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Current Record Range of Laplace Distributed Data with Application on Saudi Arabia Industrial Data

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Abstract: In this study, we investigated the current record which is a method of selecting upper and lower records simultaneously and then subtracting them to calculate the current record range. The cumulative distribution functions of the current record range from any distribution will be introduced along with the probability and cumulative distribution functions for such records when the data are Laplace distributed. In addition, closed forms of the moments of the current record range were found along with a recurrence form for it. Real Laplace-distributed data were used to show the effectiveness of the new results, along with a conclusion section.

Keywords: Current record range; Laplace distribution; Moments; Recurrence relations; Industrial data

1 Introduction

There is no doubt that industry is one of the most important components of the renaissance of nations. The Kingdom of Saudi Arabia has given great importance to industry, as it has made it one of its national priorities, that is, the ”leadership in energy and industry”. Since statistics has a major role in all aspects of life, including dealing with industrial data in terms of describing and analyzing it, which contributes greatly to understanding and extracting information from it, as well as predicting its future results, we had to clarify a new method of analysis data that will have many applications in industry, that is the current record.

The current record is a new way to choose an upper and lower record simultaneously invented by Houchens [11]. And when subtracting them, you can get the current record range, which can define so many things, such as whether a product is on spec if its measure is lower than it, or off spec if its measure is above it. So, we can say that current records and current record range have useful applications in the industrial field. Also, it can be useful whenever we are interested in the range of some data, such as weather, energy levels, and engineering data.

Many authors studied some aspects of the current records, like Aldallal [6], who studied the current record and current record range when the original data follow the generalized exponential distribution and found many useful recurrence relations for its moment-generating functions. Also, you can find similar results by Barakat et al. [8] but for other distributions. Raqab [12] established bound expectations for current record range and record increment with bounded support, and later in [14], he made some inequalities about it. While Barakat et al. [9] established an algorithm from which we can find exact prediction intervals for future current records and current record range when it follows any continuous distribution. Ahmadi and Balakrishnan [3] used the current records to find an outer and inner prediction interval for order statistics, and Chahkandi and Ahmadi [10] did the same to predict the intervals for the k records. Finding distribution-free prediction intervals for future current record statistics is a prominent issue that has been tackled by Raqab [13]. Later, Ahmadi and Balakrishnan [2] did the same but for quantile intervals. A new generalized type of current record data was introduced by Ahmadi et al. [4], which they named the Current K-records and found their use in the distribution-free confidence intervals. Another study on the current record range was introduced by Ahmadi and Balakrishnan [5] by finding its confidence intervals for some quantiles in terms of it. And after that they studied the preservation of some reliability properties for it which cited in [1].

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In this paper, we will denote the lower current record by L_m^{c*} and the upper current record by U_m^{c*} for the sequence $\{X_m\}$ when the mth record of any kind is detected. Noticed that, $L_{m+1}^{c*} = L_m^{c*}$ if $U_{m+1}^{c*} > U_m^{c*}$ and $U_{m+1}^{c*} = U_m^{c*}$ if $L_{m+1}^{c*} < L_m^{c*}$. That is, the lower current record value is the lowest observation seen to date at the time when the mth record (of either kind) is observed. We can conclude that, $U_0^{c*} = L_0^{c*} = X_1$. For $m \geq 1$, the interval (L_m^{c*}, U_m^{c*}) will be called the record coverage. Current record range will be calculated by $R_m^{c*} = U_m^{c*} - L_m^{c*}$. A new lower current record or upper current record will be created once a new current record range is set.

The probability distribution function (PDF) of L_m^{c*}, U_m^{c*} and R_m^{c*}, based on any cumulative distribution function (CDF) $F(x)$, were introduced by Houchens [11] in the following formulas

$$f_{L_m^{c*}}(x) = 2^m f(x)\Big[1 - F(x)\sum_{\kappa=0}^{m-1}\frac{[-\log F(x)]^\kappa}{\kappa!}\Big], \tag{1}$$

$$f_{U_m^{c*}}(x) = 2^m f(x)\Big[1 - \bar{F}(x)\sum_{\kappa=0}^{m-1}\frac{[-\log \bar{F}(x)]^\kappa}{\kappa!}\Big], \tag{2}$$

and

$$f_{R_m^{c*}}(r) = \frac{2^m}{(m-1)!}\int_{-\infty}^{\infty} f(r+x)f(x)\Big[-\log\left(1 - F(r+x) + F(x)\right)\Big]^{m-1}dx,\ 0 < r < \infty, \tag{3}$$

where $\bar{F}(x) = 1 - F(x)$.

As mentioned before that the current record has some usage in the industrial field, and because some of this industrial field data follows the Laplace distribution, we had to make this study. We will study the current record data when it originally follows the Laplace distribution function $La(\theta,\lambda)$. The PDF and CDF of the Laplace distribution function are given respectively by:

$$f(x|\theta,\lambda) = \begin{cases} \frac{1}{2\lambda}e^{-\frac{x-\theta}{\lambda}}, & x \geq \theta \\ \frac{1}{2\lambda}e^{-\frac{-x+\theta}{\lambda}}, & x < \theta, \end{cases} \tag{4}$$

$$F(x|\theta,\lambda) = \begin{cases} 1 - \frac{1}{2}e^{-\frac{x-\theta}{\lambda}}, & x \geq \theta \\ \frac{1}{2}e^{-\frac{-x+\theta}{\lambda}}, & x < \theta. \end{cases} \tag{5}$$

Where θ is a location parameter, and $\lambda > 0$ is a scale parameter. In our study we will focus on the $x \geq \theta$ scenario.

In Section 2, we will introduce a new theory for which we can find the CDF of the current record range from any continuous distribution. After that, we will take the Laplace distribution function as an application on it and find its CDF and PDF. The moments and a recurrence relation of the current record range under a Laplace originally distributed data was introduced in Section 3. To show how to apply the results of this paper, Section 4 contains some real data to clarify that. Section 5 has some conclusion remarks on the paper.

2 CDF and PDF of Current Record Range

In this section, we will try to find the CDF of a current record range from any continuous distribution function. After that, a closed form for the PDF of the current record range will be presented. All when the original data follows a Laplace distribution $X \sim La(\theta,\lambda)$.

Theorem 2.1. For any $m > 1$, the CDF of R_m^{c*}, is given, by

$$F_{R_m^{c*}}(r) = \sum_{\kappa=0}^{m-1}\frac{2^m}{\kappa!}\int_{-\infty}^{\infty}(1 + F(x) - F(r+x))[-\log(1 + F(x) - F(r+x))]^\kappa f(x)dx. \tag{6}$$

Proof. To prove (6), we use the relation (3) and start with the evident relation

$$F_{R_m^{c*}}(r) = \frac{2^m}{(m-1)!}\int_{-\infty}^{\infty}\int_0^r[-\log(1 - F(v+x) + F(x))]^{m-1}f(v+x)f(x)dvdx.$$

Upon taking the transformation $F(v+x) = z$, where $f(v+x)dv = dz$, we get

$$F_{R_m^{c*}}(r) = \frac{2^m}{(m-1)!}\int_{-\infty}^{\infty}\int_{F(x)}^{F(r+x)}[-\log(1 - z + F(x))]^{m-1}f(x)dzdx.$$

Now, let

$$I = \int_{F(x)}^{F(r+x)} [-\log(1-z+F(x))]^{m-1} dz.$$

By taking the transformation $t = -\log(1+F(x)-z)$, where $dt = \frac{1}{1+F(x)-z} dz$, we get

$$I = \int_0^{-\log(1+F(x)-F(r+x))} t^{m-1} e^{-t} dt.$$

Thus, we have

$$F_{R_m^{c*}}(r) = 2^m \int_{-\infty}^{\infty} f(x) \int_0^{-\log(1+F(x)-F(r+x))} \frac{t^{m-1} e^{-t}}{(m-1)!} dt dx.$$

Upon using the relation

$$\int_0^{\tau} \frac{x^{m-1} e^{-x}}{(m-1)!} dx = \sum_{\kappa=0}^{m-1} \frac{e^{-\tau} \tau^{\kappa}}{\kappa!}, \tag{7}$$

we get (6). Thus, the theorem is proved.

Now, we will use the result of Theorem 2.1 to find the CDF and PDF of the current record range when the original data follows a Laplace distribution.

Theorem 2.2. The CDF of current record range of Laplace distributed data $F_{R_m^{c*}}(r)$ and it PDF $f_{R_m^{c*}}(r)$ for $m > 1$ will be given by

$$F_{R_m^{c*}}(r) = \sum_{\kappa=0}^{m-1} \frac{2^{m-\kappa-1}}{\kappa!} \sum_{p=0}^{\infty} \frac{a_p(\kappa)}{2^p} \left[\frac{(1-e^{-r/\lambda})^{\kappa+p}}{\kappa+p+1} - \frac{(1-e^{-r/\lambda})^{\kappa+p+1}}{2(\kappa+p+2)} \right], \tag{8}$$

where $a_p(\kappa)$ is the coefficient of the logarithmic expansion introduced by Balakrishnan and Cohen [7]

$$[-ln(1-t)]^i = \left(\sum_{p=1}^{\infty} \frac{t^p}{p} \right)^i = \sum_{p=0}^{\infty} a_p(i) t^{i+p}, \quad |t| < 1, \tag{9}$$

and

$$f_{R_m^{c*}}(r) = \sum_{\kappa=0}^{m-1} \frac{2^{m-\kappa-1}}{\lambda \kappa!} \sum_{p=0}^{\infty} \frac{a_p(\kappa)}{2^p} \left[\frac{(\kappa+p)(1-e^{-r/\lambda})^{\kappa+p-1} e^{-r/\lambda}}{\kappa+p+1} - \frac{(\kappa+p+1)(1-e^{-r/\lambda})^{\kappa+p} e^{-r/\lambda}}{2(\kappa+p+2)} \right]. \tag{10}$$

Proof. First, we will start with (4) and (5) in (6) to get

$$F_{R_m^{c*}}(r) = \sum_{\kappa=0}^{m-1} \frac{2^m}{2\lambda \kappa!} \int_{\theta}^{\infty} \left(1 - \frac{1}{2} \xi e^{-\frac{x-\theta}{\lambda}}\right) e^{-\frac{x-\theta}{\lambda}} [-\log(1 - \frac{1}{2} \xi e^{-\frac{x-\theta}{\lambda}})]^{\kappa} dx,$$

where $\xi = 1 - e^{-r/\lambda}$. After that we will make substitute $z = \frac{1}{2} e^{-\frac{x-\theta}{\lambda}}$, we get

$$F_{R_m^{c*}}(r) = \sum_{\kappa=0}^{m-1} \frac{2^m}{\kappa!} \int_0^{\frac{1}{2}} (1-\xi z)[-\log(1-\xi z)]^{\kappa} dz,$$

upon using relation (9), we reach

$$F_{R_m^{c*}}(r) = \sum_{\kappa=0}^{m-1} \frac{2^m}{\kappa!} \sum_{p=o}^{\infty} a_p(\kappa) \int_0^{\frac{1}{2}} (1-\xi z)[\xi z]^{\kappa+p} dz.$$

Finally, with some routine calculations, we find (8). (10) is obviously the first derivative of (8).

Another but closed form of the PDF of the current record range can be given in Theorem 2.3. Although we will need the result of the following lemma in the proof.

lemma 2.1. For $n > 1$, the following integration will equal:

$$\int_0^a e^{-x} x^{n-1} (1-e^{-x}) dx = \Gamma(n) - \Gamma(n,a) + 2^{-n} \Big(\Gamma(n,2a) - \Gamma(n) \Big), \tag{11}$$

where $\Gamma(n,a)$ in the incomplete gamma function.

Theorem 2.3. For $m \geq 2$, the PDF of the current record range of originally Laplace distributed data can be described using the following closed form

$$f_{R_m^{c*}}(r) = \frac{2^m e^{-r/\lambda}}{(m-1)!\lambda(1-e^{-r/\lambda})^2}\Bigg[\Gamma(m) - \Gamma\Big(m, -Log\frac{1}{2}\Big[1+e^{-r/\lambda}\Big]\Big) + 2^{-m}\Big(\Gamma(m, -2\,Log\frac{1}{2}\Big[1+e^{-r/\lambda}\Big]) - \Gamma(m)\Big)\Bigg]. \tag{12}$$

Proof. By substituting (4) and (5) in (3), we get

$$f_{R_m^{c*}}(r) = \frac{2^m e^{-r/\lambda}}{(m-1)!(2\lambda)^2}\int_\theta^\infty \Big(e^{-\frac{x-\theta}{\lambda}}\Big)^2\Bigg[-Log\Big[1-\frac{1}{2}e^{-\frac{x-\theta}{\lambda}}(1-e^{-r/\lambda})\Big]\Bigg]^{m-1} dx$$

By taking some transformations such as $\iota = e^{-\frac{x-\theta}{\lambda}}$ and $\varpi = -Log[1-\frac{1}{2}\nu\,\iota]$, where $\nu = 1-e^{-r/\lambda} \geq 0$, we reach

$$f_{R_m^{c*}}(r) = \frac{2^m e^{-r/\lambda}}{(m-1)!\lambda(\nu)^2}\int_0^{-Log[1-\frac{1}{2}\nu]} e^{-\varpi}(\varpi)^{m-1}(1-e^{-\varpi})d\varpi \tag{13}$$

Upon using formula (11) from Lemma 2.1 in (13) we reach (12).

3 Moments and Recurrence Relation

We will establish a theory to find the moments of current record range $\Psi_{R_m^{c*}}^{(i)} = E(R_m^{c*})^i, m = 2,3,4,...$ where i is a positive integer represents the ordinal of the moment. Also, a recurrence relation between current record range moments will be presented.

Theorem 3.1. The i^{th} moment of the moments of current record range $\Psi_{R_m^{c*}}^{(i)}$ and for $m > 1$ will equal

$$\Psi_{R_m^{c*}}^{(i)} = \lambda^i\Gamma(i+1)\sum_{\kappa=0}^{m-1}\frac{2^{m-\kappa-1}}{\kappa!}\sum_{p=0}^{\infty}\frac{a_p(\kappa)}{2^p}\Bigg[\frac{\kappa+p}{\kappa+p+1}\sum_{j=0}^{\kappa+p-1}(-1)^j\binom{\kappa+p-1}{j}(1+j)^{-(i+1)} - \frac{(\kappa+p+1)}{2(\kappa+p+2)}\sum_{j=0}^{\kappa+p}(-1)^j\binom{\kappa+p}{j}(1+j)^{-(i+1)}\Bigg]. \tag{14}$$

Proof. We will use the well-known formula to find the i^{th} moment

$$\Psi_{R_m^{c*}}^{(i)} = \int_0^\infty r^i f_{R_m^{c*}}(r)\,dr,$$

then we will substitute (10) in it to get

$$\Psi_{R_m^{c*}}^{(i)} = \sum_{\kappa=0}^{m-1}\frac{2^{m-\kappa-1}}{\lambda\,\kappa!}\sum_{p=0}^{\infty}\frac{a_p(\kappa)}{2^p}\Bigg[\frac{\kappa+p}{\kappa+p+1}\int_0^\infty r^i e^{-r/\lambda}(1-e^{-r/\lambda})^{k+p-1}dr - \frac{(\kappa+p+1)}{2(\kappa+p+2)}\int_0^\infty r^i e^{-r/\lambda}(1-e^{-r/\lambda})^{k+p}dr\Bigg]. \tag{15}$$

Let

$$\Upsilon(i) = \int_0^\infty r^i e^{-r/\lambda}(1-e^{-r/\lambda})^{\kappa+p}dr,$$

using the following binomial expansion in the last integration

$$(1-e^{-r/\lambda})^{\kappa+p} = \sum_{j=0}^{\kappa+p}\binom{\kappa+p}{j}(-1)^j e^{-j\,r/\lambda},$$

we reach

$$\Upsilon(i)=\sum_{j=0}^{\kappa+p}\binom{\kappa+p}{j}(-1)^j\int_0^{\infty}r^i e^{-(j+1)r/\lambda}dr=\lambda^{i+1}\Gamma(i+1)\sum_{j=0}^{\kappa+p}\binom{\kappa+p}{j}(-1)^j(1+j)^{-(i+1)}. \tag{16}$$

And by substituting the result of (16) into (15) and with some simplifications, we conclude (14).

lemma 3.1. A recurrence relation between $\Psi^{(i)}_{R^{c*}_{m+1}}$ and $\Psi^{(i)}_{R^{c*}_{m}}$ can be given by

$$\begin{aligned}\Psi^{(i)}_{R^{c*}_{m+1}}-2\Psi^{(i)}_{R^{c*}_{m}}=\frac{\lambda^i\Gamma(i+1)}{m!}\sum_{p=0}^{\infty}\frac{a_p(m)}{2^p}\Bigg[&\frac{m+p}{m+p+1}\sum_{j=0}^{m+p-1}(-1)^j\binom{m+p-1}{j}(1+j)^{-(i+1)}\\&-\frac{(m+p+1)}{2(m+p+2)}\sum_{j=0}^{m+p}(-1)^j\binom{m+p}{j}(1+j)^{-(i+1)}\Bigg]\end{aligned} \tag{17}$$

Proof. By replacing m with $m+1$ in (14) and with some calculations, we get (15).

Using equations (14) and (17), we conducted the mean (1^{st} moment) of current record range $\Psi_{R^{c*}_m}$ for different values of λ and m. And the results have been recorded in Table 1.

Table 1: Mean of current record range $\Psi_{R^{c*}_m}$

λ / m	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3
2	0.349821	0.437276	0.524732	0.612187	0.699642	0.787098	0.874553	0.962008	1.04946
3	0.942015	1.17752	1.41302	1.64853	1.88403	2.11953	2.35504	2.59054	2.82605
4	1.98287	2.47859	2.97431	3.47003	3.96575	4.46146	4.95718	5.4529	5.94863
5	3.99586	4.99483	5.9938	6.99276	7.99173	8.99069	9.98966	10.9886	11.9876
6	7.99915	9.99894	11.9987	13.9985	15.9983	17.9981	19.9979	21.9977	23.9975
7	15.9998	19.9998	23.9998	27.9997	31.9997	35.9997	39.9996	43.9996	47.9995

As we can see, when the value of λ increases, the value of the mean increases. Also, when the order of the current record is enlarged, the value of the mean is enlarged too. All of the previous is a normal thing, and it's a good indication that the formulas are working fine.

4 Numerical Analysis(Data Description and Distribution Fitting)

This study analyzes industrial production data related to economic activities within the industrial sector of the Kingdom of Saudi Arabia. The data were obtained from the *Industrial Production Survey*, conducted by the General Authority for Statistics (GaStat). The main objectives of this survey are:

– To compile the Industrial Production Index (IPI);
– To provide short-term data on the industrial sector and its contribution to economic development.

The survey covers the following economic activities classified under the industrial sector:

– Mining and quarrying;
– Manufacturing;
– Electricity and gas supply;
– Water supply, sanitation, waste management, and treatment.

The dataset is publicly accessible at the official GaStat website: https://www.stats.gov.sa/statistics-tabs?tab=436312&category=123454.

For this study, we focused specifically on the *manufacturing industries*, using monthly data from the period **January 2023 to February 2025**. The observed values are:

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121.62228, 118.80192, 122.40066, 113.65555, 115.26614, 115.97938, 117.73552,
117.89077,117.68901, 120.17991, 120.45594, 119.03682, 121.83467, 124.72838,
126.22889, 129.51602,126.32376, 124.22691, 123.87777, 123.62504, 122.25780,
126.09280, 125.56340, 126.67740, 127.12330, 128.66340.
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These data were modeled using the Laplace distribution, defined by the CDF given in (5). The parameters of the distribution were estimated using the method of Maximum Likelihood Estimation (MLE). The resulting estimates are:

–Location parameter (θ) = **122.3292**
–Scale parameter (λ) = **3.5632**

To evaluate the goodness-of-fit, the Kolmogorov–Smirnov (KS) and Anderson–Darling (AD) tests were applied. The results are as follows:

–**Kolmogorov–Smirnov test**: $D = 0.11398$, p-value $= 0.8507$
–**Anderson–Darling test**: $A_n = 0.49078$, p-value $= 0.7547$

Both tests indicate that the Laplace distribution provides an adequate fit to the data, as the high p-values suggest no significant deviation from the assumed distribution. Also, Figure 1, which represents the CDF of the data's empirical distribution versus the CDF of the fitted Laplace distribution, indicates the same thing.

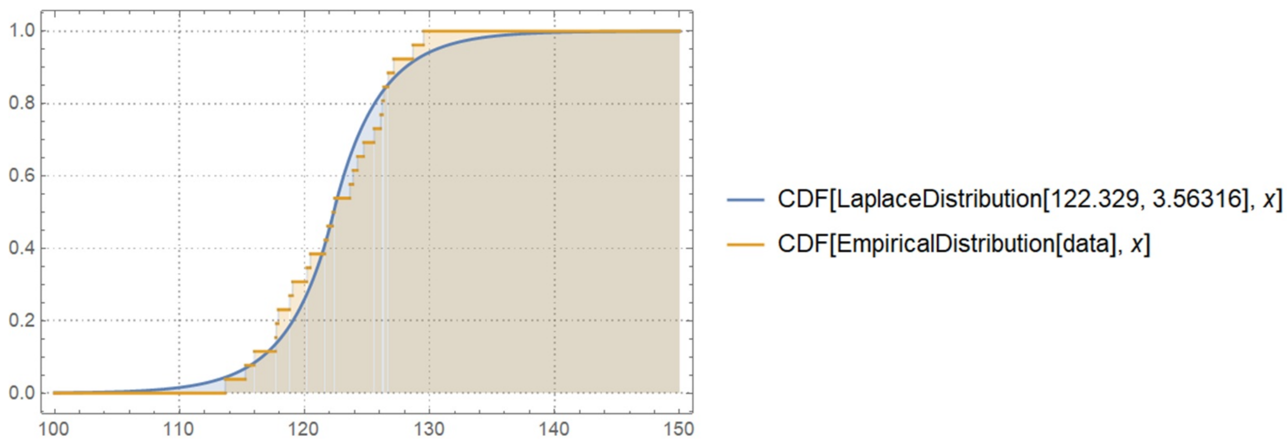

Fig. 1: Empirical CDF vs Fitted Laplace CDF

Now we will extract the L_m^{c*}, U_m^{c*}, and R_m^{c*} from the previous data and score them in Table 2.

Table 2: The L_m^{c*}, U_m^{c*} and R_m^{c*} of the data

m	1	2	3	4	5	6	7
U_m^{c*}	121.62228	121.62228	122.40066	122.40066	124.72938	126.22889	129.51602
L_m^{c*}	121.62228	118.80192	118.80192	113.65555	113.65555	113.65555	113.65555
R_m^{c*}	0	2.820366	3.59874	8.74511	11.07383	12.57334	15.86047

One can easily now find the CDF and PDF of R_m^{c*} by substituting with the previously estimated value of λ and θ in equations (8) and (9), which will be a great help in the future when dealing with this data. Also, one can find a very helpful formula from which we can calculate the i^{th} moment for any value of m under the same values of λ and θ in (14) and its recurrence relation in (15).

5 Conclusion

Predicting a future value or explaining a certain outcome is one of the applications in which you will need the PDF and CDF of the historical data to do so. In this paper, we found the PDF and CDF of the current record range when the original data follows the Laplace distribution. Also, the moments of the current record range for the same distribution were calculated with a recurrence relation for it. As the current record range has an application in the industrial field, a numerical example involving real industrial data was presented to show the uses of the formulas presented.

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