Advanced Engineering Technology and Application
An International Journal

http://dx.doi.org/10.18576/aeta/140101

A Model of Distribution of Temperature in A Ventilated Facades

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Received: 12 Dec. 2024, Revised: 22 Feb. 2025, Accepted: 30 Mar. 2025.
Published online: 1 May. 2025.

Abstract: In this paper we present a model for analysis of distribution of temperature in space and time between a wall of a building and the facing of a facade with account possible convection of movement of air. In the framework of the considered model one can take into account native convection of air and possibility to take into account forced convection. Based on the model we analyzed the considered distribution of temperature. Also, we present an analytical approach for analysis of the considered distribution to increase of predictability of the heat transport with account native and forced convection. The approach gives a possibility to take into account changes of parameters in space and time, as well as the nonlinearity of the considered processes.

Keywords: ventilated facade; model of heat transport; analytical approach for modeling.

1 Introduction

In civil engineering one can find increasing of interest to constructions with ventilated facades. This type of facades are used as decorative construction and as additional thermal and wind defending. Ventilated facades are defending of insulation, which adjoin to the wall of the building from the effects of the environment, could be defended as ventilated facades. At the same time large convective flows decreasing the insulating role of these facades. In this situation it is attracted an interest to formulate a model for prognosis of processes in ventilated facades (heat transport, convection, ...). Now a series of computational works [1-8] have already been published. However, development of existing models and methods for their analysis still attracted an interest now. Structure of the considered ventilated facade shown in Fig. 1. On this figure the Oxz plane coincides with a building wall, which has a temperature T_{in}. This temperature could be changed with time (changing of temperature in the room). Outer wall of the facade (at the distance L_y from the internal wall with room) with the ambient temperature T_{ext} could be also changed with time with changing of environment. Main aim of this paper is formulated a model for prognosis of heat transport in the considered ventilated facade with account of the natural and forced convection. An accompanying aim of this paper is selection of an analytical approach for analysis of the distribution to increase of predictability of the heat transport.

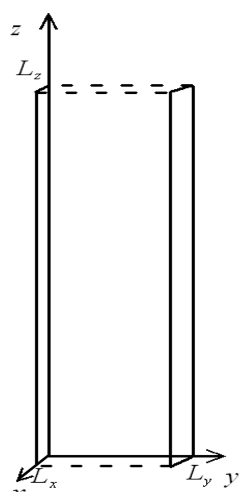

Fig. 1: Structure of considered ventilated facade

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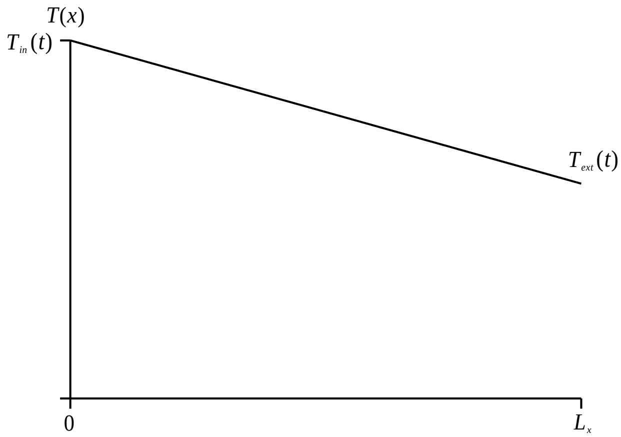

Fig. 2: Structure of initial distribution of temperature

2 Method of solution

To solve the considered aims we calculate distribution of temperature in the considered ventilated facade in space and time. The required distribution of temperature was calculated by solution of the following the second Fick's law

$$c\frac{\partial T\left(x,y,z,t\right)}{\partial t}=$$
$$=div\left\{\lambda\cdot grad\left[T\left(x,y,z,t\right)\right]-\left[\vec{v}\left(x,y,z,t\right)+\vec{\bar{v}}\left(x,y,z,t\right)\right]\cdot c\cdot\rho\cdot T\left(x,y,z,t\right)\right\}. \quad (1)$$

Here c is the heat capacity of the system; $T(x,y,z,t)$ is the distribution of temperature in space and time; x, y, z are the spatial coordinates; t is the time; ρ is the distribution of the concentration of particles of air in space and time; λ is the coefficient of thermal conductivity, the value of which is determined by the ratio: $\lambda=\bar{v}\bar{l}c_v\rho/3$, where $\bar{l}$ is the mean free path of gas molecules between collisions, $\bar{v}$ is the modulus of the root-mean-square velocity of the gas molecules, which equal to $\bar{v}=\sqrt{2kT/m}$, k is the Boltzmann constant; $\vec{v}$ is the velocity of air under influence of forced convection; m is the mass of the molecule. It is known that the overwhelming majority in the composition of air of atmosphere is nitrogen. In this situation we neglect by other components of air of atmosphere in comparison with nitrogen. Boundary and initial conditions for Eq. (1) could be written as

$T(0,y,z,t) = T(L_x,y,z,t) = T(x,y,0,t) = T(x,y,L_z,t) = T(x,L_y,z,t) = T_{ext}(t)$, $T(x,0,z,t) = T_{in}(t)$,

$T(x,y,z,t) = T_{ext}(0) = T_0$. (2)

Here $T_{ext}(t)$ is the ambient temperature, $T_{in}(t)$ temperature of wall of building. Taking into account dependence of the coefficient of thermal conductivity and the root- mean-square velocity of movement of gas molecules on temperature leads to Eq. (1) to the following form

$$c\frac{\partial T\left(x,y,z,t\right)}{\partial t}=\frac{\bar{l}c_v\rho}{9}\sqrt{\frac{8k}{m}}\left[\frac{\partial^2 T^{3/2}\left(x,y,z,t\right)}{\partial x^2}+\frac{\partial^2 T^{3/2}\left(x,y,z,t\right)}{\partial y^2}+\frac{\partial^2 T^{3/2}\left(x,y,z,t\right)}{\partial z^2}\right]-$$
$$-\left(c\sqrt{\frac{2k}{m}}+v_x\right)\frac{\partial\left[T^{3/2}\left(x,y,z,t\right)\cdot\rho\right]}{\partial x}-\left(c\sqrt{\frac{2k}{m}}+v_y\right)\frac{\partial\left[T^{3/2}\left(x,y,z,t\right)\cdot\rho\right]}{\partial y}-$$
$$-\left(c\sqrt{\frac{2k}{m}}+v_z\right)\frac{\partial\left[T^{3/2}\left(x,y,z,t\right)\cdot\rho\right]}{\partial z}. \quad (1a)$$

Next the equation has been solved by method of averaging functional corrections [9] with a decreased quantity of iteration steps [10]. In the framework of this method initial approximation of temperature has been choose as solution of this boundary-value problem without accounting of transient processes: $T_0(y,t) =[T_{in}(t)-T_{ext}(t)]\ (1-y/L_y)+T_{ext}(t)$ (see Fig. 2). This relation does not depend on the x and z coordinates due to the symmetry of the system. The above relation has been substituted into the right-hand side of equation (1a). The Substitution leads to obtaining of the first-order approximation of the considered temperature in the following form

$$c\frac{\partial T_1(y,t)}{\partial t}=\left[T_{in}(t)-T_{ext}(t)\right]^2\frac{\bar{l}c_v\rho}{6L_y^2}\left(c\sqrt{\frac{2k}{m}}+v_y\right)\sqrt{\frac{2k}{m}}\left\{\left[T_{in}(t)-T_{ext}(t)\right]\left(1-\frac{y}{L_y}\right)+T_{ext}(t)\right\}^{-1/2}+$$
$$+\rho\left[T_{in}(t)-T_{ext}(t)\right]\left(c\sqrt{\frac{2k}{m}}+v_y\right)\left\{\left[T_{in}(t)-T_{ext}(t)\right]\left(1-\frac{y}{L_y}\right)+T_{ext}(t)\right\}^{1/2}. \tag{3}$$

Integration of this equation on time leads to the following result

$$T_1(y,t)=\frac{\bar{l}c_v\rho}{6L_y^2}\left(\sqrt{\frac{2k}{m}}+\frac{v_y}{c}\right)\sqrt{\frac{2k}{m}}\int\limits_0^t\left[T_{in}(\tau)-T_{ext}(\tau)\right]^2\left\{\left[T_{in}(\tau)-T_{ext}(\tau)\right]\left(1-\frac{y}{L_y}\right)+T_{ext}(\tau)\right\}^{-1/2}d\tau+$$
$$+\rho\left(\sqrt{\frac{2k}{m}}+\frac{v_y}{c}\right)\int\limits_0^t\left[T_{in}(\tau)-T_{ext}(\tau)\right]\left\{\left[T_{in}(\tau)-T_{ext}(\tau)\right]\left(1-\frac{y}{L_y}\right)+T_{ext}(\tau)\right\}^{1/2}d\tau. \tag{3a}$$

The second-order approximation of temperature could be obtained by using standard iterative procedure, i.e. by replacement of the considered temperature on the sum of not yet known average value of required approximation (α_2 in our case) and approximation with previous order and the average value α_2, i.e. $T(y,t)\to\alpha_2+T_1(y,t)$. Substitution of this sum into the right side of Eq. (3a) leads to the following result

$$\frac{\partial T_2(y,t)}{\partial t}=\frac{\bar{l}c_v\rho}{9c}\sqrt{\frac{8k}{m}}\left\{\frac{1}{\sqrt{\alpha_2+T_1(y,t)}}\left[\frac{\partial T_1(y,t)}{\partial y}\right]^2+\left[\alpha_2+T_1(y,t)\right]^{1/2}\frac{\partial^2 T_1(y,t)}{\partial^2 y}\right\}-$$
$$-\frac{3\rho}{2}\left(\sqrt{\frac{2k}{m}}+\frac{v_y}{c}\right)\sqrt{\alpha_2+T_1(y,t)}\frac{\partial T_1(y,t)}{\partial y}. \tag{4}$$

Integration of this equation on time leads to the following result

$$T_2(y,t)=\frac{\bar{l}c_v\rho}{3c}\sqrt{\frac{2k}{m}}\frac{\partial}{\partial y}\left[\int\limits_0^t\sqrt{\alpha_2+T_1(y,\tau)}\frac{\partial T_1(y,\tau)}{\partial y}d\tau\right]-$$
$$-\frac{3\rho}{2}\left(\sqrt{\frac{2k}{m}}+\frac{v_y}{c}\right)\int\limits_0^t\sqrt{\alpha_2+T_1(y,\tau)}\frac{\partial T_1(y,\tau)}{\partial y}d\tau. \tag{4a}$$

Approximations of temperature with higher orders could be obtained by using analogous procedure, i.e. by replacement $T(y,t)$ on $\alpha_n+T_{n-1}(y,t)$ (n is the order of required approximation) in the right side of Eq. (1a). In this paper the second-order approximation of distribution of temperature in the framework of method of averaging functional corrections has been calculated. The approximation is usually enough good approximation for obtaining of qualitative results and obtaining some quantitative results. The results of analytical calculations were verified by comparing them with the results of numerical simulation. Next let us determine components of speed of air at forced convection $\vec{v}$. We determine the considered components as solution on Navier-Stocks equation [11]

$$\frac{\partial \vec{v}}{\partial t}+\left(\vec{v}\cdot\nabla\right)\vec{v}=-\nabla\left(\frac{P}{\rho}\right)+\nu\,\Delta\vec{v}, \tag{5}$$

where P is pressure in the considered system. Equations for projections of the considered speed could be written as

$$\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}=\nu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right), \tag{5a}$$

$$\frac{\partial v_y}{\partial t}+v_y\frac{\partial v_y}{\partial x}=\nu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right), \tag{5b}$$

$$\frac{\partial v_z}{\partial t}+v_z\frac{\partial v_z}{\partial x}=-\frac{\partial}{\partial z}\left(\frac{P}{\rho}\right)+\nu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right). \tag{5c}$$

Boundary and initial conditions for above equations are

$v_x(x,y,z,0) = 0$, $v_y(x,y,z,0) = 0$, $v_z(x,y,z,0) = V_0$, $v_x(0,y,z,t) = v_x(L_x,y,z,t) = 0$,

$v_y(x,0,z,t) = v_y(x,L_y,z,t) =0$, $v_z(x,y,0,t) = v_z(x,y,L_z,t) = V_0$.

We assume, that movement of air is vertical along the axis Oz. We solved the equations (5) by method of averaging of functional corrections [9,10]. In this case we consider average values of the considered projections α_{1x}, α_{1y}, α_{1z} as initial approximations. Substitution of the considered average values in right sides of Eq. (5) gives a possibility to obtain the first-order approximations of the considered projections v_{1x}, v_{1y}, v_{1z} in the following form

$$v_{1x}(x,y,z,t) = 0,\quad v_{1y}(x,y,z,t) = 0,\quad v_{1z}=V_0-\frac{\partial}{\partial z}\int_0^t\frac{P}{\rho}\,d\tau. \tag{6}$$

The second-order approximations of projections of the considered speed of air were obtained by the following standard replacement [9,10] of the required functions in the right sides of Eqs. (5): $v_x\to\alpha_{2x}+v_{1x}$, $v_y\to\alpha_{2y}+v_{1y}$, $v_z\to\alpha_{2z}+v_{1z}$. Equations for required approximations of the considered projections could be written as

$$\frac{\partial v_{2x}}{\partial t}=\nu\,\frac{\partial^2 v_{1z}}{\partial z^2},\quad \frac{\partial v_{2y}}{\partial t}=\nu\,\frac{\partial^2 v_{1z}}{\partial z^2},\quad \frac{\partial v_{2z}}{\partial t}=\nu\,\frac{\partial^2 v_{1z}}{\partial z^2}-\left(\alpha_{2z}+v_{1z}\right)\frac{\partial v_{1z}}{\partial x}-\frac{\partial}{\partial z}\left(\frac{P}{\rho}\right). \tag{7}$$

Integration of Eqs. (7) on time with account the first-order approximations of the considered projections lead to the following result

$$v_{2x}=\int_0^t\nu\,\frac{\partial^2 v_{1z}}{\partial z^2}\,d\tau,\quad v_{2y}=\int_0^t\nu\,\frac{\partial^2 v_{1z}}{\partial z^2}\,d\tau,$$

$$v_{2z}=\int_0^t\nu\,\frac{\partial^2 v_{1z}}{\partial z^2}\,d\tau-\int_0^t\left(\alpha_{2z}+v_{1z}\right)\frac{\partial v_{1z}}{\partial x}\,d\tau-\int_0^t\frac{\partial}{\partial z}\left(\frac{P}{\rho}\right)d\tau+V_0. \tag{8}$$

Approximations of speed of air at forced convection with higher orders could be obtained by using analogous procedure, i.e. by replacement $v_s(x,y,z,t)$ on $\alpha_{ns}+v_{n-1}(x,y,z,t)$ (n is the order of required approximation) in the right side of Eq. (1a). In this paper the second-order approximation of distribution of the speed in the framework of method of averaging functional corrections has been calculated. The approximation is usually enough good approximation for obtaining of qualitative results and obtaining some quantitative results. The results of analytical calculations were verified by comparing them with the results of numerical simulation.

3 Discussion

In this section we analyzed heat transfer in a ventilated facade by relations, which were calculated. Figs. 3 and 4 show spatial distributions of temperature at different values of temperature of external wall of building. Fig. 3 show distribution of temperature in facade at increasing of temperature of the external building wall. Increasing of number of curves corresponds to increasing of the temperature. Fig. 4 show distribution of temperature in the facade for decreasing of temperature of the building wall. Increasing of number of curves corresponds to decreasing of the temperature. The obtained distributions of temperature more precisely in comparison with analogous distributions of temperature without nonlinearity. In this situation we obtain more adequate model of heat transport with possibility to calculated results with higher speed in comparison with recently introduced approaches.

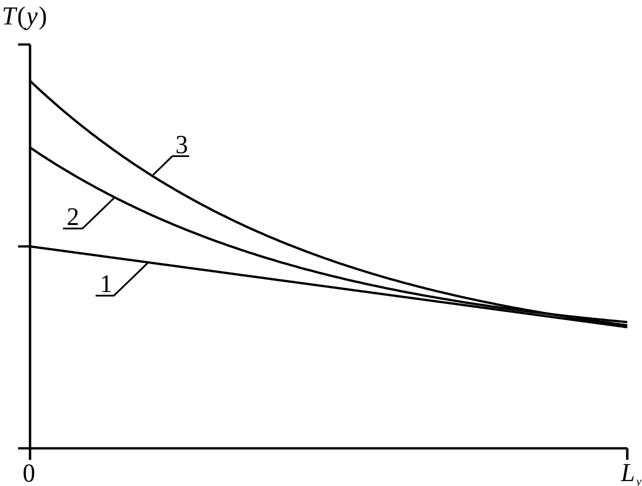

Fig. 3: Increasing of curve numbers corresponds to increasing temperature of the building wall

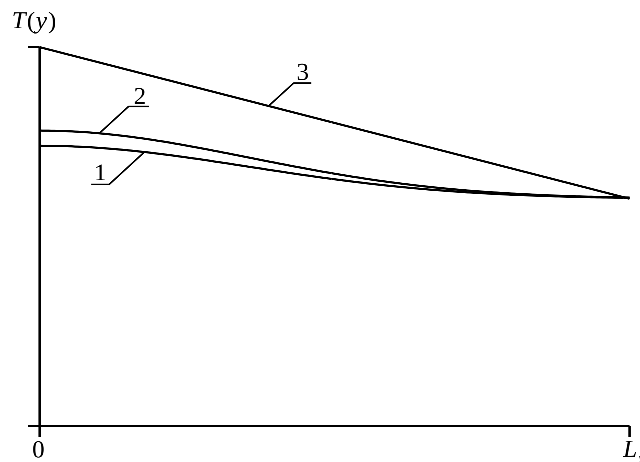

Fig. 4: Increasing of curve numbers corresponds to decreasing temperature of the building wall

4 Conclusion

In this paper we analyzed spatio-temporal distribution of temperature in a ventilated facade. The analysis gives us a possibility to make a prognosis of the design of a building with a more comfortable temperature for its users. Convective heat transfer analysis also allows for the removal of excess moisture from the building. We present an analytical approach for analysis of distribution of temperature between a wall of a building and the facing of the facade with account convection movement of air (including native and forced convections), possibility of changing of temperature on boundaries of the considered facade and nonlinearity of heat transport.

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