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Dynamics of Ultrashort Optical Pulses in Nonlinear Media Interplay

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Abstract: In this paper, we apply the extended Fan's sub equation method to analyze the optical solitons solutions of the coupled nonlinear Schrödinger-Poisson system. The interplay of self-phase modulation, cross-phase modulation, and dispersion governs the pulse dynamics in nonlinear media, where the propagation of ultrashort optical pulses can be best explained by the coupled nonlinear Schrödinger-Poisson system. An effective analytical tool for solving a variety of nonlinear partial differential equations is the extended Fan sub equation method, which is used in our paper. Explicit analytic optical solitons solutions can be determined by this method, which also introduces an interesting auxiliary function and simplifies the coupled nonlinear Schrödinger-Poisson system. We construct optical soliton solutions, including bright, dark, and singular types Â to illustrate the method Â efficacy. It is possible to better comprehend the underlying nonlinear phenomena in optical systems by using the derived Â optical Â soliton solutions, which are remarkably stable and robust. In short, this research provides a new approach for studying optical Â solitons solutions inside the coupled nonlinear Schrödinger-Poisson system framework and clarifies the complex interplay Â between dispersive and nonlinear effects that influence the behavior of optical pulses. The obtained optical Â solitons solutions Â have the potential to further our understanding of nonlinear optics and enable in the development of new, more useful photonic devices. The solutionsÂ presentedÂ here provide a solid basis for future research in the fields of nonlinear optics and plasma physics, and they make progress toward the current study of coupled nonlinear systems soliton dynamics.

Keywords: Coupled nonlinear Schrödinger-Poisson system, optical solitons, extended Fan's sub equation method, analytical solutions, soliton dynamics.

1 Introduction

The significance of studying optical soliton solutions in coupled nonlinear systems to a variety of domains, such as nonlinear optics and plasma physics, has attracted a lot of attention [21]. To represent the complex interaction between nonlinearity and external perturbations, the coupled nonlinear Schrödinger-Poisson system is used in this context. This system appears in a number of physical phenomena, including optical fiber communication systems and plasma waves in semiconductor devices [1–20]. The present article investigates the extended Fan sub-equation method for optical soliton solutions of the coupled nonlinear Schrödinger-Poisson system. The evolution of complex wave functions in the presence of both nonlinear interactions and an external potential is described by the coupled Schrödinger-Poisson system of partial differential equations. The system can

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be expressed mathematically as follows:

$$\begin{cases} -i\frac{\partial \psi}{\partial t} = -\Delta \psi + \phi(x)\psi, \\ -\Delta \phi = |\psi|^2. \end{cases} \quad (1)$$

Physical meaning of the Schrödinger-Poisson System: The complex-valued function $\psi(x,t)$ represents the wave function of a quantum particle or a field in a nonlinear medium. The equation describes the temporal evolution of the wave function, governed by its kinetic energy $-\Delta \psi$ and potential energy $\phi(x)\psi$, here Δ is a laplacian operator. The wave function density determines the potential $\phi(x)$ self-consistently, establishing a feedback loop between the behavior of the particles and their surroundings. The electrostatic potential produced by the charge density of the particle or field is represented by the real-valued function $\phi(x)$. The Poisson equation connects the electrostatic potential and the probability density of the wave function through the source term $|\psi|^2$. This equation describes how the charge distribution of the particle or field interacts with and changes the surrounding environment. The system models circumstances in which the particle or field has a large impact on the surroundings, leading to self-interaction and nonlinear behavior. The Schrödinger equation is fundamental for describing quantum systems, governing wave-like behavior and probability distributions. The Poisson equation, which relates charge density and electric potential, comes from electrostatics. There is a wealth of research on the coupled nonlinear Schrödinger-Poisson system in the context of several physical phenomena. The system appears in nonlinear optics when optical pulse propagation in fiber optic communication networks is studied [32–37].

A helpful method Â for finding exact solutions to nonlinear partial differential equations is the extended Â Fan's sub-equation method, which was introduced recently. Applying it to the coupled nonlinear Schrödinger-Poisson system provides a promising way to get solutions for optical soliton dynamics and comprehend how an external potential affects them. With this study, we want to contribute to the literature by precisely solving the coupled nonlinear Schrödinger-Poisson system with the extended Fan sub-equation method [38–47]. This investigation of soliton dynamics not only broadens our theoretical knowledge but also has applications in nonlinear optics and plasma physics for system design and optimization.

By combining the coupled Schrödinger Poisson system system and the extended Fan's sub equation method, we aim to:

uncover new optical soliton solutions, expand the known repertoire of optical soliton solutions for the coupled Schrödinger Poisson system system, gain deeper insights, reveal the influence of key system parameters on optical soliton dynamics, contribute to optical soliton applications, provide potential avenues for utilizing newfound optical soliton properties in advanced photonic devices.

2 Analysis of the Schrödinger Poisson system by extended Fan's sub equation method

In this section the focus is going to construct the optical solitons solutions to the following Schrodinger Poisson system in the form

$$\begin{cases} -i\frac{\partial \psi}{\partial t} = -\Delta \psi + \phi(x)\psi, \\ -\Delta \phi = |\psi|^2, \end{cases} \quad (2)$$

using the definition of Δ the system can be converted into single equation which is

$$-i\frac{\partial^3 \psi}{\partial x^2 \partial t} + \frac{\partial^4 \psi}{\partial x^4} - 2\phi^{'}(x)\frac{\partial \psi}{\partial x} - \phi(x)\frac{\partial^2 \psi}{\partial x^2} + |\psi|^2\psi = 0. \quad (3)$$

Making wave transformation

$$\psi(x,t) = U(\xi) \times e^{i(\kappa x - \omega t)}, \quad (4)$$

where $\psi(x,t)$ is the complex wave function, and v is the velocity of the wave, κ and ω are arbitrary constants. Substituting Eq. 4 into Eq.3, and separating the real and imaginary parts of the equation gives a pair of relations. Following is the constraint condition for imaginary part

$$v = \frac{2}{\kappa}\Big(2\kappa^2 - \omega - \phi(x)\Big), \quad (5)$$

and the real part is

$$U^{iv}(\xi) + [2v\kappa - 6\kappa^2 - \omega - \phi(x)]U^2(\xi) - [2\phi'(x)]U'(\xi) + [\kappa^4 + \omega\kappa^2 + \phi(x)\kappa^2]U(\xi) + U^3(\xi) = 0. \quad (6)$$

The homogeneous balance principle is used to find the value of the positive integer N, i.e, by balancing between the highest order derivatives and the nonlinear terms appearing in nonlinear ordinary differential equation. More precisely, if the degree of $U(\xi)$ is $\mathtt{deg}[U(\xi)] = N$, then the degree of the other terms will be expressed as follows:

$$\mathtt{deg}\left[\frac{d^qU(\xi)}{d\xi^q}\right] = N+q,$$

$$\mathtt{deg}\left[(U(\xi))^p\left(\frac{dqU(\xi)}{d\xi^q}\right)^s\right] = Np + s(N+q).$$

Applying balance principle to the Eq.6

$$\mathtt{deg}[U^{iv}] = N+4 = \mathtt{deg}[U^3] = 3N, \quad (7)$$

which leads to $N = 2$. Eq.6 has the following formal solution

$$U(\xi) = \beta_0 + \beta_1\eta(\xi) + \beta_2\eta^2(\xi), \quad (8)$$

where β_0, β_1 and β_2 are to be determined later and $\eta(\xi)$ satisfies the following general elliptic equation [38–41],

$$\eta'^2 = \left(\frac{d\eta(\zeta)}{d\zeta}\right)^2 = \mu_0 + \mu_1\eta(\zeta) + \mu_2\eta^2(\zeta) + \mu_3\eta^3(\zeta) + \mu_4\eta^4(\zeta). \qquad (9)$$

Substituting Eq.8 along with Eq.9 into Eq.6, and comparing the coefficients of like terms, we obtained system of algebraic equations. This system of algebraic equations is solved with the help of Maple then

$$\beta_0 = \frac{(80\mu_4\mu_2 - 15\mu_3^2)\sqrt{-30}}{120\mu_4}, \quad \beta_1 = \sqrt{-30}\mu_3, \quad \beta_2 = 2\sqrt{-30}\mu_4. \qquad (10)$$

Substituting Eq.10 along with Eq.8 into Eq.4, we get the following optical soliton solutions of Eq.2

Case I:

If $\mu_0 = r^2$, $\mu_1 = 2rp$, $\mu_2 = 2rq + p^2$, $\mu_3 = 2pq$, $\mu_4 = q^2$, then η is one of the 24 $\eta_1^l (l = 1,2,\cdots,24)$. For instance, if we take $l = 1,3,5,7,9,11$ then optical soliton solutions [38–41] for the Eq.2 are

Type I: When $p^2 - 4qr > 0$ and $pq \neq 0, (qr \neq 0)$ then

$$\psi_{1,1}(x,t) = \left\{\beta_0 + \beta_1\left(-\frac{1}{2q}\left(p + \sqrt{\Omega_1}\tanh\left(1/2\sqrt{\Omega_1}\xi\right)\right)\right) + \beta_2\left(-\frac{1}{2q}\left(p + \sqrt{\Omega_1}\tanh\left(1/2\sqrt{\Omega_1}\xi\right)\right)\right)^2\right\} \times e^{i\kappa x - \omega t}$$

where $\Omega_1 = p^2 - 4qr$.

$$\psi_{1,3}(x,t) = \left\{\beta_0 + \beta_1\left(-\frac{1}{2q}\left(p + \sqrt{\Omega_1}\left(\tanh\sqrt{\Omega_1}\xi \pm i\,\mathrm{sec}h\sqrt{\Omega_1}\right)\right)\right) + \beta_2\left(-\frac{1}{2q}\left(p + \sqrt{\Omega_1}\left(\tanh\sqrt{\Omega_1}\xi \pm i\,\mathrm{sec}h\sqrt{\Omega_1}\right)\right)\right)^2\right\} \times e^{i\kappa x - \omega t}$$

$$\psi_{1,5}(x,t) = \left[\beta_0 + \beta_1\left(-\frac{1}{4q}\left(2p + \sqrt{\Omega_1}\left(\tanh\left(\frac{\sqrt{\Omega_1}\xi}{4}\right) + \coth\left(\frac{\sqrt{\Omega_1}\xi}{4}\right)\right)\right)\right) + \beta_2\left(-\frac{1}{4q}\left(2p + \sqrt{\Omega_1}\left(\tanh\left(\frac{\sqrt{\Omega_1}\xi}{4}\right) + \coth\left(\frac{\sqrt{\Omega_1}\xi}{4}\right)\right)\right)\right)^2\right] \times e^{i(\kappa x - \omega t)},$$

$$\psi_{1,7}(x,t) = \left[\beta_0 + \beta_1\left(+\frac{1}{2q}\left(-p - \frac{\sqrt{(B^2-A^2)\Omega_1} + A\sqrt{\Omega_1}\sinh\sqrt{\Omega_1}\xi}{A\cosh\sqrt{\Omega_1}\xi + B}\right)\right) + \beta_2\left(\frac{1}{2q}\left(-p - \frac{\sqrt{(B^2-A^2)\Omega_1} + A\sqrt{\Omega_1}\sinh\sqrt{\Omega_1}\xi}{A\cosh\sqrt{\Omega_1}\xi + B}\right)\right)^2\right] \times e^{i(\kappa x - \omega t)},$$

$$\psi_{1,9}(x,t) = \left[\beta_0 + \beta_1\left(\frac{-2r\sinh\left(\frac{\sqrt{\Omega_1}\xi}{2}\right)}{p\sinh\left(\frac{\sqrt{\Omega_1}\xi}{2}\right) - \sqrt{\Omega_1}\cosh\left(\frac{\sqrt{\Omega_1}\xi}{2}\right)}\right) + \beta_2\left(\frac{-2r\sinh\left(\frac{\sqrt{\Omega_1}\xi}{2}\right)}{p\sinh\left(\frac{\sqrt{\Omega_1}\xi}{2}\right) - \sqrt{\Omega_1}\cosh\left(\frac{\sqrt{\Omega_1}\xi}{2}\right)}\right)^2\right] \times e^{i(\kappa x - \omega t)},$$

$$\psi_{1,11}(x,t) = \left\{\beta_0 + \beta_1\left(\frac{2r\sinh(\sqrt{\Omega_1}\xi)}{-p\sinh(\sqrt{\Omega_1}\xi) + \sqrt{\Omega_1}\cosh(\sqrt{\Omega_1}) \pm (\sqrt{\Omega_1})}\right) + \beta_2\left(\frac{2r\sinh(\sqrt{\Omega_1}\xi)}{-p\sinh(\sqrt{\Omega_1}\xi) + \sqrt{\Omega_1}\cosh(\sqrt{\Omega_1}) \pm (\sqrt{\Omega_1})}\right)^2\right\} \times e^{i(\kappa x - \omega t)},$$

where $\beta_0 = \frac{(80\mu_4\mu_2 - 15\mu_3^2)\sqrt{-30}}{120\mu_4}$, $\beta_1 = \sqrt{-30}\mu_3$, $\beta_2 = 2\sqrt{-30}\mu_4$.

Type II: When $4qr - p^2 < 0$ and $pq \neq 0, (qr \neq 0)$ then η one of the twelve that are left $eta_1^l (l = 13,14,\cdots,24)$. For instance, if we take $l = 13,15,16,18,20,24$ then

$$\psi_{1,13}(x,t) = \left\{\beta_0 + \beta_1\left(\frac{1}{2q}\left(-p + \sqrt{\Omega_2}\tan\left(\frac{\sqrt{\Omega_2}\xi}{2}\right)\right)\right) + \beta_2\left(\frac{1}{2q}\left(-p + \sqrt{\Omega_2}\tan\left(\frac{\sqrt{\Omega_2}\xi}{2}\right)\right)\right)^2\right\} \times e^{i(\kappa x - \omega t)},$$

where $\Omega_2 = 4qr - p^2$.

$$\psi_{1,15}(x,t) = \left\{\beta_0 + \beta_1\left(\frac{1}{2q}\left(p + \sqrt{\Omega_2}\left(\tan(\sqrt{\Omega_2}\xi) \pm \sec(\sqrt{\Omega_2})\right)\right)\right) + \beta_2\left(\frac{1}{2q}\left(p + \sqrt{\Omega_2}\left(\tan(\sqrt{\Omega_2}\xi) \pm \sec(\sqrt{\Omega_2})\right)\right)\right)^2\right\} \times e^{i(\kappa x - \omega t)},$$

$$\psi_{1,16}(x,t) = \left\{\beta_0 + \beta_1\left(-\frac{1}{2q}\left(-p + \sqrt{\Omega_2}\left(\cot(\sqrt{\Omega_2}\xi) \pm \csc(\sqrt{\Omega_2})\right)\right)\right) + \beta_2\left(-\frac{1}{2q}\left(-p + \sqrt{\Omega_2}\left(\cot(\sqrt{\Omega_2}\xi) \pm \csc(\sqrt{\Omega_2})\right)\right)\right)^2\right\} \times e^{i(\kappa x - \omega t)},$$

$$\psi_{1,18}(x,t)=\left\{\beta_0+\beta_1\left(\frac{1}{2q}\left(-p+\frac{\pm\sqrt{(A^2-B^2)(\Omega_2)}-A\sqrt{\Omega_2}\cos\sqrt{\Omega_2}\xi}{A\sin\sqrt{\Omega_2}\xi+B}\right)\right)+\beta_2\left(\frac{1}{2q}\left(-p+\frac{\pm\sqrt{(A^2-B^2)(\Omega_2)}-A\sqrt{\Omega_2}\cos\sqrt{\Omega_2}\xi}{A\sin\sqrt{\Omega_2}\xi+B}\right)\right)^2\right\}\times e^{i(\kappa x-\omega t)},$$

where A and B are non-zero real constants, that fulfill $A^2-B^2>0$.

$$\psi_{1,20}(x,t)=\left[\beta_0+\beta_1\left(\frac{2r\cos\left(\frac{\sqrt{\Omega_2}\xi}{2}\right)}{\sqrt{\Omega_2}\sin\left(\frac{\sqrt{\Omega_2}\xi}{2}\right)+p\cos\left(\frac{\sqrt{\Omega_2}\xi}{2}\right)}\right)+\beta_2\left(\frac{2r\cos\left(\frac{\sqrt{\Omega_2}\xi}{2}\right)}{\sqrt{\Omega_2}\sin\left(\frac{\sqrt{\Omega_2}\xi}{2}\right)+p\cos\left(\frac{\sqrt{\Omega_2}\xi}{2}\right)}\right)^2\right]\times e^{i(\kappa x-\omega t)},$$

$$\psi_{1,24}(x,t)=\left[\beta_0+\beta_1\left(\frac{4r\cos\left(\frac{\sqrt{\Omega_2}\xi}{4}\right)\sin\left(\frac{\sqrt{\Omega_2}\xi}{4}\right)}{-2p\cos\left(\frac{\sqrt{\Omega_2}\xi}{4}\right)\sin\left(\frac{\sqrt{\Omega_2}\xi}{4}\right)+2\sqrt{\Omega_2}cos^2\left(\frac{\sqrt{\Omega_2}\xi}{4}\right)-\sqrt{\Omega_2}}\right)\right.$$
$$\left.+\beta_2\left(\frac{4r\cos\left(\frac{\sqrt{\Omega_2}\xi}{4}\right)\sin\left(\frac{\sqrt{\Omega_2}\xi}{4}\right)}{-2p\cos\left(\frac{\sqrt{\Omega_2}\xi}{4}\right)\sin\left(\frac{\sqrt{\Omega_2}\xi}{4}\right)+2\sqrt{\Omega_2}cos^2\left(\frac{\sqrt{\Omega_2}\xi}{4}\right)-\sqrt{\Omega_2}}\right)^2\right]\times e^{i(\kappa x-\omega t)},$$

where $\beta_0=\frac{\left(80\mu_4\mu_2-15\mu_3{}^2\right)\sqrt{-30}}{120\mu_4}$, $\beta_1=\sqrt{-30}\mu_3$, $\beta_2=2\sqrt{-30}\mu_4$.

Case II

If $\mu_0=r^2$, $\mu_1=2rp$, $\mu_2=0$, $\mu_3=2pq$, $\mu_4=q^2$, $p^2=-2rq$, then η is one of the twelve $\eta_l^{II}(l=1,2,3,\cdots 12)$.

Type I: When $qr<0$ and $(qr\neq 0)$, for instance, if we take $l=1,3,5,7,8,12$ then optical soliton solutions of 2 are.

$$\psi_{2,1}(x,t)=$$
$$\left\{\beta_0+\beta_1\left(-\frac{1}{2q}\left(\pm\sqrt{-2qr}+\sqrt{-6qr}\tanh\left(\frac{\sqrt{-6qr}\xi}{2}\right)\right)\right)+\beta_2\left(-\frac{1}{2q}\left(\pm\sqrt{-2qr}+\sqrt{-6qr}\tanh\left(\frac{\sqrt{-6qr}\xi}{2}\right)\right)\right)^2\right\}\times e^{i(\kappa x-\omega t)},$$

$$\psi_{2,3}(x,t)=\left\{\beta_0+\beta_1\left(-\frac{1}{2q}\left(\pm\sqrt{-2qr}+\sqrt{-6qr}\tanh\left(\sqrt{-6qr}\xi\right)\pm isech\left(\sqrt{-6qr}\xi\right)\right)\right)+\right.$$
$$\left.\beta_2\left(-\frac{1}{2q}\left(\pm\sqrt{-2qr}+\sqrt{-6qr}\tanh\left(\sqrt{-6qr}\xi\right)\pm isech\left(\sqrt{-6qr}\xi\right)\right)\right)^2\right\}\times e^{i(\kappa x-\omega t)}$$

$$\psi_{2,5}(x,t)=\left\{\beta_0+\beta_1\left(-\frac{1}{4q}\left(\pm 2\sqrt{-2qr}+\sqrt{-6qr}\left(\tanh\left(\sqrt{-6qr/4}\xi\right)+\coth\left(\sqrt{-6qr/4}\xi\right)\right)\right)\right)\right.$$
$$\left.+\beta_2\left(-\frac{1}{4q}\left(\pm 2\sqrt{-2qr}+\sqrt{-6qr}\left(\tanh\left(\sqrt{-6qr/4}\xi\right)+\coth\left(\sqrt{-6qr/4}\xi\right)\right)\right)\right)^2\right\}\times e^{i(\kappa x-\omega t)},$$

$$\psi_{2,7}(x,t)=\left\{\beta_0+\beta_1\left(\frac{1}{2q}\left(\mp\sqrt{-2qr}-\frac{\sqrt{(B^2-A^2)(-6qr)}+A\sqrt{-6qr}\sinh\sqrt{-6qr}\xi}{A\cos\sqrt{-6qr}\xi+B}\right)\right)\right.$$
$$\left.+\beta_2\left(\frac{1}{2q}\left(\mp\sqrt{-2qr}-\frac{\sqrt{(B^2-A^2)(-6qr)}+A\sqrt{-6qr}\sinh\sqrt{-6qr}\xi}{A\cos\sqrt{-6qr}\xi+B}\right)\right)^2\right\}\times e^{i(\kappa x-\omega t)},$$

$$\psi_{2,8}(x,t)=\left\{\beta_0+\beta_1\left(\frac{2r\cosh\sqrt{-6qr}\xi}{\sqrt{-6qr}\sinh\left(\sqrt{-6qr}\xi\right)\mp\sqrt{-2qr}\cosh\left(\sqrt{-6qr}\xi\right)\pm i\sqrt{-6qr}}\right)\right.$$
$$\left.+\beta_2\left(\frac{2r\cosh\sqrt{-6qr}\xi}{\sqrt{-6qr}\sinh\left(\sqrt{-6qr}\xi\right)\mp\sqrt{-2qr}\cosh\left(\sqrt{-6qr}\xi\right)\pm i\sqrt{-6qr}}\right)^2\right\}\times e^{i(\kappa x-\omega t)},$$

$$\psi_{2,12}(x,t)=\left\{\beta_0+\beta_1\left(\frac{4r\cosh\left(\frac{\sqrt{-6qr}\xi}{4}\right)\sinh\left(\frac{\sqrt{-6qr}\xi}{4}\right)}{\mp2\sqrt{-2qr}\cosh\left(\frac{\sqrt{-6qr}\xi}{4}\right)\sinh\left(\frac{\sqrt{-6qr}\xi}{4}\right)+2\sqrt{-6qr}\cosh^2\left(\frac{\sqrt{-6qr}\xi}{4}\right)-\sqrt{-6qr}}\right)\right.$$
$$\left.+\beta_2\left(\frac{4r\cosh\left(\frac{\sqrt{-6qr}\xi}{4}\right)\sinh\left(\frac{\sqrt{-6qr}\xi}{4}\right)}{\mp2\sqrt{-2qr}\cosh\left(\frac{\sqrt{-6qr}\xi}{4}\right)\sinh\left(\frac{\sqrt{-6qr}\xi}{4}\right)+2\sqrt{-6qr}\cosh^2\left(\frac{\sqrt{-6qr}\xi}{4}\right)-\sqrt{-6qr}}\right)^2\right\}\times e^{i(\kappa x-\omega t)},$$

where $\beta_0=\frac{\left(80\mu_4\mu_2-15\mu_3{}^2\right)\sqrt{-30}}{120\mu_4}$, $\beta_1=\sqrt{-30}\mu_3$, $\beta_2=2\sqrt{-30}\mu_4$, $v=\frac{6\kappa^2+\omega+\phi(x)}{2\kappa}$.

Case III:
If $\mu_0=\mu_1=0,\mu_2,\mu_3,\mu_4$, are arbitrary constants then η is one of the ten η_l^{III} $(l=1,2,3,...,10.)$.
Type 3. For instance, if we take $l=3$, then $\mu_2=4,\mu_3=-\frac{4(2b+d)}{a},\mu_4=\frac{c^2+4b^2+4bd}{a^2}$;, the optical soliton solution of 2 is

$$\psi_{3,3}(x,t)=\left\{\frac{2/3\sqrt{-30}\left(c^2+b^2+4bd-3bd-3/4d^2\right)}{a^2}+\frac{-4\sqrt{-30}\,(2b+d)}{a}\left(\frac{a\,sech^2\xi}{b+c\,sech\,\xi}\right)+\right.$$
$$\left.\frac{8\left(1/4c^2+b^2+bd\right)\sqrt{-30}}{a^2}\left(\frac{a\,sech^2\xi}{b+c\,sech\xi}\right)\right\}\times e^{i(\kappa x-\omega t)},$$

a,b,c, and d are arbitrary constants.
Case IV:
If $\mu_1=\mu_3=0,\mu_0,\mu_2,\mu_4$, are arbitrary constants then η is one of the sixteen $\eta_l^{IV}l(l=1,2,3,...,16)$.

For instance, if we take $l=13,\mu_0=\frac{1}{4},\mu_2=\frac{1-2m^2}{2}$ and $\mu_4=\frac{1}{4}$;then the following optical soliton solutions are obtained.

$$\psi_{4,13}(x,t)=\left\{1/3\left(1-2m^2\sqrt{-30}\right)+1/2\sqrt{-30}\left((ns\xi\pm cs\xi)\right)^2\right\}\times e^{i(\kappa x-\omega t)}, \tag{11}$$

in the limiting case when $m\to1$ the solution 11 becomes the combined optical soliton solution

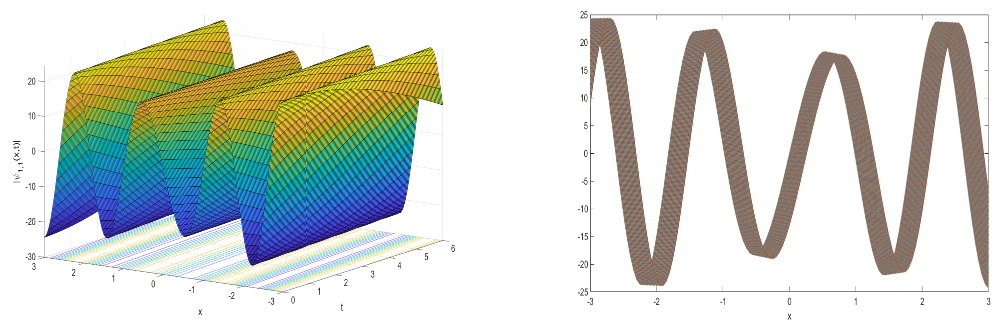
$$\psi_{4,13,1}(x,t)=\left\{1/3\left(1-2m^2\right)\sqrt{-30}+1/2\sqrt{-30}\left((\coth\pm\csc h\xi)\right)^2\right\}\times e^{i(\kappa x-\omega t)},$$

when $m\to0$, in this case we have periodic singular solution

$$\psi_{4,13,2}(x,t)=\left\{1/3\left(1-2m^2\right)\sqrt{-30}+1/2\sqrt{-30}\left(\csc\pm\cot\xi\right)^2\right\}\times e^{i(\kappa x-\omega t)}.$$

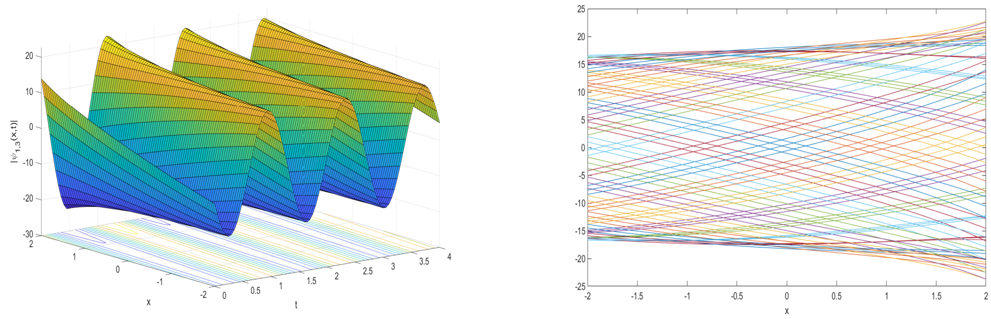
Case V:
If $\mu_2=\mu_4=0,\mu_0,\mu_1,\mu_3$, are arbitrary constants then the system does not admit a solution of this group.

3 Graphical Representations



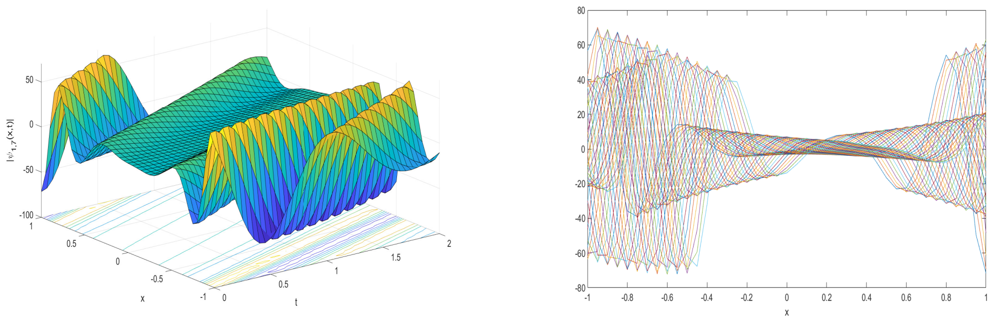
(a) (b)

Fig. 1: 3-D and 2-D line plot of $|\psi_{1,1}(x,t)|$ for $v=0.075$, $p=3.05$, $q=1.35$, $r=1.25$, $\kappa=4.05$, $\omega=0.22$, $\mu_2=1.03$, $\mu_3=1.4$, $\mu_4=1.8$, $\phi(x)=1$.



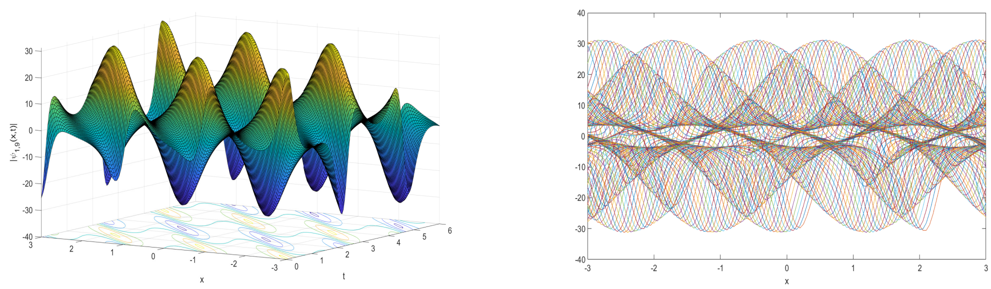
(a) **(b)**

Fig. 2: 3-D and 2-D line plot of $|\psi_{1,3}(x,t)|$ for $v = 0.25$, $p = 3.25$, $q = 2.15$, $r = 0.015$, $\kappa = 0.25$, $\omega = 4.53$, $\mu_2 = 5.02$, $\mu_3 = 3.5$, $\mu_4 = 3.7$, $\phi(x) = 1$.



(a) **(b)**

Fig. 3: 3-D and 2-D line plot of $|\psi_{1,7}(x,t)|$ for $v = 0.25$, $p = 3.05$, $q = 1.35$, $r = 1.25$, $\kappa = 8.085$, $\omega = 3.53$, $\mu_2 = 1.01$, $\mu_3 = 0.6$, $\mu_4 = 1.8$, $A = 2$, $B = 3$, $\phi(x) = 1$.



(a) **(b)**

Fig. 4: 3-D and 2-D line plot of $|\psi_{1,9}(x,t)|$ for $v = 0.85$, $p = 4.05$, $q = 1.35$, $r = 1.25$, $\kappa = 2.85$, $\omega = 0.83$, $\mu_2 = 1.02$, $\mu_3 = 0.5$, $\mu_4 = 1.7$, $A = 2$, $B = 3$, $\phi(x) = 1$

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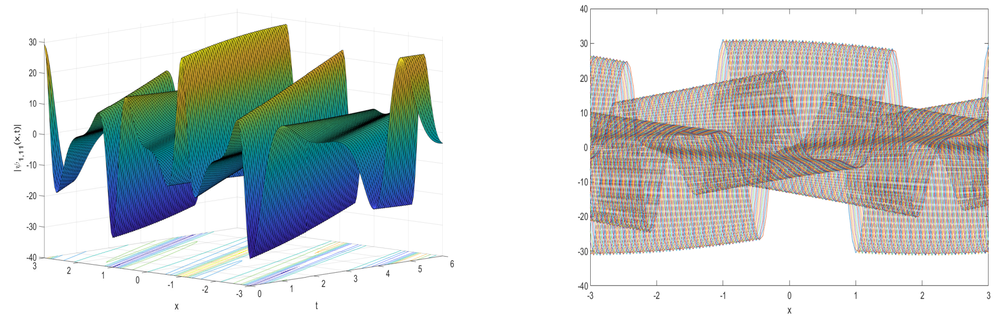
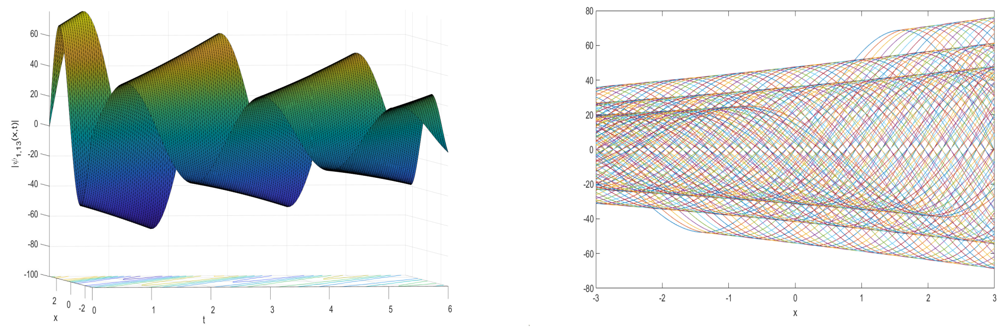


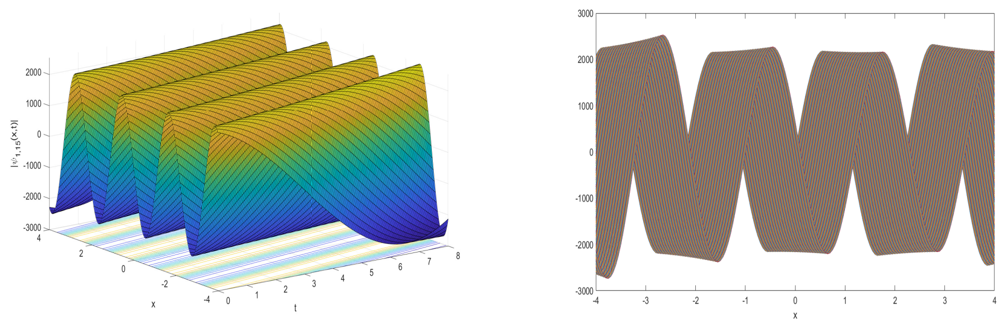
(a) **(b)**

Fig. 5: 3-D and 2-D line plot of $|\psi_{1,11}(x,t)|$ for $v = 0.45$, $p = 5.05$, $q = 1.35$, $r = 1.25$, $\kappa = 1.45$, $\omega = 0.45$, $\mu_2 = 1.02$, $\mu_3 = 0.5$, $\mu_4 = 1.7, A = 2, B = 3$, $\phi(x) = 1$.



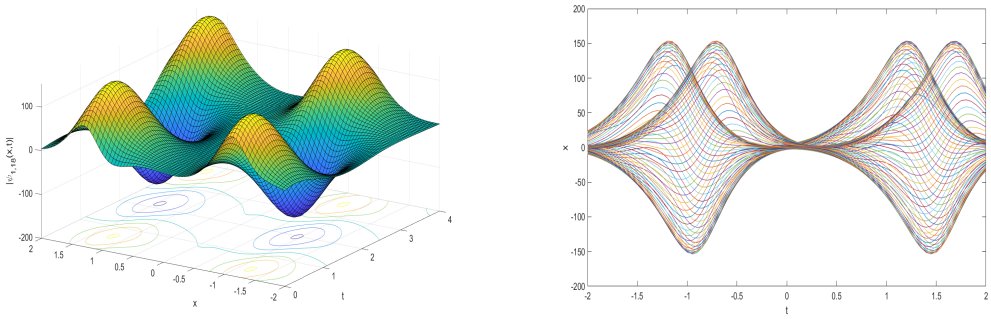
(a) **(b)**

Fig. 6: 3-D and 2-D line plot of $|\psi_{1,13}(x,t)|$ for $v = 0.85$, $p = 0.15$, $q = 0.05$, $r = 0.025$, $\kappa = 1.05$, $\omega = 2.73$, $\mu_2 = 1.02$, $\mu_3 = 1.5$, $\mu_4 = 2.7$, $\phi(x) = 1$.



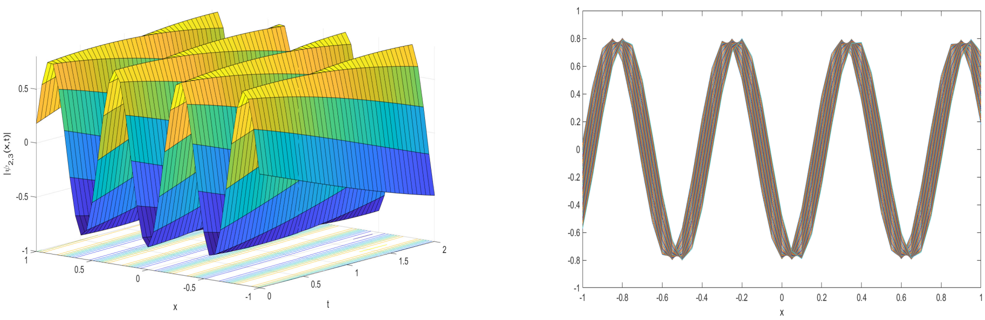
(a) **(b)**

Fig. 7: 3-D and 2-D line plot of $|\psi_{1,15}(x,t)|$ for $v = 0.45$, $p = 0.10$, $q = 0.15$, $r = 0.025$, $\kappa = 2.85$, $\omega = 0.43$, $\mu_2 = 1.02$, $\mu_3 = 0.5$, $\mu_4 = 21.7$, $\phi(x) = 1$.



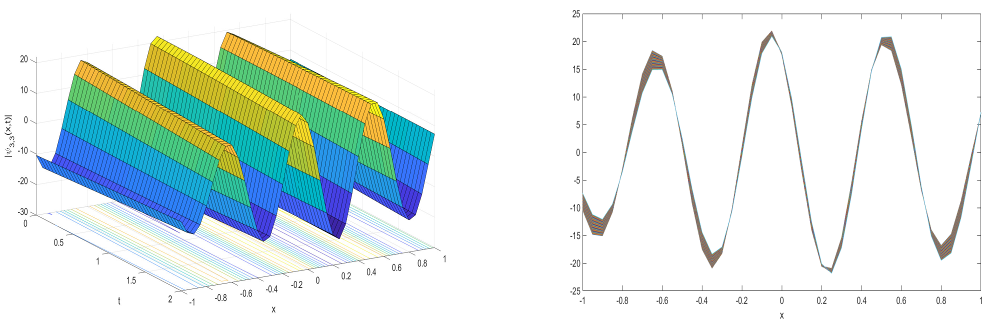
(a) **(b)**

Fig. 8: 3-D and 2-D line plot of $|\psi_{1,18}(x,t)|$ for $v = 0.25$, $p = 2.10$, $q = 3.25$, $r = 3.25$, $\kappa = 0.85$, $\omega = 0.43$, $\mu_2 = 0.12$, $\mu_3 = 0.5$, $\mu_4 = 1.7, A = 3, B = 2$, $\phi(x) = 1$.



(a) **(b)**

Fig. 9: 3-D and 2-D line plot of $|\psi_{2,3}(x,t)|$ for $v = 0.45$, $p = 0.25$, $q = 0.015$, $r = 0.025$, $\kappa = 10.85$, $\omega = 0.43$, $\mu_0 = r^2$, $\mu_1 = 2rp$, $\mu_2 = 2rq + p^2$, $\mu_3 = 2pq$, $\mu_4 = q^2$, $\phi(x) = 1$.



(a) **(b)**

Fig. 10: 3-D and 2-D line plot of $|\psi_{3,3}(x,t)|$ for $v = 0.15$, $p = 0$, $q = 0.15$, $r = 0.25$, $\kappa = 10.85$, $\omega = 0.043$, $\mu_2 = 1$, $\mu_2 = -2ca^{-1}$, $\mu_4 = c^2 - b^2a^2$, $\phi(x) = 1$.

4 Discussion and Conclusion

In conclusion, this research has provided a comprehensive investigation into the optical soliton solutions of the coupled nonlinear Schrödinger-Poisson system using the extended Fan's sub-equation method. The study sheds light on the rich dynamics of the system and explores the influence of external potential on optical soliton profiles. Furthermore, the analysis has been extended to include the examination of dark soliton and bright soliton solutions within the context of the coupled system. The obtained optical soliton solutions exhibit intriguing features, capturing the coexistence of dark and bright solitons in the nonlinear evolution of the system. Dark solitons, characterized by localized depressions in the amplitude, and bright solitons, featuring localized peaks, demonstrate the system's ability to support a diverse range of nonlinear structures. The stability and robustness of these optical soliton solutions underscore their significance in understanding the nonlinear dynamics of coupled systems. Dark solitons, known for their unique phase features and potential applications in signal processing, introduce an additional layer of complexity to the soliton dynamics in the coupled system. The interplay between dark and bright solitons reveals intricate interactions that can be harnessed for controlling and manipulating optical pulses in fiber optic communication systems. The findings of this study contribute to the existing literature on optical soliton dynamics in coupled nonlinear systems, providing exact optical soliton solutions for the Schrödinger-Poisson system. The inclusion of dark and bright solitons enhances our understanding of the system's nonlinear behavior and offers new perspectives for applications in plasma physics and nonlinear optics. In practical terms, the obtained optical soliton solutions may find application in the design and optimization of optical communication systems, where the control and manipulation of solitons are crucial for signal transmission and information processing. The insights gained from this research pave the way for future investigations into more complex coupled systems and underscore the importance of analytical methods, such as the extended Fan's sub-equation method, in unveiling the intricate dynamics of nonlinear phenomena.

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