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A Numerical Investigation of Systems of Partial Differential Equations Under Non-Singular Kernel

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Abstract: This article investigates systems of nonlinear fractional partial differential equations (NFPDEs) utilizing the Shehu Adomian decomposition method (STDM) with a non-singular kernel. The STDM is combining the Shehu transform method with the Adomian decomposition method, providing exact and analytical solutions of systems of nonlinear fractional partial differential equations. The convergence and existence results for the suggested technique are presented. Two numerical examples are used to demonstrate the reliability and efficacy of proposed technique using Matlab software. The findings demonstrate that accurate and reliable approximations can be achieved in only a few terms.

Keywords: Fractional partial differential equations, Caputo-Fabrizio derivative, existence, fixed point theory, Adomian polynomial, Shehu transform.

1 Introduction

Fractional-order partial differential equations (PDEs) are increasingly recognized for their importance in developing methods for nonlinear models. Solutions to nonlinear PDEs of arbitrary order play a pivotal role in understanding the characteristics and behavior of complex problems in applied mathematics and technology. Despite their significance, deriving analytical solutions for these equations remains a formidable challenge. Over the past three decades, there has been a significant focus on initiating and studying a variety of numerical techniques ([1], [2], [3], [4], [5], [6], [7], [8], [9], [10]).

Integral transformations are undeniably one of the most beneficial and efficient techniques in theoretical and applied mathematics, finding numerous applications in fields such as biology ([11], [12], [13]), electrodynamics [14], mechanics [15], biotechnology [16], chaos theory [17], and others. In recent years, various integral transforms, such as the Laplace transform ([18], [19]), Elzaki transform ([20], [21]), Yang integral transform [22], Aboodh transform [23], Sumudu transform [24], Shehu transform [25] etc. have been employed for solving different physical models.
Over the past decades, various fractional operators have been introduced to deepen the understanding of model dynamics. These include the Riemann-Liouville (RL) and Caputo derivatives ([26], [27], [28],[30], [31], [32], [33], [34]), as well as the Caputo–Fabrizio (CF) ([35], [36], [37], [38]) and Atangana–Baleanu [39] operators. In 2015, Caputo and Fabrizio [35] proposed a novel nonlinear fractional derivative, now known as the Caputo–Fabrizio derivative operator. That same year, Losada and Nieto [40] explored several of its fundamental properties. Owing to its flexibility and unique characteristics, numerous studies have since been conducted on various fractional partial differential systems. The objective of this study is to investigate the numerical and analytical solutions of systems of nonlinear fractional partial differential equations (NFPDEs). We employed the Shehu transform decomposition method (STDM) to obtain numerical and graphical solutions for various applications of the systems of nonlinear fractional partial differential

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equations. To reduce computational complexity and intricacy, we proposed the STDM, which combines the Shehu transform and the ADM. This approach was specifically designed to solve the time-fractional systems of nonlinear fractional partial differential equations, which serves as the primary motivation for this research. The proposed technique constructs a convergent series as the solution, ensuring both accuracy and efficiency.

2 Preliminaries

This section provides key definitions, which are essential for understanding the subsequent results.

Definition 1. *The Caputo fractional derivative of $\Phi \in C^m_{-1}$ is defined as ([14], [15])*

$$ {}^{C}D^{\zeta}_{\eta}\Phi(\xi,\eta) = \begin{cases} \frac{\partial^p \Phi(\xi,\eta)}{\partial x^p}, & \zeta = p \in \aleph, \\ \frac{1}{\Gamma(p-1)} \int\limits_0^{\eta} (\eta-\phi)^{p-1-\zeta} \frac{\partial^p \Phi(\xi,\phi)}{\partial \phi^p} d\phi, & p-1<\zeta \le p. \end{cases} \tag{1}$$

Definition 2. *The Caputo–Fabrizio fractional-order derivative (CF) is defined [19] as follows:*

$$ {}^{CF}{}_0D^{\zeta}_{\eta}\Phi(\xi,\eta) = \frac{1}{(1-\zeta)} \int_0^{\eta} \exp\left(\frac{-\zeta(\eta-\phi)}{\zeta-1}\right) \Phi'(\xi,\phi) d\phi, \tag{2}$$

such that $0<\zeta\le 1$.

Definition 3. *S. Maitama and W. Zaho [41] developed a noval transform of exponential order function $\Phi(\eta)$ over the set of $\mathscr{A}$,*

$$\mathscr{A} = \left\{ \Phi(\eta) : \exists\, \xi_1, \xi_1, \xi_2 > 0,\ |\Phi(\eta)| < \xi_1 e^{\frac{|\eta|}{\xi_j}},\ if\ \eta \in (-1)^j \times [0,\infty) \right\}$$

by to integral

$$ S[\Phi(\eta)] = \mathrm{T}(\nu,\rho) = \int\limits_0^{\infty} \Phi(\xi)\, e^{\frac{-\nu\eta}{\rho}} d\eta,\ \nu>0,\ \eta>0\,. \tag{3}$$

Remark. If $\rho = 1$, then ST becomes Laplace's transform, and also for $\nu = 1$, this transform converts into Yang's integral transform [8].

Definition 4. *The Shehu transform (ST) for fractional CF derivative (2) is given as*

$$ S\left({}^{CF}_{0}D^{\zeta}_{\eta}[\Phi(\xi,\eta)]\right) = \frac{1}{1-\zeta+\zeta\left(\frac{\rho}{\nu}\right)} \left(V(\nu,\rho) - \frac{\rho}{\nu}\Phi(0)\right), \tag{4}$$

here, $V(\nu,\rho)$ is ST of $\Phi(\xi,\eta)$.

3 Proposed Methodology

Let's consider a fractional nonlinear partial differential equation:

$$ {}^{CF}D^{\zeta}_{\eta}\Phi(\xi,\eta) = \Re\,\Phi(\xi,\eta) + \aleph\,\Phi(\xi,\eta) + P(\xi,\eta),\ m-1<\zeta\le m, \tag{5}$$

with initial condition

$$ \Phi(\xi,\eta) = \phi(\xi), \tag{6}$$

where ${}^{CF}D^{\zeta}_{\eta} = \frac{\partial^{\zeta}}{\partial \eta^{\zeta}}$ represents the fractional CF derivative of order ζ, $\Re$ and $\aleph$ are stand for linear and nonlinear functions respectively, and P denotes the source term.
Applying the ST to both sides of equation (5), we obtain

$$ S\left[{}^{CF}D^{\zeta}_{\eta}\Phi(\xi,\eta)\right] = S[\Re\Phi(\xi,\eta)] + S[\aleph\,\Phi(\xi,\eta)] + S[P(\xi,\eta)]\,. \tag{7}$$

Utilizing the differentiation property of the ST, we obtain

$$S[\Phi(\xi,\eta)] = \left(\tfrac{\rho}{\upsilon}\right)[\Phi(\xi,0)] + \left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S[P(\xi,\eta)] + \left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S[\Re[\Phi(\xi,\eta)] + \aleph[\Phi(\xi,\eta)]]. \tag{8}$$

Now, applying the inverse ST to both sides of the equation (8), we obtain

$$\Phi(\xi,\eta) = S^{-1}\left[\left(\tfrac{\rho}{\upsilon}\right)[\Phi(\xi,0)]\right] + S^{-1}\left[\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S[P(\xi,\eta)]\right] + S^{-1}\left[\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S[\Re[\Phi(\xi,\eta)] + \aleph[\Phi(\xi,\eta)]]\right]. \tag{9}$$

Let, $\Phi(\xi,\eta)$ has infinite series solution as

$$\Phi(\xi,\eta) = \sum_{\tau=0}^{\infty}\Phi_\tau(\xi,\eta), \tag{10}$$

and the nonlinear term $\aleph\Phi(\xi,\eta)$ is expressed as

$$\aleph\Phi(\xi,\eta) = \sum_{\tau=0}^{\infty}A_\tau\ , \tag{11}$$

where A_τ is the Adomian polynomial, given by

$$A_\tau = \frac{1}{\Gamma(\tau+1)}\left[\frac{d^\tau}{d\Pi^\tau}\left\{\aleph\left(\sum_{\iota=0}^{\infty}\Pi^\iota\xi_\iota, \sum_{\iota=0}^{\infty}\Pi^\iota\eta_\iota\right)\right\}\right]_{\Pi=0}. \tag{12}$$

Using equations (10) and (11) in equation (9), we get

$$\sum_{\ell=0}^{\infty}\Phi(\xi,\eta) = \Phi(\xi,0) + S^{-1}\left[\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S[P(\xi,\eta)]\right] + S^{-1}\left[\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S\left[\Re\Phi\left(\sum_{\tau=0}^{\infty}\xi_\tau, \sum_{\tau=0}^{\infty}\eta_\tau\right) + \sum_{\tau=0}^{\infty}A_\ell\right]\right]. \tag{13}$$

From equation (13), we get

$$\begin{aligned}
\Phi_0(\xi,\eta) &= \Phi(\xi,0) + S^{-1}\left[\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S[P(\xi,\eta)]\right],\\
\Phi_1(\xi,\eta) &= S^{-1}\left[\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S[\Re\Phi(\xi_0,\eta_0) + A_0]\right],\\
\Phi_{\tau+1}(\xi,\eta) &= S^{-1}\left[\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S[\Re\Phi(\xi_\tau,\eta_\tau) + A_\tau]\right],\ \tau \geq 1.
\end{aligned} \tag{14}$$

4 convergence analysis

In this section, we illustrate the uniqueness and convergence of the $STDM_{CF}$ solution.

Theorem 1.*The $STDM_{CF}$ solution of equation (5) is unique, when* $0 < (\Delta_1+\Delta_2)(1-\zeta+\eta\zeta) < 1$.

*Proof.*Let $B = (C[\mathrm{I}], \|.\|)$ be the Banach space with the norm $\|\Phi\| = \max_{(\xi,\eta)\in \mathrm{I}}|\Phi(\xi,\eta)|$, for all continuous function on I. Let T : B $\to$ B is a nonlinear mapping, where

$$\Phi_{\ell+1}^{CF}(\xi,\eta) = \Phi_0(\xi,\eta) + S^{-1}\left(\left(1-\zeta+\zeta\left(\frac{\rho}{\upsilon}\right)\right)S[\Re[\Phi_\ell(\xi,\eta)] + \Re[\Phi_\ell(\xi,\eta)]]\right), \ell \geq 0. \tag{15}$$

Suppose that $|\Re(\Phi(\xi,\eta)) - \Re(\Phi^*(\xi,\eta))| < \Delta_1|\Phi(\xi,\eta) - \Phi^*(\xi,\eta)|$ and $|\aleph(\Phi(\xi,\eta)) - \aleph(\Phi^*(\xi,\eta))| < \Delta_2|\Phi(\xi,\eta) - \Phi^*(\xi,\eta)|$, where Δ_1 and Δ_2 are Lipschitz constants and Φ and Φ^* are real-valued continuous functions.

$$\begin{aligned}
\left\|\mathrm{T}\Phi(\xi,\eta) - \mathrm{T}\tilde{\Phi}(\xi,\eta)\right\| &\leq \max_{(\xi,\eta)\in I}\Big|S^{-1}\left[\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S\left[\Re\Phi(\xi,\eta) - \Re\tilde{\Phi}(\xi,\eta)\right]\right.\\
&\quad \left.+\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S\left[\aleph\Phi(\xi,\eta) - \aleph\tilde{\Phi}(\xi,\eta)\right]\right],\\
&\leq \max_{(\xi,\eta)\in I}(\Delta_1+\Delta_2)\left[S^{-1}\left[\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S\left[\Phi(\xi,\eta) - \tilde{\Phi}(\xi,\eta)\right]\right]\right],\\
&\leq (\Delta_1+\Delta_2)\left[S^{-1}\left[\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S\left\|\Phi(\xi,\eta) - \tilde{\Phi}(\xi,\eta)\right\|\right]\right],\\
&= (\Delta_1+\Delta_2)(1-\zeta+\eta\zeta)\left\|\Phi(\xi,\eta) - \tilde{\Phi}(\xi,\eta)\right\|.
\end{aligned}$$

T is contraction as $0 < (\Delta_1+\Delta_2)(1-\zeta+\eta\zeta) < 1$. Hence, the solution of (5) is unique from Banach fixed point theorem

Theorem 2. *The $STDM_{CF}$ solution of equation (5) is convergent.*

Proof. Let $\Phi_m = \sum_{r=0}^{m} \Phi_r(\xi,\eta)$. To prove Φ_m is a Cauchy sequence in B. Let

$$\begin{aligned}
\left\|\Phi_m - \Phi_p\right\| &= \max_{\eta \to I}\left|\sum_{r=p+1}^{\infty}\Phi_r\right|, \quad p=1,2,...\\
&\le \max_{\eta \to I}\left|S^{-1}\left[\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S\left[\sum_{r=p+1}^{\infty}\left(\Re\Phi_{r-1}+\aleph\Phi_{r-1}\right)\right]\right]\right|,\\
&= \max_{\eta \to I}\left|S^{-1}\left[\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S\left[\sum_{r=p+1}^{m-1}\left(\Re\Phi_{r}+\aleph\Phi_{r}\right)\right]\right]\right|,\\
&\le \max_{\eta \to I}\left|S^{-1}\left[\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S\left[\left(\Re\left(\Phi_{m-1}\right)-\Re\left(\Phi_{p-1}\right)\right)\right.\right.\right.\\
&\quad \left.\left.\left.+\left(\aleph\left(\Phi_{m-1}\right)-\aleph\left(\Phi_{p-1}\right)\right)\right]\right]\right|,\\
&\le \theta_1\left|S^{-1}\left[\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S\left[\left(\Re\left(\Phi_{m-1}\right)-\Re\left(\Phi_{p-1}\right)\right)\right]\right]\right|\\
&\quad +\theta_2\left|S^{-1}\left[\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S\left[\left(\Re\left(\Phi_{m-1}\right)-\Re\left(\Phi_{p-1}\right)\right)\right]\right]\right|\\
&= \left(\theta_1+\theta_2\right)\left(1-\zeta+\eta\zeta\right)\left\|\Phi_{m-1}-\Phi_{p-1}\right\|.
\end{aligned}$$

Let $m=p+1$, then
$\left\|\Phi_{p+1}-\Phi_p\right\| \le \theta\left\|\Phi_p-\Phi_{p-1}\right\| \le \theta^2\left\|\Phi_{p-1}-\Phi_{p-2}\right\| \le .... \le \theta^p\left\|\Phi_1-\Phi_0\right\|,$
where $\theta=\left(\theta_1+\theta_2\right)\left(1-\zeta+\eta\zeta\right)$.
Similarly, we have

$$\begin{aligned}
\left\|\Phi_m-\Phi_p\right\| &\le \left\|\Phi_{p+1}-\Phi_p\right\|+\left\|\Phi_{p+2}-\Phi_{p+1}\right\|+.....+\left\|\Phi_m-\Phi_{m-1}\right\|,\\
&\le \left(\theta^p+\theta^{p+1}+....+\theta^{m-1}\right)\left\|\Phi_1-\Phi_0\right\|,\\
&\le \theta^p\left(\tfrac{1-\theta^{m-p}}{1-\theta}\right)\left\|\Phi_1\right\|.
\end{aligned}$$

As $o<\theta<1$, we get $1-\theta^{m-p}<1$, so
$\left\|\Phi_m-\Phi_p\right\| \le \left(\frac{\theta^p}{1-\theta}\right)\max_{\eta\to I}\left\|\Phi_1\right\|$.
Since $\left\|\Phi_1\right\|<\infty$, $\left\|\Phi_m-\Phi_p\right\| \to 0$ as $p\to\infty$. As a result, Φ_m is a Cauchy sequence in B, implying that the series Φ_m is convergent.

5 Applications of system of FPDEs

Application 1: Consider the system of FPDEs. as:

$$\begin{aligned}
&{}^{CF}D_{\eta}^{\zeta}\omega(\xi,\eta)-\tfrac{\partial\vartheta}{\partial\xi}+\omega+\vartheta=0,\\
&{}^{CF}D_{\eta}^{\zeta}\vartheta(\xi,\eta)-\tfrac{\partial\omega}{\partial\xi}+\omega+\vartheta=0, \quad 0<\zeta\le 1,
\end{aligned} \tag{16}$$

subject to initial conditions

$$\omega(\xi,0)=\sinh(\xi), \qquad \vartheta(\xi,0)=\cosh(\xi). \tag{17}$$

By applying the Shehu transform (ST) to both sides of equation (16) and simplifying, we obtain:

$$\begin{aligned}
S[\omega(\xi,\eta)] &= \left(\tfrac{\rho}{\upsilon}\right)[\omega(\xi,0)]+\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S\left[\tfrac{\partial\vartheta}{\partial\xi}-\omega-\vartheta\right],\\
S[\vartheta(\xi,\eta)] &= \left(\tfrac{\rho}{\upsilon}\right)[\vartheta(\xi,0)]+\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)L\left[\tfrac{\partial\omega}{\partial\xi}-\omega-\vartheta\right].
\end{aligned} \tag{18}$$

Applying the inverse ST to both sides of equation (18), we obtain:

$$\begin{aligned}
\omega(\xi,\eta) &= \omega(\xi,0)+S^{-1}\left[\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S\left[\tfrac{\partial\vartheta}{\partial\xi}-\omega-\vartheta\right]\right],\\
\vartheta(\xi,\eta) &= \vartheta(\xi,0)+S^{-1}\left[\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right)S\left[\tfrac{\partial\omega}{\partial\xi}-\omega-\vartheta\right]\right].
\end{aligned} \tag{19}$$

Table-I: The absolute error between the exact and approximate solution of ω_{STDM} at $\xi = 0.3, 0.5, 0.7$, and $0.01 \leq \eta \leq 0.05$ for application 1.

ξ	η	$\lvert\omega_{Exact} - \omega_{appr}\rvert$ $\zeta = 0.6$	$\lvert\omega_{Exact} - \omega_{appr}\rvert$ $\zeta = 0.8$	$\lvert\omega_{Exact} - \omega_{appr}\rvert$ $\zeta = 1$
	0.01	$1.04628031 \times 10^{-2}$	1.0462464×10^{-3}	1.6×10^{-9}
	0.02	$1.04706498 \times 10^{-2}$	1.0470109×10^{-3}	6.1×10^{-9}
0.3	0.03	$1.04779121 \times 10^{-2}$	1.0477241×10^{-3}	1.38×10^{-8}
	0.04	1.0484799×10^{-2}	1.0484038×10^{-3}	2.44×10^{-8}
	0.05	$1.04914077 \times 10^{-2}$	1.0490603×10^{-3}	3.81×10^{-8}
	0.01	$1.12864199 \times 10^{-2}$	1.1286062×10^{-3}	2.6×10^{-9}
	0.02	$1.12948871 \times 10^{-2}$	1.1294338×10^{-3}	1.04×10^{-8}
0.5	0.03	$1.13027259 \times 10^{-2}$	1.1302079×10^{-3}	2.34×10^{-8}
	0.04	$1.13101617 \times 10^{-2}$	1.1309479×10^{-3}	4.17×10^{-8}
	0.05	$1.13172993 \times 10^{-2}$	1.1316648×10^{-3}	6.51×10^{-8}
	0.01	$1.25630004 \times 10^{-2}$	1.256261×10^{-3}	3.8×10^{-9}
	0.02	$1.25724281 \times 10^{-2}$	1.2571849×10^{-3}	1.52×10^{-8}
0.7	0.03	1.2581158×10^{-2}	1.2580512×10^{-3}	3.42×10^{-8}
	0.04	1.2589441×10^{-2}	1.258881×10^{-3}	6.07×10^{-8}
	0.05	$1.25973939 \times 10^{-2}$	1.259687×10^{-3}	9.48×10^{-8}

Let, the functions $\omega(\xi,\eta)$ and $\vartheta(\xi,\eta)$ have infinite series solutions as:

$$\omega(\xi,\eta) = \sum_{\tau=0}^{\infty} \omega_\tau(\xi,\eta), \qquad \vartheta(\xi,\eta) = \sum_{\tau=0}^{\infty} \vartheta_\tau(\xi,\eta), \tag{20}$$

Furthermore, equation (19) can be rewritten as:

$$\begin{aligned} \sum_{\tau=0}^{\infty} \omega_\tau(\xi,\eta) &= \omega(\xi,0) + S^{-1}\left[\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right) S\left[\sum_{\tau=0}^{\infty} \frac{\partial \vartheta_\tau}{\partial \xi} - \sum_{\tau=0}^{\infty} \omega_\tau - \sum_{\tau=0}^{\infty} \vartheta_\tau\right]\right], \\ \sum_{\tau=0}^{\infty} \vartheta_\tau(\xi,\eta) &= \vartheta(\xi,0) + S^{-1}\left[\left(1-\zeta+\zeta\left(\tfrac{\rho}{\upsilon}\right)\right) S\left[\sum_{\tau=0}^{\infty} \frac{\partial \vartheta_\tau}{\partial \xi} - \sum_{\tau=0}^{\infty} \omega_\tau - \sum_{\tau=0}^{\infty} \vartheta_\tau\right]\right]. \end{aligned} \tag{21}$$

Finally, We have recurrence relations as:
$\omega_0(\xi,\eta) = \sinh(\xi)$,
$\vartheta_0(\xi,\eta) = \cosh(\xi)$,
$\omega_1(\xi,\eta) = -\cosh(\xi)(1-\zeta+\eta\zeta)$,
$\vartheta_1(\xi,\eta) = -\sinh(\xi)(1-\zeta+\eta\zeta)$,
$\omega_2(\xi,\eta) = \sinh(\xi)\left(1-2\zeta+\zeta^2+2\eta\zeta(1-\zeta)+\frac{\zeta^2\eta^2}{2}\right)$,
$\vartheta_2(\xi,\eta) = \cosh(\xi)\left(1-2\zeta+\zeta^2+2\eta\zeta(1-\zeta)+\frac{\zeta^2\eta^2}{2}\right)$,
similarly, we obtain next terms in the same manner. Hence, the approximate solution of (16) is given as

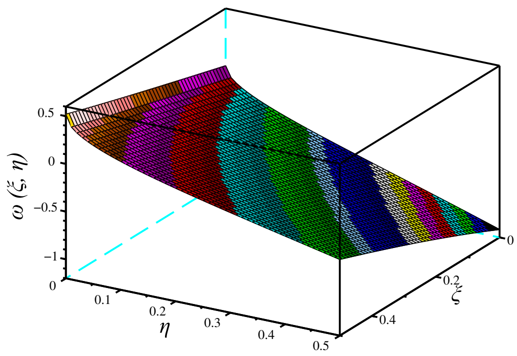
$$\begin{aligned} \omega(\xi,\eta) &= \sinh(\xi) - \cosh(\xi)(1-\zeta+\eta\zeta) \\ &\quad + \sinh(\xi)\left(1-2\zeta+\zeta^2+2\eta\zeta(1-\zeta)+\frac{\zeta^2\eta^2}{2}\right) - \ldots.., \\ \vartheta(\xi,\eta) &= \cosh(\xi) - \sinh(\xi)(1-\zeta+\eta\zeta) \\ &\quad + \cosh(\xi)\left(1-2\zeta+\zeta^2+2\eta\zeta(1-\zeta)+\frac{\zeta^2\eta^2}{2}\right) - \ldots.., \end{aligned} \tag{22}$$

specifically, when $\zeta = 1$, the solution of (16) is

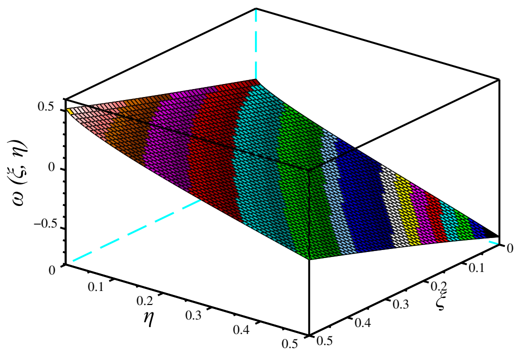
$$\begin{aligned} \omega(\xi,\eta) &= \sinh((\xi-\eta)), \\ \vartheta(\xi,\eta) &= \cosh((\xi-\eta)). \end{aligned} \tag{23}$$

Application 2 Consider the system of non-linear FPDEs. as:

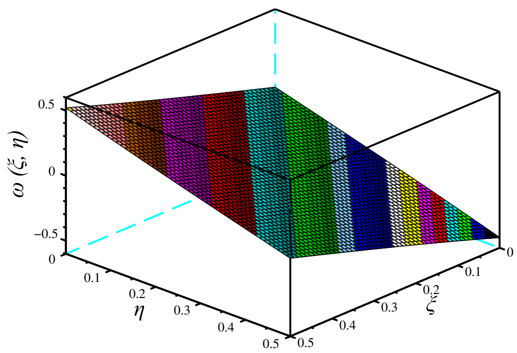
$$\begin{aligned} {}^{CF}D_\eta^\zeta \omega(\xi,\eta) &= \frac{\partial^2\omega}{\partial\xi^2} + 2\omega\frac{\partial\omega}{\partial\xi} - \omega\frac{\partial\vartheta}{\partial\xi} - \vartheta\frac{\partial\omega}{\partial\xi}, \\ {}^{CF}D_\eta^\zeta \vartheta(\xi,\eta) &= \frac{\partial^2\vartheta}{\partial\xi^2} + 2\vartheta\frac{\partial\vartheta}{\partial\xi} - \omega\frac{\partial\vartheta}{\partial\xi} - \vartheta\frac{\partial\omega}{\partial\xi}, \quad 0<\zeta\leq 1, \end{aligned} \tag{24}$$



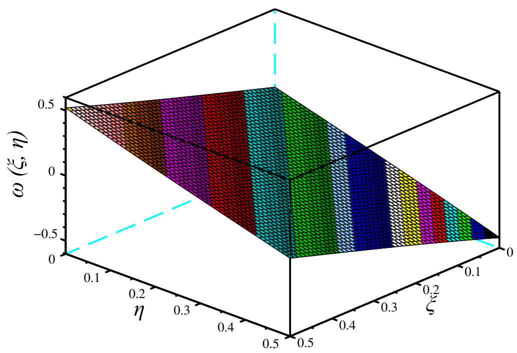
(a) $\zeta = 0.50$



(b) $\zeta = 0.75$



(c) $\zeta = 1$



(d) Exact Sol.

Fig. 1: $3-D$ nature of approximate solutions of $\omega(\xi,\eta)$ with distinct values of ζ Vs. exact solution for application 1.

subject to initial conditions

$$\omega(\xi,0) = \sin(\xi), \qquad \vartheta(\xi,0) = \sin(\xi). \tag{25}$$

By applying the Shehu transform (ST) to both sides of equation (24) and simplifying, we obtain:

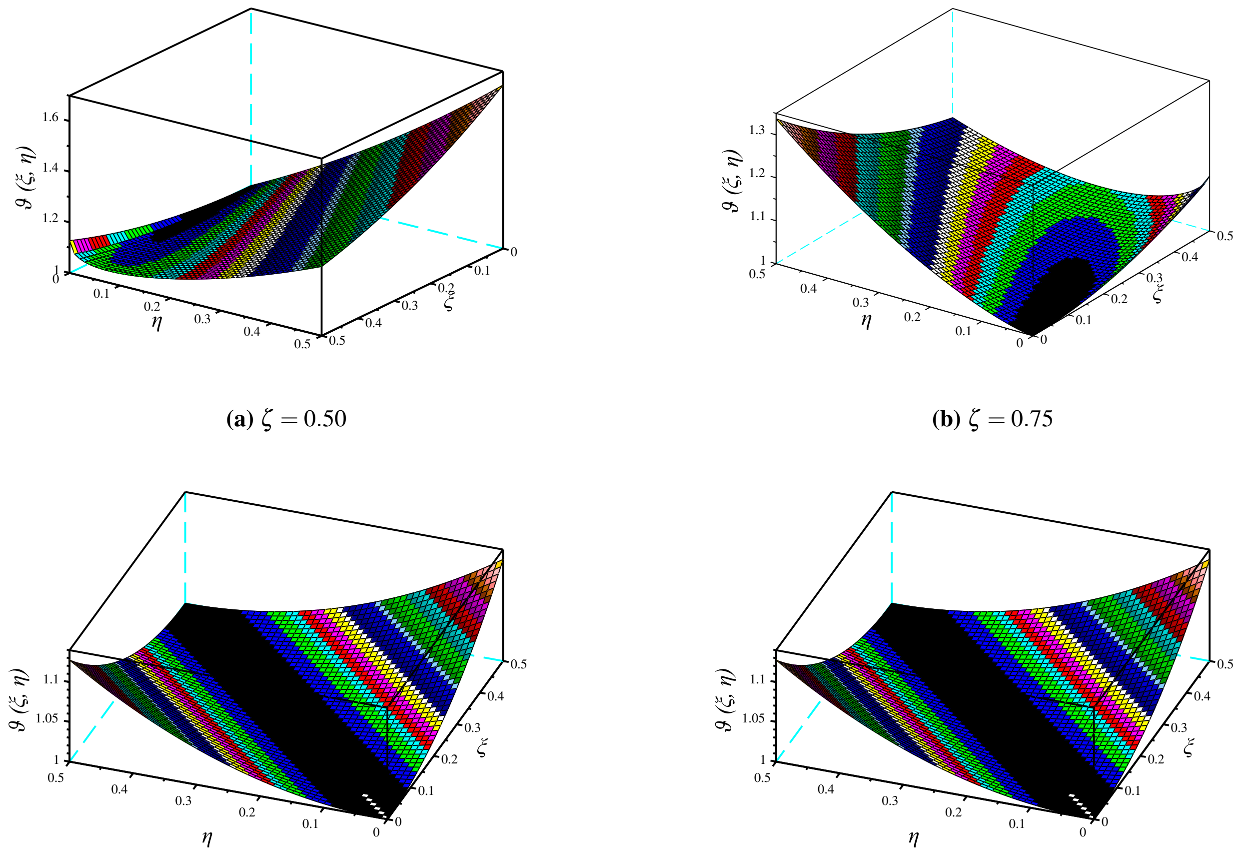
$$\begin{aligned} S[\omega(\xi,\eta)] &= \left(\frac{\rho}{\upsilon}\right)\omega(\xi,0) + \left(1-\zeta+\zeta\left(\frac{\rho}{\upsilon}\right)\right) S\left[\frac{\partial^2\omega}{\partial\xi^2} + 2\omega\frac{\partial\omega}{\partial\xi} - \omega\frac{\partial\vartheta}{\partial\xi} - \vartheta\frac{\partial\omega}{\partial\xi}\right], \\ S[\vartheta(\xi,\eta)] &= \left(\frac{\rho}{\upsilon}\right)\vartheta(\xi,0) + \left(1-\zeta+\zeta\left(\frac{\rho}{\upsilon}\right)\right) S\left[\frac{\partial^2\vartheta}{\partial\xi^2} + 2\vartheta\frac{\partial\vartheta}{\partial\xi} - \omega\frac{\partial\vartheta}{\partial\xi} - \vartheta\frac{\partial\omega}{\partial\xi}\right]. \end{aligned} \tag{26}$$

Applying the inverse ST to both sides of equation (26), we obtain:

$$\begin{aligned} \omega(\xi,\eta) &= \omega(\xi,0) + S^{-1}\left[\left(1-\zeta+\zeta\left(\frac{\rho}{\upsilon}\right)\right) S\left[\frac{\partial^2\omega}{\partial\xi^2} + 2\omega\frac{\partial\omega}{\partial\xi} - \omega\frac{\partial\vartheta}{\partial\xi} - \vartheta\frac{\partial\omega}{\partial\xi}\right]\right], \\ \vartheta(\xi,\eta) &= \vartheta(\xi,0) + S^{-1}\left[\left(1-\zeta+\zeta\left(\frac{\rho}{\upsilon}\right)\right) S\left[\frac{\partial^2\vartheta}{\partial\xi^2} + 2\vartheta\frac{\partial\vartheta}{\partial\xi} - \omega\frac{\partial\vartheta}{\partial\xi} - \vartheta\frac{\partial\omega}{\partial\xi}\right]\right]. \end{aligned} \tag{27}$$

Let, the functions $\omega(\xi,\eta)$ and $\vartheta(\xi,\eta)$ have infinite series solutions as:

$$\omega(\xi,\eta) = \sum_{\tau=0}^{\infty}\omega_\tau(\xi,\eta), \qquad \vartheta(\xi,\eta) = \sum_{\tau=0}^{\infty}\vartheta_\tau(\xi,\eta), \tag{28}$$



(a) $\zeta = 0.50$ **(b)** $\zeta = 0.75$

(c) $\zeta = 1$ **(d)** Exact Sol.

Fig. 2: $3-D$ nature of approximate solutions of $\vartheta(\xi,\eta)$ with distinct values of ζ Vs. exact solution for application 1.

and the nonlinear terms are decomposed using the Adomian polynomials as :

$$\begin{aligned} 2\omega\frac{\partial\omega}{\partial\xi} &= \sum_{\tau=0}^{\infty} A_\tau, & \omega\frac{\partial\vartheta}{\partial\xi} &= \sum_{\tau=0}^{\infty} B_\tau, \\ \vartheta\frac{\partial\omega}{\partial\xi} &= \sum_{\tau=0}^{\infty} C_\tau, & 2\vartheta\frac{\partial\vartheta}{\partial\xi} &= \sum_{\tau=0}^{\infty} D_\tau, \end{aligned} \tag{29}$$

calculating, we get

$A_0 = 2\omega_0\frac{\partial\omega_0}{\partial\xi}$,

$A_1 = 2\left(\omega_0\frac{\partial\omega_1}{\partial\xi} + \omega_1\frac{\partial\omega_0}{\partial\xi}\right)$,

$A_2 = 2\left(\omega_0\frac{\partial\omega_2}{\partial\xi} + \omega_1\frac{\partial\omega_1}{\partial\xi} + \omega_2\frac{\partial\omega_1}{\partial\xi}\right)$,

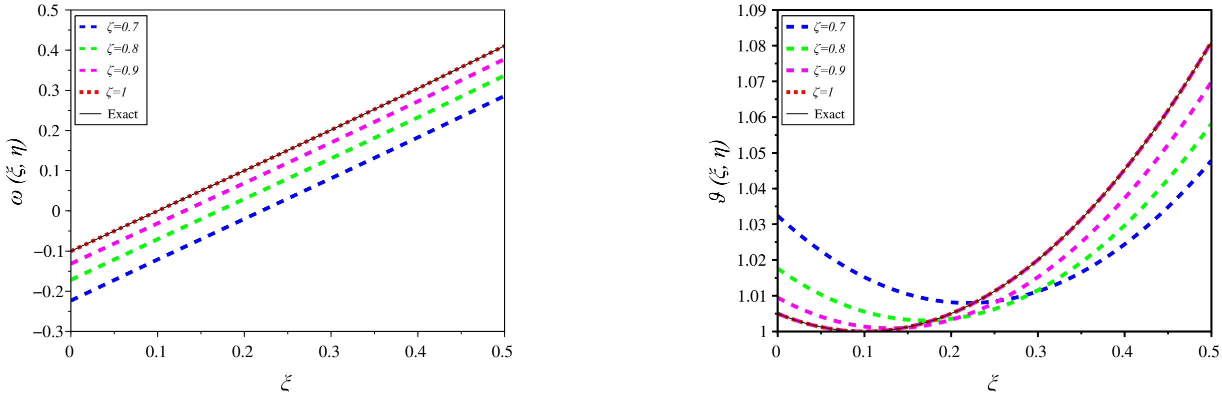
$A_3 = 2\left(\omega_0\frac{\partial\omega_3}{\partial\xi} + \omega_1\frac{\partial\omega_2}{\partial\xi} + \omega_2\frac{\partial\omega_1}{\partial\xi} + \omega_3\frac{\partial\omega_0}{\partial\xi}\right)$,

$B_0 = \omega_0\frac{\partial\vartheta_0}{\partial\xi}$,

$B_1 = \omega_0\frac{\partial\vartheta_1}{\partial\xi} + \omega_1\frac{\partial\vartheta_0}{\partial\xi}$,

$B_2 = \omega_0\frac{\partial\vartheta_2}{\partial\xi} + \omega_1\frac{\partial\vartheta_1}{\partial\xi} + \omega_2\frac{\partial\vartheta_0}{\partial\xi}$,

$B_3 = \omega_0\frac{\partial\vartheta_3}{\partial\xi} + \omega_1\frac{\partial\vartheta_2}{\partial\xi} + \omega_2\frac{\partial\vartheta_1}{\partial\xi} + \omega_3\frac{\partial\vartheta_0}{\partial\xi}$,



(a) **(b)**

Fig. 3: $2-D$ nature of approximate solutions of (16) with distinct values of ζ Vs. exact solution.

Table-II: The absolute error between the exact and approximate solution of ϑ_{STDM} at $\xi = 0.3, 0.5, 0.7$, and $0.01 \leq \eta \leq 0.05$ for application 1.

ξ	η	$\lvert\omega_{Exact} - \omega_{appr}\rvert$ $\zeta = 0.6$	$\lvert\omega_{Exact} - \omega_{appr}\rvert$ $\zeta = 0.8$	$\lvert\omega_{Exact} - \omega_{appr}\rvert$ $\zeta = 1$
	0.01	3.047951×10^{-3}	3.047890×10^{-4}	5.0×10^{-9}
	0.02	3.050252×10^{-3}	3.050270×10^{-4}	2.1×10^{-8}
0.3	0.03	3.052391×10^{-3}	3.052580×10^{-4}	4.7×10^{-8}
	0.04	3.054431×10^{-3}	3.054900×10^{-4}	8.4×10^{-8}
	0.05	3.056399×10^{-3}	3.057250×10^{-4}	1.31×10^{-7}
	0.01	5.215653×10^{-3}	5.215530×10^{-4}	6.0×10^{-9}
	0.02	5.219579×10^{-3}	5.219490×10^{-4}	2.3×10^{-8}
0.5	0.03	5.223223×10^{-3}	5.223280×10^{-4}	5.1×10^{-8}
	0.04	5.226691×10^{-3}	5.227010×10^{-4}	9.0×10^{-8}
	0.05	5.230029×10^{-3}	5.230720×10^{-4}	1.41×10^{-7}
	0.01	7.592677×10^{-3}	7.592480×10^{-4}	6.0×10^{-9}
	0.02	7.598386×10^{-3}	7.598180×10^{-4}	2.5×10^{-8}
0.7	0.03	7.603682×10^{-3}	7.603610×10^{-4}	5.6×10^{-8}
	0.04	7.608716×10^{-3}	7.608910×10^{-4}	1.0×10^{-7}
	0.05	5.230029×10^{-3}	7.614140×10^{-4}	1.57×10^{-7}

$$
\begin{aligned}
C_0 &= \vartheta_0 \frac{\partial \omega_0}{\partial \xi}, \\
C_1 &= \vartheta_0 \frac{\partial \omega_1}{\partial \xi} + \vartheta_1 \frac{\partial \omega_0}{\partial \xi}, \\
C_2 &= \vartheta_0 \frac{\partial \omega_2}{\partial \xi} + \vartheta_1 \frac{\partial \omega_1}{\partial \xi} + \vartheta_2 \frac{\partial \omega_0}{\partial \xi}, \\
C_3 &= \vartheta_0 \frac{\partial \omega_3}{\partial \xi} + \vartheta_1 \frac{\partial \omega_2}{\partial \xi} + \vartheta_2 \frac{\partial \omega_1}{\partial \xi} + \vartheta_3 \frac{\partial \omega_0}{\partial \xi}, \\
D_0 &= 2\vartheta_0 \frac{\partial \vartheta_0}{\partial \xi}, \\
D_1 &= 2\left(\vartheta_0 \frac{\partial \vartheta_1}{\partial \xi} + \vartheta_1 \frac{\partial \vartheta_0}{\partial \xi}\right), \\
D_2 &= 2\left(\vartheta_0 \frac{\partial \vartheta_2}{\partial \xi} + \vartheta_1 \frac{\partial \vartheta_1}{\partial \xi} + \vartheta_2 \frac{\partial \vartheta_1}{\partial \xi}\right), \\
D_3 &= 2\left(\vartheta_0 \frac{\partial \vartheta_3}{\partial \xi} + \vartheta_1 \frac{\partial \vartheta_2}{\partial \xi} + \vartheta_2 \frac{\partial \vartheta_1}{\partial \xi} + \vartheta_3 \frac{\partial \vartheta_0}{\partial \xi}\right),
\end{aligned}
$$

Furthermore, equation (27) can be rewritten as:

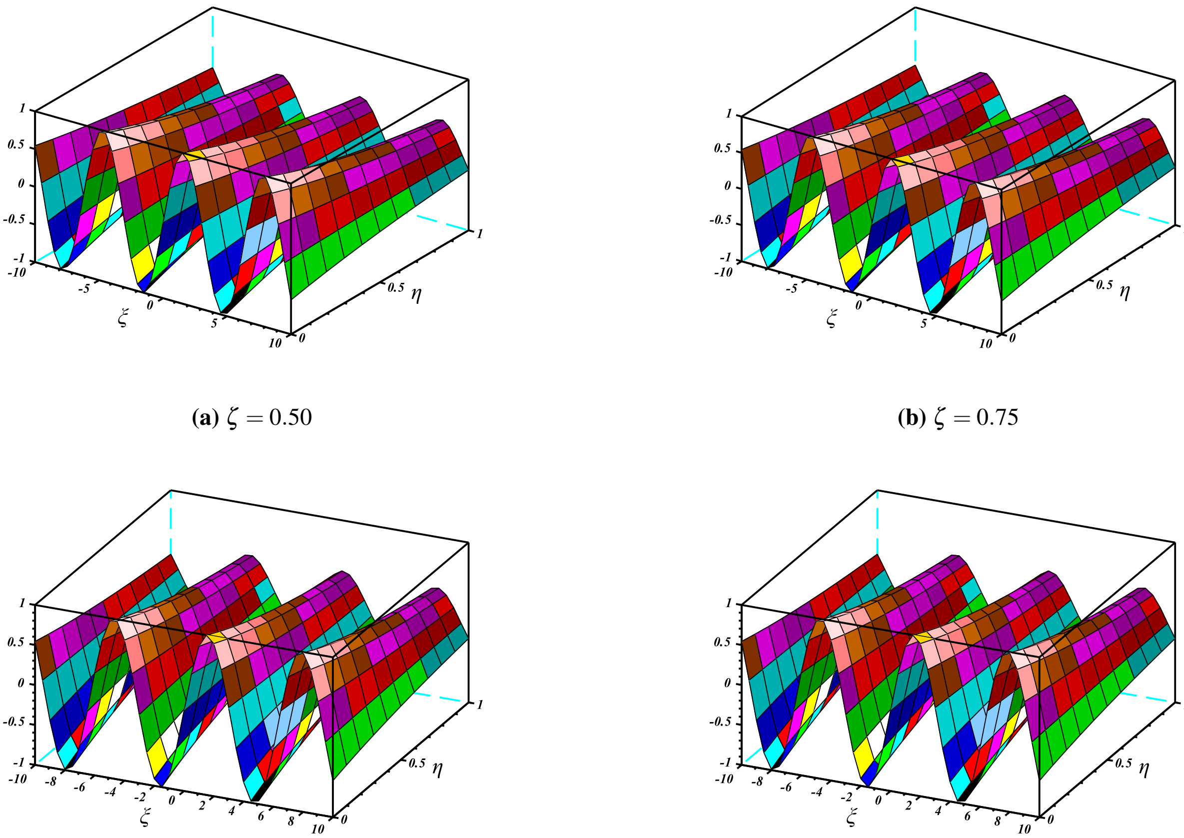
$$
\begin{aligned}
\sum_{\tau=0}^{\infty} \omega_\tau(\xi,\eta) &= \omega(\xi,0) + S^{-1}\left[\left(1-\zeta+\zeta\left(\frac{\rho}{\upsilon}\right)\right) S\left[\sum_{\tau=0}^{\infty} \frac{\partial^2 \omega_\tau}{\partial \xi^2} + \sum_{\tau=0}^{\infty} A_\tau - \sum_{\tau=0}^{\infty} B_\tau - \sum_{\tau=0}^{\infty} C_\tau\right]\right], \\
\sum_{\tau=0}^{\infty} \vartheta_\tau(\xi,\eta) &= \vartheta(\xi,0) + S^{-1}\left[\left(1-\zeta+\zeta\left(\frac{\rho}{\upsilon}\right)\right) S\left[\sum_{\tau=0}^{\infty} \frac{\partial^2 \vartheta_\tau}{\partial \xi^2} + \sum_{\tau=0}^{\infty} D_\tau - \sum_{\tau=0}^{\infty} B_\tau - \sum_{\tau=0}^{\infty} C_\tau\right]\right].
\end{aligned} \tag{30}
$$

Finally, We have recurrence relations as:
$\omega_0(\xi,\eta)=\sin(\xi)$,
$\vartheta_0(\xi,\eta)=\sin(\xi)$,
$\omega_1(\xi,\eta)=-\sin(\xi)(1-\zeta+\eta\zeta)$,
$\vartheta_1(\xi,\eta)=-\sin(\xi)(1-\zeta+\eta\zeta)$,
$\omega_2(\xi,\eta)=\sin(\xi)\left(1-2\zeta+\zeta^2+2\eta\zeta(1-\zeta)+\frac{\zeta^2\eta^2}{2}\right)$,
$\vartheta_2(\xi,\eta)=\sin(\xi)\left(1-2\zeta+\zeta^2+2\eta\zeta(1-\zeta)+\frac{\zeta^2\eta^2}{2}\right)$,
similarly, we obtain next terms in the same manner. Hence, the approximate solution of (16) is given as

$$\begin{aligned}\omega(\xi,\eta)&=\sin(\xi)-\sin(\xi)(1-\zeta+\eta\zeta)+\sin(\xi)\left(1-2\zeta+\zeta^2+2\eta\zeta(1-\zeta)+\frac{\zeta^2\eta^2}{2}\right)-..,\\ \vartheta(\xi,\eta)&=\sin(\xi)-\sin(\xi)(1-\zeta+\eta\zeta)+\sin(\xi)\left(1-2\zeta+\zeta^2+2\eta\zeta(1-\zeta)+\frac{\zeta^2\eta^2}{2}\right)-.,\end{aligned} \tag{31}$$

specifically, when $\zeta=1$, the solution of (24) is

$$\begin{aligned}\omega(\xi,\eta)&=\sin(\xi)e^{-n},\\ \vartheta(\xi,\eta)&=\sin(\xi)e^{-n}.\end{aligned} \tag{32}$$



(a) $\zeta=0.50$ **(b)** $\zeta=0.75$

(c) $\zeta=1$ **(d)** Exact Sol.

Fig. 4: $3-D$ nature of approximate solutions of both $\omega(\xi,\eta)$ and $\vartheta(\xi,\eta)$ with distinct values of ζ Vs. exact solution for application 2.

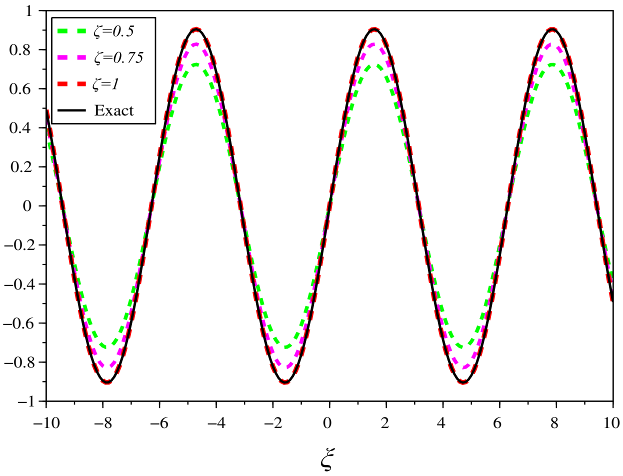


Fig. 5: $2-D$ nature of approximate solutions of both $\omega(\xi,\eta)$ and $\vartheta(\xi,\eta)$ with distinct values of ζ Vs. exact solution for application 2.

Table III: The error between the exact and fourth order approximation solution of ω_{LADM} and ϑ_{LADM} at $\eta = 0.01, 0.05, 0.1$, and $-10 \leq \xi \leq 10$ for application 2.

ξ	η	*Exact*	*Approximate* $\zeta = 0.9$	*Approximate* $\zeta = 1$	$\lvert\omega_{Exact} - \omega_{appr}\rvert$ $\zeta = 1$
-10		0.53860801	0.53860801	0.53513757	$4.52526905 \times 10^{-13}$
-5		0.94938282	0.94938282	0.94326562	$7.97695243 \times 10^{-13}$
0	0.01	0	0	0	0
5		−0.94938282	−0.94938282	−0.94326562	$7.97695243 \times 10^{-13}$
10		−0.53860801	−0.53860801	−0.53513757	$4.52526905 \times 10^{-13}$
-10		0.51748889	0.51748889	0.50729786	$1.40499934 \times 10^{-9}$
-5		0.91215699	0.91215699	0.89419367	$2.47653631 \times 10^{-9}$
0	0.05	0	0	0	0
5		−0.91215699	−0.91215699	−0.89419367	$2.47653631 \times 10^{-9}$
10		−0.51748889	−0.51748889	−0.50729786	$1.40499934 \times 10^{-9}$
-10		0.49225066	0.4922507	0.477697	$4.45901684 \times 10^{-8}$
-5		0.86767056	0.86767064	0.84201742	$7.85973081 \times 10^{-8}$
0	0.1	0	0	0	0
5		−0.86767056	−0.86767064	−0.84201742	$7.85973081 \times 10^{-8}$
10		−0.49225066	−0.4922507	−0.477697	$4.45901684 \times 10^{-8}$

6 Results and Discussions

Tables 1-II present the simulation results in terms of absolute errors at $\zeta = 0.6, 0.8, 1$ for different values of ξ and η, while Table III provides approximate results at $\zeta = 0.9, 1$ and absolute error at $\zeta = 1$ for equations (24) and (16), respectively. Figures 1 and 2 illustrate the 3D plots of the approximate and exact solutions for application 1. Figure 3 further explores the 2D nature of the approximate solution for different values of ζ and the exact solution of equation (16).

Additionally, Figure 4 depicts the 3D plots of the approximate and exact solutions for application 2, while figure 5 presents the 2D representation of the approximate solution for different values of ζ and the exact solution of application 2. The graphical results indicate that the approximate solutions closely align with the exact solutions at $\zeta = 1$.

7 Conclusion

In this paper, we successfully applied a hybrid approach, STDM, to analyze systems of partial differential equations (PDEs) with a non-singular kernel. The uniqueness and existence of the proposed method are established using the fixed-point theorem. To demonstrate the accuracy and reliability of the approach, we solved two applications of PDE systems. The results indicate that accurate and reliable approximations can be obtained with only a few terms. Furthermore, the findings confirm that the proposed technique is effective for the numerical and graphical analysis of both linear and nonlinear PDE systems.

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