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A Novel Limit of Bipolar K-Q Group Acting on ϑ-Subgroup, Normal Subgroup, and Homomorphism of ϑ-Fuzzy Subgroup

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Abstract: In this paper, we introduce and define the concept of a Bipolar K-Q Group acting on a ϑ-fuzzy subset, along with its associated algebraic properties. The study begins by examining the fundamental notion of a Bipolar K-Q Group acting on ϑ-fuzzy subgroups, shedding light on their structural and theoretical attributes. We then extend the discussion to Bipolar K-Q Groups acting on ϑ-fuzzy cosets, providing a detailed analysis of their algebraic properties and significance. These concepts are instrumental in exploring the interplay between group theory and fuzzy logic. Furthermore, we present a new notation: the Bipolar K-Q Group acting on a ϑ-fuzzy normal subgroup. This notion is further investigated in the context of quotient groups, particularly with respect to homomorphisms. We demonstrate various group-theoretical properties, including normality, compatibility, and structural relationships, to highlight the robustness of this framework. The paper provides a comprehensive foundation for understanding the algebraic behavior of these new group structures, offering insights into their broader theoretical and practical applications. By combining the principles of fuzzy logic and algebraic groups, this work advances the mathematical study of uncertainty and creates opportunities for future research in related areas of Mathematics.

Keywords: Fuzzy Sets, Fuzzy Subgroup, Bipolar K-Q Group Acting on ϑ-fuzzy subset, Bipolar K-Q Group Acting on ϑ-fuzzy subgroup, Bipolar K-Q Group Acting on ϑ-fuzzy normal subgroup, Bipolar K-Q Group Acting on ϑ-fuzzy coset.

1 Introduction

Let ${\xi_T}^K{}_0*$ be an Bipolar acting defined as
Ajmal [1] described the new notation of Homomorphism of groups, Correspondence theorem and fuzzy quotient groups in 1994. In 2005, Fuzzy group theory developed by Mordeson [2]. Fuzzy normal subgroups and fuzzy cosets introduced by Mukherjee [3] in 1984. Gupta [4] initiated by the new concept of Theory of T-norms and fuzzy inference methods in 1984. In Das [5], developed the concept of Fuzzy groups and level subgroups in 1981. Khare S S[6] described the concept of Fuzzy Homomorphism and Algebraic Structures in 1993. In 1979, Fuzzy groups redefined introduced by Sherwood H [7]. Rosenfeld A [8] first introduced by Fuzzy Groups in 1971. Abdul Salam [9] described the new notation of A new Construction of a group acting on fuzzy algebraic

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structure in 2016. In 1982, Fuzzy Sets and Possibility theory initiated by Yager R R [10]. Tarnauceanu [11] described the new concept of Classifying fuzzy normal subgroups of finite groups in 2015. In 1965, Fuzzy Sets first introduced by Zadeh [12]. In Nagarajan [13], developed by the concept of a new structure and construction of Q-fuzzy groups in 2009.

2 Preliminaries

Definition 1
Let X be a non-empty set. A fuzzy subset μ of the set X is a mapping μ: X $\to$ [0,1].
Definition 2
Let G be any group. A mapping μ: G $\to$ [0,1] is a fuzzy group if

1.μ(xy) $\geq$ minμ(x),μ(y)
2.$\mu(x^{-1})$=μ(x), for all x, y $\in$ G

Definition 3
Let Q and G be a set and a group respectively. A mapping μ: G $\times$ Q $\to$ [0,1] is called Q-fuzzy set in G. For any Q-fuzzy set μ in G and t $\in$ [0,1] we define the set
U(μ; t) = x *in* G / μ(x,q) $\geq$ t, q $\in$ Q
Which is called an upper cut of "μ" and can be used to characterize μ.
Definition 4 Let G and Q be any two nonempty sets and κ *in* [0,1] and A be a Q-FSb of a set G. The fuzzy set A^κ of G is called the κ-Q-FSb of G is defined by:
$A^\kappa(\theta$,q) = (A(θ,q),κ), $\forall$ θ $\in$ G and q *in* Q.

3 A Novel Limit of Bipolar Group Acting on Subgroup and Normal subgroup

Definition (3.1)
Let
${\xi_T}^K{}_0*$ be an Bipolar acting defined as ${\xi_T}^{K-}{}_0*$: ([0,1] $\times$ [0,1]) $\times$ Q) $\to$ [0,1]
and ${\xi_T}^{K+}{}_0*$: ([0,1] $\times$ [0,1]) $\times$ Q) $\to$ [0,1] by ${\xi_T}^{K-}{}_0 * ((a_1,a_2),q)$=$\{{\xi_T}^-{}_0 * ((1-a_1),q),k) \wedge {\xi_T}^-{}_0 * (((1-a_2),q),k)\}$
and ${\xi_T}^{K+}{}_0 * ((a_1,a_2),q)$=$\{{\xi_T}^+{}_0 * ((1-a_1),q),k) \vee {\xi_T}^+{}_0 * ((1-a_2),q),k)\}$
Infect ${\xi_T}^K{}_0*$ admits the properties given below:

1.**(a)** ${\xi_T}^{K-}{}_0*((a_1,a_2),q) = {\xi_T}^{K-}{}_0*((a_2,a_1),q)$
(b) ${\xi_T}^{K+}{}_0*((a_1,a_2),q) = {\xi_T}^{K+}{}_0*((a_2,a_1),q)$
2.**(a)** ${\xi_T}^{K-}{}_0*((a_1,1),q) = {\xi_T}^{K-}{}_0*((1,a_1),q) = {\xi_T}^{K-}{}_0*(((a_1,1),q),k) = 0$
(b) ${\xi_T}^{K+}{}_0*((a_1,1),q) = {\xi_T}^{K+}{}_0*((1,a_1),q) = {\xi_T}^{K-}{}_0*(((a_1,1),q),k) = 0$

Definition(3.2)
Let X and Q be any two non-empty set and $k,\vartheta \in [0,1]$.
Let ${\xi_0}^{\vartheta-}{}_k* : (X\times Q) \to [0,1]$
and ${\xi_0}^{\vartheta+}{}_k* : (X\times Q) \to [0,1]$ is a Bipolar K-Q Group.

1.${\xi_0}^{\vartheta-}{}_k*(a_1*s,q) = \{1-{\xi_0}^-*((1,a_1)*s,q),k)\wedge((1-\vartheta),q),k)\}, \forall a_1 \in [0,1], q \in Q and k \in [0,1]$
2.${\xi_0}^{\vartheta+}{}_k*(a_1*s,q) = \{1-{\xi_0}^+*((1,a_1)*s,q),k)\wedge((1-\vartheta),q),k)\}$

Example(3.2.1)
Let X = Set of Young People define K-Q Group Acting ϑ-fuzzy subset of X

$$ {\xi_0}^{\vartheta-}{}_k*(a_1*s,q) = \begin{cases} 1, & if a_1 < 26 \\ \frac{45-a_1}{15}, & if 26 \leq a_1 \leq 45 \\ 0, & if a_1 > 45 \end{cases} $$

$$ *(a*s,q) = \begin{cases} 1, & \text{if } a < 30 \\ \frac{55-a}{15}, & \text{if } 30 a 55 \\ 0, & \text{if } a > 55 \end{cases} $$

Take ϑ = 0.8 and $a_1 = 30$, we have ${\xi_0}^{\vartheta}{}_k*(a_1*s,q) = 0.8$

Definition 1(3.3). *Let X and Q be any two non-empty set, G be a group and ,k [0,1]. Let : (G × Q) → [0,1] and : (G × Q) → [0,1] be a Bipolar K-Q Group Acting -fuzzy subset of G and operating on S. Then * is called Bipolar K-Q Group Acting -fuzzy subgroup of G if its following conditions:*

[(i)]
(a)((aa) * s),q) {*((a * s,q),k) *((a * s,q),k)}*
(b)((aa) * s),q) {*((a * s,q),k) *((a * s,q),k)}*
*(ii)(a) *((a¹ * s),q) *((a * s,q),k)*
*(b) *((a¹ * s),q) *((a * s,q),k)*

[Continue with Example 3.3.1]

Example 1(3.3.1). Let X and Q be any two non-empty set and * be a Bipolar K-Q Group Acting fuzzy subset of G. Let G = {1,a,a,aa} defined as:

[(a)]
1.*(1 * s,q) = 0.8 and *(a * s,q) = *(a * s,q) = *(aa * s,q) = 0.7
2.*(1 * s,q) = 0.9 and *(a * s,q) = *(a * s,q) = *(aa * s,q) = 0.8

Define Bipolar K-Q Group Acting -fuzzy subset * of G, for = 0.6 as follows:
[(a)]*(1 * s,q) = 0.2 and *(a * s,q) = *(a * s,q) = *(aa * s,q) = 0.1 *(1 * s,q) = 0.3 and *(a * s,q) = *(a * s,q) = *(aa * s,q) = 0.2

Clearly, * is Bipolar K-Q Group Acting -fuzzy subgroup of G.

2.**Definition 2(3.2).** *Let X and Q be any two non-empty set and k, [0,1]. Let : (X × Q) → [0,1] and : (X × Q) → [0,1] is a Bipolar K-Q Group Acting subset of * of X denotes the Bipolar K-Q Group Acting -fuzzy subset of X and defined as follows that:*

[(a)]

1.(a * s,q) = {1 - *((1,a) * s,q),k) ((1 -),q),k)}, a [0,1], q Q and k [0,1]*
2.(a * s,q) = {1 - *((1,a) * s,q),k) ((1 -),q),k)}*

Example 2(3.2.1). Let X = {Set of Young People} define K-Q Group Acting -fuzzy subset of X

$$*(a*s,q) = \begin{cases} 1, & \text{if } a < 26 \\ \frac{45-a}{15}, & \text{if } 26a45 \\ 0, & \text{if } a > 45 \end{cases}$$

$$*(a*s,q) = \begin{cases} 1, & \text{if } a < 30 \\ \frac{55-a}{15}, & \text{if } 30a55 \\ 0, & \text{if } a > 55 \end{cases}$$

Take = 0.8 and a = 30, we have *(a * s,q) = 0.8

Definition 3(3.3). *Let X and Q be any two non-empty set, G be a group and ,k [0,1]. Let : (G × Q) → [0,1] and : (G × Q) → [0,1] be a Bipolar K-Q Group Acting -fuzzy subset of G and operating on S. Then * is called Bipolar K-Q Group Acting -fuzzy subgroup of G if its following conditions:*
[(i)]
(a)((aa) * s),q) {*((a * s,q),k) *((a * s,q),k)}*
(b)((aa) * s),q) {*((a * s,q),k) *((a * s,q),k)}*
*(ii)(a) *((a¹ * s),q) *((a * s,q),k)*
*(b) *((a¹ * s),q) *((a * s,q),k)*

[Continue with Example 3.3.1]

Example 3(3.3.1). Let X and Q be any two non-empty set and * be a Bipolar K-Q Group Acting fuzzy subset of G. Let G = {1,a,a,aa} defined as:
[(a)]
1.*(1 * s,q) = 0.8 and *(a * s,q) = *(a * s,q) = *(aa * s,q) = 0.7
2.*(1 * s,q) = 0.9 and *(a * s,q) = *(a * s,q) = *(aa * s,q) = 0.8

Define Bipolar K-Q Group Acting -fuzzy subset * of G, for = 0.6 as follows:
[(a)]*(1 * s,q) = 0.2 and *(a * s,q) = *(a * s,q) = *(aa * s,q) = 0.1 *(1 * s,q) = 0.3 and *(a * s,q) = *(a * s,q) = *(aa * s,q) = 0.2

Clearly, * is Bipolar K-Q Group Acting -fuzzy subgroup of G.

1.**Proposition 1(3.4).** *Let * be Bipolar K-Q Group Acting -fuzzy subgroup of (S,) in acting on S. Then the following statements holds:*
[(i)]
(a)(a * s,q) *((e * s,q),k)*
(b)(a * s,q) *((e * s,q),k)*
*(ii)(a) *(aa¹ * s,q) = *((e * s,q),k) = *(a * s,q) = *(a * s,q)*
*(b) *(aa¹ * s,q) = *((e * s,q),k) = *(a * s,q) = *(a * s,q), a,a [0,1], q Q and k [0,1]*

Proof.[(i)]
Since *(a * s,q) = *((e * s,q),k) and also

$$*(a*s,q) = \{*((a*s,q),k)*((a*s,q),k)\}$$

= {*((a * s,q),k) *((a * s,q),k)} = *((a * s,q),k) *(a * s,q) = *((e * s,q),k), a [0,1], q Q and k [0,1]
(b)Since *(a * s,q) = *((e * s,q),k) and also

$$*(a*s,q) = \{*((a*s,q),k)*((a*s,q),k)\}$$

= {*((a * s,q),k) *((a * s,q),k)} = *((a * s,q),k) *(a * s,q) = *((e * s,q),k), a [0,1], q Q and k [0,1]
(ii)(a)

$$*(aa*s,q) = *((aa*s,q)(aa*s,q))$$

{*((aa¹ * s,q),k) *((a¹a * s,q),k)} {*((e * s,q),k) *((a * s,q),k)} *(a * s,q) *((a * s,q),k)

(1)

Similarly,

$$*(a*s,q) = *(aaa*s,q)$$

{*((aa¹ * s,q),k) *((a * s,q),k)} {*((e * s,q),k) *((a * s,q),k)} *(a * s,q) *((a * s,q),k)

(2)

From (1) and (2) *(a * s,q) = *((a * s,q),k)

Note 1(3.5).
[(i)]
(a)*(a * (st),q) {*((a * s,q),k) *((a * t,q),k)}
(b)*(a * (st),q) {*((a * s,q),k) *((a * t,q),k)} [Continue with remaining notes...]

Theorem 1(3.6). *Every Bipolar K-Q Group Acting fuzzy subgroup is also Bipolar K-Q Group Acting -fuzzy subgroup of G.*

*Proof.*Let * be a Bipolar K-Q Group Acting -fuzzy subgroup of G. Now,
[(a)]*((aa) * s,q) {(1 - *((aa * s,q),k)) ((1 -),q),k)} {1 - (*((a * s,q),k) *((a * s,q),k)) ((1 -),q),k)} = {(1 - *((a * s,q),k)) ((1 -),q),k)) (1 - * ((a * s,q),k) ((1 -),q),k))} = {*((a * s,q),k) *((a * s,q),k)} *((aa) * s,q) {*((a * s,q),k) *((a * s,q),k)}

1.**Theorem 2(3.7).**
***Let** A and B be any two non-empty sets, G be a group, and let μ and ν be Bipolar Group Acting fuzzy subgroups of G with operators on G such that*

$$\mu,\nu : G \to [0,1]$$

and let $\omega : G \to [0,1]$ be defined as

$$\omega(x) = \min\{\mu(x),\nu(x)\}, \quad \forall x \in G.$$

Then ω is also a Bipolar Group Acting fuzzy subgroup of G.

Proof:
(a)Let $x,y \in G$. Then,

$$\omega(xy) = \min\{\mu(xy), \nu(xy)\}.$$

Since μ and ν are Bipolar Group Acting fuzzy subgroups, we have:

$$\mu(xy) \geq \min\{\mu(x), \mu(y)\}, \quad \nu(xy) \geq \min\{\nu(x), \nu(y)\}.$$

Thus,

$$\omega(xy) \geq \min\{\omega(x), \omega(y)\}.$$

(b)Let $x \in G$. Then,

$$\omega(x^{-1}) = \min\{\mu(x^{-1}), \nu(x^{-1})\}.$$

Since μ and ν are Bipolar Group Acting fuzzy subgroups, we have:

$$\mu(x^{-1}) = \mu(x), \quad \nu(x^{-1}) = \nu(x).$$

Therefore,

$$\omega(x^{-1}) = \min\{\omega(x)\}.$$

Hence, ω is also a Bipolar Group Acting fuzzy subgroup of G.
[3.8] ***Let*** *A and B be any two non-empty sets, G be a group, and let μ and ν be Group Acting fuzzy subgroups of G with operators on G. Then their sum*

$$(\zeta_0^- k*(*))^{\vartheta-}(a_1'a_2' * s,q) \geq \{(\zeta_0^- k*(*))^{\vartheta-}(a_1' * s,q) \wedge (\zeta_0^- k*(*))^{\vartheta-}(a_2' * s,q)\}$$

is also a Bipolar Group Acting fuzzy subgroup of G.

Proof: Let $x,y \in G$. Since μ and ν are Group Acting fuzzy subgroups of G, we have

$$\mu(xy) \geq \min\{\mu(x), \mu(y)\}, \quad \nu(xy) \geq \min\{\nu(x), \nu(y)\}.$$

Thus,

$$(\zeta_0^- k*(*))^{\vartheta-}(a_1'a_2' * s,q) \geq \{(\zeta_0^- k*(*))^{\vartheta-}(a_1' * s,q) \wedge (\zeta_0^- k*(*))^{\vartheta-}(a_2' * s,q)\}$$

(i)For all $x \in G$,

$$(\mu+\nu)(e) = \max\{\mu(e), \nu(e)\} = 1.$$

(ii)(a) For all $x \in G$,

$$(\mu+\nu)(x^{-1}) = \max\{\mu(x^{-1}), \nu(x^{-1})\}.$$

Since $\mu(x^{-1}) = \mu(x)$ and $\nu(x^{-1}) = \nu(x)$, it follows that

$$(\mu+\nu)(x^{-1}) = (\mu+\nu)(x).$$

(b) Since the conditions for a fuzzy subgroup hold, we conclude that

$\mu+\nu$ *is a Bipolar Group Acting fuzzy subgroup of G.*

Corollary 1(3.9). *The intersection of any finite number of Bipolar K-Q Group Acting $\vartheta - fuzzysubgroupofGisalsoBipolarK - QGroupActing\vartheta - fuzzysubgroupofG$.*

Proposition 2(3.10). *Let X and Q be any two non-empty set, G be a group and $\vartheta, k \in [0,1]$and $*$ $beaBipolarK - QGroupActing\vartheta - fuzzysubsetsuchthat$ $*(a_1 * s,q) = *((a_1^{-1} * s,q), \forall a_1 \in G, q \in Q$ and $s \in S$ and $*$*

$$(a_1 * s,q) = *((a_1^{-1} * s,q), \forall a_1 \in G, q \in Q \text{ and } s \in S$$

Let $(n,q) \geq (m,q), where k, \vartheta \in [0,1] and (m,q) = \sup\{*(a_1 * s,q), \forall a_1 \in G, q \in Q$ and $s \in S\}$ and

$$(m,q) = \inf\{*(a_1 * s,q), \forall a_1 \in G, q \in Q \text{ and } s \in S\}$$

Then $*$ is also Bipolar K-Q Group Acting $\vartheta - fuzzysubgroupofG$.
Proof

(a)We have (n,q) $\geq$ (m,q) $\Rightarrow$ $(\vartheta,q) \geq sup\{{\xi_0}^{\vartheta-}{}_k * (a_1 * s,q), \forall a_1 \in G, q \in Q$ and $s \in S\}$
So, we have ${\xi_0}^{\vartheta-}{}_k * (a_1 * s,q) = \{(1-{\xi_0}^- * ((a_1 * s,q),k)) \wedge ((1-\vartheta),q),k)\}$
Also,
${\xi_0}^{\vartheta-}{}_k * ((a_1a_2) * s,q) \geq {\xi_0}^{\vartheta-} * ((a_1 * s,q),k) \wedge {\xi_0}^{\vartheta-} * ((a_2 * s,q),k)\}$
(b)Similarly for positive membership function:
We have (n,q)$\geq$(m,q)
$\Rightarrow (\vartheta,q) \leq inf\{{\xi_0}^{\vartheta+}{}_k * (a_1 * s,q), \forall a_1 \in G, q \in Q, s \in S\}$
${\xi_0}^{\vartheta+}{}_k * (a_1 * s,q) = \{(1-{\xi_0}^+ * ((a_1 * s,q),k)) \vee ((1-\vartheta),q),k)\}$
Also,
${\xi_0}^{\vartheta+}{}_k * (a_1a_2)*,q) \leq \{{\xi_0}^{\vartheta+} * ((a_1 * s,q),k) \vee {\xi_0}^{\vartheta+}{}_k * ((a_2 * s,q),k)\}$
Therefore, ${\xi_0}^{\vartheta+}{}_k*$ is Bipolar $K-Q$ Group Acting ϑ fuzzy subgroup of G.

Definition(3.13)
Let ${\xi_0}^{\vartheta-}{}_k*$ be Bipolar $K-Q$ Group Acting ϑ fuzzy subgroup of G. Let Bipolar $K-Q$ Group Acting ϑ fuzzy left coset $(a_1 * s,q){\xi_0}^{\vartheta-}{}_k*$ and $(a_1 * s,q){\xi_0}^{\vartheta+}{}_k*$ be defined as follows:
$(a_1 * s,q){\xi_0}^{\vartheta-}{}_k * (x * s,q) = \{1-{\xi_0}^-{}_k * ((a_1^{-1}x * s,q),k) \wedge ((1-\vartheta),q),k)\}$
and
$(a_1 * s,q){\xi_0}^{\vartheta+}{}_k * (x * s,q) = \{1-{\xi_0}^+{}_k * ((a_1^{-1}x * s,q),k) \vee ((1-\vartheta),q),k)\}$, $\forall a_1, x \in G, q \in Q$.

Definition (3.14) The group $G/{\xi_0}^{\vartheta}{}_k*$ of Bipolar $K-Q$ Group Acting ϑ fuzzy coset of a Bipolar K-Q Group Acting ϑ fuzzy normal subgroup of G is called Bipolar $K-Q$ Group Acting ϑ Quotient group of G by ${\xi_0}^{\vartheta}{}_k*$.

theorem (3.15) Every fuzzy normal subgroup is also Bipolar $K-Q$ Group Acting ϑ fuzzy normal subgroup of G.

proof

(a)Let ${\xi_0}^{\vartheta}{}_{k*}$ is Bipolar $K-Q$ Group Acting ϑ fuzzy normal subgroup of G
$\Rightarrow (a_1*s,q){\xi_0}^{\vartheta-}{}_{k*} = {\xi_0}^{\vartheta-}{}_{k}*(a_1*s,q)$
Then for any $a_1, x \in G$ and $q \in Q$
We have,
${\xi_0}^{\vartheta-}{}_{k}*(a_1^{-1}x*s,q) = {\xi_0}^{\vartheta-}{}_{k}*(xa_1^{-1}*s,q)$
So,
${\xi_0}^{\vartheta-}{}_{k}*(a_1^{-1}x*s,q) = \{1-{\xi_0}^{-}*((a_1^{-1}x*s,q),k) \wedge ((1-\vartheta),q),k)\}$
$= \{1-{\xi_0}^{-}*((xa_1^{-1}*s,q),k) \wedge ((1-\vartheta),q),k)\}$
$\Rightarrow (a_1*s,q){\xi_0}^{\vartheta}{}_{k}*(x*s,q) = {\xi_0}^{\vartheta}{}_{k}*(a_1*s,q)(x*s,q), \forall a_1, x \in G$ and $q \in Q$

(b)Let ${\xi_0}^{\vartheta}{}_{k*}$ is Bipolar $K-Q$ Group Acting ϑ fuzzy normal subgroup of G
$\Rightarrow (a1*s,q){\xi_0}^{\vartheta+}{}_{k*} = {\xi_0}^{\vartheta+}{}_{k}*(a_1*s,q)$
Then for any $a_1, x \in G$ and $q \in Q$
We have,
${\xi_0}^{\vartheta+}{}_{k}*(a_1^{-1}x*s,q) = {\xi_0}^{\vartheta+}{}_{k}*(xa_1^{-1}*s,q)$
So,
${\xi_0}^{\vartheta+}{}_{k}*(a_1^{-1}x*s,q) = \{1-{\xi_0}^{+}*((a_1^{-1}x*s,q),k) \vee ((1-\vartheta),q),k)\}$
$= \{1-{\xi_0}^{+}*((xa_1^{-1}*s,q),k) \vee ((1-\vartheta),q),k)\}$
$\Rightarrow (a_1*s,q){\xi_0}^{\vartheta+}{}_{k}*(xasts,q) = {\xi_0}^{\vartheta+}{}_{k}*(a_1*s,q)(x*s,q), \forall a_1, x \in G, q \in Q$

Hence ${\xi_0}^{\vartheta}{}_{k*}$ is also Bipolar $K-Q$ Group Acting ϑ fuzzy normal subgroup of G.

Proposition (3.16)
Let ${\xi_0}^{\vartheta}{}_{k*}$ be Bipolar $K-Q$ Group Acting ϑ fuzzy normal subgroup of G. Then:
${\xi_0}^{\vartheta-}{}_{k}*((b^{-1}a_1b*s,q)) = {\xi_0}^{\vartheta-}{}_{k}*(a_1*s,q)$ or ${\xi_0}^{\vartheta-}{}_{k}*(a_1a_2*s,q) = {\xi_0}^{\vartheta-}{}_{k}*(a_2a_1*s,q)$
and
${\xi_0}^{\vartheta+}{}_{k}*((b^{-1}a_1b*s,q)) = {\xi_0}^{\vartheta+}{}_{k}*(a_1*s,q)$ or ${\xi_0}^{\vartheta+}{}_{k}*(a_1a_2*s,q) = {\xi_0}^{\vartheta+}{}_{k}*(a_2a_1*s,q)$
$\forall a_1, a_2 \in G$ and $q \in Q$.

proof

(a)Let ${\xi_0}^{\vartheta}{}_{k*}$ be Bipolar $K-Q$ Group Acting ϑ fuzzy normal subgroup of G.
Then, we have
$(a_1*s,q){\xi_0}^{\vartheta-}{}_{k*} = {\xi_0}^{\vartheta-}{}_{k}*(a_1*s,q)$. $\forall a_1 \in G$ and $q \in Q$
Consequently, we have
${\xi_0}^{\vartheta-}{}_{k}*((a_2a_1)^{-1}*s,q) = {\xi_0}^{\vartheta-}{}_{k}*((a_1a_2)^{-1}*s,q)$
Hence,${\xi_0}^{\vartheta-}{}_{k}*(a_1a_2*s,q) = {\xi_0}^{\vartheta-}{}_{k}*(a_2a_1*s,q)$.

(b)Let ${\xi_0}^{\vartheta}{}_{k*}$ be Bipolar $K-Q$ Group Acting ϑ fuzzy normal subgroup of G.
Then, we have
$(a_1*s,q){\xi_0}^{\vartheta+}{}_{k*} = {\xi_0}^{\vartheta+}{}_{k}*(a_1*s,q)$. $\forall a_1 \in G$ and $q \in Q$.
Consequently, we have
${\xi_0}^{\vartheta+}{}_{k}*((a_2a_1)^{-1}*s,q) = {\xi_0}^{\vartheta+}{}_{k}*((a_1a_2)^{-1}*s,q)$
Hence,
${\xi_0}^{\vartheta+}{}_{k}*(a_1a_2*s,q) = {\xi_0}^{\vartheta+}{}_{k}*(a_2a_1*s,q)$.

Theorem (3.17)
Let ${\xi_0}^{\vartheta}{}_{k*}$ be Bipolar $K-Q$ Group.
(a)**(i)**${\xi_0}^{\vartheta-}{}_{k}*(a_2a_1*s,q) = {\xi_0}^{\vartheta-}{}_{k}*(a_1a_2*s,q)$
(ii)${\xi_0}^{\vartheta+}{}_{k}*(a_2a_1*s,q) = {\xi_0}^{\vartheta+}{}_{k}*(a_1a_2*s,q)$
(b)**i**${\xi_0}^{\vartheta-}{}_{k}*(a_1a_2a_1^{-1}*s,q) = {\xi_0}^{\vartheta-}{}_{k}*(a_2*s,q)$
(ii)${\xi_0}^{\vartheta+}{}_{k}*(a_1a_2a_1^{-1}*s,q) = {\xi_0}^{\vartheta+}{}_{k}*(a_2*s,q)$
(c)**(i)**${\xi_0}^{\vartheta-}{}_{k}*(a_1a_2a_1^{-1}*s,q) \geq {\xi_0}^{\vartheta-}{}_{k}*(a_2*s,q)$
(ii) ${\xi_0}^{\vartheta+}{}_{k}*(a_1a_2a_1^{-1}*s,q) \leq {\xi_0}^{\vartheta+}{}_{k}*(a_2*s,q)$
(d)**(i)** ${\xi_0}^{\vartheta-}{}_{k}*(a_1a_2a_1^{-1}*s,q) \geq {\xi_0}^{\vartheta-}{}_{k}*(a_2*s,q)$
(ii) ${\xi_0}^{\vartheta+}{}_{k}*(a_1a_2a_1^{-1}*s,q) \leq {\xi_0}^{\vartheta+}{}_{k}*(a_2*s,q)$
$\forall\ a_1, a_2 \in G$ and $q \in Q$.

Theorem (3.18)
Let ${\xi_0}^{\vartheta}{}_{k*}$ be Bipolar $K-Q$ Group Acting ϑ fuzzy subgroup of G. Let $(n,q) \geq (m,q)$, where $\vartheta \in [0,1]$ and
(i) $(m,q) = Sup\{{\xi_0}^{\vartheta-}{}_{k}*(a_1*s,q), a_1 \in G$ and $q \in Q\}$ and
(ii) $(m,q) = inf\{{\xi_0}^{\vartheta-}{}_{k}*(a_1*s,q), a_1 \in G$ and $q \in Q\}$
then
${\xi_0}^{\vartheta}{}_{k*}$ is also Bipolar $K-Q$ Group Acting ϑ fuzzy normal subgroup of G.

Proof

(a)Let $(n,q) \geq (m,q)$
So, $(n,q) \geq Sup\{{\xi_0}^{\vartheta-}{}_{k}*(a_1*s,q), \forall\ a_1 \in G, q \in Q$ and $s \in S\}$
$\Rightarrow (n,q) \geq {\xi_0}^{\vartheta-}{}_{k}*(a_1*s,q), \forall\ a_1 \in G, q \in Q$ and $s \in S$
We have,
${\xi_0}^{\vartheta-}{}_{k}*(a_1^{-1}a_2*s,q) = {\xi_0}^{\vartheta-}{}_{k}*(xa_1^{-1}*s,q)$
Thus,
${\xi_0}^{\vartheta-}{}_{k}*(a_1a_2*s,q) \geq \{{\xi_0}^{\vartheta-}{}_{k}*((a_1*s,q),k) \wedge {\xi_0}^{\vartheta-}{}_{k}*((a_2*s,q),k)\}$

(b)Let $(n,q) \geq (m,q)$
So, $(n,q) \leq inf\{{\xi_0}^{\vartheta+}{}_{k}*(a_1*s,q), \forall\ a_1 \in G, q \in Q$ and $s \in S\}$
$\Rightarrow (n,q) \geq {\xi_0}^{\vartheta+}{}_{k}*(a_1*s,q), \forall\ a_1 \in G, q \in Q$ and $s \in S$
Thus,
${\xi_0}^{\vartheta+}{}_{k}*(a_1a_2*s,q) \leq \{{\xi_0}^{\vartheta+}{}_{k}*((a_1*s,q),k) \vee {\xi_0}^{\vartheta+}{}_{k}*((a_2*s,q),k)\}$

Hence ${\xi_0}^{\vartheta}{}_{k*}$ is also Bipolar $K-Q$ Group Acting ϑ fuzzy normal subgroup of G.

Theorem 3(3.19). *Let * be Bipolar K-Q Group Acting* $\vartheta - fuzzynormalsubgroupofG.Thenthesetdefineas$ *[(i)]*

$G_* = \{(a_1 * s, q) \in G : *(a_1 * s, q) = *(e * s, q)\}GG_* = \{(a_1 * s, q) \in G : *(a_1 * s, q) = *(e * s, q)\}G$

Proof(Proof of Theorem 3.19).
[(i)]
1.Let $G_* benonemptysetande \in G$
Let $a_1, a_2 \in G_*$

$$*(a_1 a_2^{-1} * s, q)\{*((a_1 * s, q), k) \wedge *((a_2^{-1} * s, q), k)\}$$

Since, * is Bipolar K-Q Group Acting $\vartheta - fuzzynormalsubgroupofG$
$*(a_1 * s, q) = *(a_2^{-1} a_1 a_2 * s, q)$
Therefore, we have

$$*(a_1 * s, q) * (a_2 * s, q)$$

Similarly, we have

$$*(a_2 * s, q) * (a_1 * s, q)$$

Hence $*(a_2 * s, q) = *(a_1 * s, q)$
Now we prove, $G_* G$.
Let $a_1 \in G_* and a_2 \in G$.
We have,

$$*(a_2^{-1} a_1 a_2 * s, q) = *(a_1 * s, q) = *(e * s, q)$$

Consequently, we have $a_2^{-1}, a_1, a_2 \in G_*$
2.Let $G_* benonemptysetande \in G$
Let $a_1, a_2 \in G_*$

$$*(a_1 a_2^{-1} * s, q)\{*((a_1 * s, q), k) \vee *((a_2^{-1} * s, q), k)\}$$

Since, * is Bipolar K-Q Group Acting $\vartheta - fuzzynormalsubgroupofG$
$*(a_1 * s, q) = *(a_2^{-1} a_1 a_2 * s, q)$
Therefore, we have

$$*(a_1 * s, q) * (a_2 * s, q)$$

Similarly, we have

$$*(a_2 * s, q) * (a_1 * s, q)$$

Hence $*(a_2 * s, q) = *(a_1 * s, q)$
Now we prove, $G_* G$.
Let $a_1 \in G_* and a_2 \in G$.
We have,

$$*(a_2^{-1} a_1 a_2 * s, q) = *(a_1 * s, q) = *(e * s, q)$$

Consequently, we have $a_2^{-1}, a_1, a_2 \in G_*$

4 Homomorphism of Bipolar Group Acting on Subgroup and Normal subgroup

Theorem 4(4.1). *Let $\zeta_0^- k^* : (G \times Q) \to G'$ and $\zeta_0^+ k^* : (G \times Q) \to G'$ be bijective homomorphisms of a group G into a group G'. If * is a Bipolar K-Q Group Acting on a ϑ-fuzzy subgroup of G, then the homomorphic image $\zeta_{0k}^*(^*)$ is also a Bipolar K-Q Group Acting on a ϑ-fuzzy subgroup of G'.*

*Proof.*Given that * be Bipolar K-Q Group Acting $\vartheta - fuzzysubgroupofG$.
[(i)]Let $a_1', a_2' \in G', q \in Q$ and $K \in [0,1] We have, a_1, a_2 \in G, K \in [0,1]$ and $q \in Q such that \zeta_0^- k * (a_1 * s, q) = (a_1' * s, q)$ and $\zeta_0^- k * (a_2 * s, q) = (a_2' * s, q)$ Now, equation equation

$$\begin{aligned}(\zeta_0^- k * (*))^{\vartheta -}(a_1' a_2' * s, q) \geq \\ \{(\zeta_0^- k * (*))^{\vartheta -}(a_1' * s, q) \wedge \\ (\zeta_0^- k * (*))^{\vartheta -}(a_2' * s, q)\}\end{aligned}$$

$$= \{1 - \zeta_0^- * (*)((a_1 a_2 * s, q), k) \wedge ((1 - \vartheta), q), k)\}$$

$$= *(a_1 a_2 * s, q)$$

$\{*((a_1 * s, q), k) \wedge *((a_2 * s, q), k)\}$
$= \{(\zeta_0^- k * (*))^{\vartheta -}(a_1 * s, q) \wedge (\zeta_0^- k * (*))^{\vartheta -}(a_2 * s, q)\}$
Consequently,

$$\begin{aligned}(\zeta_0^- k * (*))^{\vartheta -}(a_1' a_2' * s, q) \geq \\ \{(\zeta_0^- k * (*))^{\vartheta -}(a_1' * s, q) \wedge \\ (\zeta_0^- k * (*))^{\vartheta -}(a_2' * s, q)\}\end{aligned}$$

Also,

$$\begin{aligned}(\zeta_0^- k * (*))^{\vartheta -}(a_1' a_2' * s, q) \geq \\ \{(\zeta_0^- k * (*))^{\vartheta -}(a_1' * s, q) \wedge \\ (\zeta_0^- k * (*))^{\vartheta -}(a_2' * s, q)\}\end{aligned}$$

$= *(a^{-1} * s, q) = *(a * s, q) = (\zeta_0^- k * (*))^{\vartheta -}(a' * s, q)$
1.The proof for positive membership function follows similarly.

Theorem 5(4.2). *Let $\zeta_0^- k^* : (G \times Q) \to G'$ and $\zeta_0^+ k^* : (G \times Q) \to G'$ be bijective homomorphisms of a group G into a group G'. If * is a Bipolar K-Q Group Acting on a ϑ-fuzzy normal subgroup of G, then the homomorphic image $\zeta_{0k}^*(^*)$ is also a Bipolar K-Q Group Acting on a ϑ-fuzzy normal subgroup of G'.*

*Proof.*Given that * is a Bipolar K-Q Group Acting on a ϑ-fuzzy normal subgroup of G.
Let $a'_1, a'_2 \in G'$, $q \in Q$, and $K \in [0,1]$. Then there exist unique elements $a_1, a_2 \in G$ such that:

$$\zeta_0^- k^*(a_1 * s, q) = (a'_1 * s, q), \quad \zeta_0^- k^*(a_2 * s, q) = (a'_2 * s, q).$$

Now, we compute:

$$\begin{aligned}
(\zeta_0^- k^*(^*))^{\vartheta -}(a'_1 a'_2 * s, q) = \\
\{1 - (\zeta_0^- k^*(^*)((a'_1 a'_2 * s, q), k)) \\
\wedge ((1-\vartheta), q), k)\} \\
= \{1 - \zeta_0^- k^*(^*)(\zeta_0^- k^*(a_1 * s, q) \\
\zeta_0^- k^*(a_2 * s, q)) \wedge ((1-\vartheta), q), k)\} \\
=^* (a_1 a_2 * s, q) \geq \\
\{^*((a_1 * s, q), k) \\
\wedge^* ((a_2 * s, q), k)\}
\end{aligned}$$

Similar proof follows for the positive membership function. Therefore, $\zeta_{0k}^*(^*)$ is a Bipolar K-Q Group Acting on a ϑ-fuzzy normal subgroup of G'.

Theorem 6(4.3). *Let $\zeta_0^- k^* : (G \times Q) \to G'$ and $\zeta_0^+ k^* : (G \times Q) \to G'$ be homomorphisms of a group G into a group G'. If * is a Bipolar K-Q Group Acting on a ϑ-fuzzy subgroup of G', then the preimage $\zeta_{0k}^{*\,-1}(^*)$ is also a Bipolar K-Q Group Acting on a ϑ-fuzzy subgroup of G.*

*Proof.*Given that * is a Bipolar K-Q Group Acting on a ϑ-fuzzy subgroup of G', we consider:
Let $a_1, a_2 \in G$, $K \in [0,1]$, and $q \in Q$. We have:

$$\begin{aligned}
(\zeta_0^- k^{*-1}(^*))^{\vartheta}(a_1 a_2 * s, q) &= \zeta_0^- k^{*-1}(^{\vartheta})(a_1 a_2 * s, q) \\
&=^{\vartheta} (\zeta_0^- k^*(a_1 a_2 * s, q)) \\
&=^{\vartheta} (\zeta_0^- k^*(a_1 * s, q)\zeta_0^- k^*(a_2 * s, q)) \\
&\geq \{^{\vartheta}(\zeta_0^- k^*(a_1 * s, q)) \wedge^{\vartheta} (\zeta_0^- k^*(a_2 * s, q))\}.
\end{aligned}$$

Thus, the closure property holds.
For inverses:

$$\begin{aligned}
(\zeta_0^- k^{*-1}(^*))^{\vartheta}(a^{-1} * s, q) &= \zeta_0^- k^{*-1}(^{\vartheta})(a^{-1} * s, q) \\
&=^{\vartheta} (\zeta_0^- k^*(a^{-1} * s, q)) \\
&=^{\vartheta} ((\zeta_0^- k^*(a * s, q))^{-1}) \\
&=^{\vartheta} (\zeta_0^- k^*(a * s, q)).
\end{aligned}$$

Thus, $(\zeta_0^- k^{*-1}(^*))^{\vartheta}(a^{-1} * s, q) = (\zeta_0^- k^{*-1}(^*))^{\vartheta}(a * s, q)$. Similar proof follows for the positive membership function.

Hence, $\zeta_{0k}^{*\,-1}(^*)$ is a Bipolar K-Q Group Acting on a ϑ-fuzzy subgroup of G.

Theorem 7(4.4). *Let $\zeta_0^- k* : (GQ)ßG' and \zeta_0^+ k* : (GQ)ßG' beahomomorphismofagroupGintoagroupG'.If * beBipolarK - QGroupActing\vartheta - fuzzynormalsubgroupofG'thenthepreimage\zeta_{0k} *^{-1} (*)isBipolarK - QGroupActing\vartheta - fuzzynormalsubgroupofG$.*

*Proof.*Let $*$ be Bipolar K-Q Group Acting $\vartheta - fuzzynormalsubgroupofG'$.
[(i)]Let $a_1, a_2 \in G, K \in [0,1]$ and $q \in QThus, *(a_1 a_2 * s, q) = *(\zeta_0^- k * (a_1 a_2 a_1^{-1} * s, q)) \{*((a_2^{-1} a_1 * s, q), k) \wedge *((a_2 * s, q), k)\} = *(a_2^{-1} a_1 a_2 * s, q) = (a_2 * s, q) * (a_3 * s, q) \quad Let a_1, a_2 \in G, K \in [0,1]$ and $q \in QHence, \zeta_{0k} *^{-1} (*)$ is Bipolar K-Q Group Acting ϑ-fuzzy normal subgroup of G.

1. **Theorem 8(4.5).** *Let $*$ be Bipolar K-Q Group Acting $\vartheta - fuzzynormalsubgroupofGanda_1, a_2 \in G, K \in [0,1]$ and $q \in Q.If$*
   [(i)]
*1.$(a_1 * s, q)* = (a_2 * s, q) *$ and $(a_2 * s, q)* = (y * s, q) * (a_1 * s, q)* = (a_2 * s, q) *$ and $(a_2 * s, q)* = (y * s, q)*$*
   then
   *[(i)]$(a_1 a_2 * s, q)* = (xy * s, q) *$ and $(a_1 a_2 * s, q)* = (xy * s, q)*$*

2. *Proof(Proof of Theorem 4.5).*
   [(i)]
   1.Suppose that $(a_1 * s, q)* = (x * s, q) * and (a_2 * s, q)* = (y * s, q) *$
   By theorem (3.20), we have $(a_1^{-1} a_2 * s, q) \in G_* and (a_2^{-1} y * s, q) \in G_*$
   Since, Bipolar K-Q Group Acting $\vartheta - fuzzynormalsubgroupofG$
   $*(a_1 * s, q) = *(a_2^{-1} a_1 a_2 * s, q)$
   Therefore, we have

$$*(a_1 * s, q) * (a_2 * s, q)$$

Similarly, we have

$$*(a_2 * s, q) * (a_1 * s, q)$$

Hence $*(a_1 a_2 * s, q) = *(xy * s, q)$
2.Similar proof follows for positive membership functions.

5 Conclusion

In the present research, we established the new notations of Bipolar K-Q Group Acting on ϑ-fuzzy subset, Bipolar K-Q Group Acting on ϑ-fuzzy subgroup, and addressed different results and algebraic characteristics. We investigated the effect of these algebraic characteristics on the concept of Bipolar K-Q Group Acting on ϑ-fuzzy cosets.

Furthermore, we introduced a new notation of Bipolar K-Q Group Acting on ϑ-fuzzy normal subgroup and quotient group with respect to group homomorphism. In future research, we aim to expand this notion to intuitionistic fuzzy sets, bipolar fuzzy sets, anti-fuzzy sets, and neutrosophic fuzzy sets, as well as explore their various algebraic aspects.

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