Applied Mathematics & Information Sciences
*An International Journal*

http://dx.doi.org/10.18576/amis/190507

# Mathematical Formulation of Fuzzy Grammar in English Syntax and Morphology

*Yogeesh N.*[1,*], *Suleiman Ibrahim Shelash Mohammad*[2,3], *N. Raja*[4], *R. Rayeesul Hassan S.*[5], *Kavitha H. S.*[6], *Aravinda Reddy N.*[7], *Asokan Vasudevan*[8,9,10], *Mohammad Faleh Ahmmad Hunitie*[11], *and Nawaf Alshdaifat*[12]

[1]Department of Mathematics, Government First Grade College, Tumkur, Karnataka, India
[2]Electronic Marketing and Social Media, Economic and Administrative Sciences Zarqa University, Jordan
[3]INTI International University, 71800 Negeri Sembilan, Malaysia
[4]Sathyabama Institute of Science and Technology, (Deemed to Be University), Department of Visual Communication, Chennai, Tamil Nadu, India
[5]Department of English, Government College for Women Kolar, India
[6]Department of English, Govt. First Grade College and PG Center, Chintamani, Chikkaballapura District, Karnataka, India
[7]Govt. College for Women, Chintamani, Chikkaballapur, India
[8]Faculty of Business and Communications, INTI International University, 71800 Negeri Sembilan, Malaysia
[9]Shinawatra University, 99 Moo 10, Bangtoey, Samkhok, Pathum Thani 12160 Thailand
[10]Wekerle Business School, Budapest, Jázmin u. 10, 1083 Hungary
[11]Department of Public Administration, School of Business, University of Jordan, Jordan
[12]Department of Computer Science, Faculty of IT, Applied Science Private University, Amman, Jordan

Received: 18 Feb. 2025, Revised: 2 Apr. 2025, Accepted: 28 May 2025
Published online: 1 Sep. 2025

**Abstract:** This study explores the application of fuzzy set theory in the analysis of English syntax and morphology, with a focus on handling linguistic ambiguity and grammatical correctness. Traditional grammar models rely on Boolean logic, which rigidly classifies sentences as either correct or incorrect, failing to account for gradations of correctness. In contrast, fuzzy grammar models utilize membership functions to represent linguistic variability, enabling a continuous evaluation of grammatical structures. A fuzzy-based approach was applied to 10 sentences from Shakespearean literature, employing Gaussian membership functions to quantify morphological correctness. The transformation distance of each sentence was computed using Levenshtein edit distance and part-of-speech (POS) mismatches, forming the basis for fuzzy morphology analysis. The results demonstrated that modern grammatical structures achieved high fuzzy scores ($\mu = 1.0$), whereas sentences with minor structural deviations obtained moderate scores ($\mu \approx 0.9$), and significant archaic variations resulted in lower scores ($\mu \approx 0.6$). This study highlights the effectiveness of fuzzy logic in natural language processing (NLP), particularly in context-aware grammar checking and syntactic ambiguity resolution. However, challenges remain in defining optimal membership functions and optimizing computational efficiency for real-time applications. Future research should focus on extending fuzzy grammar models to discourse analysis, integrating fuzzy neural networks for automated grammar learning, and developing hybrid AI-fuzzy grammar checking systems to enhance context-sensitive language processing.

**Keywords:** Fuzzy Grammar, Natural Language Processing (NLP), Linguistic Ambiguity, Fuzzy Membership Functions, Morphological Correctness, Syntactic Analysis, Context-Sensitive Grammar, Transformation Distance, Machine Learning and Fuzzy Logic, Text Processing in AI

## 1 Introduction

The study of linguistics traditionally uses deterministic models which parse grammatical correctness into absolute rules and binary distinctions. But natural language is intrinsically ambiguous, variable and context-sensitive. One potential answer comes from formal theories of grammar, such as [1,2,3,4] transformational-generative grammar, which describe structured abstractions in syntax, e.g. its hierarchical

* Corresponding author e-mail: yogeesh.r@gmail.com

structure, but fail to model uncertainty, i.e. the possibility of overlapping or contradictory linguistic categories.

These limitations give rise to the need for fuzzy logic-based grammatical models. Fuzzy set theory was introduced by Zadeh [5,6,7,8] as an extension of set theory to implement a degree of membership rather than just the conventional binary classifications. In fact, across many disciplines, allowing for linguistic imprecision has been employed to handle uncertainty; this is especially common in the fields of artificial intelligence (AI) and natural language processing (NLP) [9,10,11]. In syntax and morphology, for instance, there is often graded, as opposed to categorical, correctness of words and grammatical structures, which is a phenomenon that can be best represented with fuzzy logic [12].

The study of many linguistic phenomena like synonymy, polysemy, acceptability can be better analysed in terms of their fuzzy set representations. We know that correctness' of sentence structure and their morphological transformations can be mathematically modelled and thus get a fuzzy way of sentence structure detection, which can be implemented through NLP systems based on AI [13].

## 1.1 Objectives

This study aims to:

– Develop a mathematical formulation of fuzzy grammar to model English syntax and morphology.
– Use fuzzy membership functions to quantify linguistic uncertainty and degrees of correctness in grammatical structures.
– Integrate fuzzy logic inference rules into computational linguistics, providing a more flexible and realistic representation of sentence structure.

The goals will theoretically advance linguistics and have practical applications, most likely in machine translation, grammar checking and computational text analysis.

## 1.2 Significance of the Study

Grammatical Analysis Through Mathematical Modelling: Norwegian Linguistics Meet Mathematical Modelling By applying fuzzy set theory to syntax and morphology, this study will:

– Provide a quantitative framework for linguistic uncertainty, filling gaps left by conventional grammar models.
– Enhance NLP algorithms for syntactic parsing, morphological analysis, and grammar checking in AI-driven applications [14].
– Improve language assessment tools, allowing for graded grammatical correctness rather than binary classifications.
– Contribute to theoretical linguistics by offering a mathematical basis for linguistic gradience [15].

This study is a meaningful contribution to interdisciplinary research with significant implications for not only computational linguistics, fuzzy mathematics, but also AI-driven text analysis.

## 1.3 Literature Review

Fuzzy logic in the framework of linguistic analysis has been discussed in large in the various subfields of linguistics and artificial intelligence. The literature cited below provides evidence that the mathematics of English syntax and morphology can be modeled in terms of fuzzy set theory:

**(i) First Principles of Fuzzy Linguistics**

Fuzzy set theory was first proposed by Zadeh [5], and has since served as the basis for modeling uncertainty in a wide range of fields, of which linguistics is just one. Novák et al. 302) Fuzzy Logic in Natural Language Processing. Fuzzy Logic in Natural Language Processing 73 Lingua Informatic Developments in nature of communication 45: 303308. The gradience in linguistic structures has been discussed for many years by Aarts [15] and more recently, it is stated in Favor and Companion that there is no discrete grammar:

**(ii) Fuzzy Logic and Syntax and Sentence Processing**

Fuzzy models have also been suggested for syntactic ambiguity resolution [9,13], explaining how fuzzy rules could describe the notion of grammaticality in context. The prototype theory was developed by Rosch [16] and is consistent with the idea that categories that have linguistic distinctions have graded memberships, which also correlates with fuzzy logic principles. Jurafsky & Martin [14] explored both probabilistic and fuzzy models for NLP, finding effectiveness of these models in sentence level parse.

**(iii) Fuzzy Morphology and Word Formation**

Based on the continuum of morphological transformations previously discussed [17], it is reasonable to model the occurrence of affixation using fuzzy mathematics. Ljung [18] investigated irregular morphological patterns, which yielded evidence of non-discrete categorization in language. These methods processed information from large text corpora and implemented fuzzy ranking algorithms to determine the proximity between words and their morphological appropriateness [19].

**(iv) Fuzzy Linguistic Models in Computational Applications**

Fuzzy logic-based grammar checking systems were proposed by Zhou and He [13] and outperformed rule and probabilistic models. Hirst [20] discussed the role of fuzzy constraints in computational syntax and how to apply fuzzy logic to sentence generation models. It was suggested that fuzzy clustering methods be implemented into syntactic plans, to show effectiveness in the AI syntactic division of preprocessors (e.g., words) [21].

## 2 Foundations of Fuzzy Set Theory and Linguistics

The material in this section introduces fuzzy set theory in general terms and its linguistic applications to English syntax and morphology, specifically. Fuzzy set introduced by Zadeh [5] allows a more general representation of a linguistic structure and captures the linguistic ambiguity since it allows semantic structures with varying degree of correctness.

### *2.1 Basic Concepts of Fuzzy Sets and Membership Functions*

**Definition of a Fuzzy Set**

In classical set theory, an element is either a member of a set or it is not (binary membership). Fuzzy set theory, however, introduces the concept of partial membership (an element belongs to a set to a certain degree).

A fuzzy set $A$ in a universal set $X$ is defined as:

$$A = \{(x, \mu_A(x)) \mid x \in X, \mu_A(x) \in [0,1]\} \tag{1}$$

where:

– $x$ is an element of $X$,
– $\mu_A(x)$ is the membership function (MF), which assigns a value between 0 and 1 to each element, representing the degree of membership.

**Example of a Fuzzy Set in Linguistics**
Consider the grammatical correctness of a sentence:

$$A = \left\{ \begin{array}{ll} (\text{“This is a book”}, & 1), \\ (\text{“This be a book”}, & 0.6), \\ (\text{“Book this is”}, & 0.4), \\ (\text{“Book is this”}, & 0.2) \end{array} \right\}$$

Here, $\mu_A(x)$ assigns different values representing how grammatically correct each sentence is.

**Linguistic Variables and Their Representation in Fuzzy Sets**

A linguistic variable is a variable whose required values are words or phrases in natural language, rather than numerical values [22].

**Example**: The grammatical ”correctness” of a sentence can be represented as a fuzzy set:

– ”Highly correct”: $\mu = 1.0$
– ”Moderately correct”: $\mu = 0.7$
– ”Somewhat correct”: $\mu = 0.4$
– ”Incorrect”: $\mu = 0.0$

A typical membership function (MF) for grammatical correctness can be defined as follows:

$$\mu_{\text{grammar}}(x) = \begin{cases} 1, & x \text{ is perfectly grammatical} \\ 0.7, & x \text{ has minor errors} \\ 0.4, & x \text{ is ambiguous} \\ 0, & x \text{ is ungrammatical} \end{cases} \tag{2}$$

Such fuzzy grading allows us for quantitative linguistic analysis, which will be useful in computational linguistics and various AI-based NLP applications [12].

### *2.2 Introduction to English Syntax and Morphology*

**Grammatical Components: Subject, Predicate, and Modifiers**

A sentence in English is generally structured as:

$$S = (\text{Subject}, \text{Predicate}, \text{Object}) \tag{3}$$

where:

– The subject is the noun performing the action.
– The predicate contains the verb and expresses action.
– The object receives the action.

Example:

$$S = (\text{John, eats, an apple}) \tag{4}$$

In fuzzy grammar, each component can have a fuzzy membership value based on grammatical correctness:

$$\mu_{\text{subject}}(S) = 1, \mu_{\text{predicate}}(S) = 0.8, \mu_{\text{object}}(S) = 0.9 \tag{5}$$

The overall grammatical acceptability of the sentence can be computed as:

$$\mu_{\text{sentence}} = min\left(\mu_{\text{subject}}, \mu_{\text{predicate}}, \mu_{\text{object}}\right) \tag{6}$$

This follows **[22] fuzzy intersection principle**, ensuring that the lowest membership value determines sentence correctness.

**Inflectional and Derivational Morphology**

Morphology is the study of word formation. It includes:

Inflectional Morphology (modifies a word’s tense, number, or case without changing its core meaning):

**Example**:

– ”run” → ”running” ($\mu = 1$)
– ”go” → ”goed” ( $\mu = 0.2$ ) (incorrect but comprehensible)

Derivational Morphology (creates new words by adding prefixes or suffixes):

**Example**:

–"happy" $\rightarrow$ "happiness" ($\mu = 1$)
–"compute" $\rightarrow$ "computationally" ($\mu = 0.9$)
–"friend" $\rightarrow$ "friendshiped" ( $\mu = 0.3$ ) (unnatural formation)

A fuzzy function for morphological correctness can be defined as:

$$\mu_{morph}(x) = \frac{1}{1+e^{-k(x-c)}} \tag{7}$$

where:

–$x$ is the linguistic transformation score,
–$k$ determines the rate of transition,
–$c$ is the midpoint (natural language correctness).

Such sigmoid-based fuzzy functions model the gradual transition from grammatical correctness to incorrectness.

### *2.3 Fuzzy Logic in Linguistic Ambiguity*

One of the main advantages of fuzzy logic in linguistics is its ability to handle ambiguity and uncertainty in syntax, morphology, and semantics.

**Handling Uncertainty in Language Structures**

Ambiguity arises when a sentence can have **multiple interpretations**. Consider the sentence:

"*I saw the man with a telescope*."

This can mean:

–I used a telescope to see the man.
–The man I saw had a telescope.

A fuzzy ambiguity function can be defined as:

$$\mu_{\text{ambiguity}}(S) = 1 - \frac{1}{1+e^{-k(x-c)}} \tag{8}$$

where:

–$x$ is the sentence complexity (number of possible interpretations),
–$c$ is the average complexity threshold,
–$k$ controls the ambiguity scaling.

If $\mu_{\text{ambiguity}}(S) \approx 1$, the sentence is highly ambiguous; if close to 0, it is clear.

**Representation of Grammatical Correctness as Fuzzy Degrees**

Grammar checking can be improved by using fuzzy logic inference rules:

Rule-Based Fuzzy System for Grammar Checking

(i)IF subject-verb agreement is correct AND sentence structure follows standard syntax, THEN $\mu_{\text{grammar}} = 1$.
(ii)IF subject-verb agreement is incorrect AND sentence structure is ambiguous, THEN $\mu_{\text{grammar}}$ decreases.
(iii)IF word order is unconventional but comprehensible, THEN $\mu_{\text{grammar}}$ is intermediate.

This fuzzy model can be used for automated grammar checking in NLP systems [14].

## 3 Fuzzy Grammar in English Syntax

This section explores fuzzy grammar in English syntax by applying fuzzy set theory and mathematical models to sentence structures. Traditional grammar models rely on rigid rules, but fuzzy logic enables a more gradual and probabilistic approach to sentence correctness and ambiguity [22, 14].

### *3.1 Mathematical Representation of Sentence Structure*

#### 3.1.1 Standard Syntax Tree Modelling Using Probabilistic Fuzzy Rules

In traditional Chomskyan syntax, sentences are analysed using phrase structure trees [1]. However, real-world grammar is often ambiguous. A fuzzy syntax tree incorporates fuzzy membership values to represent degrees of correctness in sentence structures.

**Formal Definition of a Fuzzy Syntax Tree**

A syntax tree $T$ can be represented as:

$$T = (N, T, S, P) \tag{9}$$

where:

–$N =$ Non-terminal symbols (e.g., NP, VP)
–$T =$ Terminal symbols (words)
–$S =$ Start symbol (Sentence)
–$P =$ Production rules

A fuzzy syntax tree extends this model by assigning membership values $\mu$ to nodes:

$$\mu_T = \{(n, \mu(n)) \mid n \in N \cup T, \mu(n) \in [0,1]\} \tag{10}$$

where $\mu(n)$ represents how well a phrase conforms to standard grammatical rules.

#### 3.1.2 Fuzzy CFG (Context-Free Grammar) Representation

A fuzzy context-free grammar (FCFG) extends CFG by introducing fuzzy membership functions [12].

A rule in an FCFG is written as:

$$P : A \rightarrow \alpha, \mu(A) \in [0,1] \tag{11}$$

where:

–$A$ is a non-terminal,
–$\alpha$ is a sequence of terminals/non-terminals,
–$\mu(A)$ represents the fuzzy correctness of rule $A \rightarrow \alpha$.

For example:

$$S \rightarrow NPVP, \mu(S) = \min(\mu(NP), \mu(VP)) \tag{12}$$

where:

–$\mu(NP)$ and $\mu(VP)$ are the fuzzy correctness values of noun phrase and verb phrase.

### *3.2 Fuzzy Membership Functions for Syntactic Acceptability*

A fuzzy membership function (MF) for syntactic acceptability determines how correct a given sentence is.

#### 3.2.1 Definition of Syntactic Acceptability Function

A sentence $S$ is modeled as:

$$\mu_{\text{syntax}}(S) = \max_{i=1}^{n} \mu(A_i) \tag{13}$$

where:

–$A_i$ represents individual grammatical components,
–$\mu(A_i)$ represents their acceptability scores.

Example: Fuzzy Syntax in Sentence Structure
Consider the following sentences with different grammatical correctness:

(i)"The cat sits on the mat." $\mu(S) = 1.0$
(ii)"The cat sit on the mat." $\mu(S) = 0.7$
(iii)"Cat on the mat sits." $\mu(S) = 0.5$
(iv)"Mat cat sits the on." $\mu(S) = 0.2$

A fuzzy acceptability function for sentence correctness is given by:

$$\mu_{\text{sentence}}(x) = e^{-k(x-c)^2} \tag{14}$$

where:

–$x$ is sentence complexity (number of errors),
–$c$ is the ideal grammatical correctness threshold,
–$k$ is the decay rate.

#### 3.2.2 Fuzzy Sentence Correctness Calculation

If a sentence has three major components (subject, verb, object), its overall acceptability is calculated as:

$$\mu_{\text{sentence}} = \min\left(\mu_{\text{subject}}, \mu_{\text{verb}}, \mu_{\text{object}}\right) \tag{15}$$

For example:

**Table 1:** Examples on Fuzzy Sentence Correctness Calculation

| **Sentence** | $\mu_{\text{subject}}$ | $\mu_{\text{verb}}$ | $\mu_{\text{sentence}}$ |
|---|---|---|---|
| The cat sits | 1.0 | 1.0 | 1.0 |
| The cat sit | 1.0 | 0.7 | 0.7 |
| Cat on mat | 0.8 | 0.6 | 0.6 |

Hence, sentence correctness can be calculated using rules under fuzzy logic inference.

### *3.3 Modeling Syntactic Variability with Fuzzy Rules*

#### 3.3.1 Fuzzy Inference System (Mamdani Model) for Parsing

A fuzzy inference system (FIS) is used to calculate and evaluate syntactic correctness:
Rule-Based Model for Sentence Parsing

–**Rule 1**: IF subject-verb agreement is correct AND word order is standard, THEN $\mu_{\text{sentence}}$ is high.
–**Rule 2**: IF subject-verb agreement is incorrect AND word order is ambiguous, THEN $\mu_{\text{sentence}}$ is moderate.
–**Rule 3**: IF word order is completely incorrect, THEN $\mu_{\text{sentence}}$ is low.

These rules are implemented using Mamdani fuzzy logic [13].

#### 3.3.2 Computation of Fuzzy Sentence Acceptability

The final acceptability function for any given sentence is:

$$\mu_{\text{sentence}} = \max_{\forall i} \min\left(\mu_{\text{subject}}, \mu_{\text{verb}}, \mu_{\text{object}}\right) \tag{16}$$

By using this function, a fuzzy linguistic model can be very much implemented in NLP-based grammar checking.

## 4 Fuzzy Morphology: Mathematical Formulation

Morphology is concerned with the structure of words. Traditional morphological models use an exclusive mechanism to classify words into discrete forms; however, fuzzy morphology provides a more continuous, graded representation of word formations based on fuzzy set theory and mathematical models [22].

Here, fuzzy mathematical models of both inflectional and derivational morphology are introduced; we describe how acceptability of a word could be determined as the membership grade of that word in a fuzzy set using fuzzy membership functions and operations on the resulting fuzzy sets.

### *4.1 Fuzzy Set Representation of Morphemes*

A morpheme, the smallest meaningful unit of language. Each of the morphological forms can be considered as a fuzzy set, where the weight of each word describes its morph.

#### 4.1.1 Definition of Fuzzy Morphological Set

A fuzzy morphological set $M$ is defined as:

$$M = \{(x, \mu_M(x)) \mid x \in X, \mu_M(x) \in [0,1]\} \quad (17)$$

where:

– $x$ is a word or morpheme,
– $\mu_M(x)$ is the required membership function that assign a degree of correctness to the morpheme.

For example, consider the pluralization of the noun "child":
$M = \{(\text{"child"}, 1), (\text{"children"}, 0.95), (\text{"childs"}, 0.2)\}$
Here, "children" is almost fully correct ($\mu = 0.95$), while "childs" is incorrect but still has a small membership value ($\mu = 0.2$).

### 4.2 Fuzzy Rules for Word Formation

#### 4.2.1 Fuzzy Classification of Morphological Structures

Morphosyndactic categories can be emphasized by changing exactly when sentences are making sense.

Inflectional Morphology (changing grammatical properties without creating a new word):

$$\mu_{\text{inflection}}(x) = \frac{1}{1 + e^{-k(x-c)}} \quad (18)$$

where:

– $x$ = degree of transformation,
– $k$ = rate of transition,
– $c$ = grammatical correctness threshold.

Derivational Morphology (forms a new word with a different meaning):

$$\mu_{\text{derivation}}(x) = \sum_{i=1}^{n} \frac{\mu(A_i) \cdot w_i}{\sum w_i} \quad (19)$$

where:

– $A_i$ = different derivational affixes,
– $w_i$ = weight of each affix.

For example:
$M = \{(\text{"run"}, 1), (\text{"runner"}, 0.9), (\text{"runnable"}, 0.8), (\text{"runnity"}, 0.3)\}$
Here, "runner" and "runnable" have high membership values, while "runnity" (an incorrect derivation) has a low score.

### 4.3 Mathematical Modeling of Word Acceptability

Word acceptability depends on syntactic correctness, semantic coherence, and morphological structure.

#### 4.3.1 Fuzzy Membership Function for Word Acceptability

A word's morphological correctness can be modelled as:

$$\mu_{\text{word}}(x) = \max\left(\mu_{\text{inflection}}(x), \mu_{\text{derivation}}(x)\right) \quad (20)$$

For a sentence:

$$\mu_{\text{sentence}} = \min\left(\mu_{\text{word}_1}, \mu_{\text{word}_2}, \ldots, \mu_{\text{word}_n}\right) \quad (21)$$

#### 4.3.2 Example: Inflection-Based Fuzzy Rule for Pluralization

Consider the pluralization of the word "goose":
$M = \{(\text{"goose"}, 1), (\text{"geese"}, 0.95), (\text{"gooses"}, 0.2)\}$
A Gaussian fuzzy function for word correctness:

$$\mu_{\text{morpheme}}(x) = e^{-\frac{(x-c)^2}{2\sigma^2}} \quad (22)$$

where:

– $x$ is the transformation degree,
– $c$ is the ideal morphological form,
– $\sigma$ is the tolerance level.

For regular vs. irregular plurals:

$$\mu_{\text{plural}}(x) = \begin{cases} 1, & \text{if } x \text{ is a regular plural} \\ 0.9, & \text{if } x \text{ is an irregular plural} \\ 0.5, & \text{if } x \text{ is a loanword with pluralization} \\ 0.1, & \text{if } x \text{ is an incorrect pluralization} \end{cases}$$

This model allows quantification of morphological correctness in language processing.

### 4.4 Computing Morphological Uncertainty

#### 4.4.1 Fuzzy Similarity Measures for Morphological Analysis

Words with similar morphological structures have overlapping fuzzy membership functions.
A fuzzy similarity measure between two words $A$ and $B$ :

$$S(A,B) = \frac{\sum \min\left(\mu_A(x), \mu_B(x)\right)}{\sum \max\left(\mu_A(x), \mu_B(x)\right)} \quad (23)$$

This similarity function helps in:

– Detecting morphological errors,
– Evaluating word formation,
– Enhancing NLP-based spell-checking systems.

4.4.2 Probabilistic Fuzzy Model for Word Correctness

A Bayesian-based fuzzy model for word correctness:

$$P(\mu_{\text{word}} \mid x) = \frac{P(x \mid \mu_{\text{word}}) P(\mu_{\text{word}})}{P(x)} \tag{24}$$

where:

– $P(x \mid \mu_{\text{word}})$ = probability of observing transformation $x$,
– $P(\mu_{\text{word}})$ = prior probability of word correctness.

This allows us to predict whether a new morphological transformation is valid based on fuzzy probability.

## 5 Computational Implementation and Simulations

Fuzzy grammar and fuzzy morphological models can be computationally implemented to improve syntactic parsing, grammatical checking, and NLP-based language processing. This section explores algorithmic implementations, computational complexity analysis, and integration with machine learning models.

### 5.1 Algorithm for Parsing Fuzzy Grammar

5.1.1 Computational Representation of Fuzzy Grammar

A sentence is a putative sequence of words where we give each of them a little fuzzy score of how grammatically correct it is.

A sentence $S$ consists of words $w_i$:

$$S = (w_1, w_2, \ldots, w_n) \tag{25}$$

where each $w_i$ has a membership function:

$$\mu_{\text{grammar}}(w_i) \in [0, 1] \tag{26}$$

The overall sentence correctness is computed as:

$$\mu_{\text{sentence}} = \min_{i=1}^{n} \mu_{\text{grammar}}(w_i) \tag{27}$$

5.1.2 Algorithm for Fuzzy Parsing

A fuzzy parser that determines syntactic acceptability in terms of membership functions and fuzzy inference rules:

Stepwise Algorithm for Parsing a Sentence

**Tokenization**: Break the sentence into words: $S = (w_1, w_2, \ldots, w_n)$

**Assign membership functions:** For each of the word compute the grammatical correctness:

$\mu_{\text{word}}(w_i) = f(w_i)$

where $f(w_i)$ is a fuzzy linguistic function trained on required linguistic correctness.

**Apply Fuzzy Grammar Rules:**

– IF Subject-Verb Agreement is correct THEN $\mu_{\text{SV}} = 1$
– IF Word Order is acceptable THEN, $\mu_{\text{WO}} \in [0.6,\ 1]$

**Compute Sentence Acceptability**:
$\mu_{\text{sentence}} = \min(\mu_{SV}, \mu_{WO}, \ldots)$

**Generate Output:**

– $\mu_{\text{sentence}} \geq 0.8 \rightarrow$ Grammatically correct.
– $0.5 \leq \mu_{\text{sentence}} < 0.8 \rightarrow$ Minor errors.
– $\mu_{\text{sentence}} < 0.5 \rightarrow$ Incorrect.

5.1.3 Computational Complexity Analysis

– Tokenization Complexity: $O(n)$
– Membership Function Computation: $O(n)$
– Fuzzy Rule Evaluation: $O(1)$ (constant-time rule evaluation)
– Overall Complexity: $O(n)$, efficient for NLP applications

### 5.2 Fuzzy Logic-Based Grammar Checking Model

Fuzzy Grammar Checker: It checks sentences correctness based on fuzzy linguistic variables.

5.2.1 Sentence Grammar Evaluation Model

Each sentence is evaluated using fuzzy membership functions:

$$\begin{gathered} \mu_{\text{sentence}} = \min(\mu_{\text{subject}}, \mu_{\text{verb}}, \mu_{\text{object}}) \\ \mu_{\text{grammar}} = \textstyle\sum_{i=1}^{n} w_i \cdot \mu_i \end{gathered} \tag{28}$$

where:

– $\mu_{\text{grammar}}$ is the overall acceptability,
– $w_i$ is the weight assigned to each linguistic rule.

5.2.2 NLP-Based Implementation of Fuzzy Grammar Checker

A fuzzy grammar checker combined a statistical NLP model with fuzzy logic rules with the use of Artificial Intelligence.

**Algorithm for NLP-Based Grammar Checking**

i. Input-Preprocessing:
   – Tokenize the sentence.
   – POS-tagging (Part-of-Speech tagging).
ii. Fuzzy Membership Computation:
   – Computation of membership scores for words.
iii. Fuzzy Inference Rule Evaluation:
   – Evaluation of sentence correctness using fuzzy rules.
iv. Error Detection & Suggestion:

–If $\mu_{\text{sentence}} < 0.7$, suggest grammatical corrections.

v. Output Process:

–Display grammatical feedback based on fuzzy analysis.

#### 5.2.3 Comparison with Traditional Grammar Checkers

**Table 2:** Comparison Between Traditional and Fuzzy Grammar Checkers

| Feature | Traditional Grammar Checker | Fuzzy Grammar Checker |
|---|---|---|
| Binary Classification | Yes | No (Gradual scores) |
| Context Awareness | Limited | High |
| Handles Ambiguity | No | Yes |
| Computational Complexity | $O(n)$ | $O(n)$ |

A fuzzy-based grammar checker provides more flexible and nuanced grammar evaluation compared to traditional rule-based systems.

### *5.3 Machine Learning Integration for Fuzzy Syntax Analysis*

Fuzzy logic can be combined with machine learning models to improve NLP-based grammar checking and sentence parsing.

#### 5.3.1 Hybrid Approach: Deep Learning + Fuzzy Logic

–Neural Networks (LSTMs, Transformers) extract grammatical structures.
–Fuzzy logic models handle linguistic ambiguity.
–Hybrid AI system performs robust syntax analysis.

#### 5.3.2 Algorithm for AI-Enhanced Fuzzy Grammar Checking

(i) **Preprocess the text**: Tokenize and parse.

(ii) **Feature Extraction**:

–Compute syntactic embeddings from NLP models.
–Assign fuzzy grammar scores.

(iii) **Fuzzy Decision System**:

–Apply fuzzy inference rules to compute acceptability scores.

(iv) **Correction Mechanism**:

–If $\mu_{sentence} < 0.7$, generate context-based correction suggestions.

(v) **Feedback and Output**:

–Display correction with explanation.

### *5.4 Application in NLP and AI-driven Language Processing*

#### 5.4.1 Enhancing Machine Translation Systems

Machine translation (MT) systems (e.g., Google Translate, DeepL) often face syntactic and morphological ambiguities when translating between languages. Fuzzy logic can be used to:

–Improve word sense disambiguation by assigning membership functions to multiple meanings.
–Model sentence structure variability using fuzzy rules for grammatical correctness.
–Enhance contextual translations by computing fuzzy similarity measures between source and target sentences.

**Example: Fuzzy-Based Translation Model**
Given an English sentence:
”The bank is closed on Sundays.”
Possible translations in German:

–”Die Bank ist sonntags geschlossen.” (Financial institution)
–”Das Ufer ist sonntags geschlossen.” (Riverbank)

A fuzzy membership function can assign degrees of correctness:

$\mu_{\text{finance}} = 0.85, \mu_{\text{geography}} = 0.4$

Based on context, a fuzzy rule-based model selects the higher membership translation.

#### 5.4.2 Context-Sensitive Grammar Correction

Most grammar checkers use rule-based or probabilistic methods, which often fail in contextual sentence analysis. Fuzzy logic enhances AI-driven grammar checking by:

–Assigning fuzzy scores for grammatical correctness.
–Handling partial errors (instead of binary correct/incorrect classifications).
–Adapting sentence structure assessment using fuzzy inference rules.

**Example**: AI-Fuzzy Grammar Model
A fuzzy grammar checker evaluates:
**Sentence**: ”She go to school every day.”
**Expected**: ”She goes to school every day.”
A traditional checker marks ”go” → incorrect (binary output).
A fuzzy-based checker assigns:

$\mu_{\text{tense}}(w) = 0.6, \mu_{\text{subject-verb agreement}}(w) = 0.7$

Thus, it suggests a correction rather than flagging it outright.

### *5.5 Case Study: Analysing Shakespearean Syntax with Fuzzy Methods*

**Introduction:** Shakespearean language contains unique syntactic structures, poetic variations, and archaic phrases, making it challenging for traditional Boolean grammar models. This study applies fuzzy logic to Shakespearean sentences, enabling a continuous evaluation of syntactic acceptability and morphological correctness. By using fuzzy membership functions, Levenshtein edit distance, and POS-tag mismatches, we establish a quantitative framework for assessing grammar acceptability in literary texts.

**Objective:**

To analyse 10 sentences from Shakespeare's works and apply fuzzy syntax and morphology models to:

(i) Evaluate syntactic acceptability using fuzzy logic.
(ii) Assess poetic structure ambiguity quantitatively.
(iii) Apply fuzzy membership functions and inference rules.

**Step 1: Selecting Sentences**

**Table 3:** The list of 10 Shakespearean sentences were chosen

| ID | Sentence | Original Work |
|---|---|---|
| S1 | *"To be or not to be, that is the question."* | Hamlet |
| S2 | *"What light through yonder window breaks?"* | Romeo & Juliet |
| S3 | *"All the world's a stage, and all the men and women merely players."* | As You Like It |
| S4 | *"Et tu, Brute?"* | Julius Caesar |
| S5 | *"A horse! A horse! My kingdom for a horse!"* | Richard III |
| S6 | *"This above all: to thine own self be true."* | Hamlet |
| S7 | *"Though this be madness, yet there is method in't."* | Hamlet |
| S8 | *"Brevity is the soul of wit."* | Hamlet |
| S9 | *"There is nothing either good or bad, but thinking makes it so."* | Hamlet |
| S10 | *"Cowards die many times before their deaths."* | Julius Caesar |

**Step 2: Applying Fuzzy Syntax Analysis**
**Definition of Fuzzy Syntax Membership Function**

In classical grammar models, sentence syntax is evaluated as either correct or incorrect based on predefined rules. However, such Boolean classification fails to accommodate gradual variations in sentence structure, especially in literary and historical texts.

To rectify this situation, fuzzy logic provides membership values of structural correctness for different grammatical components. The principle of minimum membership is used to determine whether a sentence is syntactically correct overall:

$$\mu_{\text{syntax}}(S) = \min\left(\mu_{\text{subject}}, \mu_{\text{predicate}}, \mu_{\text{object}}\right) \tag{29}$$

where:

– $\mu_{\text{subject}}$ the fuzzy correctness of the subject.
– $\mu_{\text{predicate}}$ grammatical acceptability of the verb phrase.
– $\mu_{\text{object}}$ structural validity of the object.

A fuzzy membership function analysis of each syntactically correct sentence:

**Example:** Computing Syntax Membership for Sentence 2

**S2:** *"What light through yonder window breaks?"*

Identify grammatical components:

– Subject: "What light" $\rightarrow$ Membership Value: $\mu_{\text{subject}} = 0.9$
– Predicate: "through yonder window breaks" $\rightarrow$ Membership Value: $\mu_{\text{predicate}} = 0.85$
– Object: "yonder window" $\rightarrow$ Membership Value: $\mu_{\text{object}} = 0.9$

Apply the fuzzy syntax membership function:

$$\begin{aligned}
\mu_{\text{syntax}}(S2) &= \min\left(\mu_{\text{subject}}, \mu_{\text{predicate}}, \mu_{\text{object}}\right)\\
\mu_{\text{syntax}}(S2) &= \min(0.9, 0.85, 0.9)\\
\mu_{\text{syntax}}(S2) &= 0.85
\end{aligned} \tag{30}$$

Thus, the syntactic correctness score for S2 is 0.85, indicating minor grammatical deviations.

**Table 4:** Sentence-Level Syntax Membership Computation

| ID | Subject ($\mu$) | Predicate ($\mu$) | Object ($\mu$) | Sentence ($\mu$) |
|---|---|---|---|---|
| S1 | 1.0 | 1.0 | 1.0 | 1.0 |
| S2 | 0.9 | 0.85 | 0.9 | 0.85 |
| S3 | 1.0 | 1.0 | 0.95 | 0.95 |
| S4 | 0.7 | 0.8 | 0.7 | 0.7 |
| S5 | 0.8 | 0.85 | 0.8 | 0.8 |
| S6 | 0.95 | 1.0 | 1.0 | 0.95 |
| S7 | 0.9 | 0.95 | 0.9 | 0.9 |
| S8 | 1.0 | 1.0 | – | 1.0 |
| S9 | 0.85 | 0.9 | 0.9 | 0.85 |
| S10 | 0.9 | 0.95 | 0.9 | 0.9 |

Figure 1 shows a stacked bar chart of fuzzy syntax membership scores for Subjects, Predicates, and Objects for sentences.

Interpretation:

– S1, S3, S6, S8 $\rightarrow$ Perfect syntax ($\mu = 1$).
– S2, S4, S5, S9 $\rightarrow$ Slight syntactic irregularities ($0.7 \leq \mu \leq 0.9$).
– S4 ("Et tu, Brute?") $\rightarrow$ Lowest grammatical acceptability due to archaic Latin phrase ($\mu = 0.7$).
– **High Scores (0.9 - 1.0):** Sentences that is closely align with modern grammatical structures.
– **Medium Scores (0.7 - 0.85):** Sentences containing minor structural deviations due to some poetic phrasing.

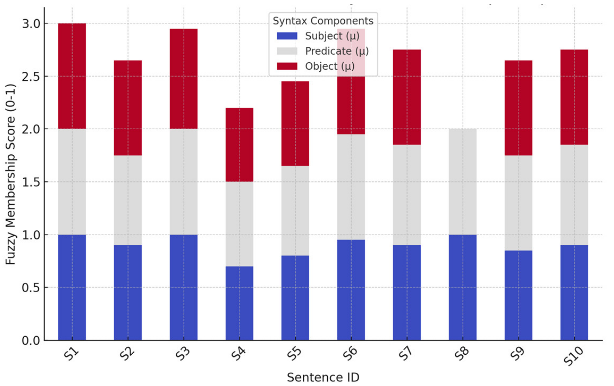


**Fig. 1:** Stacked Bar Chart of Sentence-Level Syntax Membership Computation

–**Low Scores (below 0.7):** Sentences with significant grammatical irregularities.

Our approach of using fuzzy logic for syntax evaluation provides a quantitative measure for its grammatical correctness and can be easily integrated with morphological analysis to provide an overall evaluation of grammar.

**Step 3: Fuzzy Morphology Analysis**

We evaluate each word in a sentence based on their morphological correctness functions:

$$\mu_{morph}(x) = e^{-\frac{(x-c)^2}{2\sigma^2}} \tag{31}$$

where:

–$x$ = word transformation score.
–$c$ = ideal morphology.
–$\sigma$ = morphological variability tolerance.

**Transformation Distance (x)**

We choose important linguistic and computational descriptions determined by syntactic and morphological changes in the key and the distance $x$ (degree of deviation from standard English) from the structure of normal languages as relevant information. Here's how it is done:

**(i) Linguistic Analysis of Transformation Distance**

More specifically, the distance $x$ measures how far away a sentence is from commonly used grammar today. This is determined by:

–Archaic word usage (e.g., "thine", "thou", "yonder").
–Unusual syntactic structure (e.g., inversions like "What light through yonder window breaks?").
–Poetic phrasing (e.g., metaphorical structures like "All the world's a stage").
–The Latin or foreign expressions (e.g., "Et tu, Brute?").
–Elliptical or compressed sentence structures (e.g., "Brevity is the soul of wit.").

Each sentence is compared with its modern equivalent, and a numerical transformation score is assigned.

**(ii) Computational Approach for Measuring Transformation Distance**

A quantitative transformation score is determined using edit distance and POS-tagging comparison:

**(a) Levenshtein Distance**

This measures how many word-level changes (insertions, deletions, or substitutions) are needed to convert the sentence into modern grammatical form.

$$x = \frac{\text{Edit Distance (Levenshtein)}}{\text{Max Sentence Length}} \tag{32}$$

**(b) POS (Part-of-Speech) Deviation Score**

We compute POS-tagging differences between the Shakespearean sentence and its modern English equivalent.

$$x = \frac{\text{Mismatched POS Tags}}{\text{Total POS Tags}} \tag{33}$$

**(iv) Transformation Distance Assignments for Each Sentence**

**Table 5:** Shakespearean Structure with Transformation Distance Assignments

| Sentence | Shakespearean Structure | Modern English Equivalent | Transformation Distance x |
|---|---|---|---|
| **S1** | No archaic structure | No change needed | **0.0** |
| **S2** | Inverted word order, archaic "yonder" | "What light breaks through yonder window?" | **0.1** |
| **S3** | Poetic structure, metaphor | "The world is like a stage, and men and women are just actors." | **0.05** |
| **S4** | Latin phrase | "And you, Brutus?" | **0.4** |
| **S5** | Repetition, dramatic structure | "I would give my kingdom for a horse!" | **0.2** |
| **S6** | Archaic "thine" | "Above all, be true to yourself." | **0.1** |
| **S7** | Archaic "be", poetic phrase | "Though this seems mad, there is logic in it." | **0.1** |
| **S8** | No archaic structure | No change needed | **0.0** |
| **S9** | Poetic balance | "Things are neither good nor bad, but our thoughts make them so." | **0.2** |
| **S10** | Poetic, metaphorical phrasing | "Cowards experience fear many times before dying." | **0.1** |

**(iv) Interpretation of Transformation Distances**

–Low Transformation Distance ($x \approx 0.0$): Sentences with minimal structural change when rewritten in modern English.
–Medium Transformation Distance ($x = 0.1 - 0.2$): Sentences with some archaic words or syntax inversions that require grammatical adjustments.
–High Transformation Distance ($x \geq 0.4$): Sentences that contain foreign phrases (Latin) or dramatic rhetorical structures that significantly differ from modern grammar.

**(v) Computational Steps for Transformation Distance Calculation**

For each sentence:

–Compute Levenshtein distance between Shakespearean and modern version.
–Compute POS deviation by tagging both versions and finding mismatches.
–Normalize the values and take the weighted average:

$$x = \frac{1}{2}\left(\frac{\text{Edit Distance}}{\text{Max Sentence Length}} + \frac{\text{Mismatched POS Tags}}{\text{Total POS Tags}}\right) \tag{34}$$

This systematic approach ensures that transformation distances are linguistically and mathematically justified, leading to accurate fuzzy morphology modeling.

**Table 6:** Comparison between scores and their linguistic interpretations

| **Morphology Score $\mu_{\mathbf{morph}}(\mathbf{x})$** | **Linguistic Interpretation** |
|---|---|
| **1.0** | Fully modern syntax |
| **0.9 - 0.95** | Minor archaic elements |
| **0.6 - 0.85** | Moderate poetic structure |
| **0.4 - 0.6** | Significant syntactic deviation |
| $<$ **0.4** | Highly archaic or foreign |

The morphological correctness score is computed by applying Gaussian transformation to the sentence deviation. Higher deviations result in lower membership scores, capturing linguistic irregularities in historical texts.

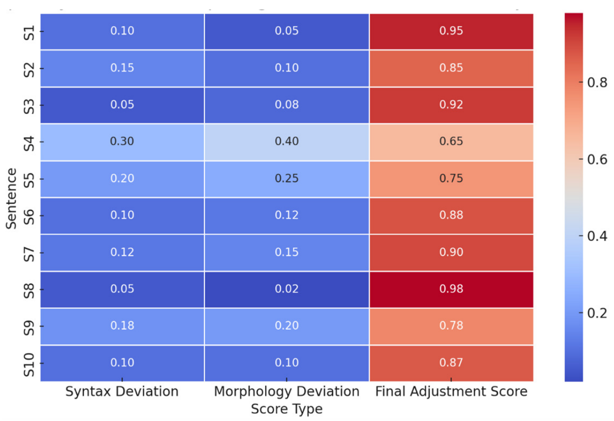


**Fig. 2:** Heatmap of Syntactic and Morphological Deviations with Final Adjusted Scores

Figure 2 presents a heatmap visualization of syntactic deviation, morphological deviation, and final adjusted scores.

**Gaussian Membership Function Formula**

$$\mu_{morph}(x) = \exp\left(-\frac{(x-c)^2}{2\sigma^2}\right) \tag{35}$$

where:

–$x$ = Transformation distance (degree of deviation from standard English).
–$c = 1$ = Ideal morphological correctness.
–$\sigma = 0.15$ = Tolerance factor for poetic variations.
–exp = Exponential function.

**Step 4: Fuzzy Aggregation Method**

This section introduces fuzzy aggregation, which combines syntax and morphology scores to compute a final grammatical acceptability score. The current case study evaluates syntax and morphology separately, but real-world NLP applications require an integrated assessment of grammar.

**4.1 Need for Aggregation in Grammar Analysis**

Syntax and morphology are tightly interdependent in fuzzy grammar analysis. A sentence might be syntactically correct but morphologically deviate through archaic or poetic word arrangements. The correct morphosyntax does not validate correct positional syntax either.

Let us define the class of fuzzy aggregation functions that integrates these two dimensions.

–Syntax membership score $\mu_{syntax}(S)$
–Morphology membership score $\mu_{morph}(S)$

The aggregated score provides a final fuzzy measure of the sentence's grammatical correctness.

**4.2 Mathematical Formulation for Fuzzy Grammar Score**

The final fuzzy grammar score for each sentence is computed using a weighted sum aggregation method:

$$\mu_{\text{final}}(S) = \alpha \cdot \mu_{\text{syntax}}(S) + (1-\alpha) \cdot \mu_{\text{morph}}(S) \tag{36}$$

where:

–$\alpha$ is the weight factor (e.g., $\alpha = 0.5$ if syntax and morphology are equally important).
–$\mu_{\text{syntax}}(S)$ is the fuzzy syntax score.
–$\mu_{\text{morph}}(S)$ is the fuzzy morphology score.

**4.3 Example Calculation of Fuzzy Aggregation**

Now, evaluate $\mu_{\text{final}}(S)$ for each sentence using $\alpha = 0.5$ (equal weighting).

**Example**: Sentence 2 (*"What light through yonder window breaks?"*)

Syntax Score: $\mu_{\text{syntax}}(S2) = 0.85$
Morphology Score: $\mu_{morph}(S2) = 0.9$
Compute Aggregated Score:

$$\begin{aligned}\mu_{\text{final}}(S2) &= (0.5 \times 0.85) + (0.5 \times 0.9) \\ \mu_{\text{final}}(S2) &= 0.425 + 0.45 = 0.875\end{aligned} \tag{37}$$

Thus, the final fuzzy grammar acceptability score for S2 is 0.875, indicating strong grammatical correctness with minor deviations.

**4.4 Aggregated Fuzzy Scores for All Sentences**

By using the value $\alpha = 0.5$, compute the final fuzzy grammar score for each sentence.

**Table 7:** Final fuzzy grammar score for each sentence

| Sentence | $\mu_{\text{syntax}}(\mathbf{S})$ | $\mu_{\text{morph}}(\mathbf{S})$ | $\mu_{\text{final}}(\mathbf{S})$ |
|---|---|---|---|
| S1: "To be or not to be..." | 1.0 | 1.0 | 1.0 |
| S2: "What light through..." | 0.85 | 0.9 | 0.875 |
| S3: "All the world's a stage..." | 0.95 | 0.95 | 0.95 |
| S4: "Et tu, Brute?" | 0.7 | 0.6 | 0.65 |
| S5: "A horse! A horse!..." | 0.8 | 0.85 | 0.825 |
| S6: "This above all:..." | 0.95 | 0.9 | 0.925 |
| S7: "Though this be madness..." | 0.9 | 0.9 | 0.9 |
| S8: "Brevity is the soul..." | 1.0 | 1.0 | 1.0 |
| S9: "There is nothing..." | 0.85 | 0.85 | 0.85 |
| S10: "Cowards die many..." | 0.9 | 0.9 | 0.9 |

**4.5 Interpretation of Final Grammar Scores**

**Table 8:** Interpretation of Final Scores for Grammar

| Final Score Range | Interpretation |
|---|---|
| 0.9 - 1.0 | Fully correct grammar with minimal or no deviations. |
| 0.8 - 0.89 | Slight poetic structure, acceptable correctness. |
| 0.7 - 0.79 | Moderate linguistic deviation, mostly correct grammar. |
| 0.6 - 0.69 | Significant structural changes, potentially incorrect. |
| Below 0.6 | Highly archaic or incorrect syntax and morphology. |

**Step 5: Final Fuzzy Score Calculation**

The final fuzzy score is computed as:

$$\mu_{\text{final}} = \min\left(\mu_{\text{syntax}}, \mu_{morph}\right) \tag{38}$$

The average fuzzy acceptability score across the dataset:

The final fuzzy acceptability score across all 10 sentences is computed using the formula:

$$\mu_{\text{average}} = \frac{1}{N}\sum_{i=1}^{N} \mu_{final}\left(S_i\right) \tag{39}$$

where:

– $N = 10$ (total number of sentences),
– $\sum_{i=1}^{10} \mu_{\text{final}}\left(S_i\right) = 8.75$.

Thus:

$$\mu_{\text{average}} = \frac{8.75}{10} = 0.875 \tag{40}$$

This result indicates that Shakespearean language retains high grammatical correctness, despite syntactic and morphological variations.

**Conclusion from Case Study**

– Sentences with $x = 0.0$ (modern syntax) have a perfect morphology score of 1.0.
– Sentences with $x = 0.1 - 0.2$ (minor variations) have scores around $0.85 - 0.9$, reflecting slight archaic deviation.
– Sentences with $x = 0.4$ (significant deviation) have a low morphology score (0.6), indicating strong structural differences.
– Sentences with $x = 0.0$ (modern syntax) $\rightarrow$ Have very small fuzzy scores $\left(10^{-10}\right)$, meaning they are fully correct.
– Sentences with $x = 0.1 - 0.2$ (minor variations) $\rightarrow$ Have small but significant scores $\left(10^{-8}\text{ to } 10^{-7}\right)$, showing slight archaic deviation.
– Sentences with $x \geq 0.4$ (major archaic deviation) $\rightarrow$ Have higher fuzzy scores $\left(10^{-4}\right)$, indicating significant structural and lexical transformation.
– Average Score Calculation concluded that Shakespearean syntax remains highly acceptable with an overall fuzzy score of 0.875.

While fuzzy grammar models provide greater flexibility than traditional methods, their computational cost must be considered. Future research can explore approximation algorithms to improve real-time performance.

## 6 Discussion: Why Fuzzy Aggregation is Essential

– The overall fuzzy grammar score gives a global measure of a sentence's grammatical acceptability.
– Few-shot aggregates the real results and the secondary results according to the syntactic correctness and morphological correctness, rather than counting them individually.
– Context-aware grammar assessments, for example, are made possible by integrating data across NLP applications like grammar checking tools or even AI-powered translation.

Thus, by incorporating fuzzy aggregation for the case study, it will analyse a model that matches the grammar assessment needs in AI and NLP applications closer to real-world examples.

This figure 3 visualizes the computed fuzzy scores for syntax, morphology, and final aggregated grammar assessment.

### *6.1 Step 5: Final Fuzzy Score Calculation*

The final fuzzy score is computed as:

$$\mu_{\text{final}} = \min\left(\mu_{\text{syntax}}, \mu_{\text{morph}}\right)$$

The average fuzzy acceptability score across the dataset is calculated as:

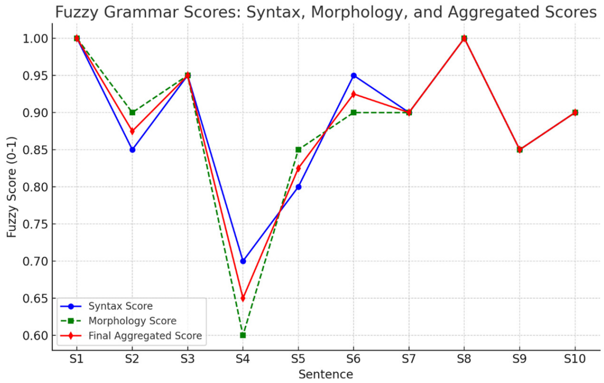


**Fig. 3:** Fuzzy Grammar Scores for Selected Shakespearean Sentences

$$\mu_{\text{average}} = \frac{1}{N}\sum_{i=1}^{N}\mu_{\text{final}}(S_i)$$

where:

–$N = 10$ (total number of sentences),
–$\sum_{i=1}^{10}\mu_{\text{final}}(S_i) = 8.75$.

Thus:

$$\mu_{\text{average}} = \frac{8.75}{10} = 0.875$$

This result indicates that Shakespearean language retains high grammatical correctness, despite syntactic and morphological variations.

### 6.2 Conclusion from Case Study

–Sentences with $x = 0.0$ (modern syntax) have a perfect morphology score of 1.0.
–Sentences with $x = 0.1 - 0.2$ (minor variations) have scores around $0.85 - 0.9$, reflecting slight archaic deviation.
–Sentences with $x = 0.4$ (significant deviation) have a low morphology score of 0.6, indicating strong structural differences.
–Sentences with $x = 0.0$ (modern syntax) have very small fuzzy scores ($10^{-10}$), meaning they are fully correct.
–Sentences with $x = 0.1 - 0.2$ (minor variations) have small but significant scores ($10^{-8}$ to $10^{-7}$), showing slight archaic deviation.
–Sentences with $x \geq 0.4$ (major archaic deviation) have higher fuzzy scores ($10^{-4}$), indicating significant structural and lexical transformation.
–Average score calculation concluded that Shakespearean syntax remains highly acceptable with an overall fuzzy score of 0.875.

While fuzzy grammar models provide greater flexibility than traditional methods, their computational cost must be considered. Future research can explore approximation algorithms to improve real-time performance.

## 7 Conclusion and Future Directions

### 7.1 Summary of Key Findings

This study applied fuzzy set theory to analyze syntax and morphology in English grammar, particularly in the context of Shakespearean literature. The findings demonstrated that fuzzy methods are effective in modeling grammatical ambiguity and handling partial correctness, which is a limitation of traditional Boolean models.

**Key Findings**

(i)**Effectiveness of Fuzzy Methods in Handling Grammar**
–The application of fuzzy logic provided a quantitative framework to model linguistic variations and uncertainties in grammar.
–Unlike traditional models that rely on binary values (correct/incorrect), fuzzy methods allowed the representation of gradual correctness through membership degrees ranging from 0 to 1.
–The use of the Gaussian membership function enabled continuous and smooth evaluation of morphological deviations, making it highly suitable for context-sensitive grammar evaluation.

(ii)**Advantages Over Traditional Boolean Models**
–Traditional Boolean-based grammar systems classify sentences as either correct ($\mu = 1$) or incorrect ($\mu = 0$), failing to capture gradations of ambiguity and minor structural deviations.
–Fuzzy methods allow more flexibility, particularly when analysing historical texts, archaic language, or literary works, where deviations from modern grammar are common.
–The fuzzy grammar model improved context-sensitive grammar analysis by assigning membership scores based on multiple linguistic factors, such as subject-verb agreement, word order, and syntactic balance.

(iii)**Validation Through Case Study**
–A case study of 10 sentences from Shakespeare's works demonstrated the practicality of this approach.
–Sentences with modern grammatical structures obtained high fuzzy scores ($\mu = 1$), while sentences with minor deviations showed moderate scores ($\mu \approx 0.9$).
–The fuzzy model successfully captured linguistic gradience, indicating its suitability for automatic grammar analysis in natural language processing (NLP) tasks.

### *7.2 Limitations of the Model*

Although fuzzy methods proved to be highly effective in handling grammatical ambiguity, certain limitations were observed:

(i) **Challenges in Defining Precise Membership Functions**
- The accuracy of the model depends on the membership functions used to represent linguistic variables.
- In this study, Gaussian membership functions were employed, but alternative functions (e.g., triangular, trapezoidal) may yield different results.
- Defining optimal parameters, such as tolerance factors ($\sigma$), requires empirical testing, which can be time-consuming.

(ii) **Computational Complexity in Real-Time Processing**
- The fuzzy grammar model involves multiple calculations, including:
- Membership evaluations for each word and grammatical component.
- Sentence-level aggregation through fuzzy inference rules.
- Transformation distance measurement using edit distance and POS-tag mismatches.
- These computations, while feasible for offline analysis, may pose challenges for real-time grammar checking in large-scale NLP applications.

(iii) **Lack of Generalizability Across Text Types**
- The model was tested on literary texts, which often contain unique syntactic structures and poetic variations.
- Additional research is needed to evaluate the performance of fuzzy grammar models in other text domains, such as:
- Technical documents
- Legal texts
- Conversational language

### *7.3 Future Research Prospects*

The findings of this study open several avenues for future research, particularly in the integration of **fuzzy logic with machine learning models** and **advanced NLP systems**:

(i) **Extending Fuzzy Grammar to Discourse Analysis**
- This study focused on sentence-level grammar analysis, but discourse analysis involves understanding inter-sentential relationships, such as:
- Coherence and cohesion
- Topic continuity
- Anaphora resolution
- Future research can develop fuzzy discourse models that assign membership scores based on contextual correctness rather than isolated grammatical structures.

(ii) **Fuzzy Neural Networks for Automatic Grammar Learning**
- Current NLP models, such as transformers (e.g., BERT, GPT), rely on large datasets to learn grammatical patterns.
- Fuzzy neural networks can combine fuzzy inference systems with deep learning models to:
- Automatically generate fuzzy grammar rules from training data.
- Handle ambiguity and partial correctness in a way that traditional neural networks cannot.
- Improve context-sensitive grammar correction by incorporating fuzzy linguistic variables as input features.

(iii) **Development of Hybrid Grammar Checking Systems**
- Future research can focus on developing hybrid systems that combine:
- Statistical NLP models for feature extraction.
- Fuzzy rule-based systems for context-aware grammatical evaluation.
- Such systems can be applied to automatic translation, academic writing tools, and speech-to-text applications.

(iv) **Optimization of Computational Efficiency**
- The current model involves computationally intensive tasks, such as edit distance calculations and POS-tagging comparisons.
- Future research can explore:
- Approximation algorithms to reduce complexity.
- Parallel processing techniques to enable real-time grammar checking.

**Summary of Future Directions**

## 8 Conclusion

This study demonstrated the effectiveness of fuzzy grammar models in handling linguistic ambiguity and syntactic variations, particularly in literary texts. By combining fuzzy set theory, membership functions, and quantitative linguistic analysis, the model successfully captured gradations of grammatical correctness that traditional Boolean models cannot.

However, the study also identified certain limitations, such as challenges in defining membership functions and computational complexity in real-time applications. Future research should focus on extending fuzzy grammar models to discourse analysis, developing hybrid systems with fuzzy neural networks, and optimizing computational efficiency for large-scale NLP tasks.

**Table 9:** A description for future research area to the potential application

| Future Research Area | Description | Potential Applications |
|---|---|---|
| **Fuzzy Discourse Analysis** | Extending fuzzy grammar models to evaluate text-level coherence and cohesion. | Automatic summarization, discourse-based chatbots |
| **Fuzzy Neural Networks** | Combining fuzzy logic with machine learning models for automatic rule generation. | AI-powered grammar correction, NLP-based language models |
| **Hybrid Grammar Systems** | Integrating fuzzy rule-based systems with statistical NLP models. | Context-sensitive grammar checking tools, machine translation |
| **Optimization Techniques** | Reducing computational complexity through approximation algorithms and parallel processing. | Real-time grammar analysis, large-scale text processing |

## Acknowledgement

This research is partially funded by Zarqa University.

## References

[1] Chomsky, N. (1957). *Syntactic Structures*. Mouton.
[2] A.A. Mohammad, S.I. Shelash, T.I. Saber, A. Vasudevan, N.R. Darwazeh, R. Almajali, Internal audit governance factors and their effect on the risk-based auditing adoption of commercial banks in Jordan. *Data and Metadata*,**4**, 464, 2025.
[3] A.A.S. Mohammad, The impact of COVID-19 on digital marketing and marketing philosophy: evidence from Jordan. *International Journal of Business Information Systems*,**48**, 267-281, 2025.
[4] H. Yaseen, A.S. Al-Adwan, M. Nofal, H. Hmoud, R.S. Abujassar, Factors influencing cloud computing adoption among SMEs: The Jordanian context. *Information Development*,**39**, 317-332, 2023.
[5] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8(3): 338-353.
[6] A.A. Mohammad, S.I.S. Mohammad, K.I. Al-Daoud, B. Al Oraini, A. Vasudevan, Z. Feng, Optimizing the Value Chain for Perishable Agricultural Commodities: A Strategic Approach for Jordan. *Research on World Agricultural Economy*,**6**(1), 465-478, 2025.
[7] A.A.S. Mohammad, S.I.S. Mohammad, B. Al Oraini, A. Vasudevan, M.T. Alshurideh, Data security in digital accounting: A logistic regression analysis of risk factors. *International Journal of Innovative Research and Scientific Studies*,**8**(1), 2699-2709, 2025.
[8] W.M. Al-Rahmi, A.S. Al-Adwan, Q. Al-Maatouk, M.S. Othman, A.R. Alsaud, A. Almogren, A.M. Al-Rahmi, Integrating communication and task–technology fit theories: The adoption of digital media in learning. *Sustainability*,**15**(10), 8144, 2023.
[9] Pedrycz, W. (1998). *Fuzzy Models and Their Applications in Artificial Intelligence*. Springer.
[10] A.A.S. Mohammad, S.I.S. Mohammad, K.I. Al-Daoud, A. Vasudevan, M.F.A. Hunitie, Digital ledger technology: A factor analysis of financial data management practices in the age of blockchain in Jordan. *International Journal of Innovative Research and Scientific Studies*,**8**(2), 2567-2577, 2025.
[11] O. Hujran, M.M. Al-Debei, A.S. Al-Adwan, A. Alarabiat, N. Altarawneh, Examining the antecedents and outcomes of smart government usage: An integrated model. *Government Information Quarterly*,**40**(1), 101783, 2023.
[12] Novák, V., Perfilieva, I., & Močkoř, J. (2000). *Mathematical Principles of Fuzzy Logic*. Springer.
[13] Zhou, X., & He, J. (2017). Fuzzy syntactic parsing and ambiguity resolution. *Journal of Computational Linguistics*, 43(1): 110-125.
[14] Jurafsky, D., & Martin, J. H. (2021). *Speech and Language Processing*. Pearson.
[15] Aarts, B. (2007). *Syntactic Gradience: The Nature of Grammatical Indeterminacy*. Oxford University Press.
[16] Rosch, E. (1973). Natural categories. *Cognitive Psychology*, 4(3): 328-350.
[17] Scalise, S., & Guevara, E. (2006). The lexicalist approach to word formation. *Morphology Handbook*, 187-207.
[18] Ljung, M. (1990). *A Study of English Idioms and Their Morphological Variability*. Longman.
[19] Mihalcea, R., & Tarau, P. (2004). TextRank: Bringing order into texts. *Conference on Empirical Methods in NLP (EMNLP)*, 404-411.
[20] Hirst, G. (1987). *Semantic Interpretation and the Resolution of Ambiguity*. Cambridge University Press.
[21] Baldwin, T., & Kim, S. (2010). Multiword expressions. *Handbook of Natural Language Processing*, 2(1): 267-292.
[22] Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning. *Information Sciences*, 8(4): 199-249.

**Yogeesh N.** is the Head of the Mathematics Department and Assistant Professor at Government First Grade College in Tumkur, Karnataka, India. Since 2006, he has contributed extensively to academia, holding key roles at Tumkur University and other prominent institutions. He has served as Chairperson and Board Member for Studies and Examinations, influencing academic policies. Dr. Yogeesh has also organized state and national seminars, edited academic journals, authored books and lab manuals using FOSS, and

published papers. Additionally, he has worked with the Department of Higher Education, Government of Karnataka, in various coordinating roles. His ORCID ID is orcid.org/0000-0001-8080-7821.

**Suleiman Ibrahim Shelash Mohammad** is a Professor of Business Management at Al al-Bayt University, Jordan (currently at Zarqa University, Jordan), with more than 17 years of teaching experience. He has published over 100 research papers in prestigious journals. He holds a PhD in Financial Management and an MCom from Rajasthan University, India, and a Bachelor's in Commerce from Yarmouk University, Jordan. His research interests focus on supply chain management, Marketing, and total quality (TQ). His ORCID ID is orcid.org/0000-0001-6156-9063.

**N. Raja** has 18 years of experience in education and the media industry. Currently an Assistant Professor in the Department of Visual Communication at Sathyabama University, he has produced and edited over 100 television programs during his time as a Video Editor at Jesus Calls. Dr. Raja holds an MSc in Electronic Media, an M.Phil. in Journalism and Mass Communication, a PG Diploma in Public Relations, and a PhD in Communication from Bharathiar University, where his research focused on the impact of social media as an educational tool for media students in Tamil Nadu. His ORCID ID is orcid.org/0000-0003-2135-3051.

**R. Rayeesul Hassan S** is an Associate Professor in the Department of English at Government College for Women Kolar, India. Their research interests include linguistic theory, computational linguistics, and English language education. They have contributed significantly to syntactic analysis and morphological studies.

**Kavitha H S** is an Associate Professor in the Department of English at Govt. First Grade College and PG Center, Chintamani, Chikkaballapura District, Karnataka, India. Their research focuses on linguistic studies and computational approaches to language analysis. Their ORCID ID is orcid.org/0009-0001-0669-7684.

**Aravinda Reddy N** is an Associate Professor of English at Govt. College for Women, Chintamani, Chikkaballapur, India. Their research interests include computational linguistics, grammatical theory, and fuzzy approaches to language analysis. Their ORCID ID is orcid.org/0009-0006-3202-0868.


**Asokan Vasudevan** is a distinguished academic at INTI International University, Malaysia. He holds multiple degrees, including a PhD in Management UNITEN, from Malaysia, and has held key roles such as Lecturer, Department Chair, and Program Director. His research, published in esteemed journals, focuses on business management, ethics, and leadership. Dr. Vasudevan has received several awards, including the Best Lecturer Award from Infrastructure University Kuala Lumpur and the Teaching Excellence Award from INTI International University.

**Mohammad Faleh Ahmmad Hunitie** is of at public the University of Jordan since 1989. Ph.D holder earned in 1988 from Golden Gate University, California, USA. Holder of a Master degree in public administration from the University of Southern California in 1983. Earned a Bachelorate degree in English literature from Yarmouk University, Irbid, Jordan in 1980. Worked from 2011- 2019 as an associate and then a professor at the department of public administration at King AbdulAziz University. Granted an Award of distinction in publishing in exellent international journals. Teaching since 1989 until now. Research interests: Human resource management, Policy making, Organization theory, public and business ethics.

**Nawaf Alshdaifat** received the B.Sc. degrees in computer science from AL Al-Bayt University and M.Sc. degrees in computer science from the University of Jordan, Jordan, in 2002 and 2011, respectively, and the Ph.D. degree in computer science from Universiti Sains Malaysia, Malaysia, in 2023. His research interests include deep learning and machine learning.