Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.18576/jsap/140505

Theoretical Analysis of the New Extended Exponential-Linear Distribution with Numerical Application

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Received: 2 May 2025, Revised: 22 Jul. 2025, Accepted: 29 Aug. 2025
Published online: 1 Sep. 2025

Abstract: Traditional extensions of the exponential distribution, while aiming to capture more complex hazard rate behaviors and improve data fit, often introduce additional parameters that complicate crucial statistical processes such as estimation, inference, and model selection. This complexity is particularly evident with small or restricted datasets, where overparameterization can lead to identifiability issues and increased computational challenges. Addressing this critical need for parsimonious yet flexible models, this study proposes and explores the novel one-parameter "Extended Exponential-Linear" (EE-L) distribution. Unlike classical extensions, the EE-L enhances modeling flexibility for data exhibiting heavier tails or increasing hazard rates by multiplicatively modifying the exponential kernel with a linear function of the variable, thus preserving a single-parameter structure for easier estimation and interpretation. Statistical properties of the EE-L distribution, including its moments, hazard rate function, and quantile function, are derived. Simulation results demonstrate that parameter estimates for the EE-L distribution remained within 10 percent of actual values across various sample sizes (1000, 500, 200, 100, 50), with the exception of very small samples (20). Furthermore, its application to life testing data revealed that the maximum likelihood parameter estimate was not significantly different from the true parameter value. Finally, a comparative analysis of a waiting time dataset demonstrated the EE-L distribution's superior fitting performance compared to the exponential, inverse exponential, modified exponential, and Lomax distributions.

Keywords: Exponential distribution, exponential-linear distribution, reliability life testing, censored data

1 Introduction

The exponential distribution is a common model in probability and statistics used frequently in various domains including reliability engineering, survival analysis, and queueing theory [1,2]. Its mathematical tractability and memoryless characteristics make it a good starting point for representing non-negative continuous data. Elementary studies [3,4] defined the exponential distribution as a reasonable model for populations marked by skewed data, as noted in Equation (1).

$$f_X(x) = \lambda e^{-\lambda x},\ x > 0,\ \lambda > 0, \tag{1}$$

where X denotes a random variate from an exponential population, with λ being a scale parameter (the rate parameter). Despite the exponential distribution's utility in modeling skewed data, its inherent assumption of a constant hazard rate often proves restrictive in practice, particularly when analyzing phenomena that exhibit varying risk over time. This significant theoretical limitation has, in fact, motivated the development of numerous extensions that introduce additional parameters to capture more complex behaviors such as increasing, decreasing, or bathtub-shaped hazard functions [5,6].

These extended distributions provide the necessary flexibility to accurately model a broader spectrum of real-world phenomena with varying risk characteristics. Among these common extensions in literature are the Weibull distribution, the Gamma distribution and the generalized exponential family [7,8,9]. Notably, the Weibull distribution introduces a shape parameter to model monotone hazard rates while according to [8], the Gamma distribution generalizes the

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exponential by adding a shape parameter controlling skewness and tail behavior. Similarly, [7] incorporated additional parameters to the exponential distribution to increase its flexibility, particularly in modeling varied hazard functions.

While these modified models effectively address the exponential distribution's restrictive constant hazard rate assumption by offering increased fit and interpretability for diverse real-world phenomena, their enhanced flexibility comes with significant practical and theoretical trade-offs. Specifically, the inclusion of additional parameters, while improving model fidelity, typically complicates crucial statistical processes, like parameter estimation, inference, and model selection. This complexity is particularly pronounced in scenarios involving small samples or restricted data, where the estimation procedures may become unstable or unreliable [10]. Furthermore, this pursuit of greater flexibility through added parameters potentially contributes to issues of identifiability (where multiple parameter sets could produce similar model fits). It inherently increases the computational challenges associated with model fitting and analysis [11]. Thus, addressing one theoretical limitation introduces a new set of challenges related to model complexity and robustness.

Consequently, there is a need for parsimonious models that can enhance the flexibility of the exponential distribution without increasing the number of parameters. Such models may preserve the simplicity and interpretability of the exponential distribution while tolerating more realistic hazard rate shapes and tail behaviours. Therefore, this present study proposes and explores a novel one-parameter distribution, termed the "Extended Exponential-Linear" (EE-L) distribution. Unlike classical extensions that introduce additional shape parameters, the EE-L distribution achieves its enhanced flexibility by multiplicatively modifying the exponential kernel with a linear function of the variable. This innovative approach maintains the advantageous single-parameter structure, thereby facilitating simpler estimation and interpretation. The study further aims to derive the complete statistical properties of this novel EE-L distribution, including its moments, hazard rate function, and quantile function. It will finally assess its performance through comprehensive simulation studies.

The pursuit of enhanced distributional flexibility while maintaining parsimony has fueled considerable knowledge expansion, leading to numerous studies that demonstrate high interest in proposing novel methods to expand the family of standard probability distributions [12,13,14]. This widespread effort highlights the ongoing academic endeavor to develop models that provide improved data fit and realistic hazard rate behaviors without succumbing to the pitfalls of excessive parametric complexity. This wave of innovation includes the introduction of various new "generators" and families of distributions, such as the new generalised class of distributions [15], McDonald-G (Mc-G) family [16], beta Marshal-Olkin family [17], Kumaraswamy Marshal-Olkin family [18], log-gamma-G family [19], Weibull-G family [20], the exponentiated half-logistic family [21], Lomax Generator [22], a new lifetime exponential-X family [23], new extended-F family [24], a flexible reduced logarithmic-X family [25], a new extended-family of distributions [26], odd generalized exponential-G family [27], Logistic-X family [28], the extended odd Fréchet family [29], and the truncated Cauchy power family [30], among others.

We proceed with the study by organizing the ensuing paragraphs into four sections. Immediately after the introduction is the methods section, where the EE-L distribution and its validity are established. The next section considers the statistical properties of the EE-L distribution and its parameter estimations. The fourth section presents numerical simulations and applications together with a discussion of the results. In the final section, we provide the study's conclusion.

2 Methods

2.1 The extended exponential-linear (EE-L) distribution and its validity

Let X be a non-negative random variable. If X follows the 'Extended Exponential-Linear distribution, then its probability density function (pdf) can be defined as:

$$f(x) = \frac{\beta^2(2+x)e^{-\beta x}}{2\beta+1},\ x>0,\ \beta>0. \tag{2}$$

2.2 Proof of the legitimacy of the extended exponential-linear distribution

To show that $f(x)$ is a probability density function (pdf), we need to verify two properties:

1.Non-negativity: $f(x) \geq 0$ for all $x>0$.
2.Normalization: The integral of $f(x)$ over its domain equals 1, i.e., $\int_0^\infty f(x)dx = 1$.

Step 1: Non-negativity
Given $f(x) = \frac{\beta^2(2+x)e^{-\beta x}}{2\beta+1}$, $x > 0$, $\beta > 0$.
Note that

- $\beta > 0, \beta^2 > 0$.
- $2+x > 0$ for $x > 0$.
- $e^{-\beta x} > 0$ for all x.
- The denominator $2\beta+1 > 0$.

Therefore, $f(x) \geq 0$ for all $x > 0$.
Step 2: Normalization

$$\begin{aligned}\int_0^\infty f(x)dx &= \tfrac{\beta^2}{2\beta+1}\int_0^\infty (2+x)e^{-\beta x}dx \\ &= \tfrac{\beta^2}{2\beta+1}\left[\int_0^\infty 2e^{-\beta x}dx + \int_0^\infty xe^{-\beta x}dx\right].\end{aligned}$$

It can be shown that $\int_0^\infty e^{-\beta x}dx = \frac{1}{\beta}$ and $\int_0^\infty xe^{-\beta x}dx = \frac{1}{\beta^2}$.
Therefore,

$$\begin{aligned}\int_0^\infty f(x)dx &= \tfrac{\beta^2}{2\beta+1}\left[\tfrac{2}{\beta} + \tfrac{1}{\beta^2}\right] \\ &= \tfrac{\beta^2}{2\beta+1}\left(\tfrac{2\beta+1}{\beta^2}\right) \\ &= 1.\end{aligned}$$

Conclusion: Since $f(x) \geq 0$ for all $x > 0$ and $\int_0^\infty f(x)dx = 1$, $f(x)$ is a valid probability density function.

2.3 Cumulative distribution function (cdf)

We derive the cumulative distribution function (cdf) as:

$$\begin{aligned}F(x) &= P(X \leq x) \\ &= \int_0^x f(t)dt.\end{aligned}$$

We write the integral

$$\begin{aligned}F(x) &= \int_0^x \tfrac{\beta^2(2+t)e^{-\beta t}}{2\beta+1}dt \\ &= \tfrac{\beta^2}{2\beta+1}\int_0^x (2+t)e^{-\beta t}dt.\end{aligned}$$

Split the integral

$$\int_0^x (2+t)e^{-\beta t}dt = 2\int_0^x e^{-\beta t}dt + \int_0^x te^{-\beta t}dt.$$

Compute each integral:

$$\begin{aligned}\int_0^x e^{-\beta t}dt &= \left[-\tfrac{1}{\beta}e^{-\beta t}\right]_0^x \\ &= \tfrac{1}{\beta}\left(1 - e^{-\beta x}\right).\end{aligned}$$

For $\int_0^x te^{-\beta t}dt$, we use integration by parts: Let $u = t \Rightarrow du = dt$, $dv = e^{-\beta t}dt \Rightarrow v = -\frac{1}{\beta}e^{-\beta t}$. Then

$$\begin{aligned}\int_0^x te^{-\beta t}dt &= -\tfrac{t}{\beta}e^{-\beta t}\Big|_0^x + \tfrac{1}{\beta}\int_0^x e^{-\beta t}dt \\ &= -\tfrac{tx}{\beta}e^{-\beta x} + \tfrac{1}{\beta^2}\left(1 - e^{-\beta x}\right).\end{aligned}$$

Combining the integrals, we have

$$\int_0^x (2+t)e^{-\beta t}dt = \tfrac{2}{\beta}\left(1-e^{-\beta x}\right) - \tfrac{tx}{\beta}e^{-\beta x} + \tfrac{1}{\beta^2}\left(1-e^{-\beta x}\right)$$
$$= \left(\tfrac{2}{\beta}+\tfrac{1}{\beta^2}\right)(1-e^{-\beta x}) - \tfrac{tx}{\beta}e^{-\beta x}.$$

Multiply by the constant factor

$$F(x) = \tfrac{\beta^2}{2\beta+1}\left[\left(\tfrac{2}{\beta}+\tfrac{1}{\beta^2}\right)(1-e^{-\beta x}) - \tfrac{tx}{\beta}e^{-\beta x}\right].$$

Hence, the cumulative distribution function of the distribution is

$$F(x) = 1-\left(1+\tfrac{\beta x}{2\beta+1}\right)e^{-\beta x},\ x>0, \tag{3}$$

$\lim\limits_{x\to 0} F(x)=0$, $\lim\limits_{x\to\infty} F(x)=1$ and $b\geq a \Rightarrow F(b)\geq F(a)$.

2.4 The survival and hazard functions

We derive the survival function $S(x)$, which is the probability that the event of interest has not occurred by time x, from the cdf as follows:

$$S(x) = 1-F_X(x) = \left(1+\tfrac{\beta x}{2\beta+1}\right)e^{-\beta x},\ x>0. \tag{4}$$

The hazard function $h(x)$, an instantaneous rate at which events occur, given no prior event until time x, is derived as:

$$h(x) = \tfrac{f(x)}{S(x)}.$$

Hence,

$$h(x) = \tfrac{\beta^2(2+x)}{2\beta+\beta x+1}. \tag{5}$$

The cumulative hazard rate function is then given as:

$$H(x) = \int_0^x h(t)dt$$
$$= \ln S(x).$$

Therefore,

$$H(x) = \beta x - \ln\left(1+\tfrac{\beta x}{2\beta+1}\right). \tag{6}$$

2.5 Graphical display of the probability density and hazard functions

Figure 1 and Figure 2 show the plots of the probability density functions (pdf) at different values of β. It can be observed in Figure 1 that as the value of β increases (from $\beta=0.03$ to $\beta=0.1$), the peak systematically shifts to the left (towards $x=0$). For all the plotted β values in Figure 2, the peak of the pdf occurs at $x=0$. This indicates that for larger β values, the highest probability density is at the very beginning of the distribution. As β increases, the peak height of the pdf also increases. This signifies that a larger β concentrates the probability density more intensely around its specific mode. Also, as β increases, the dispersion of the distribution becomes smaller and more concentrated around its peak, and its "tail" decays more rapidly. Conversely, smaller β values result in a much wider and flatter distribution, extending further along the x-axis before asymptotically approaching zero.

Figure 3 and Figure 4 show plots of the hazard function for different values of β. In general, as β increases, the initial hazard rate, $h(0)$, also increases. All the hazard functions are monotonically increasing, indicating that the instantaneous failure rate rises over time. This reflects an "aging" or "wear-out" behavior, where the likelihood of failure grows as the item ages. Each function exhibits asymptotic behavior, approaching the value of β as x increases. This is evident in the graph, where each curve levels off toward a horizontal line at its corresponding β.

This distribution is considered an extension of the exponential distribution. The pdf transforms the exponential by multiplying it by a linear factor $2+x$ and normalizing. This linear factor increases the weight on larger values of x, providing for greater flexibility in modeling data with heavier tails or increasing hazard rates. When the linear term $2+x$

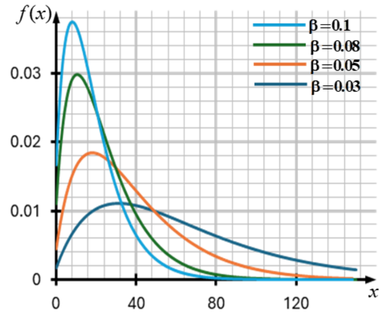

Fig. 1: Probability density function (pdf) at $\beta = 0.03, 0.05, 0.08$, and 0.1

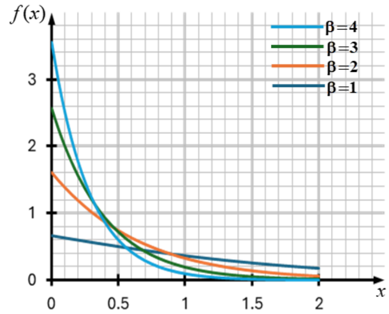

Fig. 2: Probability density function (pdf) at $\beta = 1, 2, 3$, and 4

is replaced by a constant (e.g., 1), the distribution simplifies to the exponential (up to normalization). Thus, the proposed distribution generalizes the exponential by incorporating an extra form component, making it more flexible while keeping the exponential decline.

The proposed distribution mixes the exponential decay with a linear-increasing component, allowing improved flexibility in modelling skewed data with declining tail weight. The extra term in the numerator includes a shape-modifying component that changes the early rise and decay of the distribution, in contrast to the traditional exponential distribution. This new distribution also has a structural similarity to the gamma distribution when the shape parameter approaches 2, and it also resembles the linear exponential distribution family. However, the presence of a polynomial component multiplied by an exponential kernel makes it a member of the larger family of generalized exponential models [31].

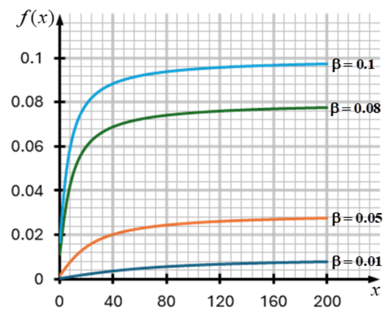

Fig. 3: Graph of hazard function ($h(x)$) at $\beta = 0.03, 0.05, 0.08$, and 0.1

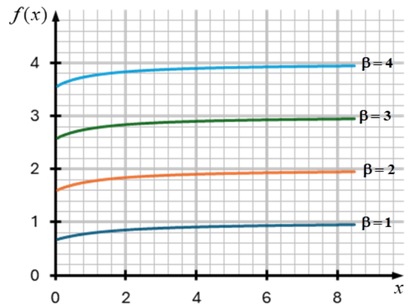

Fig. 4: Graph of hazard function ($h(x)$) at $\beta = 1,2,3$, and 4

3 Statistical properties of the new distribution

3.1 Moments and incomplete moments

The n^{th} moment of the random variable X about the origin, μ_n', is given by

$$\begin{aligned}
\mu'_n &= E(X^n) \\
&= \int_0^\infty x^n f(x)dx \\
&= \tfrac{\beta^2}{2\beta+1} \int_0^\infty x^n (2+x)e^{-\beta x}dx \\
&= \tfrac{\beta^2}{2\beta+1} \left[2\int_0^\infty x^n e^{-\beta x}dx + \int_0^\infty x^{n+1}e^{-\beta x}dx \right].
\end{aligned}$$

Recall the gamma integral: $\int_0^\infty x^k e^{-\beta x}dx = \frac{\Gamma(k+1)}{\beta^{k+1}}$. So

$$\mu_n' = \tfrac{1}{2\beta+1} \left[\tfrac{2\Gamma(n+1)}{\beta^{n-1}} + \tfrac{\Gamma(n+2)}{\beta^n} \right]. \tag{7}$$

Incomplete moments are given by

$$\begin{aligned}
\mu_n(t) &= \int_0^t x^n f(x)dx \\
&= \tfrac{\beta^2}{2\beta+1} \int_0^t x^n (2+x)e^{-\beta x}dx \\
&= \tfrac{\beta^2}{2\beta+1} \left[2\int_0^t x^n e^{-\beta x}dx + \int_0^t x^{n+1}e^{-\beta x}dx \right].
\end{aligned}$$

These integrals are incomplete gamma integrals: $\int_0^t x^k e^{-\beta x}dx = \frac{\gamma(k+1,\beta t)}{\beta^{k+1}}$, where $\gamma(s,z)$ is the lower incomplete gamma function. Therefore,

$$\mu_n(t) = \tfrac{1}{2\beta+1} \left[\tfrac{2\gamma(n+1,\,\beta t)}{\beta^{n-1}} + \tfrac{\gamma(n+2,\,\beta t)}{\beta^n} \right]. \tag{8}$$

3.2 The mean and the variance

The mean μ is the first moment:

$$\begin{aligned}
\mu &= E(X) \\
&= \tfrac{1}{2\beta+1} \left[\tfrac{2\Gamma(2)}{\beta^0} + \tfrac{\Gamma(3)}{\beta^1} \right] \\
&= \tfrac{1}{2\beta+1} \left[2 + \tfrac{2}{\beta} \right] \\
&= \tfrac{1}{2\beta+1} \left[\tfrac{2\beta+2}{\beta} \right].
\end{aligned}$$

Hence,

$$\mu = \frac{2(\beta+1)}{\beta(2\beta+1)}. \tag{9}$$

The second moment is

$$\begin{aligned}
\mu'_2 &= E(X^2) \\
&= \frac{1}{2\beta+1}\left[\frac{2\Gamma(3)}{\beta} + \frac{\Gamma(4)}{\beta^2}\right] \\
&= \frac{1}{2\beta+1}\left[\frac{4}{\beta} + \frac{6}{\beta^2}\right] \\
&= \frac{1}{2\beta+1}\left[\frac{4\beta+6}{\beta^2}\right] \\
&= \frac{4\beta+6}{\beta^2(2\beta+1)}.
\end{aligned}$$

The variance is

$$\begin{aligned}
V(X) &= \mu'_2 - \mu^2 \\
&= \frac{4\beta+6}{\beta^2(2\beta+1)} - \left(\frac{2(\beta+1)}{\beta(2\beta+1)}\right)^2.
\end{aligned}$$

Therefore,

$$V(X) = \frac{4\beta^2+8\beta+2}{\beta^2(2\beta+1)^2}. \tag{10}$$

3.3 Moment generating function

The moment generating function (MGF) $M_X(t)$ is defined as:

$$\begin{aligned}
M_X(t) &= E(e^{tX}) \\
&= \int_0^\infty e^{tx} f(x)dx \\
&= \frac{\beta^2}{2\beta+1}\int_0^\infty (2+x)e^{-(\beta-x)x}dx \\
&= \frac{\beta^2}{2\beta+1}\left[2\int_0^\infty e^{-(\beta-t)x}dx + \int_0^\infty xe^{-(\beta-t)x}dx\right] \\
&= \frac{\beta^2}{2\beta+1}\left[\frac{2}{\beta-t} + \frac{1}{(\beta-t)^2}\right],\ t<\beta.
\end{aligned}$$

Hence,

$$M_X(t) = \frac{\beta^2(2\beta-2t+1)}{(2\beta+1)(\beta-t)^2},\ t<\beta. \tag{11}$$

3.4 Order statistics

Order statistics refer to the statistics obtained from the ordered sample values. The pdf of the k^{th} order statistic $X_{(k)}$ in a sample of size n from a pdf, $f(x)$, with cdf, $F(x)$, is defined as:

$$\begin{aligned}
f_{X_{(k)}}(x) &= \frac{n!}{(k-1)!(n-k)!}[F(x)]^{k-1}[1-F(x)]^{n-k}f(x) \\
&= \frac{n!}{(k-1)!(n-k)!}\left[1-\left(1+\frac{\beta x}{2\beta+1}\right)e^{-\beta x}\right]^{k-1}\left[\left(1+\frac{\beta x}{2\beta+1}\right)e^{-\beta x}\right]^{n-k}\frac{\beta^2(2+x)e^{-\beta x}}{2\beta+1}.
\end{aligned}$$

Therefore,

$$f_{X_{(k)}}(x) = \frac{n!\beta^2}{(k-1)!(n-k)!(2\beta+1)^{n-k+1}}\left[1-\frac{\beta x+2\beta+1}{2\beta+1}e^{-\beta x}\right]^{k-1}\left[(\beta x+2\beta+1)e^{-\beta x}\right]^{n-k}(2+x)e^{-\beta x}. \tag{12}$$

3.5 Entropies

The entropy E of a continuous probability distribution is defined as: $E = -\int_0^\infty f(x)\ln f(x)dx$. Hence,

$$E = -\int_0^\infty \frac{\beta^2(2+x)e^{-\beta x}}{2\beta+1} \ln\left(\frac{\beta^2(2+x)e^{-\beta x}}{2\beta+1}\right) dx. \tag{13}$$

This integral can be computed numerically, as it may not have a simple closed-form solution.

3.6 Quantile function

Given the cumulative distribution function: $F(x) = 1-\left(1+\frac{\beta x}{2\beta+1}\right)e^{-\beta x}$, the quantile function is the inverse of the cdf: $Q(p) = F^{-1}(p)$, $0<0<1$. Thus, we solve for x in terms of p:

$$p = 1-\left(1+\frac{\beta x}{2\beta+1}\right)e^{-\beta x}$$

$$(1-p)e^{\beta x} = \left(1+\frac{\beta x}{2\beta+1}\right)$$

The following implicit equation in terms of x is derived:

$$\beta x+(2\beta+1) = (2\beta+1)(1-p)e^{\beta x}.$$

This equation involves x both inside and outside the exponential, making it transcendental and not solvable in closed form using elementary functions. We, therefore, express the solution using the Lambert W function. The Lambert W function, also known as the product logarithm, is a special mathematical function that solves equations where the unknown appears both inside and outside an exponential term. While it is not an elementary function, it is widely implemented in statistical and scientific computing software (such as R), allowing practitioners to compute quantiles without manually solving transcendental equations. Let $y = \beta x+(2\beta+1)$. Then

$$e^{\beta x} = e^{y-(2\beta+1)} = e^y e^{-(2\beta+1)}.$$

Substituting back, we have

$$\begin{aligned} y &= (2\beta+1)(1-p)e^y e^{-(2\beta+1)} \\ ye^{(2\beta+1)} &= (2\beta+1)(1-p)e^y \\ -ye^{-y} &= -(2\beta+1)(1-p)e^{-(2\beta+1)}. \end{aligned}$$

Set $z=-y$, then

$$ze^z = -(2\beta+1)(1-p)e^{-(2\beta+1)}.$$

By definition, the Lambert W functionis expressed as:

$$z = W\left(-(2\beta+1)(1-p)e^{-(2\beta+1)}\right).$$

Recall that $z=-y=-(\beta x+2\beta+1)$, and so

$$-\beta x-(2\beta+1) = W\left(-(2\beta+1)(1-p)e^{-(2\beta+1)}\right)$$

Therefore,

$$x = \frac{-(2\beta+1)-W\left(-(2\beta+1)(1-p)e^{-(2\beta+1)}\right)}{\beta}.$$

The final quantile function is

$$Q(p) = \frac{-(2\beta+1)-W\left(-(2\beta+1)(1-p)e^{-(2\beta+1)}\right)}{\beta}. \tag{14}$$

The Lambert W function is implemented in many software packages (e.g. lambertW in R). This expression gives the quantile x for any $p\in(0,1)$.

3.7 Parameter estimation

3.7.1 Maximum likelihood estimator (*MLE*)

Given the data $x_1, x_2, ..., x_n$, the likelihood function is

$$\begin{aligned} L(\beta) &= \prod_{i=1}^{n} f(x_i;\beta) \\ &= \prod_{i=1}^{n} \frac{\beta^2(2+x_i)e^{-\beta x_i}}{2\beta+1}. \end{aligned}$$

The log-likelihood function is also defined as:

$$\begin{aligned} l(\beta) &= \sum_{i=1}^{n} [2\ln\beta + \ln(2+x_i) - \beta x_i - \ln(2\beta+1)] \\ &= 2n\ln\beta + \sum_{i=1}^{n} \ln(2+x_i) - \beta\sum_{i=1}^{n} x_i - n\ln(2\beta+1). \end{aligned}$$

Derivative with respect to β:

$$\frac{dl}{d\beta} = \frac{2n}{\beta} - \sum_{i=1}^{n} x_i - \frac{2n}{2\beta+1}.$$

Set derivative to zero for *MLE*:

$$\frac{2n}{\beta} - n\bar{x} - \frac{2n}{2\beta+1} = 0.$$

Multiply by $\beta(2\beta+1)$ to clear the denominator:

$$\begin{aligned} 2(2\beta+1) - \bar{x}\beta(2\beta+1) - 2\beta &= 0 \\ 4\beta + 2 - 2\bar{x}\beta^2 - \bar{x}\beta - 2\beta &= 0 \end{aligned}$$

which gives

$$2\bar{x}\beta^2 + (\bar{x}-2)\beta - 2 = 0.$$

This is a quadratic equation in β: $a\beta^2 + b\beta + c$, where $a = 2\bar{x}, b = \bar{x}-2$, and $c = -2$. The solutions of β are given by the quadratic formula: $\beta = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$. Hence,

$$\beta = \frac{2-\bar{x}\pm\sqrt{(\bar{x}-2)^2+16\bar{x}}}{4\bar{x}}.$$

Since the product of the roots, $\frac{c}{a} = \frac{-2}{2\bar{x}} = -\frac{1}{\bar{x}}$, is negative, it follows that one root must be positive and one must be negative. Given that $\beta > 0$, we take the positive root. So, the *MLE* estimator for β is

$$\hat{\beta}_{\text{MLE}} = \frac{2-\bar{x}+\sqrt{(\bar{x}-2)^2+16\bar{x}}}{4\bar{x}}. \tag{15}$$

3.7.2 Method of moments (*MoM*) estimator

The first moment of X is:

$$E(X) = \frac{2(\beta+1)}{\beta(2\beta+1)}.$$

Set the sample mean, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, to be equal to $E(X)$.

$$\bar{x} = \frac{2(\beta+1)}{\beta(2\beta+1)}$$

$$\bar{x}\beta(2\beta+1) = 2(\beta+1).$$

Expanding and rearranging, the expression can be written as

$$2\bar{x}\beta^2 + (\bar{x}-2)\beta - 2 = 0.$$

This is the same quadratic equation as for the Maximum Likelihood Method. This means that for this particular distribution, the *MLE* and *MoM* estimators are identical. This quadratic in β can be solved explicitly as

$$\hat{\beta}_{\text{MoM}} = \frac{2-\bar{x}+\sqrt{(\bar{x}-2)^2+16\bar{x}}}{4\bar{x}}. \tag{16}$$

4 Numerical application

4.1 Simulation

According to the Probability Integral Transformation, if X is a continuous random variable with a cumulative distribution function (cdf) $F(x)$, then $U = F(X)$ follows a continuous uniform distribution over the interval $(0,1)$ [32]. The inverse transform sampling method was employed to simulate samples from the given distribution using the following steps:

1. Generate a random number u from the standard uniform distribution in the interval $[0,1]$.
2. Compute the inverse of the cdf, $F^{-1}(u)$, which is the quantile of the distribution.
3. Determine $x = F^{-1}(u)$.

The resulting values of X represent random samples from the desired distribution with the probability density function (pdf), $f(x)$. Sample sizes of 1000, 500, 200, 100, 50, and 20 were considered. Table 1 provides a comparison of the performance of MLE/MoM across these sample sizes for parameter values $\beta = 0.05, 10, 75$, and 150. The performance evaluation is based on the estimated parameter values and their absolute percentage differences from the true values.

As observed from Table 1, the MLE/MoM provided estimates that were within 10% of the actual parameter values across all sample sizes except for a small sample size of 20. Overall, the results demonstrate that the MLE/MoM estimator is reliable for moderate to large samples but can suffer from bias and variability in small samples. This highlights the importance of a sufficiently large sample size for precise parameter estimation in this distribution.

Table 1: MLE/MoM at different simulated sample sizes

	Actual: $\beta = 0.05$		Actual: $\beta = 10$		Actual: $\beta = 75$		Actual: $\beta = 150$	
Sample Size	Estimated β MoM/MLE	% Diff.	Estimated β MoM/MLE	% Diff.	Estimated β MoM/MLE	% Diff.	Estimated β MoM/MLE	% Diff.
1000	0.0496	0.9	9.962	0.4	71.145	5.1	148.367	1.1
500	0.0497	0.6	10.124	1.2	70.653	5.8	151.636	1.1
200	0.0524	4.8	9.532	4.7	80.405	7.2	139.017	7.3
100	0.0488	2.5	9.165	8.4	77.804	3.7	154.555	3.0
50	0.0490	2.0	10.825	8.3	69.392	7.5	146.545	2.3
20	0.0389	22.2	7.956	20.4	116.133	54.8	144.074	4.0

4.2 Application to life testing

Consider n samples of some manufactured components that were subjected to some reliability life tests from a specific population of interest. The random variable X of interest is the time it takes for the component to fail. Suppose the underlying failure times are $X_{(1)},\ldots,X_{(n)}$, where $X_{(i)} \le X_{(i+1)}$, $i = 1,\ldots,n-1$. Assuming that the reliability life tests conclude at the r^{th} failure, where r is less than or equal to n, the number of failures is treated as a fixed value, while the failure times are regarded as random variables.

According to [32], the likelihood function L of the first r order statistics, $X_{(1)} \le X_{(2)} \le \ldots \le X_{(r)}$, of the random variable of interest in this study can be specified as:

$$\begin{aligned} L(\beta) &= f_{X_{(1)},\ldots,X_{(r)}}\left(x_{(1)},\ldots,x_{(r)}\right) \\ &= \tfrac{n!}{(n-r)!}\left[1-F(x_{(r)})\right]^{n-r}\prod_{i=1}^{r} f(x_{(i)}) \\ &= \tfrac{n!}{(n-r)!}\left[\left(\tfrac{2\beta+\beta x_{(r)}+1}{2\beta+1}\right)e^{-\beta x_{(r)}}\right]^{n-r}\prod_{i=1}^{r}\left\{\tfrac{\beta^2\left(2+x_{(i)}\right)e^{-\beta x_{(i)}}}{2\beta+1}\right\} \\ &= \tfrac{n!}{(n-r)!}\cdot\tfrac{\beta^{2r}}{(2\beta+1)^{r+(n-r)}}\left(\prod_{i=1}^{r}(2+x_{(i)})\right).e^{-\beta\left(\sum_{i=1}^{r}x_{(i)}+(n-r)x_{(r)}\right)}.(2\beta+\beta x_{(r)}+1)^{n-r}. \end{aligned}$$

The log-likelihood function gives

$$\ln\left(L(\beta)\right) = \ln\left(\frac{n!}{(n-r)!}\right) + 2r\ln\beta - n\ln(2\beta+1) + \ln\left(\prod_{i=1}^{r}(2+x_{(i)})\right)$$
$$-\beta\left(\sum_{i-1}^{r} x_{(i)} + (n-r)x_{(r)}\right) + (n-r)\ln\left(2\beta+\beta x_{(r)}+1\right).$$

Find the derivative with respect to β

$$\frac{d\ln L}{d\beta} = \frac{2r}{\beta} - \frac{2n}{2\beta+1} - \left(\sum_{i-1}^{r} x_{(i)} + (n-r)x_{(r)}\right) + \frac{(n-r)(2+x_{(r)})}{2\beta+\beta x_{(r)}+1}.$$

Solve the likelihood equation

$$\frac{2r}{\beta} - \frac{2n}{2\beta+1} - \left(\sum_{i-1}^{r} x_{(i)} + (n-r)x_{(r)}\right) + \frac{(n-r)(2+x_{(r)})}{2\beta+\beta x_{(r)}+1} = 0. \tag{17}$$

The likelihood equation is not linear in β. Thus, we cannot determine the root explicitly using algebraic methods. We therefore resort to a numerical approach. In R, the "uniroot()" function is suitable for finding a single root of a function within a specified interval.

A sample of size 100 was simulated from the EE-L distribution using the parameter value $\beta = 0.05$. Table 2 shows the first 70 out of 100 ordered data points. It is assumed that these observations represent the outcomes of a reliability life test involving 100 devices until the failure of the 70^{th} device. On the *MLE* parameter estimates, we deduce from Equation (17); and given that $n = 100$, $r = 70$, $x_{(r)} = 49.5856$ (from Table 2) together with the "uniroot()" function in R, the value of β was estimated as $\beta_{MLE} = 0.04813$. In other words, values in Table 2 represent ordered failure times from the simulated

Table 2: Ordered data simulated from the EE-L distribution

Simulated Ordered Data							
[1]	3.7926	4.0897	4.3662	5.8803	5.9505	6.6676	8.1031
[8]	10.4429	10.5411	11.3303	12.2567	13.1069	13.3871	13.6298
[15]	14.2039	14.8874	15.0295	15.1541	15.3266	15.6665	15.7072
[22]	16.7137	17.9174	18.3817	18.6990	20.0083	20.0705	20.4284
[29]	21.2887	21.5424	21.8303	22.0174	22.8246	23.2372	23.3174
[36]	23.7597	23.7777	23.9057	24.0011	25.5058	25.5302	25.6415
[43]	26.6340	26.6929	27.4430	27.4651	27.5097	27.9717	28.1296
[50]	28.5201	29.4151	29.6169	30.1805	30.8115	30.8214	32.6889
[57]	34.3636	35.8719	37.1771	38.1047	38.2006	40.4376	41.1967
[64]	42.2768	43.5901	45.3356	45.5006	47.5298	48.5596	49.5856

reliability life test. Each entry corresponds to the time until failure for one of the devices, arranged from the earliest to the latest failure observed up to the 70^{th} failure. These ordered statistics serve as the empirical input for the likelihood estimation in Equation (17). The maximum likelihood estimates are not significantly different from the actual value of the parameters; we clarify that this conclusion is based on the numerical closeness of the estimated β to the true parameter value ($\beta = 0.05$) used in the simulation, with a relative error small enough to suggest the estimator is performing well in this scenario.

5 Application to real-life dataset

In this section, we assess the performance of the EE-L distribution by applying it to model a real-life dataset. We assessed its competitiveness by comparing it with other known probability distributions such as the exponential, inverse exponential, modified exponential, and Lomax distributions. We employed various information criteria, such as Akaike Information Criterion (AIC), Hannan-Quinn Information Criterion (HQIC), and the Bayesian Information Criterion (BIC), as well as performing the Kolmogorov-Smirnov (K-S) test to assess the goodness of fit for the considered distributions. The best distribution is determined based on having the highest p-value for the K-S test and the lowest values for AIC, BIC, HQIC, and the K-S test statistic. The dataset applied in this work consists of the waiting times (in minutes) of one hundred bank customers before they receive service. Previous studies by [33] and [34] have also analysed this dataset.

The dataset provided below includes the following waiting times (in minutes) of one hundred bank customers before they receive service:

0.8, 0.8, 1.3, 1.5, 1.8, 1.9, 1.9, 2.1, 2.6, 2.7, 2.9, 3.1, 3.2, 3.3, 3.5, 3.6, 4.0, 4.1, 4.2, 4.2, 4.3, 4.3, 4.4, 4.4, 4.6, 4.7, 4.7, 4.8, 4.9, 4.9, 5, 5.3, 5.5, 5.7, 5.7, 6.1, 6.2, 6.2, 6.2, 6.3, 6.7, 6.9, 7.1, 7.1, 7.1, 7.1, 7.4, 7.6, 7.7, 8, 8.2, 8.6, 8.6, 8.6, 8.8, 8.8, 8.9, 8.9, 9.5, 9.6, 9.7, 9.8, 10.7, 10.9, 11, 11, 11.1, 11.2, 11.2, 11.5, 11.9, 12.4, 12.5, 12.9, 13, 13.1, 13.3, 13.6, 13.7, 13.9, 14.1, 15.4, 15.4, 17.3, 17.3, 18.1, 18.2, 18.4, 18.9, 19, 19.9, 20.6, 21.3, 21.4, 21.9, 23.0, 27, 31.6, 33.1, 38.5.

The results show that the EE-L distribution is the most effective among the competing distributions based on the results presented in Tables 3 and 4. This is because it has the lowest values for AIC, BIC and HQIC, indicating better model fit. Additionally, it has the highest p-value for the K-S statistic, further supporting its superiority compared to other distributions.

Table 3: Parameter estimates for waiting time dataset

Model	Parameter	Estimate	S.E.	p-value
EEL	β	0.17612	0.01267	<0.0001
EXP	λ	0.101245	0.010125	<0.0001
IE	α	5.34761	0.53476	<0.0001
ME	α	8.3396	74.363429	0.9107
	β	4.145006	19.414514	0.8309
	λ	0.101243	0.010127	<0.0001
LOMAX	α	52.2880	54.3980	0.3364
	λ	509.0150	534.7990	0.3412

Table 4: Model selection criteria for waiting time dataset

Model	-ll	AIC	BIC	HQIC	KS_Stat	KS_pval
IE	336.5585	675.1170	677.7222	676.1714	0.167454	0.007336
Exp	329.0209	660.0418	662.6469	661.0961	0.173011	0.005025
Lomax	329.4751	662.9502	668.1605	665.0589	0.175879	0.004113
EEL	320.5352	643.0705	645.6757	644.1248	0.088083	0.419739
ME	329.0225	664.0450	671.8605	667.2081	0.173038	0.005015

The EE-L distribution outperforms the selected known distributions by capturing the behavior of the waiting time dataset. Figures 5 and 6 show the comparison of the EE-L distribution with other distributions using the waiting time dataset, based on the plot of fitted densities. The plot reveals that the EE-L distribution demonstrates a favorable and superior fit when compared to the existing distributions. The EE-L distribution better represents the dataset's flexibility

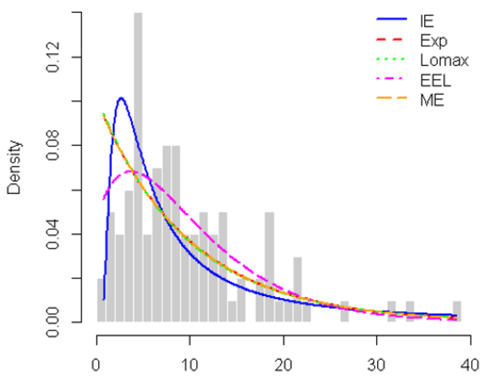

Fig. 5: Fitted pdf with the histogram for waiting time dataset

compared to other distributions, which may either be too simple or have heavy tails, such as the Lomax. The fit is validated

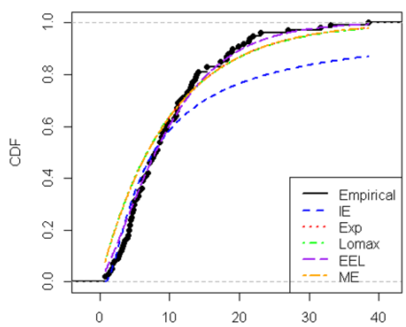

Fig. 6: Fitted cdf with empirical cdf for waiting time dataset

by the P–P plot in Figure 7, showing that the EE-L distribution aligns slightly better with the empirical distribution, particularly across the entire range.

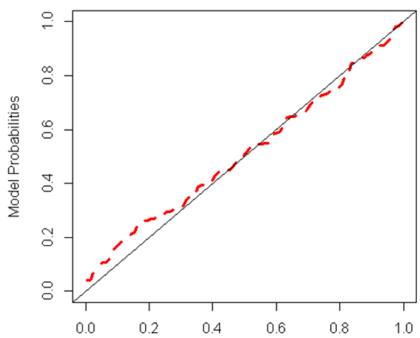

Fig. 7: P-P plots of EE-L for the waiting time dataset

6 Concluding remarks

In this paper, we present a new modification of the exponential distribution with a single parameter called the Extended Exponential-Linear distribution (EE-L). The statistical properties of the EE-L distribution are obtained, including moments, entropy measures, hazard function, survival function, order statistics, and other relevant quantities. Furthermore, the parameter of the EE-L distribution is estimated using maximum likelihood and the method of moment estimations. Simulation studies were conducted to assess the performance of the estimation technique under various scenarios. In future work, a study may be conducted to estimate the parameter of the proposed EE-L distribution using a Bayesian approach. The behavior of the hazard rate function has been investigated, and the results show that the hazard functions are monotonically increasing, indicating that the instantaneous failure rate rises over time. The proposed distribution was also applied to the bank waiting time dataset to illustrate its effectiveness. The EE-L distribution exhibited a reasonable fit to the dataset when compared to other distributions evaluated in this work for modelling real-life datasets, particularly the Exponential distribution. This suggests that the EE-L distribution may be a suitable choice for modelling datasets with varying complexity and heavy-tailed characteristics. Further studies could investigate the potential application of the EE-L distribution in other fields or industries.

Conflicts of interest statement The authors certify that they have NO affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript.

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