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# SLOWING LIGHT CONTROL FOR A SOLITON-PAIR

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**Abstract:** We investigate the propagation of a soliton-pair in 3-level atomic system out of resonance. We present an explicit analytical expression of the soliton-pair shape. We show that the speed of the soliton-pair can be controlled. Furthermore, we derive a condition for completely stopping the light.

**Keywords:** Soliton-pair, slowing light

## 1. Introduction

Considerable attention was paid in the three last decades to the non linear optical media [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19], mainly the Solitons formation in electromagnetically induced transparency medium (EIT)[20,21,22,23], as they have important physical features[24,25]. In particular, spatiotemporal solitons known as STSs [26]. They are pulses, which maintain their shape by propagating in the medium due to the balance between the group-velocity dispersion, diffraction, and nonlinear self-phase modulation. Moreover, EIT is an important tool for realization of controllable atom-light coupling, such as the manipulation of optical pulse propagation through atomic and atom-like media via slow [27,28,29] and stored light [30,31,32,22]. Among the interesting potential applications of EIT and slow light is the practical realization of a quantum memory. Electromagnetically induced transparency (EIT) has mainly been studied for three-level systems. In particular, a considerable interest has been dedicated to the Lambda-configuration where a pair of optical pulses propagates without absorption. This medium can be made experimentally [33] if, a weak signal pulse propagates through the medium pumped by a strong control field. The group velocity of the weak signal depends on the control field intensity, and can be reduced to zero. In this case the photonic information of the pulse is completely transferred to the atoms. This coherent transfer is reversible, which means that the pulse may be retrieved[34]. In another context, Park and Shin [35] showed a systematic method for constructing families of multi-component pulses in EIT media. H. Eleuch and his co-workers [36,37], derived analytical solutions of solitons and pair of solitons in dissipative resonant media.

In this paper we investigate the soliton-pair propagation in the three level dissipative media out of resonance, we elaborate an explicit analytical expression of the pair soliton shape and we derive a condition for completely stopping the light.

## 2. Model

The medium considered here is a three level atom in the $\Lambda-$configuration interacting with two non-resonant electromagnetic fields. The two atomic transitions are excited by two variable laser fields.

The three-level atom is described by quantum system with three energy levels $|0\rangle$ , $|1\rangle$ and $|2\rangle$ . The restriction to two lower energy level is valid if the frequency of the interacting waves are distant enough to all other frequencies. In this model we take into account the rates $\gamma_{1,2}$ of radiative decay from the higher level $|0\rangle$ to the levels $|1\rangle$ and $|2\rangle$ and neglecting the other dissipation effects.

This three-level system is irradiated by a light beam containing two monochromatic fields $\bar{E}_1$ and $\bar{E}_2$ which propagate with polarization adequate to couple the optical transitions ($|0\rangle \leftrightarrow |1\rangle$ and $|0\rangle \leftrightarrow |2\rangle$). We suppose that the

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levels$|1\rangle$ and $|2\rangle$ are decoupled (transition $|1\rangle \leftrightarrow |2\rangle$ negligible). In fact a free atom has at least two states at same parity between which an electrical dipole transition is not allowed. Furthermore, we assume that the two optical fields have slowly varying envelopes [34,35,36]:

$$\frac{1}{\omega_1}\left|\frac{\partial \bar{E}_1}{\partial t}\right| << \left|\bar{E}_1\right|; \qquad \frac{1}{\omega_2}\left|\frac{\partial \bar{E}_2}{\partial t}\right| << \left|\bar{E}_2\right| \tag{1}$$

In the rotating wave approximation the Hamiltonian of the system is

$$\begin{aligned} H &= H_0 + H_i \\ H_0 &= \sum_{i=1,2} \varepsilon_i a_i^+ a_i \\ H &= \sum_{i=1,2} g_i \left(a_0^+ a_i E_i + a_i^+ a_0 E_i^*\right) \end{aligned} \tag{2}$$

Where $H_0$ is the field free Hamiltonian and $H_i$ is the interaction of the atom with the electromagnetic fields. $a_i$, $a_i^+$ are respectively the annihilation and creation fermions operators of the atomic level $i$, verifying the anti-commutation relation $\left[a_i, a_j^+\right]_+ = \delta_{ij}$. The two dipole transition matrix elements which are assumed to be real are denoted by $g_1$ and $g_2$. The dynamical evolution of system is governed by the master equation

$$\frac{d}{dt}\rho = \frac{1}{i\hbar}[H,\rho] + \pounds\rho \tag{3}$$

The irreversible decay part in the system is denoted by $\pounds\rho$ and corresponds to the incoherent processes. It is given by:

$$\begin{aligned} \pounds\rho = & \frac{\gamma_1}{2}\left[a_0^+ a_1 \rho, a_1^+ a_0\right] + \frac{\gamma_1}{2}\left[a_0^+ a_1 \rho, a_1^+ a_0\right] + \\ & \frac{\gamma_2}{2}\left[a_0^+ a_2 \rho, a_2^+ a_0\right] + \frac{\gamma_2}{2}\left[a_0^+ a_2 \rho, a_2^+ a_0\right] \end{aligned} \tag{4}$$

The time-varying density matrix elements verify the following evolution equations

$$\begin{aligned} \frac{d}{dt}\rho_{10} &= i\,\omega_{10}\rho_{10} - i\,d_1 E_1^*\left(\rho_{00} - \rho_{11}\right) \\ &\quad + i d_2 \rho_{21}^* E_2^* - \left(\frac{\gamma_1+\gamma_2}{2}\right)\rho_{10} \\ \frac{d}{dt}\rho_{20} &= i\omega_{20}\rho_{20} - i\,d_2 E_2^*\left(\rho_{00} - \rho_{22}\right) \\ &\quad + i d_1 \rho_{21} E_1^* - \left(\frac{\gamma_1+\gamma_2}{2}\right)\rho_{20} \\ \frac{d}{dt}\rho_{21} &= i d_1 E_1 \rho_{20} + i d_2 \rho_{10}^* E_2^* + i\left(\omega_{20-}\omega_{10}\right)\rho_{00} \\ \frac{d}{dt}\rho_{jj} &= i\,d_j\left(E_j \rho_{j0} - E_j^* \rho_{j0}^*\right) + \gamma_j \rho_{00} \quad for\ j = 1,2 \\ \frac{d}{dt}\rho_{ij} &= \frac{d}{dt}\left(\rho_{ji}\right)^* \quad for\ i,j = 0,1,2 \\ \frac{d}{dt}\rho_{00} &= -\frac{d}{dt}\left(\rho_{11} + \rho_{22}\right) \end{aligned} \tag{5}$$

The last equation is derived from $tr(\rho) = 1$. $d_i$ are the coupling constants. $\omega_{10}$ and $\omega_{20}$ represent the two atomic transition frequencies and we have: $\omega_{10} = \frac{\varepsilon_0 - \varepsilon_1}{\hbar}$ and $\omega_{20} = \frac{\varepsilon_0 - \varepsilon_2}{\hbar}$.

$\rho_{10}$ and $\rho_{20}$ terms oscillate at the respective driving field frequency and the $\rho_{21}$ oscillate with frequency differences of the two light fields. So, we can define the slowly varying amplitudes of the off-diagonal density matrix elements $\bar{\rho_{10}}$, $\bar{\rho_{20}}$ and $\bar{\rho_{21}}$ through the relations:

$$\rho_{j0} = \bar{\rho_{j0}} \exp\left(i\omega_{j0} t\right) \quad for\ j = 1,2$$

$$\rho_{21} = \bar{\rho_{21}} \exp\left(i\left(\omega_{20} - \omega_{10}\right)t\right) \tag{6}$$

The off-diagonal elements, which describe the atomic coherences, can be decomposed into imaginary part and real parts :

$$\bar{\rho_{j0}} = \chi_{j0} + i\psi_{j0}$$

$$\bar{\rho_{21}} = \chi_{21} + i\psi_{21} \tag{7}$$

The Hermitian propriety of the density matrix ensures that the diagonal elements $\rho_{11}$, $\rho_{22}$ and $\rho_{00}$ must be real. These terms are the level populations and determine the internal energy of the atom. $\delta_1 = \omega_{10} - \omega_1$ and $\delta_2 = \omega_{20} - \omega_2$ are the detunings between the laser frequencies and the atomic transitions frequencies. We have developed in this section, the evolution of the atomic parameters. In the next section, we explore the propagation of the fields $E_1$ and $E_2$ through the medium and its spatial and temporal dynamical behavior.

## 3. Analysis of the fields propagation

This section deals with the analysis of soliton propagation in the medium described above. The signal field $E_j$ for $j = 1,2$ are described by the Maxwell equations for a slowly varying approximation (SVA) [38, 39]: $\frac{\partial \bar{E}_j}{\partial t} + c\frac{\partial \bar{E}_j}{\partial x} = ig' \bar{\rho}_{j0}$. Moreover, the condition for soliton-pair propagation is expressed as $\bar{E}_j(x,t) = \bar{E}_j(x - v_g t)$

The propagation constants of the fields $g_i'$, which are considered to be real, are given by $g_i' = \frac{2\pi}{\varepsilon_0} N g_i(\omega_2 + \delta_2)$. $\varepsilon_0$ is the vacuum electric constant, $N$ is the atomic dipole density and $c$ is the velocity of light. $v_g$ represents the group velocity of the soliton-pair. We introduce a moving coordinate $z = x - v_g t$ which propagates with the pulses's velocities. In this new moving coordinate we have $\frac{\partial}{\partial t} = -v_g \frac{\partial}{\partial z}$ and $\frac{\partial}{\partial x} = \frac{\partial}{\partial z}$. We assume in this work that the two spontaneous emission rates are approximately equal: $\gamma_1 = \gamma_2 = \gamma$.We suppose also, that $\bar{E}_2$ is real which gives $\chi_{20} = 0$. The complete set of the evolution equations for medium-fields interaction (Maxwell-Bloch equations) are obtained from Maxwell equations and the master equation

$$\begin{aligned}
\frac{d}{dz}\chi_{10} &= \frac{\delta_1}{v_g}\psi_{10} - \alpha_2\psi_{21} + \Gamma\chi_{10} \\
\frac{d}{dz}\psi_{10} &= \frac{\delta_1}{v_g}\chi_{10} - 2\alpha_1\chi_{11} - \alpha_2\chi_{21} + \Gamma\psi_{10} \\
0 &= \frac{\delta_2}{v_g}\psi_{20} + \alpha_1\psi_{21} \\
\frac{d}{dz}\psi_{20} &= \alpha_2(-1-\rho_{11}) - \alpha_1\chi_{21} + \Gamma\psi_{20} \\
\frac{d}{dz}\chi_{21} &= \alpha_1\psi_{20} + \alpha_2\psi_{10} \\
\frac{d}{dz}\psi_{21} &= \alpha_2\chi_{10} \\
\frac{d}{dz}\rho_{11} &= 2\alpha_1\psi_{10} - \Gamma(1-\rho_{11}-\rho_{22}) \\
\frac{d}{dz}\rho_{22} &= 2\alpha_2\psi_{20} - \Gamma(1-\rho_{11}-\rho_{22}) \\
\frac{d}{dz}\alpha_2 &= -\frac{d_2 g_2}{v_g(c-v_g)}\psi_{20} \\
&= -k_2\psi_{20} \\
\frac{d}{dz}\alpha_1 &= -\frac{d_1 g_1}{v_g(c-v_g)}\psi_{10} \\
&= -k_1\psi_{10}
\end{aligned} \tag{8}$$

Where $\alpha_1$ and $\alpha_2$ are both variables and related to the field amplitudes by the following expressions:

$$\alpha_1 = \frac{d_1\bar{E}_1}{v_g}; \qquad \alpha_2 = \frac{d_2\bar{E}_2}{v_g} \tag{9}$$

We deal here with the case where $\alpha_2 = A\alpha_1 = A\alpha$ which means that the two pulses constituting the amplitudes of the pair have the same shape but with different amplitudes. The new constant $\Gamma$ is defined by $\Gamma = \frac{\gamma}{v_g}$.

The optical fields 1 and 2 have slowly varying amplitudes, in this case, we can neglect the variation of the curvature and we can assume that the third and the forth order of the differentiations are negligible. After algebraic manipulations and differentiation of the Maxwell-Bloch equations, we obtain a non-linear differential equation:

$$0 = Bp^2 + Cp + Dp(\frac{dp}{d\alpha}) + E(\frac{dp}{d\alpha}) + F \tag{10}$$

where we introduce a new variable $p$ describing the field evolution of the field $\alpha$ by

$$p(\alpha) = \frac{d\alpha}{dz} \tag{11}$$

The expressions of the constants are:

$$\begin{aligned}
B &= -3 \\
C &= -\Gamma\alpha \\
D &= 4\alpha \\
E &= 4\alpha^2 \\
F &= (-A^2 - \frac{\delta_1 k_2}{\delta_2 k_1})\alpha^4
\end{aligned} \tag{12}$$

## 4. Shapes of Solitons and stopping light condition

In this section we derive an analytical explicit shape of the soliton, so we must find an explicit relation between $\alpha$ and $z$. From the definition of the $p$ function we obtain the following relation between the field amplitude $\alpha$ and the local coordinate $z$, for small field amplitude $\alpha$,we get :

$$\begin{aligned}
z &= \int \frac{d\alpha}{p} \\
&= \int \frac{d\alpha}{S_1\alpha + S_2\alpha^2} \\
&= \frac{-2}{S_1}\tanh^{-1}\left[\frac{2S_2\alpha - S_1}{S_1}\right]
\end{aligned} \tag{13}$$

where

$$\begin{aligned}
S_1 &= 4 - \Gamma \\
S_2 &= \pm X \\
&= \pm\sqrt{\frac{1}{5}(\frac{\delta_1 k_2}{\delta_2 k_1} + A^2)}
\end{aligned} \tag{14}$$

Herewith we obtain two possible expressions of the soliton pulse shapes (The plots in Fig.1 represent the two possible shapes of the solitons, the used parameters are from realistic values [40,41,42,43,44,45]) :

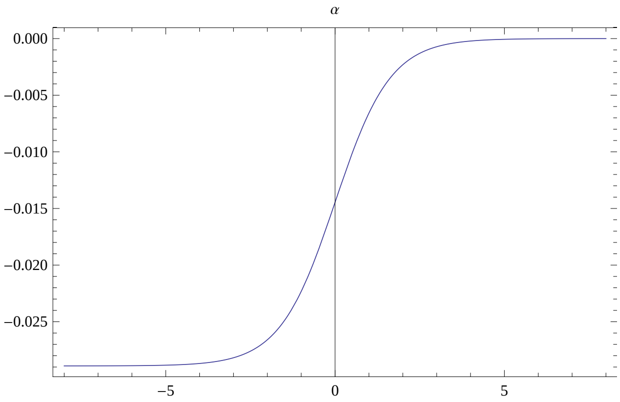
$$\begin{aligned}
\alpha_{(1)}(z) &= \frac{-S_1}{2X}(1 - \tanh(\frac{S_1 z}{2})) \\
\alpha_{(2)}(z) &= \frac{S_1}{2X}(1 - \tanh(\frac{S_1 z}{2}))
\end{aligned} \tag{15}$$

In order to calculate the velocity of the soliton, we determine, first the maximum amplitude of the soliton. It is given by the following relation:

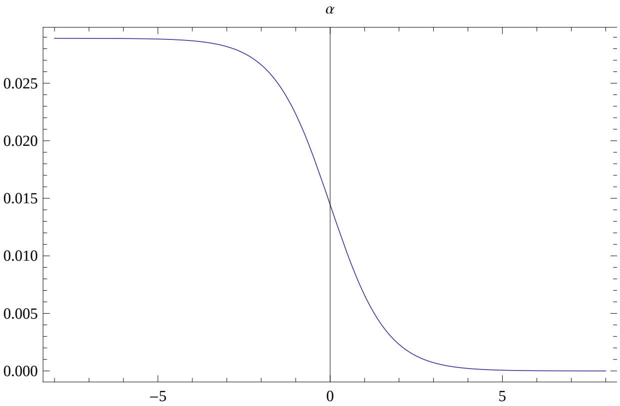
$$\alpha_{\max} = \frac{S_1}{X} = \frac{4-\Gamma}{S_2} = \frac{4-\frac{\gamma}{v_g}}{S_2} \tag{16}$$

The expression of the group velocity for the solition pair is then: $v_g = \frac{\gamma}{4 - X\alpha_{\max}}$. From the fact that $v_g \leq c$, we deduce a condition for the soliton propagation

$$\alpha_{\max} \leq \frac{4-\frac{\gamma}{c}}{X}, \tag{17}$$




(a)




(b)

**Figure 1:** Soliton shapes $\alpha_{(1)}$(a) and $\alpha_{(2)}$ (b) as function of z for the following parameters (in the SI units):$\omega_2 = 1.5*10^8; \omega_1 = 5*10^8; d2 = 3*10^{-32}; d1 = 1.5*10^{-32}; \gamma = 5*10^8; A = 1.5; g_2 = 0.15*10^8; g_1 = 0.17*10^8; \delta_1 = 0.03*10^8; \delta_2 = 3\delta_1$.

where $c$ is the velocity of light in vacuum.

Furthermore, for $\delta_1\delta_2 < 0$ another propagation condition for the soliton-pair should be verified:

$$|A| > \sqrt{-\frac{\delta_1 d_2 g_2 \omega_{20}}{\delta_2 d_1 g_1 \omega_{10}}} \tag{18}$$

The two cases where $A = \pm\sqrt{-\frac{\delta_1 d_2 g_2 \omega_{20}}{\delta_2 d_1 g_1 \omega_{10}}}$ are very interesting as the soliton's velocity vanishes which means that the soliton-pair can be stopped. This can be used to greatly reduce noise, allowing information to be transmitted more efficiently in the media[46]. Moreover, storing the energy information at a desired time, is an important challenge for the quantum information processing. In fact, storing and retrieving back a quantum state of light may destroy the information that it carries. In this Context, developing quantum memories is a way to avoid shape deformation and energy loss[46].

## 5. Conclusion

In this paper we have studied the propagation of a soliton-pair in an absorbing and a non-resonant three level atomic media in $\Lambda$ configuration. We derive an explicit analytical solution for the soliton-pair shapes. We show that the group velocity of the soliton-pair can be controlled by changing the values of the maximum amplitude and the detunings. These results are important in several physical applications such as in telecommunications. In fact, typical slow light systems exhibit loss of information through dispersion. Controlling slowing light propagation through solitons propagation is an efficient way to reduce loss of information and un-distortion.

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