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A Combined MLE and Generalized P Chart Approach to Estimate the Change Point of a Multinomial Process

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Abstract: Correct and quick identification of the change point of a process shift is very important for process improvement. Although several studies have been devoted to the estimation of the change point of the process shifts for a univariate process, little research has been done on a multivariate process, particularly for a multivariate process where the quality characteristics cannot be measured in a numerical scale. In this study, an effective approach that combines the method of maximum likelihood and the generalized p control chart is developed to estimate the starting time of a process disturbance for a multinomial process. An illustrative example is provided to show how to apply the proposed approach in practice. The positive results with the use of the proposed approach are also demonstrated by a series of simulation studies. It is found that the proposed approach has better performance than the original generalized p control chart.

Keywords: Change point, generalized P chart, MLE, multinomial process.

1 Introduction

The quality of a companys products is always the key factor for its success. Statistical process control (SPC) charts have always played an important role in industry, and are commonly used for detecting process disturbance. Although SPC charts are able to trigger a signal when disturbance have occurred in the process, they do not provide enough information to detect the root causes of an out-of-control process [1,2,3,4,5,6,7,8,9,10,11,12]. As a consequence, the issue of identification of the change point has received considerable attention recently. The change point is the starting time of the occurrence of a process disturbance, and the SPC signal indicates the time when an out-of-control state is identified by the SPC charts. The purpose of estimating the change point time is to quickly and easily identify the root causes of the underlying disturbance. The issue of estimation of the change point is very important for process monitoring and many studies have concomitantly rapidly developed around it [1,2,3,4,5,6,7,8,9,10,11,12].

It is worth noting that there are many processes in which quality characteristics cannot be measured on a continuous scale, and the quality of a product should be assessed by several correlated attributes simultaneously [5,13,14,15,16,17,18,19,20,21,22,23,24]. In addition, in many cases, binary classification is not appropriate and the quality of a product does not change abruptly from perfect to worthless. Thus, there is a need for intermediate assessments that cannot be expressed numerically, such as the quality characteristics of softness and appearance. These are associated with linguistic terms such as poor, medium, good, very good, etc. Consequently, we deal with multiple attributes and multinomial data in many quality control environments. It is very common nowadays to use multiattribute control charts for inspection of various kinds of procedures.

Although several studies have been devoted to the identification of the time of the change point for a univariate process [1,2,4,5,6,7,8,9,10,11,12], however, little research has been done on a multivariate process [3,12], particularly for a multivariate process where the

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quality characteristics cannot be measured in a continuous scale. Nedumaran et al. [3] utilized the chi-square control chart and proposed a method to estimate the time of a process mean shifts for a multivariate normal process. Recently, Hou et al. [12] proposed to apply the $|S|$ control chart and the method of maximum likelihood to identify the time of a process variance shift for a multivariate normal process. Both the studies of Nedumaran et al. [3] and Hou et al. [12] have the same assumption; that is, the process should be a measurable process, and the process distribution is a multivariate normal distribution. Therefore, in contrast to the works of Nedumaran et al. [3] and Hou et al. [12], this study is motivated to develop an approach to effectively estimate the change point of proportion shifts for a multinomial process.

The structure of this study is organized as follows. The second section addresses the proposed approach for estimating the change point of proportion shifts for a multinomial process. The third section provides an illustrative example to show how to apply the proposed method in practice. The fourth section shows results from the simulations, performed under various conditions. The final section reports the research findings and presents a conclusion to complete this study.

2 The Estimation of a Change Point

This study assumes that the multinomial process is initially in control and the sample observations come from a multinomial distribution with known proportion vector $p_0 = (p_{01},\ldots,p_{0k})$. After an unknown point in time $\tau+1$, this study assumes that the proportion vector changes from p_0 to $p_1 = (p_{11},\ldots,p_{1k})$. Let

$$X_i = [X_{i1}, X_{i2}, \ldots, X_{ik}]' \tag{1}$$

be a $k \times 1$ vector, where $X_{i1}, X_{i2}, \ldots, X_{ik}$ denote the number of observations in categories $1,2,\ldots,k$, respectively, for the i^{th} monitoring period. Reserve $i = 0$ for the base period when the process is assumed to be in control. It follows that

$$\begin{aligned}
(X_{11},\ldots,X_{1k}) &\sim mul(n_1, p_0);\\
(X_{21},\ldots,X_{2k}) &\sim mul(n_2, p_0);\\
&\vdots\\
(X_{\tau 1},\ldots,X_{\tau k}) &\sim mul(n_\tau, p_0);\\
(X_{\tau+1,1},\ldots,X_{\tau+1,k}) &\sim mul(n_{\tau+1}, p_1);\\
&\vdots\\
(X_{T,1},\ldots,X_{T,k}) &\sim mul(n_T, p_1);
\end{aligned} \tag{2}$$

where $\tau+1$ is the change point of the process, T is the signal time that the subgroup sample chi-square statistic exceeds a generalized p controls limit. To detect a multinomial process proportion shifts, Marcucci [13] proposed the following well-known chi-square statistic:

$$Z_i^2 = n_0 n_i \sum_{j=1}^{k} \frac{(p_{ij} - p_{0j})^2}{X_{ij} - X_{0j}} \tag{3}$$

where $p_{ij} = X_{ij}/n_i$ are the sample proportions. This statistic is asymptotically distributed as a chi-square distribution with $(k-1)$ degrees of freedom. The control chart that uses Z_i^2 in (3) as a monitoring statistic has an upper control limit

$$UCL = \chi_\alpha^2 (k-1) \tag{4}$$

where χ_α^2 is the upper α^{th} percentile of the chi-square distribution with $(k-1)$ degrees of freedom. If the statistic Z_i^2 is plotted out the UCL, then the process will be said to be out of control. In practice, the p_{i0} in (2) and (3) are usually estimated using a set of preliminary samples taken in the in-control based period.

The likelihood function of unknown parameters p_1 and τ can be obtained from the joint probability density function of $X_1, X_2, \ldots, X_T$, namely

$$\begin{aligned}
&L(p_1,\tau)\\
&= \prod_{i=1}^{\tau} \left\{ \frac{n_i!}{\prod_{j=1}^{k-1} x_{ij}! \left(n_i - \sum_{j=1}^{k-1} x_{ij} \right)!} \prod_{j=1}^{k-1} p_{0j}^{x_{ij}} \cdot \left(1 - \sum_{j=1}^{k-1} p_{0j} \right)^{n_i - \sum_{j=1}^{k-1} x_{ij}} \right\}\\
&\cdot \prod_{i=\tau+1}^{T} \left\{ \frac{n_i!}{\prod_{j=1}^{k-1} x_{ij}! \left(n_i - \sum_{j=1}^{k-1} x_{ij} \right)!} \prod_{j=1}^{k-1} p_{1j}^{x_{ij}} \cdot \left(1 - \sum_{j=1}^{k-1} p_{1j} \right)^{n_i - \sum_{j=1}^{k-1} x_{ij}} \right\}
\end{aligned} \tag{5}$$

Since the natural logarithm function is an increasing function, the value of the parameter that maximize the natural logarithm of the likelihood function will be the same as the one that maximize the likelihood function itself. Therefore, taking the logarithmic function of

$L(p_1,\tau)$, we have

$$\begin{aligned}
&\ln L(p_1,\tau)\\
&=\sum_{i=1}^{\tau}\left\{\ln n_i! - \sum_{j=1}^{k}\ln x_{ij}! + \sum_{j=1}^{k} x_{ij}\ln p_{0j}\right.\\
&\quad\left.+\left(n_i-\sum_{j=1}^{k-1}x_{ij}\right)\ln\left(1-\sum_{j=1}^{k-1}p_{0j}\right)\right\}\\
&\quad+\sum_{i=\tau+1}^{T}\left\{\ln n_i! - \sum_{j=1}^{k}\ln x_{ij}! + \sum_{j=1}^{k-1} x_{ij}\ln p_{1j}\right.\\
&\quad\left.+\left(n_i-\sum_{j=1}^{k-1}x_{ij}\right)\ln\left(1-\sum_{j=1}^{k-1}p_{1j}\right)\right\}\\
&=\sum_{i=1}^{T}\ln n_i! - \sum_{i=1}^{T}\sum_{j=1}^{k}\ln x_{ij}! + \sum_{i=1}^{\tau}\sum_{j=1}^{k-1} x_{ij}\ln p_{0j}\\
&\quad+\sum_{i=1}^{\tau}\left(n_i-\sum_{j=1}^{k-1}x_{ij}\right)\ln\left(1-\sum_{j=1}^{k-1}p_{0j}\right)\\
&\quad+\sum_{i=\tau+1}^{T}\sum_{j=1}^{k-1}x_{ij}\ln p_{1j}\\
&\quad+\sum_{i=\tau+1}^{T}\left(n_i-\sum_{j=1}^{k-1}x_{ij}\right)\ln\left(1-\sum_{j=1}^{k-1}p_{1j}\right) \qquad (6)
\end{aligned}$$

To find the maximum, we set the first partial derivative with respect to p_{1l} equal to zeros and obtain

$$\begin{aligned}
&\frac{\partial \ln L(p_1,\tau)}{\partial p_{1l}}\\
&=\frac{\sum_{i=\tau+1}^{T} x_{il}}{p_{1l}}+\sum_{i=\tau+1}^{T}\left(n_i-\sum_{j=1}^{k-1}x_{ij}\right)\frac{-1}{1-\sum_{j=1}^{k-1}p_{1j}}=0,\\
&\qquad\qquad l=1,2,\ldots,k-1. \qquad (7)
\end{aligned}$$

Since $\sum_{l=1}^{k} p_{1l}=1$, it can be easily proved that the maximum likelihood estimators of p_{1l} and τ are

$$\hat{p}_{1l}=\frac{\sum_{i=\tau+1}^{T} x_{il}}{\sum_{i=\tau+1}^{T} n_i},\quad l=1,2,\ldots,k \qquad (8)$$

and

$$\hat{\tau}=\arg\max_{\tau}\left\{\sum_{i=\tau+1}^{T}\sum_{j=1}^{k}x_{ij}(\ln\hat{p}_{1j}-\ln\hat{p}_{0j})\right\} \qquad (9)$$

3 An Illustrative Example

An illustrative example is simulated to show how to apply the proposed approach. Without loss of generality, we assume that each of the attribute quality characteristics is sampled from a multinomial distribution with constant sample size 100 in each monitoring period. The number of categories (k) is assumed to be 4 and the change point of the process be 11 (that is, $\tau=10$). In addition, assume that the in-control and out-of-control proportion vectors are as follows:

$$p_0=(0.25,0.25,0.25,0.25),$$
$$p_1=(0.33,0.33,0.17,0.17).$$

The significance level used in this study is 0.0027, and thus the corresponding UCL of the generalized p control chart obtained from Equation (4) is 14.17. The simulated subgroup observations are displayed in Table 1. We notice that a total of 49 subgroup observations were obtained before one of them exceeded the UCL. It can be seen that $Z_{49}^2 > UCL$, and thus, the generalized p control chart has signaled at time $T=49$. At this point, our proposed change point estimator can be applied. To utilize our proposed approach, we need to find the value of t, in the range of $0\le t<T$, which maximizes $g(\hat{p}_1,t)$, where

$$g(\hat{p}_1,t)=\sum_{i=t+1}^{T}\sum_{j=1}^{k}x_{ij}(\ln\hat{p}_{1j}-\ln p_{0j}). \qquad (10)$$

The reverse estimator $\hat{p}_{1j}$ is defined as:

$$\hat{p}_{1j}=\frac{\sum_{i=t+1}^{T} x_{ij}}{\sum_{i=t+1}^{T} n_i},\quad j=1,2,\ldots,k. \qquad (11)$$

The value of t which maximizes the $g(\hat{p}_1,t)$ values is our estimator of the last subgroup from the in-control process. As shown in Table 1, we can notice that the largest $g(\hat{p}_1,t)$ value is associated with subgroup 12. Therefore, we estimate that subgroup 13 was the first subgroup obtained from the changed process. As a consequence, subgroup 12 was the last subgroup from the in-control process. Table 1 shows how the traditional approach identified subgroup 49 as the estimate of the change point while the proposed approach identified subgroup 13 as the best. Since the real change point tool place in subgroup 11, the numerical results reveal that the proposed approach is superior to using the traditional generalized p control chart alone.

4 Simulation Studies

To evaluate the performance of the proposed approach introduced in Section 2, a series of simulations were conducted. When a signal was triggered by the generalized p control chart, the estimator introduced in Section 2 was then applied to the data to estimate the time of the change. The simulation assumed that a multinomial process is in control for the first 10 observations, and a disturbance has been introduced at time period of 11. For the first 10 subgroups, we assume that the observations followed the multinomial distribution $mul(100,(0.25,0.25,0.25,0.25))$. Starting from subgroup 11, this study assumes that sample observations followed the multinomial distribution $mul(100,(0.25+\delta,0.25+\delta,0.25-\delta,0.25-\delta))$. Without losing generality, this study considered four values of δ, and they are : $0.05, 0.10, 0.15$ and 0.20. To assess the effect of sample size for the proposed estimator, we simulated the required data with the numbers of sample sizes of $25, 50, 75, \ldots, 200$. In our simulation, 1000 runs were performed, and the estimates of T and $\hat{\tau}$ were obtained for each simulation run. The average of T and $\hat{\tau}$ for 1000 runs were recorded along with their standard errors.

Table 1: An illustrative example for computation of the proposed estimator.

Subgroup i	$X_i = [X_{i1}, X_{i2}, \ldots, X_{ik}]'$	Z_i^2	$g(\hat{p}_1, t)$
1	[23, 22, 22, 33]	1.57	154.08
2	[27, 28, 20, 25]	0.80	154.84
3	[20, 23, 27, 30]	1.17	162.11
4	[24, 23, 26, 27]	0.20	167.43
5	[22, 18, 31, 29]	2.27	176.94
6	[24, 26, 27, 23]	0.20	181.21
7	[21, 22, 28, 29]	1.01	189.95
8	[34, 23, 22, 21]	2.00	190.05
9	[31, 24, 21, 24]	1.03	191.61
10	[24, 22, 32, 22]	1.26	199.27
11	[29, 32, 17, 22]	2.87	197.42
12	[33, 22, 19, 26]	2.13	199.33
13	[41, 30, 12, 17]	10.42	190.61
14	[39, 25, 19, 17]	5.40	186.55
15	[35, 24, 18, 23]	2.91	186.03
16	[41, 32, 13, 14]	11.63	176.06
17	[35, 31, 15, 19]	5.63	170.91
18	[33, 34, 20, 13]	6.82	164.94
19	[34, 29, 14, 23]	4.86	161.89
20	[29, 35, 13, 23]	5.84	158.30
21	[31, 39, 22, 08]	12.65	150.08
22	[30, 32, 19, 19]	2.95	147.60
23	[30, 32, 19, 19]	2.95	145.15
24	[40, 28, 14, 18]	7.87	138.61
25	[31, 33, 21, 15]	4.59	134.66
26	[33, 31, 20, 16]	4.28	130.79
27	[31, 31, 19, 19]	2.92	128.39
28	[33, 35, 21, 11]	8.56	121.72
29	[36, 33, 14, 17]	7.71	114.43
30	[35, 31, 12, 22]	7.07	109.36
31	[34, 28, 18, 20]	3.24	107.08
32	[30, 35, 14, 21]	5.57	102.82
33	[39, 28, 23, 10]	9.74	96.67
34	[39, 25, 11, 25]	8.51	93.71
35	[28, 39, 15, 18]	6.87	87.78
36	[32, 33, 21, 14]	5.41	83.03
37	[28, 33, 26, 13]	5.08	81.18
38	[39, 29, 17, 15]	7.38	74.77
39	[33, 34, 13, 20]	6.82	69.07
40	[33, 29, 23, 15]	3.98	67.16
41	[34, 34, 18, 14]	6.99	60.61
42	[32, 36, 15, 17]	6.87	53.90
43	[31, 36, 15, 18]	6.27	48.04
44	[36, 33, 14, 17]	7.71	40.98
45	[35, 35, 14, 16]	8.41	33.05
46	[31, 35, 15, 19]	5.63	28.80
47	[29, 34, 20, 17]	3.75	27.50
48	[35, 35, 16, 14]	8.41	21.55
49	[35, 45, 14, 06]	22.13	–

Our simulation study focused on estimating τ, the last subgroup from the in-control process. Thus the estimate of τ should be close to 10. Table 2 displays the effect of

Table 2: Average change point estimate and standard error when used generalized p control chart and MLE under various δ.

δ	Typical approach (Generalized p chart alone)	Proposed approach (Generalized p chart/MLE)
0.05	569.89 (351.80)	10.07 (1.31)
0.10	16.38 (5.82)	9.99 (0.20)
0.15	11.12 (0.37)	9.99 (0.07)
0.20	11.00 (0.00)	10.00 (0.00)

changing δ in the case of n=100 for an illustrative example. Observing Table 2, for a process step change of magnitude $\delta = 0.05$, we noticed that the generalized p control chart, on average, triggered a signal at time 569.89. Under the same situation, the average estimated time of the proposed approach is 10.07, which is very close to the actual change point of 10. The standard error of the proposed average estimated time of the process change is also smaller. This smaller value of the standard error implies that the proposed estimator is more stable. The superior performance of the proposed approach still holds for the value of δ within the range of 0.10 to 0.20.

To study the effect of sample size on the bias and mean squared error of the proposed estimator, we carried out a series of simulations. Our results show that the biases and mean squared errors decrease as sample size increases. Table 3 and Figs. 1 and 2 give an illustrative example when $p_1 = (0.33, 0.33, 0.17, 0.17)$. By observing Table 3 and Figs. 1 and 2, it can be found that both the bias and mean squared error of the proposed estimator decreases as sample size increases. Accordingly, it seems that the proposed estimator is an asymptotic unbiased and consistent estimator for the actual change point τ. As a consequence, it can be seen that the proposed method is much more efficient in identifying the actual change point τ. Similar results were also obtained in our extensive simulation studies for other simulated data.

5 Conclusions

In order to effectively identify the change point of proportion shifts for a multinomial process, this study proposes an estimation approach that combines the MLE method and the generalized p control chart. An illustrative

Table 3: Average change point estimate and standard error when used generalized p control chart and MLE under various sample size.

n	Typical approach (Generalized p chart alone)	Proposed approach (Generalized p chart/MLE)
25	1398.31 (295.76)	9.96 (1.86)
50	320.69 (277.89)	10.01 (0.96)
75	97.84 (84.97)	10.02 (0.52)
100	40.99 (29.09)	10.01 (0.46)
125	24.38 (14.37)	10.01 (0.33)
150	17.34 (6.83)	9.98 (0.32)
175	14.78 (4.34)	9.99 (0.24)
200	13.50 (2.96)	10.00 (0.17)

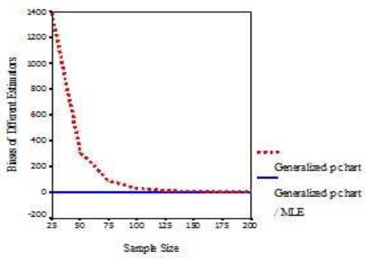

Fig. 1: Biases of two different estimators for the data simulated for various sample sizes.

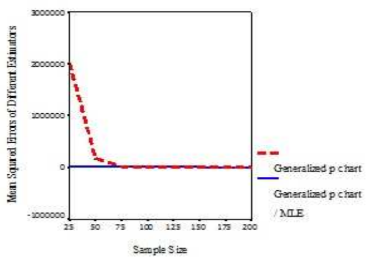

Fig. 2: Mean squared errors of two different estimators for the data simulated for various sample sizes.

example is provided to show how to apply the proposed method in practice. From the numerical result, it is found that the proposed approach has better performance than the traditional generalized p control chart alone. Since there are other types of multivariate discrete distributions, the possibility to apply the same procedure to other multivariate attribute process deserves further research.

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