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Exploring ϑ-Translation and ϑ-Multiplication: A Graphical Analysis in Fuzzy Z-Algebra and Q-Fuzzy Z-Algebra

Suleiman Ibrahim Mohammad[1,2,*], *M. Premkumar*[3], *N. Raja*[4], *V. Helan Sinthiya*[5], *K. Malathi*[6], *M. Joel Jaikumar Madhuram*[3], *P. Shanmugavel*[7], *Asokan Vasudevan*[8,9,10], *and Mohammad Faleh Ahmmad Hunitie*[11]

[1]Electronic Marketing and Social Media, Economic and Administrative Sciences Zarqa University, 13110 Zarqa, Jordan
[2]INTI International University, 71800 Negeri Sembilan, Malaysia
[3]Department of Mathematics, Sathyabama Institute of Science and Technology (Deemed to Be University) Chennai-600119, Tamilnadu, India
[4]Department of Visual Communication, Sathyabama Institute of Science and Technology, (Deemed to Be University), Chennai, Tamil Nadu, India
[5]Department of Mathematics, School of Engineering and Technology, Dhanalakshmi Srinivasan University, Trichy, Tamilnadu, India
[6]Department of Mathematics, Kongunadu College of Engineering and Technology, Trichy, Tamilnadu, India
[7]Department of Mathematics, Selvamm Arts and Science College (Autonomous), Namakkal, Tamilnadu, India
[8]Faculty of Business and Communications, INTI International University, Persiaran Perdana BBN Putra Nilai, 71800 Nilai, Negeri Sembilan, Malaysia
[9]Wekerle Business School, Budapest, Jázmin u. 10, 1083 Hungary
[10]Shinawatra University, 99 Moo 10, Bangtoey, Samkhok, Pathum Thani 12160 Thailand
[11]Department of Public Administration, School of Business, University of Jordan, Jordan

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Abstract: In this paper, we introduce new notations for algebraic structures, specifically ϑ-T and ϑ-M, and explore their properties through graphical analysis within the framework of Z-subalgebras of Z-algebras. The study provides a comprehensive discussion of the algebraic properties associated with these structures and includes illustrative examples to enhance understanding. Building on this foundation, we further extend the definitions of ϑ-T and ϑ-M to the domain of Z-ideals of Z-algebras, employing graphical analysis to demonstrate their various group-theoretical properties. These extensions offer deeper insights into the interplay between these algebraic structures and their graphical representations. Moreover, we introduce and formalize the concepts of ϑ-translations and ϑ-multiplications within the framework of Z-ideals of Q-fuzzy Z-algebras. Using graphical analysis, we provide a detailed exploration of their properties, offering a fresh perspective on how these new constructs behave within the broader context of Q-fuzzy Z-algebras. By delving into their algebraic characteristics, we reveal significant relationships and properties that contribute to the advancement of algebraic theory. The graphical approach employed throughout this paper not only facilitates a better understanding of the underlying structures but also aids in visualizing complex relationships and operations. Through rigorous analysis and the integration of examples, we highlight the utility of these notations and their potential applications in algebraic studies. This work sets the stage for further exploration and development of similar structures in advanced algebraic systems, establishing a foundation for future research in this field.

Keywords: Fuzzy Set(FS), Fuzzy Subset(FSb), Fuzzy ZIdeal (FZI), Fuzzy Z-Algebra(FZA), Fuzzy Z-Sub Algebra(FZSA), Fuzzy ϑ- Translation(Fϑ-T), Fuzzyϑ-Multiplication(Fϑ-M)

* Corresponding author e-mail: dr_sliman@yahoo.com

1 Introduction

In 1965, Zadeh L A [1], initiated by the concept of fuzzy sets. Several researchers explored on the generalization of the notion of fuzzy subset. The study of fuzzy subsets and its applications to various mathematical contents has given rise to what is now commonly called fuzzy mathematics [2,3,4]. Iseki K and Tanaka S [5,6,7], introduced the concept of an introduction to the theory of BCK-algebras in 1978. In 1980, Iseki K [8], first introduced the notation on BCI-algebras. KyoungJa Lee, Young Bae Jun and Myung Im Doh [9,10,11], introduced the concept of fuzzy translations and fuzzy multiplication of BCK/BCI-algebras in 2009 [12,13,14]. Abu Ayub Ansari and Chandramouleeswaran M [15,16,17], introduced the concept of fuzzy translation of fuzzy β-ideals of β-algebras in 2014. In 2014, Priya and Ramachandran T [18], introduced the new notation of fuzzy translation and multiplication on PS-algebras. Prasanna A, Premkumar M and Ismail Mohideen S [19] & [20] & [21] & [22] & [23], introduced the concept of fuzzy translation and multiplication on B-algebras in 2018 and also derived from Fuzzy Translation and Fuzzy Multiplication in BG - Algebras in 2019. In 2021, Premkumar [24] derived the new notation of Algebraic Properties on Fuzzy Translation and Multiplication in BP?Algebras [25,26,27]. Premkumar [19] & [20], introduced the new concept of Algebraic Properties on ω - Fuzzy Translation and Multiplication in BH-Algebras in 2020 and also derived from the concept of Characteristics of $k-Q$-Fuzzy Translation and Fuzzy Multiplication in T-Ideals in T-Algebra in 2022. Solairaju [28] described the new structure and construction of $\bar{Q}$-fuzzy groups in 2009. Sowmiya [29] & [30] & [31] & [32] initiated by the concept on Fuzzy Z-ideals in Z-algebras and also Fuzzy Algebraic Structure in Z-Algebras in 2019.

We define the new notation of Algebraic structures of ϑ-T and ϑ-M in Z-Sub Algebra of Z-Algebras. And also defined the ϑ-T and ϑ-M in Z-Ideal of Z-Algebra and discussed some of their properties and also derived new notation of Algebraic structures of ϑ-Translations and ϑ-Multiplication in Z-Sub Algebra of Q- fuzzy Z-algebras. And also defined the ϑ-translations and ϑ-Multiplications in Z-Ideal of Q-fuzzy Z-algebras and discussed some of their properties

2 Preliminaries

Definition:2.1 A Z-algebra $(\tilde{\omega},*,\theta)$ be a Z-algebra. A FSA in $\tilde{\omega}$ with a membership function $\tilde{H}_A$ is said to be a FZSA of a Z-algebra $\tilde{\omega}$ if, $\tilde{H}(\acute{r}*\check{S}) \geq \tilde{H}(\acute{r}) \wedge \tilde{H}(\check{s})$, for all $\acute{r},\check{s}$ in $\tilde{\omega}$

Definition:2.2 A Z-algebra $(\tilde{\omega},*,\theta)$ be a Z-algebra. A FSV in $\tilde{\omega}$ with a membership function $\tilde{H}_A$ is said to be a FZI of a Z-algebra $\tilde{\omega}$ if,

–$\tilde{H}(0) \geq \tilde{H}(\acute{r})$

–$\tilde{H}(\acute{r}) \geq \{\tilde{H}(\acute{r}*\check{s}) \wedge \tilde{H}(\check{s})\}$

Definition:2.3 Let $\bar{Q}$ and G a set and a group respectively. A mapping $\mu : \tilde{\omega} \times \bar{Q} \to [0,1]$ is called $\bar{Q}$-FS in G. For any $\bar{Q}$-FSμ in G and $t \in [0,1]$ we define the set

$$U(\mu;t) = \{f \in \tilde{\omega}/\mu(f,q) \geq t, q \in \bar{Q}\}$$

which is called an upper cut of ?μ? and can be use to the characterization of μ.

3 Innovative Graphical Frameworks for Understanding ϑ-Translation and ϑ-Multiplication in Fuzzy Z-Algebra and Q-Fuzzy Z-Algebra

Let, $\tilde{\omega}$ be a Z-algebra. For any $FS\tilde{H}$ of $\tilde{\omega}$ we define $T = 1 - sup\tilde{H}(\acute{r})/\acute{r} \in \tilde{\omega}$, unless otherwise we specified.

Definition:3.1 Let, $FS\tilde{H}$ be a FSb of $\tilde{\omega}$ and $\vartheta \in [0,T]$. A mapping $\tilde{H}^T_\vartheta : \tilde{\omega} \to [0,1]$ is said to be a $F\vartheta - T$ of $\tilde{H}$ if it satisfies $\tilde{H}^T_\vartheta = \tilde{H}(\acute{r}) + \vartheta, \forall \acute{r} \in \tilde{\omega}$.

Definition:3.2 Let, $FS\tilde{H}$ be a FSb of $\tilde{\omega}$ and $\vartheta \in [0,T]$. A mapping $\tilde{H}^M_\vartheta : \tilde{\omega} \to [0,1]$ is said to be a $F\vartheta - M$ of $\tilde{H}$ if it satisfies $\tilde{H}^M_\vartheta = \vartheta\ \tilde{H}(\acute{r}),\ \forall \acute{r} \in \tilde{\omega}$.

Example:3.3 Let $\tilde{\omega} = \{0,1,2,3\}$ be the set with the following table.

Table 1: Fuzzy Z-Algebra Data Set

$*$	0	1	2	3
0	0	1	2	3
1	1	0	1	1
2	2	2	0	2
3	3	3	3	0

Then $(\acute{\tilde{\omega}},*,0)$ is a $Z-$algebra.
Define $FS\tilde{H}$ is of $\acute{\tilde{\omega}}$ by

$$\tilde{H}(\acute{r}) = \begin{cases} 0.4 \text{ if } \acute{r} \neq 1, \\ 0.3 \text{ if } \acute{r} = 1 \end{cases}$$

Thus $\tilde{H}$ is a FZSA of X.
Hence $T = 1 - \sup\tilde{H}(\acute{r})/\acute{r} \in \acute{\tilde{\omega}} = 1 - 0.4 = 0.6$,
Choose $\vartheta = 0.2 \in [0,1]$ and $\vartheta = 0.3 \in [0,1]$.
Then the mapping$\tilde{H}^T_{0.2} :\to [0,1]$ is defined by

$$\tilde{H}^T_{0.2} = \begin{cases} 0.2+0.4 = 0.6, \text{ if } \acute{r} \neq 1, \\ 0.2+0.3 = 0.5, \text{ if } \acute{r} = 1 \end{cases}$$

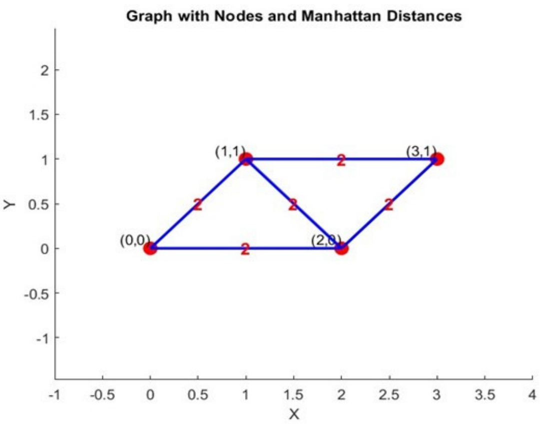

Fig. 1: Graph Representation with (x,y) Node Labels

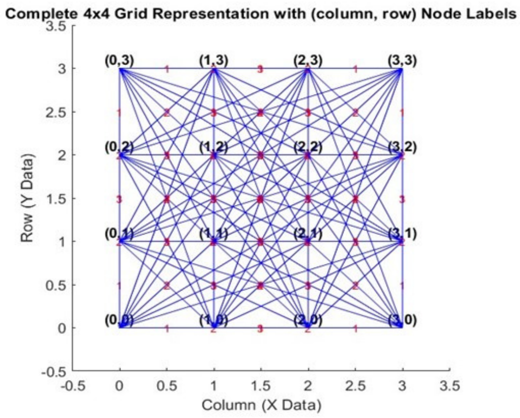

Fig. 2: Graph representation of Cayley tamale using with Fuzzy Z-Algebra

Which satisfies $\tilde{H}_{(0.2)^T}(\acute{r}) = \tilde{H}(\acute{r}) + 0.2, \forall \acute{r} \in \tilde{\omega}$, is a $0.2-T$.
The mapping$(\gamma)_((0.3)^M) : \tilde{\omega} \to [0,1]$ is defined by

$$\tilde{H}_{0.2}^{M} = \begin{cases} 0.3 \times 0.4 = 0.12, \text{ if } \acute{r} \neq 1, \\ 0.3 \times 0.3 = 0.09, \text{ if } \acute{r} = 1 \end{cases}$$

Which satisfies $\tilde{H}_((0.3)^M)(\acute{r}) = \tilde{H}(\acute{r})(0.3), \forall \acute{r} \in \tilde{\omega}$, is a $0.3M$

Theorem:3.4 If $\tilde{H}$ of $\tilde{\omega}$ is a FZSA and $\vartheta \in [0,1]$, then the F $\vartheta - T.(\tilde{H}_\vartheta)^T(\acute{r})$ of $\tilde{H}$ is also a FZSA of $\tilde{\omega}$.

Proof Let, $\acute{r}, \acute{s} \in \tilde{\omega}$ and $\vartheta \in [0,T]$
Then, $\tilde{H}(\acute{r} * \acute{s}) \geq \tilde{H}(\acute{r}) ? \tilde{H}(\acute{s})$
Now,

$$\begin{aligned} (\tilde{H}_\vartheta^T)(\acute{r} * \acute{s}) &= \tilde{H}(\acute{r} * \acute{s}) + \vartheta \\ &\geq [\tilde{H}(\acute{r}) \wedge \tilde{H}(\acute{s})] + \vartheta \\ &= [(\tilde{H}(\acute{r}) + \vartheta) \wedge (\tilde{H}(\acute{s}) + \vartheta)] \\ &= [(\tilde{H}_\vartheta^T)(\acute{r}) \wedge (\tilde{H}_\vartheta^T)(\acute{s})] \forall \acute{r}, \acute{s} \in \tilde{\omega} \end{aligned}$$

Theorem:3.5 Let,$\tilde{H}$ be a FSb of $\tilde{\omega}$ such that the $F\vartheta - T(\tilde{H}_\vartheta^T)(\acute{r})$ of $\tilde{H}$ is a FZSA of $\tilde{\omega}$, for some $\vartheta \in [0,T]$, then $\tilde{H}$ is a FZSA of $\tilde{\omega}$.

Proof Assume that $(\tilde{H}_\vartheta)^T(\acute{r})$ is a FZSA of $\tilde{\omega}$ for some $\vartheta \in [0,T]$
Let $\acute{r}, \acute{s} \in \tilde{\omega}$, we have

$$\begin{aligned} \tilde{H}(\acute{r} * \acute{s}) + \vartheta &= (\tilde{H}_\vartheta)^T(\acute{r} * \acute{s}) \\ &\geq [(\tilde{H}_\vartheta)^T(\acute{r}) \wedge (\tilde{H}_\vartheta)^T(\acute{s})] \\ &= [(\tilde{H}(\acute{r}) + \vartheta) \wedge (\tilde{H}(\acute{s}) + \vartheta)] \\ &= [\tilde{H}(\acute{r}) \wedge \tilde{H}(\acute{s})] + \vartheta \\ \Longrightarrow \tilde{H}(\acute{r} * \acute{s}) &\geq [\tilde{H}(\acute{r}) \wedge \tilde{H}(\acute{s})], \forall \acute{r}, \acute{s} \in \tilde{\omega} \end{aligned}$$

Hence, $\tilde{H}$ is FZSA of $\tilde{\omega}$.

Theorem:3.6 For any FZSA $\tilde{H}$ of $\tilde{\omega}$ and $\vartheta \in [0,1]$, if the $F\vartheta^M(\tilde{H}_\vartheta)^M(\acute{r})$ of $\tilde{H}$ is a FZSA of $\tilde{\omega}$.

Proof Let $\acute{r}, \acute{s} \in \tilde{\omega}$ and $\vartheta \in [0,T]$ Then $\tilde{H}(\acute{r} * \acute{s}) \geq \tilde{H}(\acute{r}) \wedge \tilde{H}(\acute{s})$ Now,

$$\begin{aligned} (\tilde{H}_\vartheta)^M(\acute{r} * \acute{s}) &= \vartheta \tilde{H}(\acute{r} * \acute{s}) \\ &\geq \vartheta[\tilde{H}(\acute{r}) \wedge \tilde{H}(\acute{s})] \\ &\geq [\vartheta \tilde{H}(\acute{r}) \wedge \vartheta \tilde{H}(\acute{s})] \\ &= [(\tilde{H}_\vartheta)^M(\acute{r}) \wedge (\tilde{H}_\vartheta)^M(\acute{s})] \\ \Longrightarrow (\tilde{H}_\vartheta)^M(\acute{r} * \acute{s}) &\geq [(\tilde{H}_\vartheta)^M(\acute{r}) \wedge (\tilde{H}_\vartheta)^M(\acute{s})] \end{aligned}$$

Therefore,$(\tilde{H}_\vartheta)^M$ is a FZSA of $\tilde{\omega}$.

Theorem:3.7 For any FSb,$\tilde{H}$ of $\tilde{\omega}$ and $\vartheta \in [0,1]$, if the $F\vartheta - M(\tilde{H}_\vartheta)^M(\acute{r})$ of $\tilde{H}$ is a FZSA of $\tilde{\omega}$, then so in $\tilde{H}$.

Proof Assume that,$(\tilde{H}_\vartheta)^M(\acute{r})$ of $\tilde{H}$ is a FZSA of $\tilde{\omega}$ for some $\vartheta \in [0,T]$
Let $\acute{r}, \acute{s} \in \tilde{\omega}$, we have

$$\begin{aligned} \vartheta \tilde{H}(\acute{r} * \acute{s}) &= (\tilde{H}_\vartheta)^M(\acute{r} * \acute{s}) \\ &\geq [(\tilde{H}_\vartheta)^M(\acute{r}) \wedge (\tilde{H}_\vartheta)^M(\acute{s})] \\ &= [\vartheta \tilde{H}(\acute{r}) \wedge \vartheta \tilde{H}(\acute{s})] \\ &= \vartheta[\tilde{H}(\acute{r}) \wedge \tilde{H}(\acute{s})] \\ \Longrightarrow \tilde{H}(\acute{r} * \acute{s}) &\geq \vartheta[\tilde{H}(\acute{r}) \wedge \tilde{H}(\acute{s})] \end{aligned}$$

Hence, $\tilde{H}$ is a FZSA of $\tilde{\omega}$.

Theorem:3.8 If the $F\vartheta - T(\tilde{H}_\vartheta)^T(\acute{r})$ of $\tilde{H}$ is a FZI, then it satisfies the condition

$$\tilde{H}_\vartheta)^T(\acute{s} * (\acute{r} * \acute{s})) \geq (\tilde{H}_\vartheta)^T(\acute{r}).$$

Proof

$$\begin{aligned}\tilde{H}^T_\vartheta(\acute{s}*(\acute{r}*\acute{s})) &= \tilde{H}(\acute{s}*(\acute{r}*\acute{s}))+\vartheta\\ &\geq \tilde{H}(0*(\acute{s}*(\acute{r}*\acute{s})))+\vartheta\wedge\tilde{H}(0)+\vartheta\\ &\geq \tilde{H}(0*(\acute{s}*(\acute{s}*\acute{r})))+\vartheta\wedge\tilde{H}(0)+\vartheta\\ &= \tilde{H}(0*((\acute{s}*\acute{s})*\acute{r}))+\vartheta\wedge\tilde{H}(0)+\vartheta\\ &= \tilde{H}(0*(\acute{s}*\acute{r}))+\vartheta\wedge\tilde{H}(0)+\vartheta\\ &= \tilde{H}(\acute{s}*\acute{r})*0+\vartheta\wedge\tilde{H}(0)+\vartheta\\ &\geq \tilde{H}(\acute{s}*\acute{r})*+\vartheta\wedge\tilde{H}(\acute{r})+\vartheta\\ &\geq \tilde{H}^T_\vartheta(0)\wedge(\tilde{H}^T_\vartheta(\acute{r}))\\ &= \tilde{H}^T_\vartheta(\acute{r}).\end{aligned}$$

$$\Rightarrow \tilde{H}^T_\vartheta(\acute{s}*(\acute{r}*\acute{s}))\geq(\tilde{H}^T_\vartheta(\acute{r}))\forall\acute{r},\acute{s}\in\tilde{\tilde{\omega}}$$

Theorem:3.9 If,$\tilde{H}$ is a FZI of $\tilde{\tilde{\omega}}$, then the $F\vartheta-T(\tilde{H}_\vartheta)^T(\acute{r})$ of $\tilde{H}$ is a FZI of $\tilde{\tilde{\omega}}$, for all $\vartheta\in[0,T]$.

Proof Let,$\tilde{H}$ be a FZI of $\tilde{\tilde{\omega}}$ and let $\vartheta\in[0,T]$
Then,

$$\begin{aligned}(i)(\tilde{H}_\vartheta)^T(0) &= \tilde{H}(0)+\vartheta\\ &\geq \tilde{H}(\acute{r})+\vartheta\\ &= (\tilde{H}_\vartheta)^T(\acute{r})\end{aligned}$$

$$\begin{aligned}(ii)(\tilde{H}_\vartheta)^T(\acute{r}) &= \tilde{H}(\acute{r})+\vartheta\\ &\geq \tilde{H}(\acute{r}*\acute{s})\wedge\tilde{H}(\acute{s})+\vartheta\\ &= (\tilde{H}(\acute{r}*\acute{s})+\vartheta)\wedge(\tilde{H}(\acute{s})+\vartheta)\\ &= (\tilde{H}_\vartheta)^T(\acute{r}*\acute{s})\wedge(\tilde{H}_\vartheta)^T(\acute{s})\end{aligned}$$

$$\Rightarrow (\tilde{H}_\vartheta)^T(\acute{r})\geq(\tilde{H}_\vartheta)^T(\acute{r}*\acute{s})\wedge(\tilde{H}_\vartheta)^T(\acute{s})$$

Hence, $(\tilde{H}_\vartheta)^T(\acute{r})$ of $\tilde{H}$ is a FZI of $\tilde{\tilde{\omega}},\forall\vartheta\in[0,T]$

Theorem:3.10 Let,$\tilde{H}$ is a FSb of $\tilde{\tilde{\omega}}$ such that the $F\vartheta-T(\tilde{H}_\vartheta)^T(\acute{r})$ of $\tilde{H}$ is a FZI of $\tilde{\tilde{\omega}}$, for some $\vartheta\in[0,T]$, then $\tilde{H}$ is a FZI of $\tilde{\tilde{\omega}}$.

Proof Assume that, $(\tilde{H}_\vartheta)^T$ is a FZI of $\tilde{\tilde{\omega}}$ for some $\vartheta\in[0,T]$.
Let $\acute{r},\acute{s}\in\tilde{\tilde{\omega}}$
Then,

$$\begin{aligned}\tilde{H}(0)+\vartheta &= (\tilde{H}_\vartheta)^T(0)\\ &\geq (\tilde{H}_\vartheta)^T(\acute{r})\\ &= \tilde{H}(\acute{r})+\vartheta\end{aligned}$$

And so

$$\Rightarrow \tilde{H}(0)\geq\tilde{H}(\acute{r})$$

$$\begin{aligned}\tilde{H}(\acute{r})+\vartheta &= (\tilde{H}_\vartheta)^T(\acute{r})\\ &\geq (\tilde{H}_\vartheta)^T(\acute{r}*\acute{s})\wedge(\tilde{H}_\vartheta)^T(\acute{s})\\ &= (\tilde{H}(\acute{r}*\acute{s})+\vartheta)\wedge(\tilde{H}(\acute{s})+\vartheta)\\ &= \tilde{H}(\acute{r}*\acute{s})\wedge\tilde{H}(\acute{s})+\vartheta\end{aligned}$$

and so$\tilde{H}(\acute{r})\geq(\acute{r}*\acute{s})\wedge\tilde{H}(\acute{s})$

Hence, $\tilde{H}$ is a FZI of $\tilde{\tilde{\omega}}$.

Theorem:3.11 Let,$\vartheta\in[0,T]$ and let $\tilde{H}$ be a FZI of $\tilde{\tilde{\omega}}$. If $\tilde{\tilde{\omega}}$ is a Z-algebra, then the fuzzy F $\vartheta-T(\tilde{H}_\vartheta)^T$ of $\tilde{H}$ is a FZSA of $\tilde{\tilde{\omega}}$.

Proof Let, $\acute{r},\acute{s}\in\tilde{\tilde{\omega}}$.
Now, we have

$$\begin{aligned}(\tilde{H}_\vartheta)^T(\acute{r}*\acute{s}) &= \tilde{H}(\acute{r}*\acute{s})+\vartheta\\ &\geq \tilde{H}((\acute{r}*\acute{s})*\acute{s})\wedge\tilde{H}(\acute{s})+\vartheta\\ &= \tilde{H}(\acute{s}*(\acute{r}*\acute{s}))\wedge\tilde{H}(\acute{s})+\vartheta\end{aligned}$$

by Theorem 3.7

$$\begin{aligned}&\geq \tilde{H}(0)\wedge\tilde{H}(\acute{s})+\vartheta\\ &\geq \tilde{H}(\acute{r})\wedge\tilde{H}(\acute{s})+\vartheta\\ &\geq (\tilde{H}(\acute{r})+\vartheta)\wedge(\tilde{H}(\acute{s})+\vartheta)\\ &= (\tilde{H}_\vartheta)^T(\acute{r})\wedge(\tilde{H}_\vartheta)^T(\acute{s})\end{aligned}$$

Hence, $(\tilde{H}_\vartheta)^T$ is a FZSA of $\tilde{\tilde{\omega}}$.

Theorem:3.12 If the $F\vartheta-T\tilde{H}_\vartheta)^T$ of $\tilde{H}$ is a FZSA of $\tilde{\tilde{\omega}},\vartheta\in[0,T]$, then $\tilde{H}$ is a FZSA of $\tilde{\tilde{\omega}}$.

Proof Let us assume that, $(\tilde{H}_\vartheta)^T$ of $\tilde{H}$ is a FZI of $\tilde{\tilde{\omega}}$.
Then

$$\begin{aligned}\tilde{H}(\acute{r}*\acute{s})+\vartheta &= (\tilde{H}_\vartheta)^T(\acute{r}*\acute{s})\\ &\geq (\tilde{H}_\vartheta)^T((\acute{r}*\acute{s})*\acute{s})\wedge(\tilde{H}_\vartheta)^T(\acute{s})\\ &= (\tilde{H}_\vartheta)^T(\acute{s}*(\acute{r}*\acute{s}))\wedge(\tilde{H}_\vartheta)^T(\acute{s})\\ &\geq (\tilde{H}_\vartheta)^T(0)\wedge(\tilde{H}_\vartheta)^T(\acute{s})\\ &\geq (\tilde{H}_\vartheta)^T(\acute{r})\wedge(\tilde{H}_\vartheta)^T(\acute{s})\\ &= (\tilde{H}(\acute{r})+\vartheta)\wedge(\tilde{H}(\acute{s})+\vartheta)\\ &= \tilde{H}(\acute{r})\wedge\tilde{H}(\acute{s})+\vartheta\end{aligned}$$

$$\Rightarrow \tilde{H}(\acute{r}*\acute{s})\geq\tilde{H}(\acute{r})\wedge\tilde{H}(\acute{s})$$

Hence, $\tilde{H}$ is a FZSA of $\tilde{\tilde{\omega}}$

Theorem:3.13 Let, $\tilde{H}$ is a FSb of $\tilde{\tilde{\omega}}$ such that the $F\vartheta-M(\tilde{H}_\vartheta)^M(\acute{r})$ of $\tilde{H}$ is a FZI of $\tilde{\tilde{\omega}}$, for some $\vartheta\in(0,1]$, then $\tilde{H}$ is a FZI of $\tilde{\tilde{\omega}}$.

Proof Assume that, $(\tilde{H}_\vartheta)^M$ is a FZI of $\tilde{\tilde{\omega}}$ for some $\vartheta\in[0,T]$. Let $\acute{r},\acute{s}\in\tilde{\tilde{\omega}}$

$$\begin{aligned}\vartheta\tilde{H}(\acute{r}) &= (\tilde{H}_\vartheta)^M(0)\\ &\geq (\tilde{H}_\vartheta)^M(\acute{r})\\ &= \vartheta\tilde{H}(\acute{r})\end{aligned}$$

And so

$$\Rightarrow \tilde{H}(0)\geq\tilde{H}(\acute{r})$$

$$\begin{aligned}\vartheta\tilde{H}(\acute{r}) &= (\tilde{H}_\vartheta)^M(\acute{r})\\ &\geq (\tilde{H}_\vartheta)^M(\acute{r}*\acute{s})\wedge(\tilde{H}_\vartheta)^M(\acute{s})\\ &= (\vartheta\tilde{H}(\acute{r}*\acute{s}))\wedge(\vartheta\tilde{H}(\acute{s}))\\ &= \vartheta\tilde{H}(\acute{r}*\acute{s})\wedge\tilde{H}(\acute{s})\end{aligned}$$

and so

$$\tilde{H}(\acute{r})\geq(\acute{r}*\acute{s})\wedge\tilde{H}(\acute{s})$$

Hence, $\tilde{H}$ is a FZI of $\tilde{\tilde{\omega}}$.

Theorem:3.14 If, $\tilde{H}$ is a FZI of $\tilde{\omega}$, then the $F\vartheta - M(\tilde{H}_\vartheta)^M(\acute{r})$ of $\tilde{H}$ is a FZI of $\tilde{\omega}$, for all $\vartheta \in (0,1]$.

Proof Let, $\tilde{H}$ be a FZI of $\tilde{\omega}$ and let $\vartheta \in (0,1]$
Then

$$\begin{aligned}
(\tilde{H}_\vartheta)^M(0) &= \vartheta\tilde{H}(\acute{r}) \\
&\geq \vartheta\tilde{H}(\acute{r}) \\
&= \tilde{H}^M_\vartheta(\acute{r}) \\
\Rightarrow \tilde{H}^M_\vartheta(0) &\geq (\tilde{H}_\vartheta)^M(\acute{r}) \\
(\tilde{H}_\vartheta)^M(\acute{r}) &= \vartheta\tilde{H}(\acute{r}) \\
&\geq \vartheta\tilde{H}(\acute{r}*\acute{s}) \wedge \tilde{H}(\acute{s}) \\
&= \vartheta\tilde{H}(\acute{r}*\acute{s}) \wedge \tilde{H}(\acute{s}) \\
&= (\vartheta\tilde{H}(\acute{r}*\acute{s})) \wedge (\vartheta\tilde{H}(\acute{s})) \\
&\geq (\tilde{H}_\vartheta)^M(\acute{r}*\acute{s}) \wedge (\tilde{H}_\vartheta)^M(\acute{s}) \\
((\Rightarrow \tilde{H})_\vartheta)^M(\acute{r}) &\geq (\tilde{H}_\vartheta)^M(\acute{r}*\acute{s}) \wedge (\tilde{H}_\vartheta)^M(\acute{s})
\end{aligned}$$

Hence, $(\tilde{H}_\vartheta)^M$ of $\tilde{H}$ is a FZI of $\tilde{\omega}, \forall \acute{r}, \acute{s} \in (0,1]$.

Theorem:3.15 Let, $\vartheta \in (0,1]$ and let, $\tilde{H}$ be a FZI of a Z-algebra $\tilde{\omega}$. Then the F $\vartheta - M(\tilde{H}_\vartheta)^M(\acute{r})$ of $\tilde{H}$ is a FZSA of $\tilde{\omega}$.

Proof Let, $\acute{r}, \acute{s} \in \tilde{\omega}$.
Now, we have

$$\begin{aligned}
(\tilde{H}_\vartheta)^M(\acute{r}*\acute{s}) &= \vartheta\tilde{H}(\acute{r}*\acute{s}) \\
&\geq \vartheta\tilde{H}((\acute{r}*\acute{s})*\acute{s}) \wedge \tilde{H}(\acute{s}) \\
&= \vartheta\tilde{H}(\acute{s}*(\acute{r}*\acute{s})) \wedge \vartheta\tilde{H}(\acute{s}) \\
&= \vartheta\tilde{H}(0) \wedge \tilde{H}(\acute{s}) \\
&\geq \vartheta\tilde{H}(\acute{r}) \wedge \tilde{H}(\acute{s}) \\
&\geq (\vartheta\tilde{H}(\acute{r})) \wedge (\vartheta\tilde{H}(\acute{s})) \\
&= (\tilde{H}_\vartheta)^M(\acute{r}) \wedge (\tilde{H}_\vartheta)^M(\acute{s})
\end{aligned}$$

Hence, $(\tilde{H}_\vartheta)^M$ is a FZSA of $\tilde{\omega}, \forall \acute{r}, \acute{s} \in (0,1]$.

Theorem:3.16 If the $F\vartheta - T(\tilde{H}_\vartheta)^M$ of $\tilde{H}$ is a FZSA of $\tilde{\omega}, \vartheta \in [0,1]$, then $\tilde{H}$ is a FZSA of $\tilde{\omega}$.

Theorem:Proof Let us assume that, $(\tilde{H}_\vartheta)^M$ of $\tilde{H}$ is a FZI of $\tilde{\omega}$.
Then

$$\begin{aligned}
\vartheta\tilde{H}(\acute{r}*\acute{s}) &= (\tilde{H}_\vartheta)^M(\acute{r}*\acute{s}) \\
&\geq (\tilde{H}_\vartheta)^M((\acute{r}*\acute{s})*\acute{s}) \wedge (\tilde{H}_\vartheta)^M(\acute{s}) \\
&= (\tilde{H}_\vartheta)^M(\acute{s}*(\acute{r}*\acute{s})) \wedge (\tilde{H}_\vartheta)^M(\acute{s}) \\
&= (\tilde{H}_\vartheta)^M(0) \wedge (\tilde{H}_\vartheta)^M(\acute{s}) \\
&\geq (\tilde{H}_\vartheta)^M(\acute{r}) \wedge (\tilde{H}_\vartheta)^M(\acute{s}) \\
&= (\vartheta\tilde{H}(\acute{r})) \wedge (\vartheta\tilde{H}(\acute{s}))
\end{aligned}$$

$\Rightarrow \tilde{H}(\acute{r}*\acute{s}) \geq \tilde{H}(\acute{r}) \wedge \tilde{H}(\acute{s})$

Hence, $\tilde{H}$ is a FZSA of $\tilde{\omega}$.

Theorem:3.17 Intersection and union of any two $\vartheta - T$ of a FZI of $\tilde{H}$ of $\tilde{\omega}$ is also a FZI of $\tilde{\omega}$.

Proof Let $(\tilde{H}_\vartheta)^T$ and $(\tilde{H}_\delta)^T$ be two $F\vartheta - T$ of a FZI of $\tilde{H}$ of $\tilde{\omega}$, where $\vartheta, \delta \in [0,1]$.
Assume that $\vartheta \leq \delta$. Then by theorem 3.14, $(\tilde{H}_\vartheta)^T$ and $(\tilde{H}_\delta)^T$ are FZIs of $\tilde{\omega}$.
Now,

$$\begin{aligned}
(((\tilde{H})_\vartheta)^T \cap (\tilde{H}_\delta)^T)(\acute{r}) &= (\tilde{H}_\vartheta)^T(\acute{r}) \wedge (\tilde{H}_\delta)^T(\acute{r}) \\
&= (\tilde{H}(\acute{r}) + \vartheta) \wedge (\tilde{H}(\acute{r}) + \delta) \\
&= \tilde{H}(\acute{r}) + \vartheta \\
&= \tilde{H}^T_\vartheta(\acute{r})
\end{aligned}$$

And

$$\begin{aligned}
(((\tilde{H})_\vartheta)^T \cup (\tilde{H}_\delta)^T)(\acute{r}) &= (\tilde{H}_\vartheta)^T(\acute{r}) \vee (\tilde{H}_\delta)^T(\acute{r}) \\
&= (\tilde{H}(\acute{r}) + \vartheta) \vee (\tilde{H}(\acute{r}) + \delta) \\
&= \tilde{H}(\acute{r}) + \delta \\
&= \tilde{H}^T_\delta(\acute{r})
\end{aligned}$$

Hence, $(\tilde{H}_\vartheta)^T \cap (\tilde{H}_\delta)^T$ and $(\tilde{H}_\vartheta)^T \cup (\tilde{H}_\delta)^T$ are FZIs of $\tilde{\omega}$.

4 Characteristics of Structures of $\vartheta-$Translation and $\vartheta-$Multiplication in Z-Subalgebra and ideal of Q-Fuzzy Z-Algebra

Definition:4.1 Let $\tilde{H}$ and $Q-$be two fuzzy subsets of $\tilde{\omega}$ and $\vartheta \in [0,T]$. A mapping $\tilde{H}^T_\vartheta : \tilde{\omega} * Q \to [0,1]$ is said to be a Q-fuzzy ϑ- translation of $\tilde{H}$ if it satisfies $\tilde{H}^T_\vartheta = \tilde{H}(\acute{r}, q) + \vartheta, \forall \acute{r} \in \tilde{\omega}$ and $q \in Q$.

Definition:4.2 Let $\tilde{H}$ and $Q-$be two fuzzy subsets of $\tilde{\omega}$ and $\vartheta \in [0,T]$. A mapping $\tilde{H}^M_\vartheta : \tilde{\omega} * Q \to [0,1]$ is said to be a Q-fuzzy ϑ- translation of $\tilde{H}$ if it satisfies $\tilde{H}^M_\vartheta = \tilde{H}(\acute{r}, q).\vartheta, \forall \acute{r} \in \tilde{\omega}$ and $q \in Q$.

Example:4.3 Let $\tilde{\omega} = 0,1,2,3$ be the set with the following table.

Table 2: Fuzzy Q-Algebra Data Set

$*$	0	1	2	3
0	0	1	2	3
1	1	0	1	1
2	2	2	0	2
3	3	3	3	0

Then $(\tilde{\omega}, *, 0)$ is a Z-algebra.
Define Q-fuzzy set $\tilde{H}$ is of $\tilde{\omega}$ by

$$\tilde{H}(\acute{r}) = \begin{cases} 0.4, \text{ if } \acute{r} \neq 1, \\ 0.3, \text{ if } \acute{r} = 1 \end{cases}$$

Thus $\tilde{H}$ is a Q-fuzzy Z-sub algebra of X.
Hence $T = 1 - sup\tilde{H}(\acute{r}, q)/\acute{r} \in \tilde{\omega} \& q \in Q = 1 - 0.4 = 0.6,$

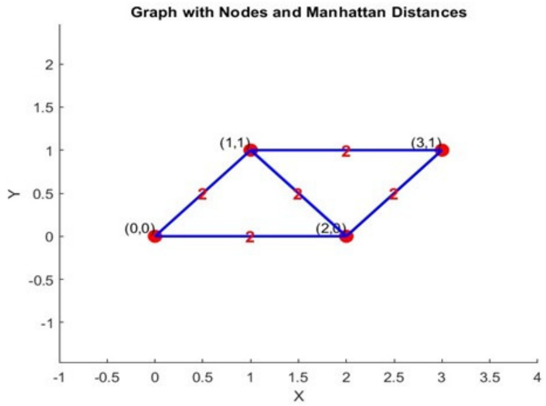

Fig. 3: Graph Representation with (x,y) Node Labels

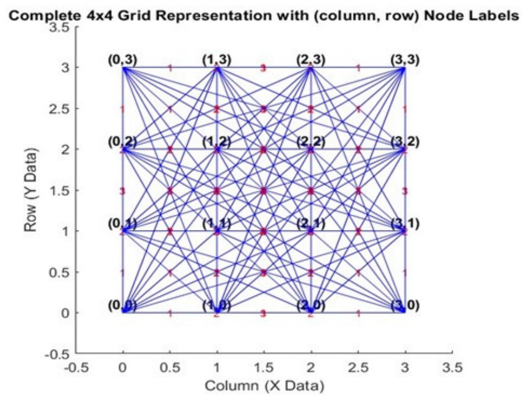

Fig. 4: Graph representation of Cayley tamale using with Fuzzy Z-Algebra

Choose $\vartheta = 0.2 \in [0,1]$ and $\vartheta = 0.3 \in [0,1]$.
Then the mapping $\tilde{H}_((0.2)^T) : \tilde{\omega} \to [0,1]$ is defined by

$$\tilde{H}_((0.2)^T) = \begin{cases} 0.2+0.4 = 0.6, \text{ if } \acute{r} \neq 1, \\ 0.2+0.3 = 0.5, \text{ if } \acute{r} = 1 \end{cases}$$

Which satisfies $\tilde{H}_((0.2)^T)(\acute{r}) = \tilde{H}(\acute{r}) + 0.2, \forall \acute{r} \in \tilde{\omega}$, is a fuzzy 0.2-translation.
The mapping $(\gamma)_((0.3)^M) : \tilde{\omega} \to [0,1]$ is defined by

$$\tilde{H}_((0.3)^M) = \begin{cases} (0.3*0.4 = 0.12, \text{ if } \acute{r} \neq 1, \\ 0.3*0.3 = 0.09, \quad \text{if } \acute{r} = 1 \end{cases}$$

Which satisfies $\tilde{H}_((0.3)^M)(\acute{r}) = \tilde{H}(\acute{r})(0.3), \forall \acute{r} \in \tilde{\omega}$ and $q \in Q$, is a fuzzy $0.3-$Multiplication.

Theorem:4.4 If $\tilde{H}$ of $\tilde{\omega}$ is a Q-fuzzy Z-sub-algebra and $\vartheta \in [0,1]$, then the Q- fuzzy ϑ-translation.$(\tilde{H}_\vartheta)^T(\acute{r},q)$ of $\tilde{H}$ is also a Q- fuzzy Z- sub algebra of $\tilde{\omega}$.

Proof Let $\acute{r},\acute{s} \in \tilde{\omega}, \vartheta \in [0,T]$ and $q \in Q$
Then, $\tilde{H}(\acute{r}*\acute{s},q) \geq \tilde{H}(\acute{r},q) \wedge \tilde{H}(\acute{s},q)$
Now,

$$\begin{aligned}(\tilde{H}_\vartheta)^T(\acute{r}*\acute{s},q) &= \tilde{H}(\acute{r}*\acute{s},q)+\vartheta \\ &\geq [\tilde{H}(\acute{r},q) \wedge \tilde{H}(\acute{s},q)]+\vartheta \\ &= [(\tilde{H}(\acute{r},q)+\vartheta) \wedge (\tilde{H}(\acute{s},q)+\vartheta)] \\ &= [(\tilde{H}_\vartheta)^T(\acute{r},q) \wedge (\tilde{H}_\vartheta)^T(\acute{s},q)].\forall \acute{r},\acute{s} \\ &\in \tilde{\omega} \text{ and } q \in Q\end{aligned}$$

Theorem:4.5 Let $\tilde{H}$ and Q be a two fuzzy subset of $\tilde{\omega}$ such that the Q- fuzzy ϑ-translation $(\tilde{H}_\vartheta)^T(\acute{r},q)$ of $\tilde{H}$ is a Q-fuzzy sub algebra of $\tilde{\omega}$, for some $\vartheta \in [0,T]$, then $\tilde{H}$ is a Q-fuzzy Z-sub algebra of $\tilde{\omega}$.

Proof Assume that $(\tilde{H}_\vartheta)^T(\acute{r},q)$ is a Q-fuzzy sub algebra of $\tilde{\omega}$ for some $\vartheta \in [0,T]$
Let $\acute{r},\acute{s} \in \tilde{\omega}$ and $q \in Q$ we have

$$\begin{aligned}\tilde{H}(\acute{r}*\acute{s},q)+\vartheta &= (\tilde{H}_\vartheta)^T(\acute{r}*\acute{s},q) \\ &\geq [(\tilde{H}_\vartheta)^T(\acute{r},q) \wedge (\tilde{H}_\vartheta)^T(\acute{s},q)] \\ &= [(\tilde{H}(\acute{r},q)+\vartheta) \wedge (\tilde{H}(\acute{s},q)+\vartheta)] \\ &= [\tilde{H}(\acute{r},q) \wedge \tilde{H}(\acute{s},q)]+\vartheta\end{aligned}$$
$$\implies \tilde{H}(\acute{r}*\acute{s},q) \geq [\tilde{H}(\acute{r},q) \wedge \tilde{H}(\acute{s},q)], \forall \acute{r},\acute{s} \in \tilde{\omega} \text{ and } q \in Q$$

Hence, $\tilde{H}$ is Q-fuzzy sub algebra of $\tilde{\omega}$.

Theorem:4.6 For any Q- fuzzy Z- sub algebra $\tilde{H}$ of $\tilde{\omega}$ and $\vartheta \in [0,1]$, if the Q-fuzzy ϑ-multiplication $(\tilde{H}_\vartheta)^M(\acute{r},q)$ of $\tilde{H}$ is a Q-fuzzy Z-sub algebra of $\tilde{\omega}$.

Proof Let $\acute{r},\acute{s} \in \tilde{\omega}, \vartheta \in [0,T]$ and $q \in Q$
Then $\tilde{H}(\acute{r}*\acute{s},q) \geq \tilde{H}(\acute{r},q) \wedge \tilde{H}(\acute{s},q)$
Now,

$$\begin{aligned}(\tilde{H}_\vartheta)^M(\acute{r}*\acute{s},q) &= \vartheta\tilde{H}(\acute{r}*\acute{s},q) \\ &\geq \vartheta[\tilde{H}(\acute{r},q) \wedge \tilde{H}(\acute{s},q)] \\ &\geq [\vartheta\tilde{H}(\acute{r},q) \wedge \vartheta\tilde{H}(\acute{s},q)] \\ &= [(\tilde{H}_\vartheta)^M(\acute{r},q) \wedge (\tilde{H}_\vartheta)^M(\acute{s},q)]\end{aligned}$$
$$\implies (\tilde{H}_\vartheta)^M(\acute{r}*\acute{s},q) \geq [(\tilde{H}_\vartheta)^M(\acute{r},q) \wedge (\tilde{H}_\vartheta)^M(\acute{s},q)]$$

Therefore, $(\tilde{H}_\vartheta)^M$ is a Q-fuzzy Z-sub algebra of $\tilde{\omega}$.

Theorem:4.7 For any fuzzy subset $\tilde{H}$ of $\tilde{\omega}, q \in Q$ and $\vartheta \in [0,1]$, if the Q-fuzzy ϑ-multiplication $(\tilde{H}_\vartheta)^M(\acute{r},q)$ of $\tilde{H}$ is a Q-fuzzy Z-sub algebra of $\tilde{\omega}$, then so is $\tilde{H}$.

Proof Assume that $(\tilde{H}_\vartheta)^M(\acute{r},q)$ of $\tilde{H}$ is a Q-fuzzy Z-sub algebra of $\tilde{\omega}$ for some $\vartheta \in [0,T]$
Let $\acute{r},\acute{s} \in \tilde{\omega}$ and $q \in Q$ we have

$$\begin{aligned}\vartheta\tilde{H}(\acute{r}*\acute{s},q) &= (\tilde{H}_\vartheta)^M(\acute{r}*\acute{s},q) \\ &\geq [(\tilde{H}_\vartheta)^M(\acute{r},q) \wedge (\tilde{H}_\vartheta)^M(\acute{s},q)] \\ &= [\vartheta\tilde{H}(\acute{r},q) \wedge \vartheta\tilde{H}(\acute{s},q)] \\ &= \vartheta[\tilde{H}(\acute{r},q) \wedge \tilde{H}(\acute{s},q)]\end{aligned}$$
$$\implies \tilde{H}(\acute{r}*\acute{s},q) \geq \vartheta[\tilde{H}(\acute{r},q) \wedge \tilde{H}(\acute{s},q)]$$

Hence, $\tilde{H}$ is a Q-fuzzy Z-sub algebra of $\tilde{\omega}$.

Theorem:4.8 If the Q-fuzzy ϑ-translation $(\tilde{H}_\vartheta)^T(\acute{r})$ of $\tilde{H}$ is a Q-fuzzy Z-Ideal, then it satisfies the condition $(\tilde{H}_\vartheta)^T(\acute{s}*(\acute{r}*\acute{s}),q) \geq (\tilde{H}_\vartheta)^T(\acute{r},q)$.

Proof

$$
\begin{aligned}
(\tilde{H}_\vartheta)^T(\acute{s}*(\acute{r}*\acute{s}),q) &= \tilde{H}(\acute{s}*(\acute{r}*\acute{s}),q)+\vartheta \\
&\geq \tilde{H}(0*(\acute{s}*(\acute{r}*\acute{s})),q) \\
&+\vartheta\wedge\tilde{H}(0,q)+\vartheta \\
&\geq \tilde{H}(0*(\acute{s}*(\acute{s}*\acute{r})),q) \\
&+\vartheta\wedge\tilde{H}(0,q)+\vartheta \\
&= \tilde{H}(0*((\acute{s}*\acute{s})*\acute{r}),q) \\
&+\vartheta\wedge\tilde{H}(0,q)+\vartheta \\
&= \tilde{H}(0*(\acute{s}*\acute{r}),q) \\
&+\vartheta\wedge\tilde{H}(0,q)+\vartheta \\
&= \tilde{H}((\acute{s}*\acute{r})*0,q) \\
&+\vartheta\wedge\tilde{H}(0,q)+\vartheta \\
&\geq \tilde{H}((\acute{s}*\acute{r})*0,q)+\vartheta\wedge\tilde{H}(\acute{r},q) \\
&+\vartheta \\
&\geq (\tilde{H}_\vartheta)^T(0,q)\wedge(\tilde{H}_\vartheta)^T(\acute{r},q) \\
&= (\{\tilde{H}_\vartheta)^T(\acute{r},q).\} \\
((\Rightarrow\tilde{H})_\vartheta)^T(\acute{s}*(\acute{r}*\acute{s}),q) &\geq (\tilde{H}_\vartheta)^T(\acute{r},q)\forall\acute{r},\acute{s}\in\tilde{\omega}\text{and } q\in Q
\end{aligned}
$$

Theorem:4.9 If $\tilde{H}$ is a Q-fuzzy Z-ideal of $\tilde{\omega}$, then the Q-fuzzy ϑ-translation $(\tilde{H}_\vartheta)^T(\acute{r},q)$ of $\tilde{H}$ is a Q-fuzzy Z-ideal of $\tilde{\omega}$, for all $\vartheta\in[0,T]$.

Proof Let $\tilde{H}$ be a Q-fuzzy Z-ideal of $\tilde{\omega}$ and let $\vartheta\in[0,T]$ and $q\in Q$
Then,

$$
\begin{aligned}
(i)\quad (\tilde{H}_\vartheta)^T(0,q) &= \tilde{H}(0,q)+\vartheta \\
&\geq \tilde{H}(\acute{r},q)+\vartheta \\
&= (\tilde{H}_\vartheta)^T(\acute{r},q) \\
(ii)\quad (\tilde{H}_\vartheta)^T(\acute{r},q) &= \tilde{H}(\acute{r},q)+\vartheta \\
&\geq \tilde{H}(\acute{r}*\acute{s},q)\wedge\tilde{H}(\acute{s},q)+\vartheta \\
&= (\tilde{H}(\acute{r}*\acute{s},q)+\vartheta)\wedge(\tilde{H}(\acute{s},q)+\vartheta) \\
&= (\tilde{H}_\vartheta)^T(\acute{r}*\acute{s},q)\wedge(\tilde{H}_\vartheta)^T(\acute{s},q) \\
\Rightarrow (\tilde{H}_\vartheta)^T(\acute{r},q) &\geq (\tilde{H}_\vartheta)^T(\acute{r}*\acute{s},q)\wedge(\tilde{H}_\vartheta)^T(\acute{s},q)
\end{aligned}
$$

Hence $(\tilde{H}_\vartheta)^T(\acute{r},q)$ of $\tilde{H}$ is a Q-fuzzy Z-ideal of $\tilde{\omega},\forall\vartheta\in[0,T]$ and $q\in Q$.

Theorem:4.10 Let $\tilde{H}$ is a fuzzy subset of $\tilde{\omega}$ and $q\in Q$ such that the Q-fuzzy ϑ-translation $(\tilde{H}_\vartheta)^T(\acute{r},q)$ of $\tilde{H}$ is a Q-fuzzy Z-ideal of $\tilde{\omega}$, for some $\vartheta\in[0,T]$, then $\tilde{H}$ is a Q-fuzzy Z-ideal of $\tilde{\omega}$.

Proof Assume that $(\tilde{H}_\vartheta)^T$ is a Q-fuzzy Z-ideal of $\tilde{\omega}$ for some $\vartheta\in[0,T]$.
Let $\acute{r},\acute{s}\in\tilde{\omega}$ and $q\in Q$
Then,

$$
\begin{aligned}
\tilde{H}(0,q)+\vartheta &= (\tilde{H}_\vartheta)^T(0,q) \\
&\geq (\tilde{H}_\vartheta)^T(\acute{r},q) \\
&= \tilde{H}(\acute{r},q)+\vartheta
\end{aligned}
$$

And so

$$
\begin{aligned}
\Rightarrow \tilde{H}(0,q) &\geq \tilde{H}(\acute{r},q) \\
\tilde{H}(\acute{r},q)+\vartheta &= (\tilde{H}_\vartheta)^T(\acute{r},q) \\
&\geq (\tilde{H}_\vartheta)^T(\acute{r}*\acute{s},q)\wedge(\tilde{H}_\vartheta)^T(\acute{s},q) \\
&= (\tilde{H}(\acute{r}*\acute{s},q)+\vartheta)\wedge(\tilde{H}(\acute{s},q)+\vartheta) \\
&= \tilde{H}(\acute{r}*\acute{s},q)\wedge\tilde{H}(\acute{s},q)+\vartheta
\end{aligned}
$$

and so$\tilde{H}(\acute{r},q)\geq(\acute{r}*\acute{s},q)\wedge\tilde{H}(\acute{s},q)$

Hence $\tilde{H}$ is a Q-fuzzy Z-ideal of $\tilde{\omega}$.

Theorem:4.11 Let $\vartheta\in[0,T],q\in Q$ and let $\tilde{H}$ be a Q-fuzzy Z-ideal of $\tilde{\omega}$. If $\tilde{\omega}$ is a Z-algebra, then the fuzzy ϑ-translation $(\tilde{H}_\vartheta)^T$ of $\tilde{H}$ is a Q-fuzzy Z-sub-algebra of $\tilde{\omega}$.

Proof Let $\acute{r},\acute{s}\in\tilde{\omega}$ and $q\in Q$
Now, we have

$$
\begin{aligned}
(\tilde{H}_\vartheta)^T(\acute{r}*\acute{s},q) &\cong \tilde{H}(\acute{r}*\acute{s},q)+\vartheta \\
&\geq \tilde{H}((\acute{r}*\acute{s},)*\acute{s},q)\wedge\tilde{H}(\acute{s},q)+\vartheta \\
&= \tilde{H}(\acute{s}*(\acute{r}*\acute{s}),q)\wedge\tilde{H}(\acute{s},q)+\vartheta \\
\text{by Theorem 3.7}& \\
&\geq \tilde{H}(0,q)\wedge\tilde{H}(\acute{s},q)+\vartheta \\
&\geq \tilde{H}(\acute{r},q)\wedge\tilde{H}(\acute{s},q)+\vartheta \\
&\geq (\tilde{H}(\acute{r},q)+\vartheta)\wedge(\tilde{H}(\acute{s},q)+\vartheta) \\
&= (\tilde{H}_\vartheta)^T(\acute{r},q)\wedge(\tilde{H}_\vartheta)^T(\acute{s},q)
\end{aligned}
$$

Hence $(\tilde{H}_\vartheta)^T$ is a Q-fuzzy Z-sub-algebra of $\tilde{\omega}$.

Theorem:4.12 If the Q-fuzzy ϑ-translation $(\tilde{H}_\vartheta)^T$ of $\tilde{H}$ is a Q-fuzzy Z-sub-algebra of $\tilde{\omega},\vartheta\in[0,T]$, then $\tilde{H}$ is a Q-fuzzy Z-sub-algebra of $\tilde{\omega}$.

Proof Let us assume that $(\tilde{H}_\vartheta)^T$ of $\tilde{H}$ is a Q-fuzzy Z-ideal of $\tilde{\omega}$ and $q\in Q$
Then

$$
\begin{aligned}
\tilde{H}(\acute{r}*\acute{s},q)+\vartheta &= (\tilde{H}_\vartheta)^T(\acute{r}*\acute{s},q) \\
&\geq (\tilde{H}_\vartheta)^T((\acute{r}*\acute{s})*\acute{s},q)\wedge(\tilde{H}_\vartheta)^T(\acute{s},q) \\
&= (\tilde{H}_\vartheta)^T(\acute{s}*(\acute{r}*\acute{s}),q)\wedge(\tilde{H}_\vartheta)^T(\acute{s},q) \\
\text{by Theorem 3.7}& \\
&\geq (\tilde{H}_\vartheta)^T(0,q)\wedge(\tilde{H}_\vartheta)^T(\acute{s},q) \\
&\geq (\tilde{H}_\vartheta)^T(\acute{r},q)\wedge(\tilde{H}_\vartheta)^T(\acute{s},q) \\
&= (\tilde{H}(\acute{r},q)+\vartheta)\wedge(\tilde{H}(\acute{s},q)+\vartheta) \\
&= \tilde{H}(\acute{r},q)\wedge\tilde{H}(\acute{s},q)+\vartheta \\
\Rightarrow \tilde{H}(\acute{r}*\acute{s},q) &\geq \tilde{H}(\acute{r},q)\wedge\tilde{H}(\acute{s},q)
\end{aligned}
$$

Hence $\tilde{H}$ is a Q-fuzzy Z-sub algebra of $\tilde{\omega}$.

Theorem:4.13 Let $\tilde{H}$ is a fuzzy subset of $\tilde{\omega}$ and $q\in Q$ such that the Q-fuzzy ϑ-Multiplication $(\tilde{H}_\vartheta)^M(\acute{r},q)$ of $\tilde{H}$ is a Q-fuzzy Z-ideal of $\tilde{\omega}$, for some $\vartheta\in[0,1]$, then $\tilde{H}$ is a Q-fuzzy Z-ideal of $\tilde{\omega}$.

Proof Assume that $(\tilde{H}_\vartheta)^M$ is a Q-fuzzy Z-ideal of $\tilde{\omega}$ for some $\vartheta\in[0,T]$.

Let $\acute{r},\acute{s} \in \tilde{\tilde{\omega}}$ and $q \in Q$

$$\begin{aligned}\vartheta\tilde{H}(\acute{r},q) &= (\tilde{H}_\vartheta)^M(0,q)\\ &\geq (\tilde{H}_\vartheta)^M(\acute{r},q)\\ &= \vartheta\tilde{H}(\acute{r},q)\end{aligned}$$

And so $\Rightarrow \tilde{H}(0,q) \geq \tilde{H}(\acute{r},q)$

$$\begin{aligned}\vartheta\tilde{H}(\acute{r},q) = (\tilde{H}_\vartheta)^M(\acute{r},q) &\\ &\geq (\tilde{H}_\vartheta)^M(\acute{r}*\acute{s},q)\wedge(\tilde{H}_\vartheta)^M(\acute{s},q)\\ &= (\vartheta\tilde{H}(\acute{r}*\acute{s},q))\wedge(\vartheta\tilde{H}(\acute{s},q))\\ &= \vartheta\tilde{H}(\acute{r}*\acute{s},q)\wedge\tilde{H}(\acute{s},q)\end{aligned}$$

And so $\Rightarrow \tilde{H}(\acute{r},q) \geq \tilde{H}(\acute{r}*\acute{s},q)\wedge\tilde{H}(\acute{s},q)$

Hence $\tilde{H}$ is a Q-fuzzy Z-ideal of $\tilde{\tilde{\omega}}$.

Theorem:4.14 If $\tilde{H}$ is a Q-fuzzy Z-ideal of $\tilde{\tilde{\omega}}$, then the Q-fuzzy ϑ-multiplication $(\tilde{H}_\vartheta)^M(\acute{r},q)$ of $\tilde{H}$ is a Q-fuzzy Z-ideal of $\tilde{\tilde{\omega}}$, for all $\vartheta \in (0,1]$.

Proof Let $\tilde{H}$ be a Q-fuzzy Z-ideal of $\tilde{\tilde{\omega}}$ and let $\vartheta \in (0,1]$ and $q \in Q$

Then

$$\begin{aligned}(\tilde{H}_\vartheta)^M(0,q) &= \vartheta\tilde{H}(\acute{r},q)\\ &\geq \vartheta\tilde{H}(\acute{r},q)\\ &= \tilde{H}^M_\vartheta(\acute{r},q)\\ \Rightarrow (\tilde{H}_\vartheta)^M(0,q) &\geq (\tilde{H}_\vartheta)^M(\acute{r},q)\\ (\tilde{H}_\vartheta)^M(\acute{r},q) &= \vartheta\tilde{H}(\acute{r},q)\\ &\geq \vartheta\tilde{H}(\acute{r}*\acute{s},q)\wedge\tilde{H}(\acute{s},q)\\ &= \vartheta\tilde{H}(\acute{r}*\acute{s},q)\wedge\tilde{H}(\acute{s},q)\\ &= (\vartheta\tilde{H}(\acute{r}*\acute{s},q))\wedge(\vartheta\tilde{H}(\acute{s},q))\\ &\geq (\tilde{H}_\vartheta)^M(\acute{r}*\acute{s},q)\wedge(\tilde{H}_\vartheta)^M(\acute{s},q)\end{aligned}$$

$$((\Rightarrow\tilde{H})_\vartheta)^M(\acute{r},q) \geq (\tilde{H}_\vartheta)^M(\acute{r}*\acute{s},q)\wedge(\tilde{H}_\vartheta)^M(\acute{s},q)$$

Hence $(\tilde{H}_\vartheta)^M$ of $\tilde{H}$ is a Q-fuzzy Z-ideal of $\tilde{\tilde{\omega}}, \forall \acute{r},\acute{s} \in [0,1]$.

Theorem:4.15 Let $\vartheta \in (0,1]$ and let $\tilde{H}$ be a Q-fuzzy Z-ideal of a Z-algebra $\tilde{\tilde{\omega}}$. Then the Q-fuzzy ϑ-multiplication $(\tilde{H}_\vartheta)^M(\acute{r})$ of $\tilde{H}$ is a Q-fuzzy Z-sub algebra of $\tilde{\tilde{\omega}}$.

Proof Let $\acute{r},\acute{s} \in \tilde{\tilde{\omega}}$ and $q \in Q$

Now, we have

$$\begin{aligned}(\tilde{H}_\vartheta)^M(\acute{r}*\acute{s},q) &= \vartheta\tilde{H}(\acute{r}*\acute{s},q)\\ &\geq \vartheta\tilde{H}((\acute{r}*\acute{s})*\acute{s}),q\wedge(\tilde{H}(\acute{s},q))\\ &= \vartheta\tilde{H}((\acute{s}*(\acute{r}*\acute{s})),q)\wedge\vartheta\tilde{H}(\acute{s},q)\\ &= \vartheta\tilde{H}(0,q)\wedge\tilde{H}(\acute{s},q)\\ &\geq \vartheta\tilde{H}(\acute{r},q)\wedge\tilde{H}(\acute{s},q)\\ &\geq (\vartheta\tilde{H}(\acute{r},q))\wedge(\vartheta\tilde{H}(\acute{s},q))\\ &= (\tilde{H}_\vartheta)^M(\acute{r},q)\wedge(\tilde{H}_\vartheta)^M(\acute{s},q)\end{aligned}$$

Hence $(\tilde{H}_\vartheta)^M$ is a Q-fuzzy Z-sub-algebra of $\tilde{\tilde{\omega}}, \forall \acute{r},\acute{s} \in (0,1]$ and $q \in Q$

Theorem:4.16 If the Q-fuzzy ϑ-translation $(\tilde{H}_\vartheta)^M$ of $\tilde{H}$ is a Q-fuzzy Z-sub-algebra of $\tilde{\tilde{\omega}}, \vartheta \in (0,1]$, then $\tilde{H}$ is a Q-fuzzy Z-sub-algebra of $\tilde{\tilde{\omega}}$.

Proof Let us assume that $(\tilde{H}_\vartheta)^M$ of $\tilde{H}$ is a Q-fuzzy Z-ideal of $\tilde{\tilde{\omega}}$ and $q \in Q$

Then

$$\begin{aligned}\vartheta\tilde{H}(\acute{r}*\acute{s},q) &= (\tilde{H}_\vartheta)^M(\acute{r}*\acute{s},q)\\ &\geq (\tilde{H}_\vartheta)^M((\acute{r}*\acute{s})*\acute{s},q)\wedge(\tilde{H}_\vartheta)^M(\acute{s},q)\\ &= (\tilde{H}_\vartheta)^M(\acute{s}*(\acute{r}*\acute{s}),q)\wedge(\tilde{H}_\vartheta)^M(\acute{s},q)\\ &= (\tilde{H}_\vartheta)^M(0,q)\wedge(\tilde{H}_\vartheta)^M(\acute{s},q)\\ &\geq (\tilde{H}_\vartheta)^M(\acute{r},q)\wedge(\tilde{H}_\vartheta)^M(\acute{s},q)\\ &= (\vartheta\tilde{H}(\acute{r},q))\wedge(\vartheta\tilde{H}(\acute{s},q))\end{aligned}$$

$$\Rightarrow \tilde{H}(\acute{r}*\acute{s},q) \geq \tilde{H}(\acute{r},q)\wedge\tilde{H}(\acute{s},q)$$

Hence, $\tilde{H}$ is a Q fuzzy Z-sub algebra of $\tilde{\tilde{\omega}}$.

Theorem:4.17 Intersection and union of any two ϑ-translation of a Q-fuzzy Z-ideal of $\tilde{H}$ of $\tilde{\tilde{\omega}}$ is also a Q-fuzzy Z-ideal of $\tilde{\tilde{\omega}}$.

Proof Let $(\tilde{H}_\vartheta)^T$ and $(\tilde{H}_\delta)^T$ be two ϑ- translations of a Q-fuzzy Z-ideal of $\tilde{H}$ of $\tilde{\tilde{\omega}}$, where $\vartheta,\delta \in [0,1]$ and $q \in Q$

Then by theorem 3.14, $(\tilde{H}_\vartheta)^T$ and $(\tilde{H}_\delta)^T$ are Q-fuzzy Z-ideals of $\tilde{\tilde{\omega}}$.

Now,

$$\begin{aligned}(((\tilde{H})_\vartheta)^T\cap(\tilde{H}_\delta)^T)(\acute{r},q) &= (\tilde{H}_\vartheta)^T(\acute{r},q)\wedge(\tilde{H}_\delta)^T(\acute{r},q)\\ &= (\tilde{H}(\acute{r},q)+\vartheta)\wedge(\tilde{H}(\acute{r},q)+\delta)\\ &= \tilde{H}(\acute{r},q)+\vartheta\\ &= \tilde{H}^T_\vartheta(\acute{r},q)\\ \text{And } \tilde{H}^T_\vartheta\cup\tilde{H}^T_\delta(\acute{r},q) &= (\tilde{H}_\vartheta)^T(\acute{r},q)\vee(\tilde{H}_\delta)^T(\acute{r},q)\\ &= (\tilde{H}(\acute{r},q)+\vartheta)\vee(\tilde{H}(\acute{r},q)+\delta)\\ &= \tilde{H}(\acute{r},q)+\delta\\ &= \tilde{H}^T_\delta(\acute{r},q)\end{aligned}$$

Hence, $\tilde{H}_(\vartheta)^T\cap\tilde{H}_(\delta)^T$ and $\tilde{H}^T_\vartheta\cup\tilde{H}^T_\delta$ are Q-fuzzy Z-ideals of $\tilde{\tilde{\omega}}$.

5 Conclusion

In the present investigation, we discussed ϑ-T and ϑ-M on Z-Algebras through FZSA, as well as other features. Also, obtained from the ϑ-T and ϑ-M on FZI of Z-Algebra. And also derived from the ϑ-Translation and ϑ-Multiplication on Z-Ideals and Z-Subalgebra of Q-Fuzzy Z-Algebra.

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Suleiman Ibrahim Mohammad is a Professor of Business Management at Al al-Bayt University, Jordan (currently at Zarqa University, Jordan), with more than 17 years of teaching experience. He has published over 100 research papers in prestigious journals. He holds a PhD in Financial Management and an MCom from Rajasthan University, India, and a Bachelor's in Commerce from Yarmouk University, Jordan. His research interests focus on supply chain management, Marketing, and total quality (TQ). His ORCID ID is orcid.org/0000-0001-6156-9063.

M. Premkumar is Assistant Professor, Department of Mathematics, Sathyabama Institute of Science and Technology (Deemed to Be University), Chennai-600119, Tamilnadu, India, with 7 years of experience. Authored more than 60 papers in various international journals published, 3 Book Chapter Published, 1 Manograph Published and 61 Research papers Presented in International Conferences India and Abroad. Acted in 4 Reputed International Journal Editorial Board Member & Acted in 7 International Journal Review Board Member. His ORCID ID is orcid.org/0000-0002-8637-063X.

N. Raja has 19 years of experience in education and the media industry. Currently an Assistant Professor in the Department of Visual Communication at Sathyabama Deemed University, he has produced and edited over 100 television programs during his time as a Video Editor at Jesus Calls. Dr. Raja holds a MSc in Electronic Media, M.Phil. in Journalism and Mass Communication, a PG Diploma in Public Relations, and a PhD in Communication from Bharathiar University, where his research focused on the impact of social media as an educational tool for media students in Tamil Nadu. His ORCID ID is orcid.org/0000-0003-2135-3051.

V. Helan Sinthiya Assistant Professor Department of Mathematics, School of Engineering and Technology, Dhanalakshmi Srinivasan University, Trichy-621112 , Tamilnadu, India. I have more than 13 years 5 months of experience, Authored more than 8 papers in various national and international journals published, 3 Book Chapter Published,4 patent published, 5 Research papers Presented in International Conferences in India , IEEE journal reviewer and acted as Resource person for various college.

K. Malathi is assistant Professor, Department of Mathematics, Kongunadu College of Engineering and Technology, Thottiam, Namakkal Main Road, Thottiam, Trichy – 621 215, Tamilnadu, India, with 10 years of experience. Authored more than 3 papers in Scopus journals published, 2 Books Chapters Published.

G. Joel Jaikumar Madhuram is an Master of Science student, Department of Mathematics, Sathyabama Institute of Science and Technology (Deemed To Be University) , Chennai-600119, Tamilnadu, India. Authored more than one paper in an international journal published, and three Research papers Presented in International Conferences India. His ORCID ID is orcid.org/0009-0004-8783-7088.

P. Shanmugavel is assistant professor, department of Mathematics, Selvamm Arts and Science College (Autonomous), Namakkal, I have 10 years' Experience in teaching. I Published 5 Papers in Scopus and one paper in web of Science.

Asokan Vasudevan is a distinguished academic at INTI International University, Malaysia. He holds multiple degrees, including a PhD in Management from UNITEN, Malaysia, and has held key roles such as Lecturer, Department Chair, and Program Director. His research, published in esteemed journals, focuses on business management, ethics, and leadership. Dr. Vasudevan has received several awards, including the Best Lecturer Award from Infrastructure University Kuala Lumpur and the Teaching Excellence Award from INTI International University. His ORCID ID is orcid.org/0000-0002-9866-4045.

Mohammad Faleh Hunitie is a professor of public administration at the University of Jordan since 1989. Ph.D holder earned in 1988 from Golden Gate University, California, USA. Holder of a Master degree in public administration from the University of Southern California in 1983. Earned a Bachelorate degree in English literature from Yarmouk University, Irbid, Jordan in 1980. Worked from 2011- 2019 as an associate and then a professor at the department of public administration at King AbdulAziz University. Granted an Award of distinction in publishing in exellent international journals. Teaching since 1989 until now. Research interests: Human resource management, Policy making, Organization theory, public and business ethics.