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# A Modified Iterative Approach Using YJ Transform for Solving Fractional PDEs

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**Abstract:** In this study, we present a new method for solving fractional partial differential equations using a transformation called the Yasser Jassim transformation, combined with the Daftardar Jafari method and the fractional Caputo operator. The proposed new method deals with linear and nonlinear equations. We proved the uniqueness and existence of this method using Grönwall's inequality and Banach's fixed-point theorem. The convergence of the approximate solutions to the exact solution was proven by tables and figures using MATLAB for the solved examples in the article, which are the fractional time-diffusion equation, the fractional Burger's equation, and the fractional Korteweg-de Vries equation.

**Keywords:** Korteweg-de Vries (KdV); Burgers' Equation; Diffusion Equation; Yasser Jassim transform (YJ); Caputo fractional derivative (CFD); Daftardar Jafari decomposition method (DJM). Fractional Partial Differential Equations.

## 1 Introduction

The subject of fractional calculus (FC) is both ancient and contemporary. It has been discussed for over three centuries, but since the 1970s, it has begun to gain more attention [1]. First, FC was considered an abstract area with just mathematical operations of little to no practical use and lacked most practical applications [2]. FC has recently been widely used in a range of applications in almost every field of science, engineering, and mathematics because of its frequent applications in fluid flow, polymer rheology, economics, biophysics, control theory, psychology, and other fields [3, 4].

Recently, there has been a lot of attention in finding effective numerical methods to simulate the fractional PDEs, such as: Reduce transform method, variational iteration method, ... [5–34]. The objective of this work is to find an analytical method that uses the fewest steps to produce an approximation that is extremely close to the exact answer while solving PDEs with CFD.

The structure of the paper is as follows. In Section 2, we present the essential definitions, properties, and mathematical tools needed for the proposed approach. Section 3 is devoted to the theoretical analysis, where we prove the existence and uniqueness of the solution using Banach's Fixed Point Theorem and Grönwall's Inequality. In Section 4, the modified method is applied to both linear and nonlinear fractional partial differential equations, including the diffusion, Burgers, and Korteweg-de Vries equations. Finally, Section 5 provides the conclusions and future directions of this study.

## 2 Preliminaries

**Definition 1.** *[35] Areal function $\varphi(x,\tau),\ x\in\mathbb{R},\ \tau>0$ is said to be in the space $C_\varepsilon,\ \varepsilon\in\mathbb{R}$ if there exists a real number $q,\ (q>\varepsilon)$, such that $\varphi(x,\tau)=\tau^q\varphi_1(x,\tau)$, where $\varphi_1(x,\tau)\in C[0,\infty]$, and it is said to be in the space $C_\varepsilon^m$ if $\varphi^{(m)}(x,\tau)\in$ $C_\varepsilon,\ m\in$ N.*

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**Definition 2.** *[35] The Riemann Liouville fractional integral operator of order* $\alpha \geq 0$, *of function* $\varphi(\tau) \in C_{\varepsilon}, \varepsilon \geq -1$ *is defined as*

$$\mathrm{I}_{\tau}^{\alpha} \varphi(\tau) = \begin{cases} \frac{1}{\Gamma(\alpha)} \int_{0}^{\tau} (\tau - \mathrm{s})^{\alpha-1} \varphi(\mathrm{~s}) \mathrm{ds}, & \alpha > 0.\tau > 0 \\ \varphi(\tau) & , \alpha = 0 \end{cases} \tag{1}$$

*where* $\Gamma(\cdot)$ *is the well-Known Gamma function.*

**Definition 3.** *[35] The Liouville-Caputo operator (LCO) with order (* $\alpha > 0$ *) of* $\varphi(\tau)$ *is defined as follows:*

$${}^{c}D_{\tau}^{\alpha} \varphi(\tau) = \begin{cases} \frac{1}{\Gamma(m-\alpha)} \int_{0}^{\tau} (\tau - s)^{m-\alpha-1} \varphi^{(m)}(s) ds, m-1 < \alpha \leq m \\ \frac{\partial^{n}}{\partial \tau^{n}} \varphi(\tau) \end{cases} \tag{2}$$

*for* $m \in N, \quad \tau > 0, \varphi \in C_{-1}^{m}$
*The following are the basic properties of the operator* ${}^{C}D_{t}^{\alpha}$ *:*

1. ${}^{C}D_{t}^{\alpha} I^{\alpha} \varphi(x, \tau) = \varphi(x, \tau)$.
2. $I^{\alpha C} D_{t}^{\alpha} \varphi(x, \tau) = \varphi(x, \tau) - \sum_{k=0}^{m-1} \frac{\tau^{k}}{k!} \varphi^{(k)}(x, 0)$.
3. ${}^{C}D_{t}^{\alpha} \tau^{\beta} = \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)} \tau^{\beta-\alpha} \quad , \alpha > 0$.

**Definition 4.** *[36] Let be* $\varphi(t)$ *a function defined for* $t > 0$, *the Yasser-Jassim Transform (YJ Transform) of* $\varphi(t)$ *is define by the following formula:*

$$\mathscr{H}[\varphi(t)] = \mathscr{K}(a) = a \int_{0}^{\infty} e^{-\frac{1}{\sqrt{a}} t} \varphi(t) dt \tag{3}$$

*Where* $a \neq 0$.

**The following are the basic properties of Yasser-Jassim Transform (YJ Transform):**

1. $\mathscr{H}[1] = a\sqrt{a}$.
2. $\mathscr{H}\left[e^{bt}\right] = a\sqrt{a} \frac{1}{1-b\sqrt{a}}$.
3. $\mathscr{H}\left[t^{\alpha}\right] = a\sqrt{a}^{\alpha+1} \Gamma(\alpha+1)$.
4. $\mathscr{H}\left[{}^{c}D_{t}^{\alpha} \varphi(t)\right] = \frac{\mathscr{K}(a)}{\sqrt{a}^{\alpha}} - \sum_{k=0}^{n-1} \frac{a}{\sqrt{a}^{\alpha-k-1}} \varphi^{(k)}(0), n-1 < \alpha \leq n$.
5. $\mathscr{H}\left[I_{t}^{\alpha} \varphi(t)\right] = \sqrt{a}^{\alpha} \mathscr{K}(a)$.

**Theorem 1.***(Banach Fixed Point) [37, 38] Let (* $X, d$ *) be a non-empty complete metric space, and let* $T : X \rightarrow X$ *be a contraction mapping; that is, there exists a constant* $L \in [0, 1)$ *such that*

$$d(T(x), T(y)) \leq d(x, y), \quad \forall x, y \in X$$

*Then, the following statements hold:*

1. *The mapping* $T$ *has a unique fixed point* $x^{*} \in X$ *such that* $T\left(x^{*}\right) = x^{*}$.
2. *For any initial point* $x_{0} \in X$, *the sequence* $\{x_{n}\}$ *defined recursively by* $x_{n+1} = T\left(x_{n}\right), n = 0, 1, 2, \ldots$, *converges to* $x^{*}$.
3. *Moreover, the convergence is at least linear, and the following error estimate holds:*

$$d\left(x_{n}, x^{*}\right) \leq \frac{L^{n}}{1-L} d\left(x_{0}, x_{1}\right), \quad \forall n \geq 0$$

**Theorem 2.***(Grönwall's Inequality) [39–45] Let* $\varphi(t)$ *be a continuous, non-negative real-valued function defined on an interval* $[0, T]$, *where* $T > 0$. *Suppose there exist constants* $C \geq 0$ *and a non-negative, continuous function* $\beta(t)$ *defined on* $[0, T]$ *such that:*

$$\varphi(t) \leq C + \int_{0}^{t} \beta(s) \varphi(s) ds, \forall t \in [0, T]$$

*Then, Grönwall's inequality guarantees that:*

$$\varphi(t) \leq C \exp\left(\int_{0}^{t} \beta(s) ds\right), \forall t \in [0, T]$$

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**Important Notes:**

1. If $C = 0$, then $\varphi(t) = 0$ for all $t \in [0, T]$.
2. The function $\beta(t)$ can be either a constant or a general non-negative continuous function.
3. This inequality is crucial for proving uniqueness and continuous dependence of solutions on initial conditions.

## 3 Fractional Yasser-Jassim Daftardar-Jafari Method (YJ-DJM).

Let us consider the fractional differential equation of the form

$$^{c}D_t^{\alpha}\varphi(x,t) + R(\varphi(x,t)) + N(\varphi(x,t)) = g(x,t), m-1 < \alpha < m, t > 0,$$

with respect to the initial conditions

$$\frac{\partial^k}{\partial t^k}\varphi(x,0) = f_k(x), \quad k = 0,1,2,\ldots,m-1 \tag{4}$$

where $^{C}D_t^{\alpha}u(x,t)$ Caputo fractional operator of order $\alpha, R$ linear operator, $N$ non linear operator, and $g(x,t)$ known source function.

The method is based on applying (YJ) transform on both sides of (4), to have:

$$\frac{\mathscr{H}(\varphi(x,t))}{(\sqrt{a})^{\alpha}} - \sum_{k=0}^{m-1}\frac{a}{(\sqrt{a})^{\alpha-k-1}}\varphi^{(k)}(0) + \mathscr{H}(R(\varphi)) + \mathscr{H}(N(\varphi)) = \mathscr{H}(g) \tag{5}$$

$$\begin{aligned}\mathscr{H}(\varphi) = \sum_{k=0}^{m-1} a(\sqrt{a})^{k+1}\varphi^{(k)}(0) + (\sqrt{a})^{\alpha}\mathscr{H}(g) - (\sqrt{a})^{\alpha}\mathscr{H}(R(\varphi))\\ - (\sqrt{a})^{\alpha}\mathscr{H}(N(\varphi))(7)\end{aligned} \tag{6}$$

Applying the invers (YJ), we have:

$$\begin{aligned}\varphi = \mathscr{H}^{-1}\left[\sum_{k=0}^{m-1} a(\sqrt{a})^{k+1}\varphi^{(k)}(0) + (\sqrt{a})^{\alpha}\mathscr{H}(g)\right] - \mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}(R(\varphi))\right]\\ - \mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}(N(\varphi))\right].\end{aligned} \tag{7}$$

Now, represent solution as an infinite series given below:

$$\varphi(x,t) = \sum_{n=0}^{\infty}\varphi_n(x,t)$$

Substituting (3) in to both sides of (7) gives:

$$\begin{aligned}\sum_{m=0}^{\infty}\varphi_m = \mathscr{H}^{-1}\left[\sum_{k=0}^{m-1} a(\sqrt{a})^{k+1}\varphi^{(k)}(0) + (\sqrt{a})^{\alpha}\mathscr{H}(g)\right]\\ - \mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left(R\left(\sum_{m=0}^{\infty}\varphi_m\right)\right)\right]\\ - \mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left(N\left(\sum_{m=0}^{\infty}\varphi_m\right)\right)\right]\end{aligned} \tag{8}$$

The nonlinear term is decomposed as in:

$$N\left(\sum_{m=0}^{\infty}\varphi_m\right) = N(\varphi_0) + \sum_{m=0}^{\infty}\left[N\left(\sum_{r=0}^{m}\varphi_r\right) - N\left(\sum_{r=0}^{m-1}\varphi_r\right)\right]. \tag{9}$$

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Substituting (9) in to (8) gives:

$$\sum_{m=0}^{\infty}\varphi_m = \mathscr{H}^{-1}\left[\sum_{k=0}^{m-1} a(\sqrt{a})^{k+1}\varphi^{(k)}(0) + (\sqrt{a})^{\alpha}\mathscr{H}(g)\right] - \mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left(R\left(\sum_{m=0}^{\infty}\varphi_m\right)\right)\right] - \mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left(N(\varphi_0) + \sum_{m=0}^{\infty}\left[N\left(\sum_{r=0}^{m}\varphi_r\right) - N\left(\sum_{r=0}^{m-1}\varphi_r\right)\right]\right)\right] \tag{10}$$

Define the recurrence relation:

$$F = \mathscr{H}^{-1}\left[\sum_{k=0}^{m-1} a(\sqrt{a})^{k+1}\varphi^{(k)}(0) + (\sqrt{a})^{\alpha}\mathscr{H}(g)\right] = \varphi_0 \tag{11}$$

$$L(\varphi_i) = -\mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left(R\left(\sum_{m=0}^{\infty}\varphi_m\right)\right)\right], \quad i \geq 1 \tag{12}$$

$$G_0 = -\mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}(N(\varphi_0))\right] \tag{13}$$

$$G_i = -\mathscr{H}^{-1}\left[\left(\sqrt{a}^{\alpha}\right)^{\mathscr{H}}\left(N\left(\sum_{r=0}^{m}\varphi_r\right) - N\left(\sum_{r=0}^{m-1}\varphi_r\right)\right)\right], i \geq 1 \tag{14}$$

Substituting (14) in to (10) gives:

$$\varphi(x,t) = F + L(\varphi) + G(\varphi) \tag{15}$$

Were

$$G(\varphi) = \sum_{i=0}^{\infty} G_i \tag{16}$$

and

$$L(\varphi) = L\left(\sum_{i=0}^{\infty}\varphi_i\right) \tag{17}$$

Moreover, the relation is defined with sequence, so that

$$\varphi_0 = F \tag{18}$$

and

$$\varphi_{i+1} = L(\varphi_i) + G_i. \tag{19}$$

Thus, the approximate solution of (3) is:

$$\varphi(x.t) = \sum_{i=1}^{\infty}\varphi_i(x.t) = \varphi_0 + \varphi_1 + \varphi_2 + \cdots$$

### *3.1 Existence of solution for YJ-DJM*

consider the nonlinear fractional differential equation of the form:

$${}^{c}D_t^{\alpha}\varphi(x,t) = A(x,t,\varphi(x,t)), \quad 0 < \alpha \leq 1, \tag{20}$$

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with respect to the initial conditions

$$\varphi(x,0) = \varphi_0(x). \tag{21}$$

Applying (YJ) transform on both sides of (20), to have:

$$\mathscr{H}(\varphi(x,t)) = (\sqrt{a})^{\alpha}\mathscr{H}(A(x,t,\varphi(x,t))) + \frac{a}{(\sqrt{a})^{\alpha-1}}\varphi_0. \tag{22}$$

We define the iterative series solution:

$$\varphi(x,t) = \sum_{n=0}^{\infty}\varphi_n(x,t). \tag{23}$$

With the recursive definitions:

$$\varphi_0(x,t) = \varphi_0(x), \tag{24}$$

$$\mathscr{H}(\varphi_1(x,t)) = (\sqrt{a})^{\alpha}\mathscr{H}(A(x,t,\varphi_0(x,t))) + \frac{a}{(\sqrt{a})^{\alpha-1}}\varphi_0(x), \tag{25}$$

$$\vdots$$

$$\mathscr{H}(\varphi_{n+1}(x,t)) = (\sqrt{a})^{\alpha}\left[\mathscr{H}(A(x,t,\Phi_n)) - \mathscr{H}(A(x,t,\Phi_{n-1}))\right]. \tag{26}$$

Were

$$\Phi_n = \sum_{k=0}^{n}\varphi_k. \tag{27}$$

We apply Th. (1) to the (DJM) iterative operator, we assume that $A(x,t,\varphi)$ is continuous in all arguments, and $A(x,t,\varphi)$ satisfies a Lipschitz condition in $\varphi$ :
$\exists L > 0$ such that $\|A(x,t,\varphi) - A(x,t,\psi)\| \leq L\|\varphi - \psi\|, \forall\varphi,\psi \in X$. Let $X = \mathrm{C}([0,T])$, the Banach space of continuous function on $[0,T]$ with the norm $|\varphi| = \sup_{t\in[0,T]}|\varphi(t)|$.
Define the (DJM) iteration operator $T$ by:

$$T(\varphi(x,t)) = \varphi_0(x) + \frac{1}{\Gamma(\alpha)}\int_0^t (t-\zeta)^{\alpha-1}A(x,\zeta,\varphi)d\zeta$$

We then build a sequence $\varphi_n$ via: $\varphi_{n+1} = T(\varphi_n)$. Let's show that this is a Cauchy sequence in $C([0,T])$, hence convergent. Let

$$h_{n+1}(t) = \varphi_{n+1} - \varphi_n = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\zeta)^{\alpha-1}\left[A(x,\zeta,\varphi_{n+1}(\zeta)) - A(x,\zeta,\varphi_{n+1}(\zeta))\right]d\zeta$$

By the Lipschitz condition:

$$|h_{n+1}(t)| = \varphi_{n+1} - \varphi_n = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\zeta)^{\alpha-1}|h_n(\zeta)|d\zeta$$

Define $H_n(t) = \sup_{s\varepsilon[0,T]}|h_n(s)|$.Then

$$H_{n+1}(t) \leq \frac{L}{\Gamma(\alpha)}\int_0^t (t-\zeta)^{\alpha-1}H_n(\zeta)d\zeta \leq \frac{Lt^{\alpha}}{\Gamma(\alpha+1)}H_n(t)$$

Let $K = \frac{Lt^{\alpha}}{\Gamma(\alpha+1)}$, if $K < 1$, we obtain: $H_{n+1}(t) \leq KH_n(t) \leq K^2H_{n-1}(t) \leq \cdots \leq K^nH_1(t)$. This shows $\varphi_n$ is a Cauchy sequence in $\mathrm{C}([0,T])$, and hence converges to a function $\varphi(t)$. The YJ-DJM generates a convergent sequence $\sum\varphi_n \to \varphi \in C([0,T])$, which is a solution of the original problem in the sense of the Caputo fractional integral equation.

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### *3.2 Uniqueness of solution for (YJ-DJM)*

consider the nonlinear fractional differential equation of the form:

$$ {}^{C}D_t^{\alpha}\varphi(x,t) = A(x,t,\varphi(x,t)), \quad 0 < \alpha \le 1, \tag{28} $$

with respect to the initial conditions

$$ \varphi(x,0) = \varphi_0(x). \tag{29} $$

Applying (YJ) transform on both sides of (20), to have:

$$ \mathscr{H}(\varphi(x,t)) = (\sqrt{a})^{\alpha}\mathscr{H}(A(x,t,\varphi(x,t))) + \frac{a}{(\sqrt{a})^{\alpha-1}}\varphi_0. \tag{30} $$

Let $\varphi(x,t)$, $\psi(x,t)$ are two solutions to the same equation with the same initial condition $\varphi_0(x)$. Then their transformed versions satisfy:

$$ \mathscr{H}(\varphi(x,t)) = (\sqrt{a})^{\alpha}\mathscr{H}(A(x,t,\varphi(x,t))) + \frac{a}{(\sqrt{a})^{\alpha-1}}\varphi_0 \tag{31} $$

$$ \mathscr{H}(\psi(x,t)) = (\sqrt{a})^{\alpha}\mathscr{H}(A(x,t,\psi(x,t))) + \frac{a}{(\sqrt{a})^{\alpha-1}}\varphi_0 \tag{32} $$

Subtracting gives:

$$ \mathscr{H}\{\varphi - \psi\} = (\sqrt{a})^{\alpha}\mathscr{H}\{A(x,t,\varphi) - A(x,t,\psi)\}. \tag{33} $$

Let $\omega(x,t) = \varphi(x,t) - \psi(x,t)$.
Substituting (34) in to (33), and take inverse transform gives:

$$ \omega(x,t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\zeta)^{\alpha-1}[A(x,\zeta,\varphi) - A(x,\zeta,\psi)]d\zeta \tag{34} $$

Take norm of (**??**)

$$ \|\omega(x,t)\| \le \frac{1}{\Gamma(\alpha)}\int_0^t (t-\zeta)^{\alpha-1}\|A(x,\zeta,\varphi) - A(x,\zeta,\psi)\|d\zeta \tag{35} $$

By assuming $A$ satisfies a Lipschitz condition:

$$ \|A(x,t,\varphi) - A(x,t,\psi)\| \le L\|\varphi - \psi\| = L\|\omega\|. $$

So:

$$ \|\omega(x,t)\| \le \frac{1}{\Gamma(\alpha)}\int_0^t (t-\zeta)^{\alpha-1}\|\omega(\zeta)\|d\zeta \tag{36} $$

Apply Grönwall's Inequality for fractional integrals, we conclude

$$ \|\omega(x,t)\| = 0 \Rightarrow \varphi(x,t) = \psi(x,t). $$

Hence, the solution is unique.

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## 4 Applications

*Example 1.*Consider the following time-fractional diffusion equation:

$$ {}^{C}D_t^{\alpha}\varphi - \varphi_{xx} = 0, \quad 0 < \alpha \leq 1, \quad x \in [0, 2\pi], t \geq 0 \tag{37} $$

With the initial condition

$$ \varphi(x,0) = \sin x, \tag{38} $$

Applying the (YJ) transform on both sides in (37), and after using the differentiation property of (YJ) transform for fractional derivative, and take $\mathscr{H}^{-1}$ to both sides, we get

$$ \varphi = \sin x + \mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left[\varphi_{xx}\right]\right]. \tag{39} $$

By (DJM) we get

$$
\begin{aligned}
F &= \sin x = \varphi_0 \\
L(\varphi_{n+1}) &= \mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left[\frac{\partial^2}{\partial x^2}\varphi_n\right]\right] \\
\varphi_1 &= \mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left[\frac{\partial^2}{\partial x^2}\varphi_0\right]\right] \\
&= \mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}[-\sin x]\right] \\
&= \frac{-t^{\alpha}}{\Gamma(\alpha+1)}\sin x \\
\varphi_2 &= \mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left[\frac{\partial^2}{\partial x^2}\varphi_1\right]\right] \\
&= \mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left[\frac{t^{\alpha}}{\Gamma(\alpha+1)}\sin x\right]\right] \\
&= \frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\sin x \\
&\vdots
\end{aligned}
$$

So, the approximate solution is:

$$ \varphi(x,t) = \sin x\left[1 - \frac{t^{\alpha}}{\Gamma(\alpha+1)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \cdots\right] \tag{40} $$

If $\alpha = 1$, we get the exact solution which is:

$$ \varphi(x,t) = \sin x e^{-t} $$

*Example 2.*Consider the following fractional KdV equation:

$$ {}^{c}D_t^{\alpha}\varphi - 3\left(\varphi^2\right)_x + \varphi_{xxx} = 0, \quad 0 < \alpha \leq 1, \quad x \in \mathbb{R}, t \geq 0. \tag{41} $$

With the initial condition

$$ \varphi(x,0) = 6x \tag{42} $$

Applying the (YJ) transform on both sides in (41), and after using the differentiation property of (YJ) transform for fractional derivative, we get

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**Table 1:** Numerical values of the approximate solution $\varphi(x,t)$ (Eq.43) for $\alpha = 0.7, 0.9$, and 1 at fixed $x = \pi/2$, varying $t$.

| $t$ | $\varphi_{0.7}$ | $\varphi_{0.9}$ | $\varphi_1$ | $\varphi_{\text{Exact}}$ | Error $_{0.7}$ | Error $_{0.9}$ | Error $_1$ |
|---|---|---|---|---|---|---|---|
| 0.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 0.00000 | 0.00000 | 0.00000 |
| 0.05556 | 0.86856 | 0.92616 | 0.94599 | 0.94596 | 0.07740 | 0.01980 | 0.00003 |
| 0.11111 | 0.80075 | 0.86751 | 0.89506 | 0.89484 | 0.09409 | 0.02733 | 0.00022 |
| 0.16667 | 0.75154 | 0.81641 | 0.84722 | 0.84648 | 0.09494 | 0.03007 | 0.00074 |
| 0.22222 | 0.71400 | 0.77124 | 0.80247 | 0.80074 | 0.08674 | 0.02950 | 0.00173 |
| 0.27778 | 0.68502 | 0.73117 | 0.76080 | 0.75747 | 0.07245 | 0.02629 | 0.00334 |
| 0.33333 | 0.66286 | 0.69573 | 0.72222 | 0.71653 | 0.05367 | 0.02080 | 0.00569 |
| 0.38889 | 0.64639 | 0.66456 | 0.68673 | 0.67781 | 0.03142 | 0.01325 | 0.00892 |
| 0.44444 | 0.63483 | 0.63742 | 0.65432 | 0.64118 | 0.00635 | 0.00376 | 0.01314 |
| 0.50000 | 0.62759 | 0.61410 | 0.62500 | 0.60653 | 0.02106 | 0.00757 | 0.01847 |

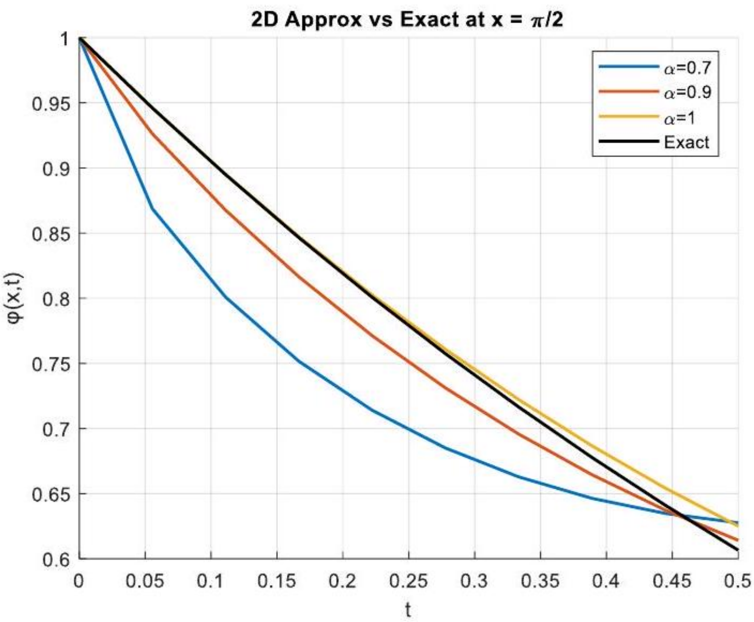


**Fig. 1:** 2D comparison of the approximate and exact solution $\varphi(x,t)$ Eq. (40) for $\alpha = 0.7, 0.9$, and 1 at fixed $x = \pi/2$, varying $t$.

$$\frac{\mathscr{H}(\varphi(x,t))}{(\sqrt{a})^{\alpha}} - \frac{a\varphi(x,0)}{(\sqrt{a})^{\alpha-1}} - \mathscr{H}\left[3\left(\varphi^2\right)_x - \varphi_{xxx}\right] = 0$$

On simplifying and using the Eq. (41), we have:

$$\mathscr{H}(\varphi) - a\sqrt{a}(6x) - (\sqrt{a})^{\alpha}\mathscr{H}\left[3\left(\varphi^2\right)_x - \varphi_{xxx}\right] = 0 \quad (46)$$

Take $\mathscr{H}^{-1}$ to both side of Eq. (46), we get

$$\varphi = 6x + \mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left[3\left(\varphi^2\right)_x - \varphi_{xxx}\right]\right] \quad (43)$$

Let

$$(48)\varphi(x,t) = \sum_{n=0}^{\infty} \varphi_n(x,t). \quad (44)$$

Substituting (**??**) in to both sides of (43) gives:

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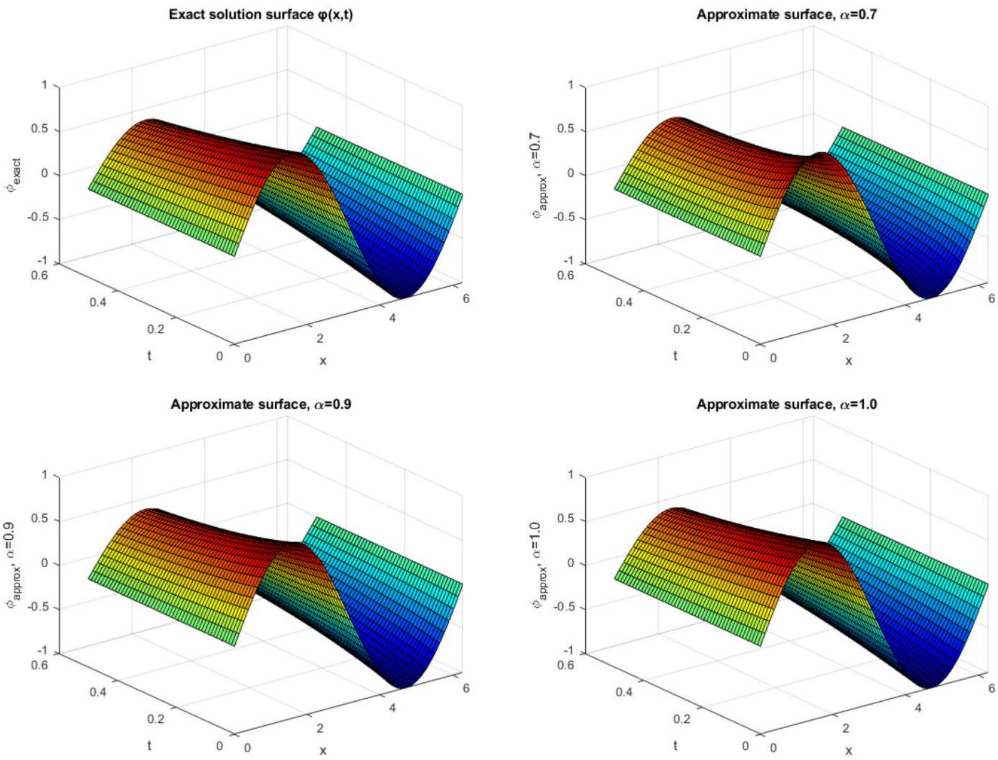


**Fig. 2:** 3D comparison of the approximate and exact solution $\varphi(x,t)$ Eq. (40) for $\alpha = 0.7, 0.9$, and 1 at fixed $x = \pi/2$, varying $t$.

$$\sum_{n=0}^{\infty} \varphi_n = 6x + \mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left[3\frac{\partial}{\partial x}\left(\sum_{n=0}^{\infty}\varphi_n\right)^2 - \frac{\partial^3}{\partial x^3}\left(\sum_{n=0}^{\infty}\varphi_n\right)\right]\right] \tag{45}$$

The nonlinear term is decomposed as in

$$3\frac{\partial}{\partial x}\left(\sum_{n=0}^{\infty}\varphi_n\right)^2 = 3\frac{\partial}{\partial x}\left\{(\varphi_0)^2 + \sum_{n=0}^{\infty}\left[\left(\sum_{i=0}^{n}\varphi_i\right)^2 - \left(\sum_{i=0}^{n-1}\varphi_i\right)^2\right]\right\}. \tag{46}$$

Substituting (46) in to both sides of (45) gives:

$$\sum_{n=0}^{\infty} \varphi_n = 6x + \mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left[3\frac{\partial}{\partial x}\left\{(\varphi_0)^2 + \sum_{n=0}^{\infty}\left[\left(\sum_{i=0}^{n}\varphi_i\right)^2 - \left(\sum_{i=0}^{n-1}\varphi_i\right)^2\right]\right\} - \frac{\partial^3}{\partial x^3}\left(\sum_{n=0}^{\infty}\varphi_n\right)\right]\right] \tag{47}$$

Then:

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$$
\begin{aligned}
&F = \varphi_0 = 6x. \\
&\varphi_1 = \mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left[3\frac{\partial}{\partial x}(\varphi_0)^2 - \frac{\partial^3}{\partial x^3}(\varphi_0)\right]\right], \\
&\quad = \mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left[6^3 x - 0\right]\right], \\
&\quad = 6^3 x\frac{t^{\alpha}}{\Gamma(\alpha+1).} \\
&\varphi_2 = \mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left[3\frac{\partial}{\partial x}\left((\varphi_0+\varphi_1)^2 - \varphi_0^2\right) - \frac{\partial^3}{\partial x^3}(\varphi_1)\right]\right], \\
&\quad = \mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left[3\frac{\partial}{\partial x}\left(\varphi_1^2 + 2\varphi_0\varphi_1\right) - 0\right]\right], \\
&\quad = \frac{6^7 x\Gamma(2\alpha+1)}{(\Gamma(\alpha+1))^2}\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + 2\left(6^5 x\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\right). \\
&\vdots
\end{aligned}
$$

The approximate solution of (40) is given by:

$$
\varphi(x,t) = 6x\left[1 + \frac{6^2 t^{\alpha}}{\Gamma(\alpha+1)} + 2\left(\frac{\left(6^2 t^{\alpha}\right)^2}{\Gamma(2\alpha+1)}\right) + \frac{\Gamma(2\alpha+1)}{(\Gamma(\alpha+1))^2}\cdot\frac{\left(6^2 t^{\alpha}\right)^3}{\Gamma(3\alpha+1)} + \cdots\right] \tag{48}
$$

The Eq. (48) is approximate solution of Eq. (40). If $\alpha = 1$, we get the exact solution which is:

$$
\varphi(x,t) = \frac{6x}{1 - 6^2 t}
$$

**Table 2:** Numerical values of the approximate solution $\varphi(x,t)$ Eq. (48) for $\alpha = 0.7, 0.9$, and 1 at fixed $x = 0.0001$, varying $t$.

| $t$ | $\varphi_{0.7}$ | $\varphi_{0.9}$ | $\varphi_1$ | $\varphi_{\text{Exact}}$ | Error $_{0.7}$ | Error $_{0.9}$ | Error $_1$ |
|---|---|---|---|---|---|---|---|
| 0 | 0.00060 | 0.00060 | 0.00060 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 0.05556 | 0.06993 | 0.01233 | 0.00580 | -0.00060 | 0.07053 | 0.01293 | 0.00640 |
| 0.11111 | 0.25340 | 0.05374 | 0.02540 | -0.00020 | 0.25360 | 0.05394 | 0.02560 |
| 0.16667 | 0.55431 | 0.13836 | 0.06900 | -0.00012 | 0.55443 | 0.13848 | 0.06912 |
| 0.22222 | 0.97559 | 0.27791 | 0.14620 | -0.00009 | 0.97567 | 0.27800 | 0.14629 |
| 0.27778 | 1.51960 | 0.48308 | 0.26660 | -0.00007 | 1.51970 | 0.48315 | 0.26667 |
| 0.33333 | 2.18840 | 0.76381 | 0.43980 | -0.00005 | 2.18850 | 0.76387 | 0.43985 |
| 0.38889 | 2.98380 | 1.12950 | 0.67540 | -0.00005 | 2.98380 | 1.12950 | 0.67545 |
| 0.44444 | 3.90710 | 1.58910 | 0.98300 | -0.00004 | 3.90720 | 1.58910 | 0.98304 |
| 0.50000 | 4.96000 | 2.15120 | 1.37220 | -0.00004 | 4.96000 | 2.15120 | 1.37220 |

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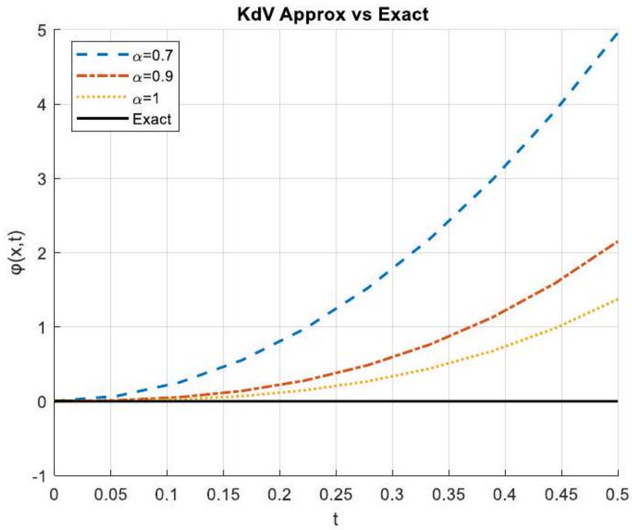


**Fig. 3:** 2D comparison of the approximate and exact solution $\varphi(x,t)$ Eq. (48) for $\alpha = 0.7, 0.9$, and 1 at fixed $x = 0.0001$, varying $t$.

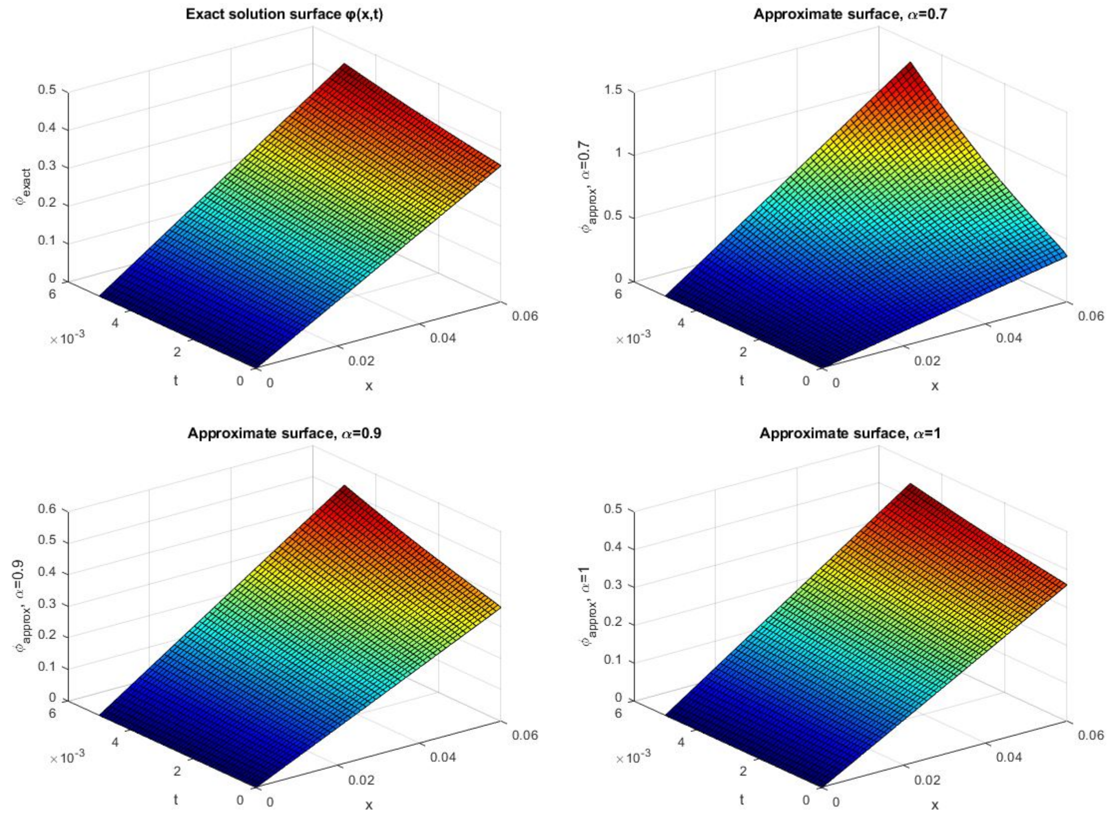


**Fig. 4:** 3D comparison of the approximate and exact solution $\varphi(x,t)$ Eq. (48) for $\alpha = 0.7, 0.9$, and 1 at fixed $x = 0.0001$, varying $t$.

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*Example 3.*Consider the following non-linear Burger's equation:

$$^{c}D_t^{\alpha}\varphi+\varphi\frac{\partial\varphi}{\partial x}=\frac{\partial^2\varphi}{\partial x^2},\quad 0<\alpha\leq 1,\quad x\in\mathbb{R},t\geq . \tag{49}$$

With the initial condition

$$\varphi(x,0)=x. \tag{50}$$

Applying the (YJ) transform on both sides in (49), and after using the differentiation property of (YJ) transform for fractional derivative, and take $\mathscr{H}^{-1}$ to both sides, we get

$$\varphi(x,t)=x+\mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left(\frac{\partial^2\varphi}{\partial x^2}\right)\right]-\mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left(\varphi\frac{\partial\varphi}{\partial x}\right)\right].$$

By (DJM) we get:

$$\begin{aligned}F&=x=\varphi_0\\ L(\varphi)&=\mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left(\frac{\partial^2\varphi}{\partial x^2}\right)\right]\\ G&=-\mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left(\varphi\frac{\partial\varphi}{\partial x}\right)\right]\end{aligned}$$

Now:

$$\begin{aligned}G_0&=-\mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left(\varphi_0\frac{\partial\varphi_0}{\partial x}\right)\right].\\ G_1&=-\mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left((\varphi_0+\varphi_1)\left(\frac{\partial\varphi_0}{\partial x}+\frac{\partial\varphi_1}{\partial x}\right)-\left(\varphi_0\frac{\partial\varphi_0}{\partial x}\right)\right)\right].\\ &\vdots\end{aligned}$$

Then

$$\begin{aligned}\varphi_0&=F=x\\ \varphi_1&=L(\varphi_0)+G_0\\ &=\mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left(\frac{\partial^2\varphi_0}{\partial x^2}\right)\right]-\mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left(\varphi_0\frac{\partial\varphi_0}{\partial x}\right)\right]\\ &=\mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}(0)\right]-\mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}(x)\right]\\ &=\frac{-xt^{\alpha}}{\Gamma(\alpha+1)}.\end{aligned}$$

$$\begin{aligned}\varphi_2=L(\varphi_1)+G_1,&\\ &=\mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left(\frac{\partial^2\varphi_1}{\partial x^2}\right)\right]-\mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left((\varphi_0+\varphi_1)\left(\frac{\partial\varphi_0}{\partial x}+\frac{\partial\varphi_1}{\partial x}\right)-\left(\varphi_0\frac{\partial\varphi_0}{\partial x}\right)\right)\right],\\ &=\mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}(0)\right]-\mathscr{H}^{-1}\left[(\sqrt{a})^{\alpha}\mathscr{H}\left(\left(x-\frac{xt^{\alpha}}{\Gamma(\alpha+1)}\right)\left(1-\frac{t^{\alpha}}{\Gamma(\alpha+1)}\right)-(x)\right)\right],\\ &=x\left[\frac{2t^{2\alpha}}{\Gamma(2\alpha+1)}-\frac{\Gamma(2\alpha+1)}{(\Gamma(\alpha+1))^2}\cdot\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}\right].\\ &\vdots\end{aligned}$$

The approximate solution of (47) is given by:

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$$\varphi(x,t) = x - \frac{xt^{\alpha}}{\Gamma(\alpha+1)} + \frac{2t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{\Gamma(2\alpha+1)}{(\Gamma(\alpha+1))^2} \cdot \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \cdots \tag{51}$$

The Eq. (51) is approximate solution to the form:

$$\varphi(x,t) = \frac{x}{1+t}$$

For $\alpha = 1$, which is the exact solution of Eq. (49) at $\alpha = 1$.

**Table 3:** Numerical values of the approximate solution $\varphi(x,t)$ Eq. (51) for $\alpha = 0.7, 0.9$, and 1 at fixed $x = 0.5$, varying $t$.

| $t$ | $\varphi_{0.7}$ | $\varphi_{0.9}$ | $\varphi_1$ | $\varphi_{\text{Exact}}$ | Error $_{0.7}$ | Error $_{0.9}$ | Error $_1$ |
|---|---|---|---|---|---|---|---|
| 0 | 0.50000 | 0.50000 | 0.50000 | 0.50000 | 0 | 0 | 0 |
| 0.11111 | 0.41555 | 0.43889 | 0.45039 | 0.45000 | 0.03445 | 0.01111 | 0.00039 |
| 0.22222 | 0.39147 | 0.40177 | 0.41175 | 0.40909 | 0.01763 | 0.00732 | 0.00266 |
| 0.33333 | 0.38381 | 0.37796 | 0.38272 | 0.37500 | 0.00881 | 0.00296 | 0.00772 |
| 0.44444 | 0.38441 | 0.36367 | 0.36191 | 0.34615 | 0.03825 | 0.01751 | 0.01576 |
| 0.55556 | 0.38926 | 0.35632 | 0.34797 | 0.32143 | 0.06783 | 0.03489 | 0.02654 |
| 0.66667 | 0.39595 | 0.35386 | 0.33951 | 0.30000 | 0.09595 | 0.05386 | 0.03951 |
| 0.77778 | 0.40282 | 0.35456 | 0.33516 | 0.28125 | 0.12157 | 0.07331 | 0.05391 |
| 0.88889 | 0.40864 | 0.35685 | 0.33356 | 0.26471 | 0.14393 | 0.09214 | 0.06886 |
| 1 | 0.41246 | 0.35932 | 0.33333 | 0.25000 | 0.16246 | 0.10932 | 0.08333 |

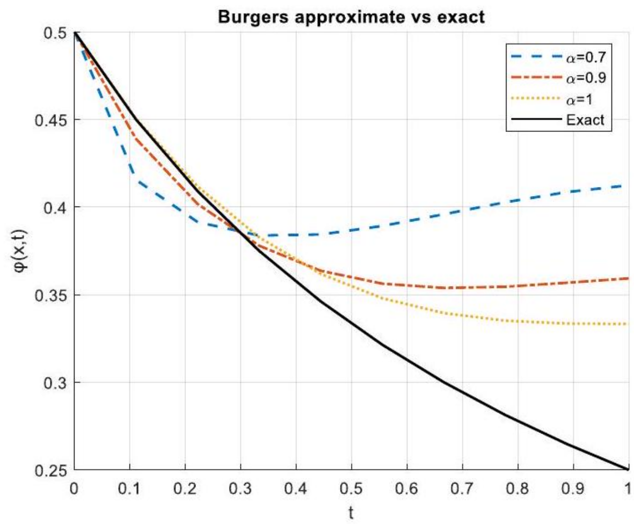


**Fig. 5:** 2D comparison of the approximate and exact solution $\varphi(x,t)$ (Eq.55) for $\alpha = 0.7, 0.9$, and 1 at fixed $x = 0.5$, varying $t$.

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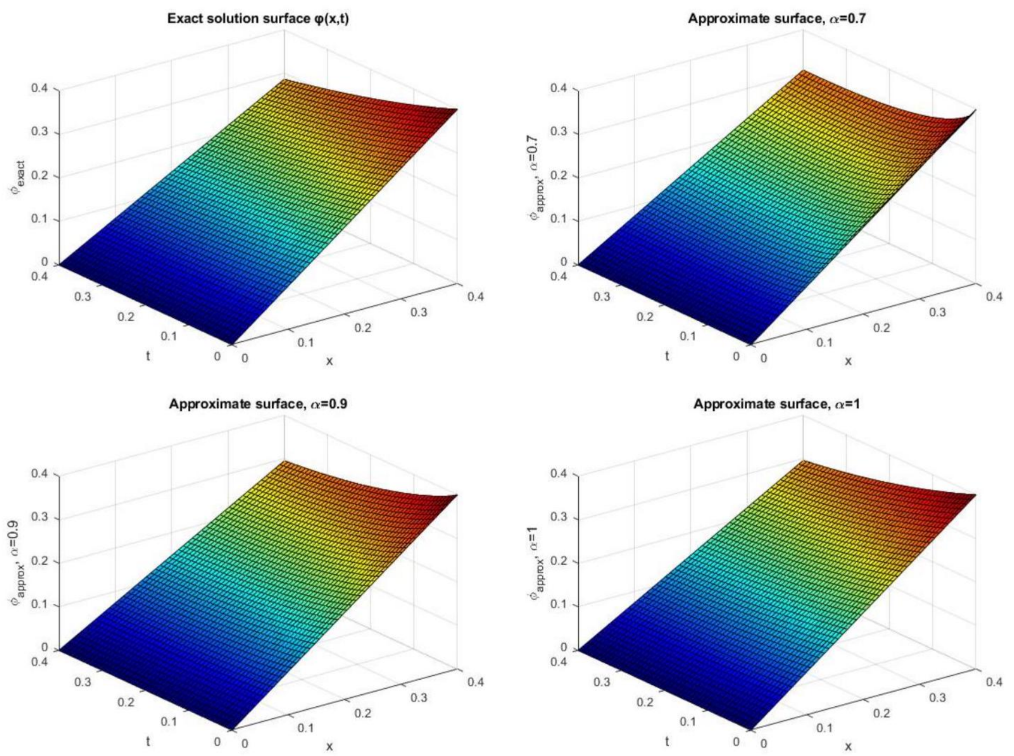


**Fig. 6:** 3D comparison of the approximate and exact solution $\varphi(x,t)$ Eq. (51) for $\alpha = 0.7, 0.9$, and 1 at fixed $x = 0.5$, varying $t$.

## 5 Conclusion

In this study, we proposed a modification of the Caputo-Daftardar-Jaafari method using the Yasser Jassim transformation to solve linear and nonlinear fractional partial differential equations. The analytical results, supported by proof, showed the existence of unique solutions, as well as the numerical results supported by tables and figures, which strengthened the validity of the proposed modified method. The modified method was applied to well-known equations, including the fractional diffusion equation, Burger's equation, and Kortog-de Vries equation, and the results showed that the method is accurate and efficient, both analytically and numerically.
Furthermore, comparative analysis with recent studies such as those by Das and Rajeev (2010), Alwehebi et al. (2023), and Veeresha et al. (2020) highlighted the simplicity and directness of the presented technique, which yields reliable solutions using fewer steps and reduced computational effort [41-43].
Recommendations:

– This method can be extended to more complex systems, including multidimensional and stochastic fractional equations.
– Future work could explore the integration of this technique with symbolic computation environments to automate and generalize its implementation.
– It is also recommended to examine the method's performance under varying fractional orders and irregular boundary conditions.

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