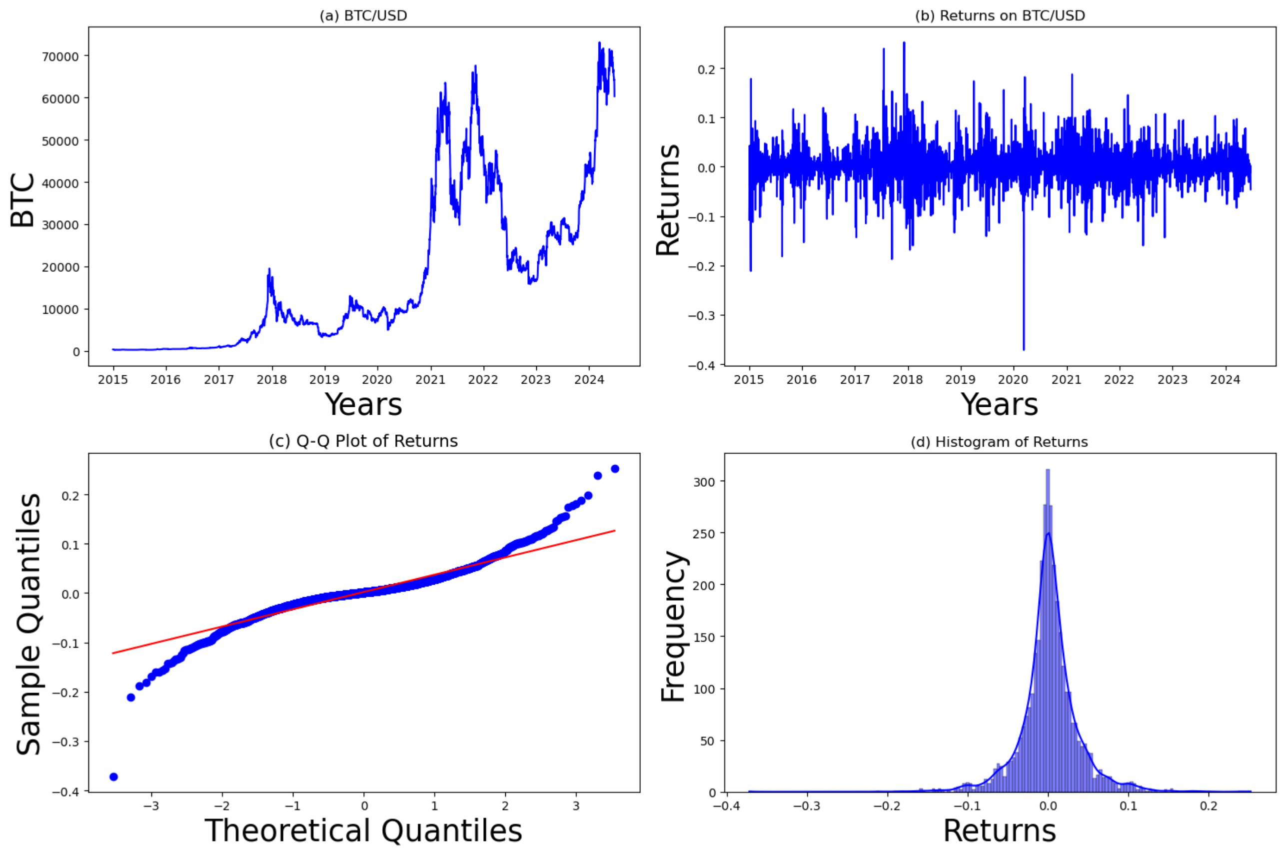

Fig. 1: Plots of adjusted closing prices for BTC/USD

Table 1: Descriptive statistics of exchange rate price returns.

	Observations	Mean	Median	Maximum	Minimum	Skewness	Kurtosis
BTC/USD	2431	0.002069	0.002100	0.225100	-0.464700	-0.854415	11.719138
ZAR/USD	2431	0.000199	-0.000300	0.046200	-0.035000	0.331632	0.975223
BWP/USD	2431	-0.000155	0.000000	0.048400	-0.053700	-0.245548	6.703611

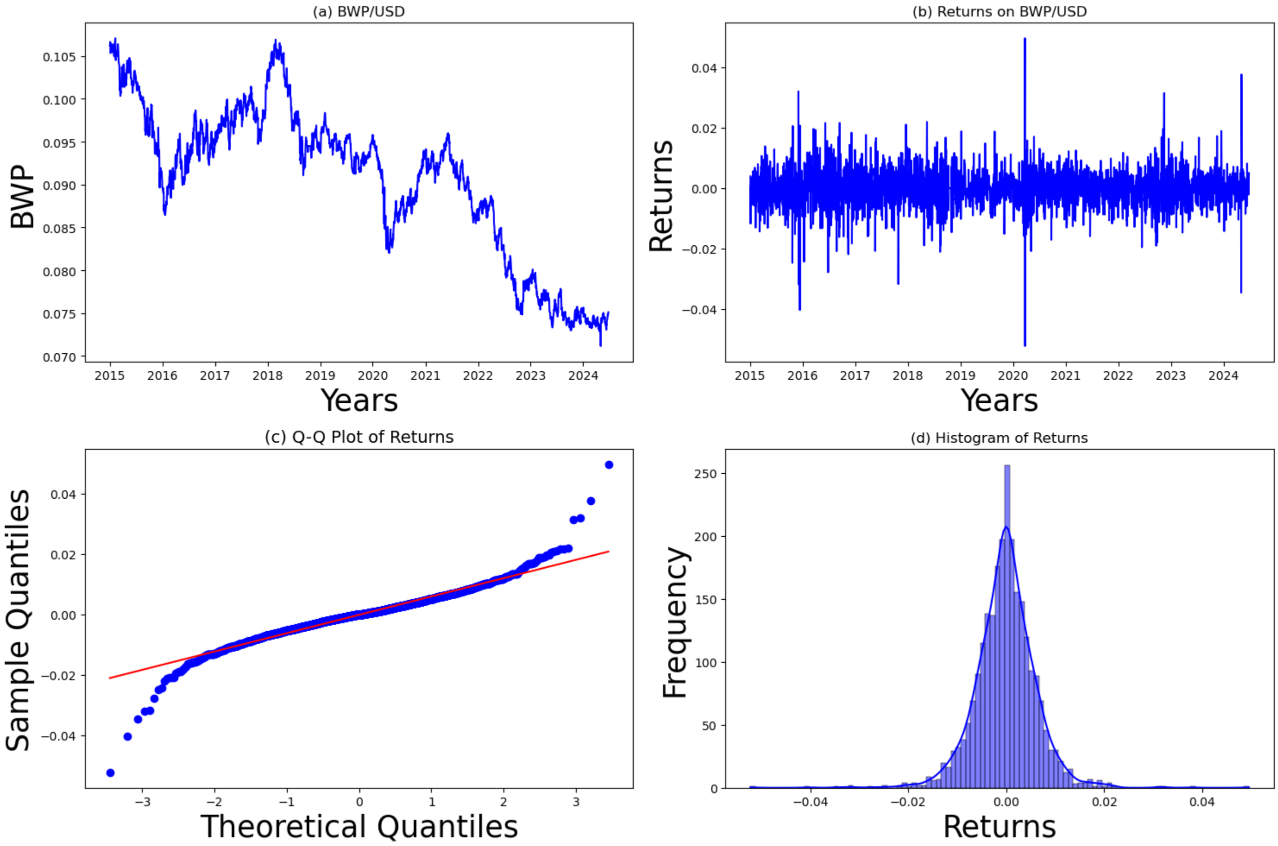

Fig. 2: Plots of adjusted closing prices for BWP/USD

Table 2 displays the results of the tests for normality, autocorrelation, and heteroskedasticity. The rejection of the null hypothesis of normality at the 5% significance level, as indicated by the Jarque-Bera test, implies that symmetric models are inappropriate for analysing the return series. The ARCH effects test led to the rejection of the null hypothesis at a 5% significance level. These findings resulted in the conclusion that ARCH effects in exchange rate returns were taken into account; therefore, we should consider the GARCH family models when analysing the aforementioned returns series. The Ljung-Box test results for ZAR/USD and BTC/USD returns reveal insignificant p-values, suggesting that the null hypothesis of no autocorrelation cannot be rejected. This implies that the returns may be considered independent and identically distributed (i.i.d.).

Table 2: Test for normality, autocorrelation and heteroscedasticity

	BTC/USD		ZAR/USD		BWP/USD	
TEST	Statistic	*p*-value	Statistic	*p*-value	Statistic	*p*-value
Jarque-Bera	14236.1433	0.0001	141.5972	0.0001	4587.199	0.0001
Ljung-Box	66.266	0.05063	63.422	0.08072	128.35	0.0001
ARCH LM Test	83.413	0.001573	141.56	0.0001	242.54	0.0001

The Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests in Table 3 indicate that the returns of the three exchange rates are stationary, as we found strong evidence against the null hypothesis that they are stationary at all significance levels of 5%. This finding confirms that these returns are appropriate for analysis using the stationary process of the regime-switching model. All assumptions are validated, indicating the data's readiness and suitability for primary analyses. Hence, we observe a benefit in modelling the returns of BWP/USD, ZAR/USD, and BTC/USD using a defined stationary MS-GARCH process.

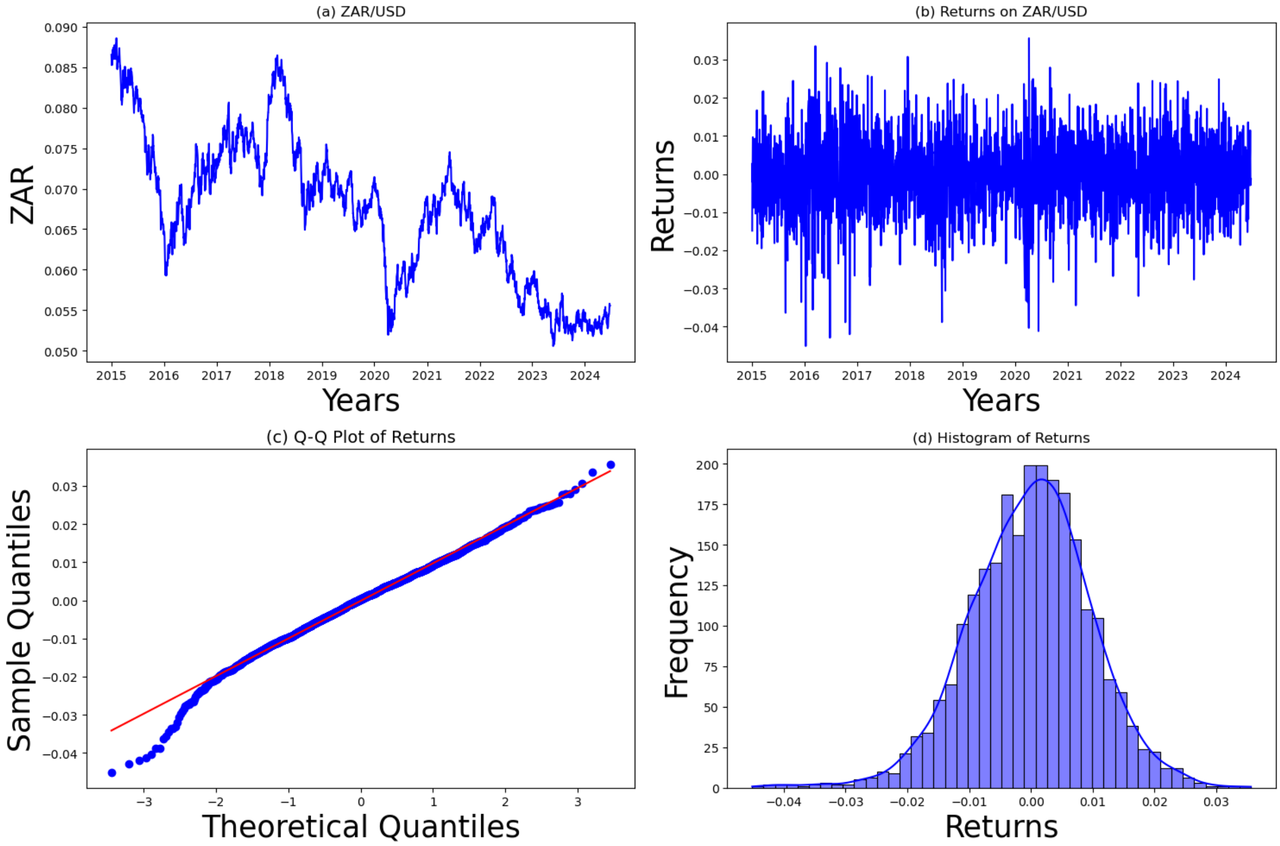

Fig. 3: Plots of adjusted closing prices for ZAR/USD

Table 3: Test for unit root and stationarity

	BTC/USD		ZAR/USD		BWP/USD	
Unit Root Test	Statistic	$CV(5\%)$	Statistic	$CV(5\%)$	Statistic	$CV(5\%)$
ADF Test	-34.7852	-1.95	-35.3157	-1.95	-38.5886	-1.95
PP Test	-50.6759	-2.863	-48.6327	-2.863	-57.0515	-2.863
Quasi-likelihood Ratio Test	28.8	13.42	15.9	13.42	32.7	13.42

3.1 Volatility Modelling with Regime Switching GARCH Process

Before estimating the model, we used a quasi-likelihood ratio statistic to check if we could use two-regime switching models for the three exchange rates, unlike [48], who used the likelihood ratio test in their research. The null hypothesis posits that the exchange rate series follows a nonlinear model with a single regime (no regime switching), whereas the alternative hypothesis allows for a model characterised by two distinct regimes. Our test results rejected the null hypothesis of no regime switching in expected returns at the significance level of 5% for all currencies used in this study. See Table 3.

In Section 2, the regime-switching models were used to conduct the analysis, which required consolidating the results obtained from the preliminary analysis. We fitted the TGARCH, eGARCH, and gjr-GARCH models that were subject to regime change using the MSGARCH package of [5]. We estimated nine models for each currency, as shown in Table 4, Table 6, and Table 5, using the student t distribution, the skewed t distribution, and the generalised error distribution for the errors. In this case, we applied the informational criteria for selecting the models to determine which provides the most accurate forecasts of the volatility of the exchange rate. These criteria also apply to the estimated predictive power of the models. [33] also used similar criteria but just took AIC and BIC into account. Taking the case of BTC/USD, the optimal error distribution for MS(2)-GJR-GARCH(1,1) is the skewed student t distribution with rank one (1). The most appropriate distribution for the BWP/USD and ZAR/USD pairs is the skewed student t distribution with rank one (1)

Table 4: The AIC, BIC and LL values for BTC/USD Returns assuming three different innovations term distributions

Student t-distribution				
Model	AIC	BIC	LL	Rank
MS(2)-EGARCH(1,1)	**-10044.825**	**-9975.2723**	**5034.4125**	3
MS(2)-TGARCH(1,1)	-10007.3215	-9937.7688	5015.6608	
MS(2)-GJRGARCH(1,1)	-9997.4358	-9927.8831	5010.7179	
Skewed Student t-distribution				
MS(2)-EGARCH(1,1)	-10056.6213	-9975.4765	5042.3107	
MS(2)-TGARCH(1,1)	-	-	-	
MS(2)-GJRGARCH(1,1)	**-10056.9189**	**-9975.7741**	**5042.4594**	1
Generalized Error-distribution				
MS(2)-EGARCH(1,1)	**-10049.9611**	**-9980.4084**	**5036.9806**	2
MS(2)-TGARCH(2,2)	-9899.38	-9829.8273	4961.69	
MS(2)-GJRGARCH(1,2)	-10015.3017	-9945.749	5019.6508	

under MS(2)-EGARCH(1,1). That being said, the top model was marked with rank 1, while the worst model was marked with rank 3.

Table 5: The AIC, BIC and LL values for BWP/USD Returns assuming three different innovations term distributions

Student t-distribution				
Model	AIC	BIC	LL	Rank
MS(2)-EGARCH(1,1)	**-15619.6171**	**-15550.0644**	**7821.8085**	3
MS(2)-TGARCH(1,1)	-15607.1384	-15537.5857	7815.5692	
MS(2)-GJRGARCH(1,1)	-15606.1271	-15536.5744	7815.0635	
Skewed Student t-distribution				
MS(2)-EGARCH(1,1)	**-15634.0204**	**-15552.8756**	**7831.0102**	1
MS(2)-TGARCH(1,1)	-15621.2284	-15540.0836	7824.6142	
MS(2)-GJRGARCH(1,1)	-15624.8879	-15543.7431	7826.4436	
Generalized Error-distribution				
MS(2)-EGARCH(1,1)	**-15620.0444**	**-15550.4917**	**7822.0222**	2
MS(2)-TGARCH(1,1)	-15118.1914	-15048.6387	7571.0957	
MS(2)-GJRGARCH(1,1)	-15604.4398	-15534.8871	7814.2199	

Table 6: The AIC, BIC and LL values for ZAR/USD Returns assuming three different innovations term distributions

Student t-distribution				
Model	AIC	BIC	LL	Rank
MS(2)-EGARCH(1,1)	**-18189.8339**	**-18120.2812**	**9106.9169**	3
MS(2)-TGARCH(1,1)	-18188.8441	-18119.2914	9106.422	
MS(2)-GJRGARCH(1,1)	-18083.1581	-18013.6054	9053.5791	
Skewed Student t-distribution				
MS(2)-EGARCH(1,1)	**-18297.017**	**-18215.8722**	**9162.5085**	1
MS(2)-TGARCH(1,1)	-18186.7925	-18105.6477	9107.3963	
MS(2)-GJRGARCH(1,1)	-18068.4321	-17987.2873	9048.216	
Generalized Error-distribution				
MS(2)-EGARCH(1,1)	**-18204.2571**	**-18134.7044**	**9114.1285**	2
MS(2)-TGARCH(1,1)	-18011.9983	-17942.4456	9017.9992	
MS(2)-GJRGARCH(1,1)	-18086.2475	-18016.6948	9055.1238	

The estimated models for the three exchange rates are presented in Tables 7 to 9. Table 7 displays the results of the optimal MS(2)-GJR-GARCH(1,1) model for BTC/USD returns. All parameter estimates demonstrate statistical significance, indicating that the low regime maintains a stable probability of 0.5655, whereas the high regime exhibits a stable probability of 0.4345. Thus, the probabilities of being in the two states are approximately 57% and 43%, respectively. The low-volatility regime is considered dominant in comparison to the high-volatility regime. The parameter estimates demonstrate that the volatility process displays heterogeneity across the two regimes, with values of $\alpha = 0.0056$ and $\alpha = 0.0002$ reported, respectively. This finding agrees with the study by [22, 45], which suggests that different regimes have different levels of volatility, long-lasting volatility, and different reactions to negative returns; hence, volatility exhibits greater persistence in regime one compared to regime two.

Table 7: MS(2)-GJR-GARCH(1,1)-Skewed Student t-distribution for BTC/USD.

	Regime 1				Regime 2			
Coefficient	Estimate	Std Error	t -value	p-value	Estimate	Std Error	t-value	p-value
ρ	0.0001	0.0000	2.1633	0.0001	0.0000	0.0000	0.9939	0.0001
γ	0.0949	0.0920	1.0314	0.0001	0.0640	0.1711	0.3743	0.0001
α	0.0056	0.0343	0.1621	0.0001	0.0002	0.0042	0.0490	0.0001
β	0.8835	0.0215	41.0180	0.0001	0.9356	0.0003	2785.5914	0.0001
μ	4.2188	0.7514	5.6147	0.0001	2.3486	0.0741	31.6971	0.0001
δ	0.8972	0.0304	29.5111	0.0001	0.9538	0.0272	35.0893	0.0001
P	0.9795	0.0093	105.2674	0.0001	0.0267	0.0070	3.7967	0.0001
Transition Matrix								
	P_{11}		0.9795			P_{12}		0.0205
	P_{21}		0.0267			P_{22}		0.9733
Stable probabilities								
			0.5655					0.4345

The persistence of volatility in the two regimes is different. The first regime reports $\alpha_{1,1} + \frac{1}{2}\alpha_{2,1} + \beta_1 \approx 0.9812$, while the second regime reports $\alpha_{1,2} + \frac{1}{2}\alpha_{2,2} + \beta_2 \approx 0.9997$. Thus, the first regime is said to be characterised by (i) low unconditional volatility, (ii) strong volatility reaction to past negative returns, and (iii) low persistence of the volatility process, while the second regime is characterised by (i) high unconditional volatility, (ii) weak volatility reaction to past negative returns, and (iii) high persistence of the volatility process. Regime one is perceived by market participants as ”tranquil market conditions” with low volatility levels, low persistence and high reaction to past negative returns. Regime two is ”turbulent market conditions” with high volatility levels and strong persistence. The leverage estimator γ is positive for both regimes, and regime two exhibits higher leverage compared to regime one (See Table 7).

Table 8: MS(2)-EGARCH(1,1)-Skewed Student t-distribution for ZAR/USD.

	Regime 1				Regime 2			
Coefficient	Estimate	Std Error	t -value	p-value	Estimate	Std Error	t-value	p-value
ρ	-0.4383	0.3305	-1.3259	0.0001	-4.7697	2.4070	-1.9816	0.0001
γ	0.0476	0.0290	1.6405	0.0001	0.0687	0.1257	0.5464	0.0001
α	0.0299	0.0157	1.9116	0.0001	0.1375	0.0736	1.8690	0.0001
β	0.9538	0.0349	27.3013	0.0001	0.4526	0.2759	1.6402	0.0001
μ	36.2945	31.9347	1.1365	0.0001	18.3480	14.8295	1.2373	0.0001
δ	1.0785	0.0446	24.1764	0.0001	1.2929	0.1056	12.2456	0.0001
P	0.9973	0.0098	101.4800	0.0001	0.0107	0.0023	4.6673	0.0001
Transition Matrix								
	P_{11}		0.9973			P_{12}		0.0027
	P_{21}		0.0107			P_{22}		0.9893
Stable probabilities								
			0.7997					0.2003

Tables 8 to 9 show the results of the best MS(2)-EGARCH (1,1) models used on the returns of ZAR/USD and BWP/USD. The findings indicate that the low regime maintains a stable probability of 0.7997, whereas the high regime exhibits a stable probability of 0.2003. We perceive the low regime as more dominant than the high volatility regime. In the analysis of the currencies, every coefficient within the conditional mean and variance equations demonstrates statistical significance. The parameter representing the conditional mean, indicated by the mean (μ), demonstrates statistical significance. The ARCH (α) and GARCH (β) parameters are important and show that there are varying levels of volatility in currency returns in all situations. Additionally, we confirmed that the data is stable by adding up the estimated ARCH and GARCH parameters for each situation and checking that their total is always less than one, specifically, $\alpha + \beta < 1$. For example, refer to [23] for more reading on data stability on the ARCH and GARCH parameters. The positive and statistically significant Student's t (δ) skewed parameters suggest that the returns deviate from a normal distribution. The gamma parameter (γ) is statistically significant, showing that negative return shocks have a bigger effect on the conditional variance for stock-in-price returns.

Table 9: MS(2)-EGARCH(1,1)-Skewed Student t-distribution for BWP/USD.

	Regime 1				Regime 2			
Coefficient	Estimate	Std Error	t -value	p-value	Estimate	Std Error	t-value	p-value
ρ	-0.1196	0.0013	-92.8650	0.0001	-8.7647	0.0183	-479.3583	0.0001
γ	0.0500	0.0054	9.1893	0.0001	-0.7130	0.0111	-64.5264	0.0001
α	-0.0171	0.0055	-3.0861	0.0001	0.9194	0.0080	114.8522	0.0001
β	0.9885	0.0001	13604.4365	0.0001	-0.1251	0.0026	-48.0924	0.0001
μ	6.3337	0.0262	241.4055	0.0001	99.6198	0.0025	40593.8376	0.0001
δ	0.9756	0.0060	162.2863	0.0001	31.3037	0.1393	224.6768	0.0001
P	0.9963	0.0014	698.9253	0.0001	0.3268	0.0000	13670.3197	0.0001
Transition Matrix								
	P_{11}		0.9963			P_{12}		0.0037
	P_{21}		0.3268			P_{22}		0.6732
Stable probabilities								
			0.9888					0.0112

The above results highlighted the superior capability of Markov-switching GARCH models to identify and distinguish between different sources of volatility clustering. [42] noted that volatility clustering has two primary sources: determination within the regime and steadiness of the regimes. This suggests that the unconditional variance is higher in one regime than in another. For ZAR/USD and BWP/USD, this implied that volatility clustering in the first regime, which is also known as the lower regime, was caused by both regime persistence and within-regime persistence and in the second regime, also known as the upper regime, the persistence of the high volatility regime is also seen. Furthermore, the transition probabilities P_{11} and P_{22} are quite high and statistically significant. This indicated the persistence of the regime [42].

3.2 Risk Estimates

Since the student-t skewed distribution coupled with the MS(2)-GJR-GARCH(1,1) and MS(2)-EGARCH(1,1) has outperformed other error distributions, the two risk measures discussed in Section 2 are computed using the MS(2)-GJR-GARCH(1,1) and MS(2)-EGARCH(1,1) for the three currencies to evaluate the risk of losses. [25], in their study, focused on both the gains and losses of the FTSE/JSE-ALSI. However, in this study, we focus only on losses. The one (1) step, five (5) steps, ten (10) steps and twenty (20) steps ahead Value-at-risk and Expected Shortfall forecasts for BTC/USD, ZAR/USD and BWP/USD are generated using the estimated regime-switching models identified as the best models. Tables 10, Table 11, and Table 12 report these results. Estimates were made at the levels of 95% and 99%, similar to the work of [46]. The negative values of the VaR and ES estimates indicate losses at the specified significance levels. When time horizons are changed, the loss and expected losses increase for all exchange rate series considered in the study. This result is consistent with that of [43], who estimated VaR at 99% for daily forecasts for a period of ten (10) days.

Table 10 shows results for VaR and ES of the BTC/USD returns at 95% and 99% levels of significance. In the one-day time horizon $h = 1$, the VaR value is about -10% and the ES value of about -14% at 99%, this implies that the expected loss for BTC/USD using VaR will not exceed 10% daily. But with the ES, the average loss (conditional on exceeding VaR) is 14.07% at 99% confidence and 8.52% at 95% confidence. The same interpretation can be made for the other horizons. It is observed that BTC/USD estimates for VaR and expected ES are not consistent across the chosen forecasting time horizons. This confirms the results by [34] that Bitcoin is generally a risky investment and thus, investors should trade with informed perspectives due to the riskiness of the BTC/USD exchange risk-return risk. This conclusion is met since the estimates of both VaR for this portfolio are higher than those of BWP/USD and ZAR/USD, respectively.

Table 11 shows the results for VaR and ES of the ZAR/USD returns at 95% and 99% levels of significance. In the one-day time horizon $h = 1$, the VaR value is about -1.89% the ES value is about -2.16% in 99%, which implies that investors

Table 10: Value at Risk and Expected Shortfall estimates for BTC/USD.

		Var		ES	
Horizon	Model	99%	95%	99%	95%
$h=1$	MS(2)-GJRGARCH(1,1)	-0.1009	-0.0547	-0.1407	-0.0852
$h=2$	MS(2)-GJRGARCH(1,1)	-0.1037	-0.0541	-0.1435	-0.0863
$h=10$	MS(2)-GJRGARCH(1,1)	-0.1101	-0.0579	-0.1762	-0.0989
$h=20$	MS(2)-GJRGARCH(1,1)	-0.1216	-0.0603	-0.1586	-0.0965

Table 11: Value at Risk (VaR) and Expected Shortfall(ES) for ZAR/USD.

		VaR		ES	
Horizon	Model	99%	95%	99%	95%
$h=1$	MS(2)-EGARCH(1,1)	-0.0189	-0.0133	-0.0216	-0.0166
$h=2$	MS(2)-EGARCH(1,1)	-0.0190	-0.0134	-0.0221	-0.0168
$h=10$	MS(2)-EGARCH(1,1)	-0.0196	-0.0136	-0.0233	-0.0175
$h=20$	MS(2)-EGARCH(1,1)	-0.0204	-0.0138	-0.0235	-0.0176

in the ZAR/USD portfolio are expecting not to lose more investment of 1.89% at the level 99% while focusing on VaR. But with ES, not more than 2.16% will be lost in this portfolio. The same interpretation can be done for the 95% level and with other time horizons.

Table 12: Value at Risk (VaR) and Expected Shortfall (ES) estimates for BWP/USD.

		VaR		BWP/USD	
Horizon	Model	99%	95%	99%	95%
$h=1$	MS(2)-EGARCH(1,1)	-0.0152	-0.0091	-0.0200	-0.0132
$h=2$	MS(2)-EGARCH(1,1)	-0.0153	-0.0091	-0.0205	-0.0131
$h=10$	MS(2)-EGARCH(1,1)	-0.0156	-0.0093	-0.0216	-0.0136
$h=20$	MS(2)-EGARCH(1,1)	-0.0157	-0.0091	-0.0196	-0.0129

Similarly, Table 12 shows the results for VaR and ES of the ZAR/USD returns at significance levels 95% and 99%. In the one-day time horizon $h=1$, the VaR value is about -1.52% and the ES value is about -2% in 99%, which implies that there is about a 2% chance that the BWP / USD exchange rate will not lose more than 2% of its value and that the breach will result in an expected loss on average of about 2% in the same time horizon. In general, the VaR and ES values at the significance level 99% are approximately 2% and about 1% at the significance level 95% throughout all time horizons. The results are consistent at both significance levels, and this speaks to the stability of the BWP / USD exchange rate and gives investors and financial players confidence in their investments.

3.3 Discussion

Assessing and measuring the risk of losses in financial returns is essential for effective risk and portfolio management. Diversification is a fundamental principle in portfolio construction, involving the strategic allocation of assets across various classes to optimise the risk-return trade-off. This study developed and applied a two-stage statistical methodology to evaluate downside risk in BTC/USD, ZAR/USD, and BWP/USD daily exchange rate portfolios. Although there is increasing interest in cryptocurrencies and emerging market currencies individually, limited research has sought to jointly analyse their volatility dynamics. This study is the first to use GMLE-based Markov-switching asymmetric GARCH models to analyse these specific currency pairs together in a unified way. A detailed initial analysis was done before estimating the model, helping to clearly understand the key features in each currency series.

The research shows that the best models for BTC/USD were MS(2)-GJR-GARCH(1,1), and for ZAR/USD and BWP/USD, it was MS(2)-EGARCH(1,1), all using the skewed Student's t-distribution. These specifications effectively capture the non-linear and asymmetric volatility dynamics typical of financial markets. Our results diverge from those of [31], who used seven error distributions, by demonstrating that fewer, well-chosen distributional assumptions, specifically those allowing for skewness and fat tails, can be equally robust. The importance of the estimated ARCH $\hat{\alpha}$ and GARCH $\hat{\beta}$ terms in all situations shows that there is conditional heteroscedasticity, which means that volatility shocks tend to last over time. Also, the positive and important asymmetry parameter (γ) suggests that negative shocks affect volatility more than positive ones, which is backed by research on financial contagion and asymmetric information.

From an economic standpoint, the volatility dynamics observed in BTC/USD reflect the speculative and sentiment-driven nature of cryptocurrency markets. The heightened downside risk and more persistent high-volatility regime suggest greater uncertainty, lower investor confidence, and potential barriers to Bitcoin's use as a medium of exchange or store of value. Conversely, the more stable volatility patterns of BWP/USD and ZAR/USD imply that these currencies, despite being from emerging economies, may offer relative insulation from systemic global shocks. This finding has meaningful implications for monetary authorities, who may leverage exchange rate regimes as tools for macroeconomic stability. The estimated transition probabilities show that Regime 1 (low volatility) is the most common for all three currencies, with slow shifts into high-volatility states, matching the findings of [34]. This pattern implies that prolonged stability is occasionally disrupted by sharp, temporary spikes in volatility, reflecting how crises manifest in currency markets. These findings also have important implications for the equity market and investor behaviour. As exchange rate volatility affects the pricing of internationally exposed firms, especially in emerging markets, persistent regime shifts can increase uncertainty in corporate earnings and cash flows. Investors in sectors related to buying and selling goods internationally, foreign loans, or global trade might have to change their risk management strategies because of exchange rate risks, as noted in [43]. Finally, risk measures such as Value-at-Risk and Expected Shortfall were forecasted at various time horizons. BTC/USD was shown to exhibit the highest level of risk across all periods, followed by ZAR/USD and then BWP/USD. The one-step-ahead forecasts were less risky across all currencies, reflecting the compounding nature of risk over time. These results reinforce the importance of understanding regime-specific risk exposures during both stable and crisis periods. For portfolio managers, this analysis highlights the value of regime-aware risk modelling when allocating capital and formulating hedging policies. The estimates of VaR and ES for the return of the BWP/USD exchange were lower for all the time horizons considered, followed by ZAR/USD and BTC/USD at the levels of significance 95% and 99%, respectively. Thus, understanding the shifts in risks during crises and non-crisis times, as revealed by these results, is beneficial for forex risk managers and investors.

4 Conclusion, Limitations, and Recommendations

This study developed and implemented a Generalised Maximum Likelihood Estimation-based Markov-switching asymmetric GARCH framework to model and forecast exchange rate volatility for BTC/USD, ZAR/USD, and BWP/USD. The GMLE approach, which optimises a simplified or approximated likelihood function, proved particularly effective for estimating MS-GARCH models that involve latent regimes and heavy-tailed error distributions. Compared to conventional Maximum Likelihood Estimation (MLE) or simulation-based methods such as Markov Chain Monte Carlo (MCMC), GMLE offers superior computational efficiency without significantly sacrificing estimation accuracy, making it especially suitable for financial time series exhibiting structural breaks and nonlinear volatility patterns.

Empirical results indicated that the MS(2)-GJR-GARCH(1,1) model best captured the volatility behaviour of BTC/USD, while MS(2)-EGARCH(1,1) models were more appropriate for ZAR/USD and BWP/USD. The use of the skewed Student's t-distribution across all models allowed for an accurate depiction of the asymmetry and fat tails observed in the data. All variance parameters were found to be statistically significant, and asymmetric volatility effects were evident, highlighting the disproportionate impact of negative shocks on volatility. The persistence of high-volatility regimes was most pronounced in BTC/USD, reinforcing its status as a high-risk, sentiment-driven asset. In contrast, BWP/USD demonstrated relatively stable volatility dynamics, making it a potentially more resilient asset for risk-averse investors.

These findings have notable implications for economic policy, financial stability, and investment strategy. For policymakers, understanding how exchange rate regimes affect volatility dynamics can inform the design of monetary policy frameworks. For investors, recognising regime-dependent volatility patterns enhances risk modelling and portfolio diversification strategies. The ability of the proposed models to forecast regime-sensitive Value-at-Risk and Expected Shortfall offers practical tools for managing tail risk and preparing for market stress events.

4.1 Limitations

Despite its contributions, this study is subject to several limitations. Firstly, although GMLE offers computational advantages, it relies on approximations of the true likelihood function, which may introduce estimation bias. A comparative analysis with simulation-based methods, such as MCMC, would help validate the robustness of the GMLE approach. Secondly, the MS-GARCH models assume regime stationarity, which may not hold during periods of severe financial stress or policy intervention. Thirdly, while the selected skewed Student's t-distribution accommodates asymmetry and leptokurtosis, it may not fully capture all distributional features present in real-world financial data. Additionally, the focus on only three currency pairs limits the generalisability of the findings. These exchange rates—BTC/USD, ZAR/USD, and BWP/USD—each operate under unique liquidity conditions and institutional settings, which could influence the observed volatility dynamics. The use of daily frequency data, although informative, excludes insights into intraday volatility fluctuations, which are particularly relevant in high-frequency trading environments. Moreover, although VaR and ES forecasts were generated, the study did not conduct formal backtesting to evaluate the reliability of these risk measures. Finally, the restriction to a two-regime model, while statistically supported, simplifies market reality. More complex models with additional regimes or time-varying transition probabilities may offer a more nuanced understanding of volatility behaviour.

4.2 Recommendations

Based on the findings and limitations of this study, several avenues for future research are recommended. First, a formal assessment of GMLE's approximation accuracy in comparison with MCMC or other exact inference methods would enhance confidence in its empirical application. Investigating alternative approximation strategies within the GMLE framework, or adopting hybrid estimation approaches may improve estimation performance. Second, robustness checks across varying model specifications, error distributions, and data frequencies would help assess the consistency and stability of the findings. Third, researchers are encouraged to apply similar models to a broader range of financial assets, including other currencies, commodities, and stock indices, to test the generalisability of the MS-GARCH-GMLE framework. Incorporating macroeconomic variables—such as interest rates, inflation, or geopolitical indicators—into the regime-switching structure could also help explain shifts between volatility regimes. Fourth, future studies should explore the use of high-frequency data to investigate intraday volatility patterns, while addressing challenges related to market microstructure noise.

Furthermore, rigorous backtesting of VaR and ES forecasts using standard validation techniques—such as the Kupiec unconditional coverage test and the Christoffersen independence test—is essential for evaluating the real-world applicability of these measures. Lastly, the extension to more sophisticated regime-switching models with time-varying transition probabilities or a greater number of regimes could offer deeper insights into the complex dynamics of financial markets, while acknowledging the trade-off between model accuracy and computational cost.

Conflict of interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

Data availability

The currency data from January 2, 2015, to February 19, 2025, was obtained from Yahoo finance website.

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