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Boundary –Layer Parameterization for Dispersion Applications

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Abstract: The first paper explains the causes and mechanisms of turbulence, as well as variances and intensities of turbulence, covariance and turbulent fluxes, gradient-transport theories, the mixing length hypothesis, the eddy diffusivities hypothesis, the Monin-Obukhov similarity theory and its values in unstable conditions, the local free-convection similarity theory, the mixed-layer and planetary boundary layer (PBL) similarity theory, boundary-layer parameterization for dispersion applications, the estimation of surface stress and heat flux, the mixing height on the PBL depth, mean wind profile, and wind speed values in unstable conditions. The three-dimensional advection equation is then solved step-by-step utilizing the power law of the vertical height. The levels of Iodine-135 that were observed and those that were anticipated are then compared using information from the Egyptian Atomic Energy Authority. Under unstable situations, the proposed model and the Gaussian model agreed on the measured concentration values of I^{135} within a factor of two. Compared to Gaussian concentrations, the recommended concentrations under unstable circumstances are substantially closer to the reported I^{135} concentration values.

Keywords: Stepwise Method; Variances and Turbulence Intensities; Covariance and Turbulent Fluxes; Heat Flux; Unstable Conditions.

Introduction

The first work covers the intensities of turbulence as well as its formation, maintenance, and variations. Covariance and turbulent fluxes, gradient-transport theories, the mixing length hypothesis, the Monin-Obukhov similarity theory, and its values in unstable circumstances are all discussed. Estimating surface stress and heat flux, mean wind profile, wind speed in unstable conditions, boundary-layer parameterization for dispersion applications, mixed-layer and PBL similarity theory, and local free-convection similarity theory are all covered in the second article. In combination with the observed concentration data for I^{135}, the entire suggested model and the Gaussian model under unstable state fell within a factor of two. the recommended concentrations under unstable circumstances are substantially closer to the reported I^{135} concentrations.

Statements and Declarations

The conditions for the existence or absence of turbulence in a stably stratified shear layer are defined by the gradient Richardson number and the form of the wind profile.

$$R_i = \frac{g}{T_v}\frac{\partial \theta_v}{\partial z}\left|\frac{\partial \overline{V}}{\partial z}\right|^{-2} \tag{1}$$

This could be compared to the ratio of buoyancy to shear force. The Richardson number, Ri=Ric <0.25, is another appropriate dynamic instability of a stably stratified flow. Thus, the main need for the dynamic instability of the lower atmosphere is Ri<0.25. Wind shear is the primary cause of dynamic instability in an otherwise statically stable layer (Ri>0). On the other hand, a dynamic instability condition also exists for any statically unstable layer (Ri<0).

According to Reynolds's original method from 1894, it has been conventional to think of different variables, including temperature and velocity, as the sum of their mean and fluctuating components as follows:

$$u = \bar{u} + \acute{u};\quad v = \bar{v} + \acute{v};\quad w = \overline{w} + \acute{w}$$

$$\theta = \bar{\theta} + \acute{\theta};\quad q = \bar{q} + \acute{q};\quad c = \bar{c} + \acute{c} \tag{2}$$

This is also known as Reynold's decomposition.

Regardless of the type of averaging used, turbulent fluctuations are the discrepancies between the instantaneous values of any variable and its mean.

$$\acute{u} = u - \bar{u};\quad \acute{v} = v - cd\bar{v};\quad \acute{w} = w - \overline{w}$$

$$\acute{\theta} = \theta - \bar{\theta};\quad \acute{q} = q - \bar{q};\quad \acute{c} = c - \bar{c} \tag{3}$$

The average of fluctuations should, by definition, equal zero ($\bar{\acute{u}} = 0, \bar{\acute{\theta}} = 0, etc$).

Variances and Intensities of Turbulence

The variances or mean-square fluctuations $\overline{\acute{u}^2}, \overline{\acute{v}^2}, \overline{\acute{w}^2}, \overline{\acute{\theta}^2}$ and so forth are the most fundamental measurements of fluctuation levels. The turbulent kinetic energy per unit mass is a single parameter that sums together all the fluctuations in velocity

$$e = 0.5\left(\overline{\acute{u}^2} + \overline{\acute{v}^2} + \overline{\acute{w}^2}\right) \tag{4}$$

Two additional equivalent measurements are standard

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deviations and root-mean-square fluctuations.

$$\sigma_u = \left(\overline{\acute{u}^2}\right)^{1/2}\ ,\ \sigma_v = \left(\overline{\acute{v}^2}\right)^{1/2},\quad \sigma_w = \left(\overline{\acute{w}^2}\right)^{1/2} \quad (5)$$

The levels of turbulence are as follows:

$$i_u = \frac{\sigma_u}{\bar{v}};\quad i_v = \frac{\sigma_v}{\bar{v}};\quad i_w = \frac{\sigma_w}{\bar{v}} \quad (6)$$

which are velocity components, or measurements of the proportionate degrees of change in different directions. Since the mean wind speed is used in the definition of turbulence intensities instead of the specific component mean velocity, i_u, i_v, and i_w are referred to as longitudinal, lateral, and vertical turbulence intensities. The following relationship between the variations of horizontal and vertical wind directions and the fluctuations of lateral and vertical velocity may be seen from the straightforward geometry of the instantaneous velocity vector shown:

$$\tan\acute{\theta} = \frac{\acute{v}}{\bar{u}+\acute{u}};\quad \tan\acute{\phi} = \frac{\acute{w}}{\bar{u}+\acute{u}} \quad (7)$$

By squaring and averaging both sides, disregarding higher-order variables, and inserting $\tan\theta' \approx \acute{\theta}$ and $\tan\phi' \approx \acute{\phi}$ in Eq. (7), one may show that: when turbulence intensities are much less than "1" (approximately ≤ 0.3).

$$\sigma_\theta = \frac{\sigma_v}{\bar{u}};\quad \sigma_\phi = \frac{\sigma_w}{\bar{u}} \quad (8)$$

The resulting lateral and vertical turbulence intensities therefore essentially match the radians-based standard deviations of the horizontal and vertical wind-direction fluctuations.

Turbulent Fluxes & Covariance

Covariance is defined as the average of the products of two variable that fluctuate. The covariance between temperature and vertical velocity variations and between different velocity components, for example, is $\overline{\acute{u}\acute{w}}, \overline{\acute{v}\acute{w}}, \overline{\acute{\theta}\acute{w}}$, and so on.

An estimate of the relationship between two variables that is more precise is given by a normalized covariance, often known as a linear correlation coefficient. As an example,

$$r_{uw} = \frac{\acute{u}\acute{w}}{\sigma_u\sigma_w},\quad r_{qw} = \frac{\acute{q}\acute{w}}{\sigma_q\sigma_w} \quad (9)$$

Schwartz's inequality states that $\overline{\acute{u}\acute{w}} < \sigma_u\sigma_w$, and so forth. The total vertical flux of the material on average is obtained by averaging the amount of material via a unit horizontal area per unit time. Perfect positive or negative correlation is shown by ±1 values.

$$\overline{cw} = \overline{(\bar{c}+\acute{c})(\bar{w}+\acute{w})} = \bar{c}\bar{w} + \overline{\acute{c}\acute{w}} \quad (10)$$

The force per unit area is equivalent to the flux, or rate at which momentum changes. Alternatively, turbulent stresses can also be thought of as turbulent fluxes of momentum, and they are as follows:

$$\tau_{zx} = -\rho\overline{\acute{w}\acute{u}}, \tau_{zy} = -\rho\overline{\acute{w}\acute{v}}, \quad (11)$$

The main topic of our study is air flows with temperature stratification and/or wind shears, which inevitably result in turbulent flux transfers and vertical homogeneities in turbulent flows. Specifically, the following formula can be used to find the momentum, heat, and water vapor vertical fluxes:

$$F = \rho\overline{\acute{u}\acute{w}}$$
$$H = \rho c_p\overline{\acute{\theta}\acute{w}} \quad (12)$$
$$E = \rho\overline{\acute{q}\acute{w}}$$

It should be noted that momentum fluxes in the lower part of the PBL are always downward (negative) due to increasing wind speed with height, whereas heat and moisture fluxes can be upward or downward depending on variations in mean potential temperature and specific humidity with length above the surface. Vertical fluxes in the convective mixed layer often peak at the surface and decrease more quickly as one ascends. At night, the fluxes are significantly weaker.

Theories of Gradient-Transport

We will provide a summary of the more basic gradient-transport theories, which are still often applied in dispersion research and air pollution meteorology.

The Eddy Diffusivities Theory

The kinetic theory of gases and experimental observations of molecular exchange processes in laminar viscous flows have shown simple proportionality between the average molecular fluxes of momentum, heat, and concentration, respectively, in the direction of fluxes. We now refer to the above flux-gradient connections as Newton's law of viscosity, Fourier's law of heat conduction, and Fick's law of mass diffusion:

$$F = -\rho\nu\frac{\partial u}{\partial z}$$
$$H = -\rho c_p k\frac{\partial\theta}{\partial z} \quad (13)$$
$$M = -D\frac{\partial c}{\partial z}$$

where D stands for mass diffusivity, k for heat diffusivity, and ν for kinematic viscosity.

Since molecular exchanges are directly equal to turbulent transport or fluxes, the relationship between them and mean gradients is also thought to exist[1]. An example of this relationship is as follows.

$$\overline{\acute{u}\acute{w}} = -k_m\frac{\partial\bar{u}}{\partial z}$$
$$\overline{\acute{\theta}\acute{w}} = -k_h\frac{\partial\bar{\theta}}{\partial z} \quad (14)$$
$$\overline{\acute{c}\acute{w}} = -k_z\frac{\partial\bar{c}}{\partial z}$$

These are known as the eddy diffusivities, k_m, k_h, and k_z, or the mass, heat, and momentum exchange coefficients, respectively. Eddies viscosity and eddy diffusivities of mass and heat may vary with respect to the PBL. Moreover, diffusivities that are horizontal, like k_x and k_y, are generally larger than those that are vertical, or "k_z". Eddy diffusivities in the convective mixed layer can be endlessly large or negative because the vertical fluxes of mass, heat, and momentum usually do not follow the mean gradients of concentration, potential temperature, and velocity.

The Theory of Mixing Length

The vertical eddy diffusivities can be expressed as follows using Prandtl's mixing length theory [2,3]:

$$K_m = l_m^2 \left|\frac{\partial \bar{V}}{\partial z}\right|$$

$$K_h = l_m l_h \left|\frac{\partial \bar{V}}{\partial z}\right| \quad (15)$$

where L_h and L_m represent the average mixing lengths for momentum and heat transmission, respectively. But the mixing-length idea also produces:

$$K_m = c_m l_m \sigma_w$$

$$K_h = c_h l_h \sigma_w \quad (16)$$

expanding Eq. (15) to incorporate turbulent diffusivities of passive, non-buoyant material in different directions, where the empirical constants c_h and c_m are used:

$$K_x = c l_x \sigma_u$$

$$K_y = c l_y \sigma_v \quad (17)$$

$$K_z = c l_z \sigma_w$$

The typical length scales of diffusion in the x, y, and z directions are represented by the values l_x, l_y, and l_z, respectively. These are called huge eddy scales, and they are also called scales of turbulence. Gradient transport theories provide the clearest and most direct links between turbulent fluxes of momentum, heat, and mass for mean gradients in velocity, temperature, and mass concentration. A gradient transport theory would not be able to predict counter-gradient fluxes without first altering the flux-gradient relations (14) to account for them. The following modification of the heat flux was proposed by[4]:

$$\overline{\acute{\theta}\acute{w}} = -k_h\left(\frac{\partial \bar{\theta}}{\partial z} - \upsilon_c\right) \quad (18)$$

where υ_c=6.5x10^{-4} Km^{-1} is a constant found empirically

It is discovered that, particularly in the neutral surface layer, the mixing length and the eddy viscosity both change linearly with height, that is:

$$l_m = kz; \qquad K_m = kzu \quad (19)$$

where the Von-Karman constant expression is k=0.4. In the event of such a flow, the turbulent quantities and mean velocity gradient should be influenced by both the height above the surface and the kinematic momentum flux, or surface stress τ_0/ρ. This similarity hypothesis is sufficient. This implies thatτ_0 can be used as a stand-in for various other possible variables, including PBL depth, geostrophic winds, and surface roughness. The following is the likely resemblance theory for the mean velocity gradient:

$$\frac{\partial \bar{u}}{\partial z} = f\left(z, \frac{\tau_9}{\rho}\right) \quad (20)$$

results in a continuous dimensionless wind shear for which "z" and $u_* = (\tau_0/\rho)^{0.5}$ are the only appropriate length and velocity scales, respectively.

$$\frac{z}{u_*}\frac{\partial \bar{u}}{\partial z} = constant = \frac{1}{k} \quad (21)$$

The logarithmic velocity profile law, which is widely recognized, can be acquired by integrating Eq. (21) concerning "z."

$$\bar{u} = \frac{u_*}{k}\, ln\frac{z}{z_0} \qquad z >> z_0 \quad (22)$$

The reason $\bar{u} = 0$ at z=z_0 is the surface roughness length indicated by z_0.

Using the same similarity hypothesis for turbulent quantities leads to the straightforward conclusion that in the neutral surface layer, the ratios $\sigma_u/u_*, \sigma_v/u_*$ and σ_w/u_* must be constants. The following are the best estimates for the normalized standard deviations:

$$\frac{\sigma_u}{u_*} \cong 2.5; \quad \frac{\sigma_v}{u_*} \cong 1.9; \quad \frac{\sigma_w}{u_*} \cong 1.3 \quad (23)$$

for neutral and stable conditions.

Theory of Monin-Obukhov Similarity

The Monin-Obukhov (M-O) similarity theory is widely accepted as being more appropriate. The basis of the similarity hypothesis proposed by[5] is the notion that gradients and turbulent properties of a stratified surface layer depend exclusively on the height "z". These include the kinematic surface stress τ_0/ρ., the buoyancy variable g/T_0, and the kinematic heat flux $H_0/\rho c_p$. The following independent scales are derived and utilized to build the dimensionless group or parameters of the M-O similarity theory after buoyancy and heat flux variables are added to the first list for the neutral surface layer:

Friction velocity: $u_* = \left(\frac{\tau_0}{\rho}\right)^{0.5}$

Friction temperature: $\theta_* = -\frac{H_0}{\rho c_p u_*}$

Hight above surface: z

Buoyancy length:

$$L = \frac{(\tau_0/\rho)^{3/2}}{k(g/T_0)(H_0/\rho c_p)} = \frac{u_*^2}{k(g/T_0)\theta_*} \quad (24)$$

The depth of the near-surface layer where shear effects are most likely to be significant under any stability condition is described by the buoyancy length scale, also known as the Obukhov length. The stability parameter, or Monin-Obukhov similarity, is defined as the ratio of the two length scales, or z/L. Comparable to the Richardson number (Ri), it measures the proportional significance of buoyancy and shear effects.

The following M-O similarity relations were obtained via dimensional analysis and application of the M-O similarity hypothesis to mean velocity and potential temperature gradient:

$$\frac{kz}{u_*}\left(\frac{\partial \bar{u}}{\partial z}\right) = \emptyset_m(\xi)$$

$$\frac{kz}{\theta_*}\left(\frac{\partial \bar{\theta}}{\partial z}\right) = \emptyset_h(\xi) \qquad (25)$$

It implies that temperature gradients and dimensionless wind shear are two specific effects of ξ=z/L. Integrating Eq. (25) in reference to "z" yields the normalized profile relations:

$$u = \frac{u_*}{k}\left(ln\frac{z}{z_0} - \psi_m(\xi)\right)$$

$$\frac{\bar{\theta}-\bar{\theta}_0}{\theta_*} = \frac{\alpha}{k}\left(ln\frac{z}{z_0} - \psi_h(\xi)\right) \qquad (26)$$

where α is a constant between 0.9 and 1, $\bar{\theta}_0$ is the potential temperature at $z=z_0$, and $\psi_m(\xi)$ and $\psi_h(\xi)$ are exclusively connected to $\phi_m(\xi)$ and $\phi_h(\xi)$, respectively.

$$\psi_m(z/L) = \int_{z_{0/L}}^{z/L}[1-\phi_m(\xi)]\frac{d\xi}{\xi}$$

$$\psi_h(z/L) = \int_{z_{0/L}}^{z/L}[1-\phi_h(\xi)]\frac{d\xi}{\xi} \qquad (27)$$

As suggested by[6,3], the profile relations (26) can be understood as simple variants of the log-law, in which the stability-dependent ψ-function decreases with increasing stability. When conditions are steady, ψ-functions are -ve and fluctuate linearly with z/L. As stability z/L increases, the temperature and wind profiles exhibit log-linear behavior before turning into linear trends. As stability gets closer to neutral and changes into instability, the wind profile's curvature rises.

The basic M.O similarity functions, $\phi_m(\xi)$ and $\phi_h(\xi)$, can be used to characterize additional variables that involve mean gradients, such as:

$$Ri = \frac{\xi\phi_h(\xi)}{\phi_m^2(\xi)}$$

$$\frac{k_m}{kzu_*} = \frac{1}{\phi_m(\xi)}$$

$$\frac{k_h}{kzu_*} = \frac{1}{\phi_h(\xi)} \qquad (28)$$

The Businger-dyer relationship is the most common and fundamental type.

$$\phi_h = \phi_m^2 = (1-15\xi)^{-0.5}, \qquad for -5 < \xi < 0$$

$$\phi_h = \phi_m = 1+5\xi, \quad for\ 1 > \xi \geq 0 \qquad (29)$$

The following are the corresponding relationships between ξ and Ri:

$$\xi = Ri, \qquad for\ Ri < 0$$

$$\xi = \frac{Ri}{1-5Ri}, \quad for\ 0 \leq \xi < 0.2 \qquad (30)$$

In extremely stable conditions that approach the critical condition (Ri) for the maintenance of turbulence, the foregoing empirical versions of the similarity function are not predicted to hold; however, the second of Eq. (30) predicts a critical value of Ri=0.2[7]. On the other hand, the following evidence-based empirical relations can be put out for unstable and convective circumstances.

$$\frac{\sigma_{u,v}}{u_*} = \left(12 - 0.5\frac{h}{L}\right)^{1/3}, \quad for\ \frac{h}{z} < 0$$

$$\frac{\sigma_w}{u_*} = 1.3\left(1 - 3\frac{h}{L}\right)^{1/3}, \quad for\ \frac{h}{z} \leq 0 \qquad (31)$$

The reliance of $\sigma(u,v)/u_*$ on h/L and the exponent one third are in agreement with the mixed-layer similarity theory prediction of the CBL.

Theory of Local Free-Convection Similarity

Obukhov's local free convective on similarity hypothesis, in which u_* is deemed irrelevant and only z, g/T_0, and $H_0/\rho c_p$ are deemed appropriate independent variables, is another similarity hypothesis of validity restricted to only vertical velocity and temperature in the free convective surface layer. The following scales for local free convection result from these.

$Length: z$

Velocity:

$$u_f = \left(\frac{g}{T_0}\frac{H_0}{\rho c_p}z\right)^{1/3} \qquad (32)$$

Temperature:$\theta_f = \left(\frac{H_0}{\rho c_p}\right)^{2/3}\left(\frac{g}{T_0}z\right)^{-1/3}$

The main theories of the local free-convection similarity theory are

$$\frac{\sigma_w}{u_f} = 1.4$$

$$\frac{\sigma_\theta}{\theta_f} = 1.3 \qquad (33)$$

Theory of Mixed-Layer and PBL Similarity

The wind layer similarity theory is the most popular theory for parameterizing diffusion and turbulence in the convective boundary layer. For the convective mixed layer, similarity hypothesis offers the following scales: z, g/T_0, Q_0, and h are the most suitable independent variables[8].

Length: h

Velocity:

$$w_* = \left(\frac{g}{T_0} Q_0 h\right)^{1/3} \quad (34)$$

Temperature: $T_* = \frac{Q_0}{w_*}$

The normalized turbulent values σ_u/w_*, σ_w/w_*, σ_θ/T_*,, and so forth are expected to be unique functions of z/h according to the mixed-layer similarity theory. Essentially, the only evidence of turbulence in the CBL is the fact that fluctuations in horizontal velocity are not always determined by z/h.

$$\frac{\sigma_u}{w_*} \approx \frac{\sigma_v}{w_*} \approx 0.6 \quad (35)$$

Boundary layer parameterization for dispersion applications

In the PBL, the vertical distribution of mean wind speed, wind direction, and turbulence controls the movement and diffusion of material released into the atmospheric boundary layer. Their reliance on atmospheric stability, which affects the rising plume or puff, is substantial. The vertical extent of the material mixing between the elevated releases and the PBL surface is typically determined by the PBL depth or mixing height. In this paper, we address a number of fundamental techniques for determining boundary layer parameters for dispersion modeling [9]. Starting with three basic parameters, we can characterize mean winds and turbulence using a variety of secondary components and scales: the PBL depth, the surface heat flux, and the surface stress or friction velocity.

The profile technique is discussed here in its most strict form, requiring measurements of the mean temperature differential between two levels (z_1 and z_2) at the surface layer and the mean wind speed at one level (z_r). The relations (17) of Monin-Obukhov similarity profiles that yield "u_*" and "θ_*" are as follows:

$$u_* = \frac{k\bar{u}_r}{ln(z/z_0) - \psi_m(z_r/L)}$$

$$\theta_* = \frac{k\Delta\bar{\theta}}{\alpha[ln(z_2/z_1) - \psi_h(z_2/L) + \psi_h(z_1/L)]} \quad (36)$$

which have empirical counterparts that are stability-dependent similarity functions with α=1 and ψ_m and ψ_h, based on Eqns. (18) and (20).

$$\psi_m - \psi_h = -\frac{5z}{L} \qquad for\ \frac{z}{L} \geq 0$$

$$\psi_m = 2ln\left(\frac{1+x}{2}\right) + ln\left(\frac{1+x^2}{2}\right) - 2tan^{-1}x + \frac{\pi}{2} \quad , \frac{z}{L} < 0 \quad (37)$$

$$\psi_h = 2ln\frac{1+x^2}{2},\ for\ \frac{z}{L} < 0$$

where, $x = (1 - 15z/L)^{1/4}$

Because L is defined by the same scales through Eq. (15), Eq. (27) can be solved iteratively for u_* and θ_*. The bulk Richardson number R_{ib} and z_r/L have a functional relationship that can be used to completely eliminate the iteration process.

$$R_{ib} = \frac{g\Delta\bar{\theta}z_r}{T_0\bar{u}_r^2} = \frac{z_0}{L}\frac{\left[ln(z_2/z_1) - \Psi_h\left(\frac{z_2}{L}\right) + \Psi_h(z_1/L)\right]}{\left[ln(z_r/z_0) - \Psi_m\left(\frac{z_r}{L}\right)\right]^2} \quad (38)$$

Concerning the allocated values z_0, z_1, z_2, and z_r.

If the wind speed difference, $\Delta\bar{u}$ between the two heights, z_2 and z_1, is known in addition to the temperature difference, then an extra gradient technique (Arya, 1988) can be applied. With the widely accepted logarithmic finite-difference approximations for the gradients [3,10], we can get the fundamental M-O relation (16).

$$u_* = \frac{k\Delta\bar{u}}{[\phi_m(z_m/L)ln(z_2/z_1)]}$$

$$\theta_* = \frac{k\Delta\bar{\theta}}{[\phi_h(z_m/L)ln(z_2/z_1)]} \quad (39)$$

The gradient Richardson number, $z_m = (z_1/z_2)^{1/2}$, can be found by taking the geometric mean of the two heights.

$$Ri_m = \frac{g}{T_0}\frac{\Delta\bar{\theta}z_m}{(\Delta\bar{u})^2} ln\left(\frac{z_2}{z_1}\right) \quad (40)$$

This has the following bearing on the M-O similarity parameter:

$$\frac{z_m}{L} = Ri_m \qquad for\ Ri_m < 0$$

$$\frac{z_m}{L} = \frac{Ri_m}{(1-5Ri_m)}, \quad for\ 0 \leq Ri_m < 0.2 \quad (41)$$

One can estimate the surface heat flux by utilizing the formula $H_0 = -\rho c_p u_*\theta_*$. Alternatively, using the gradient-transport radiation for H_0 and E_0 and assuming eddy diffusivity for heat and water vapor, one can compute the Bowen ratio, $B = H_0/L_eE_0$.

$$B = \frac{c_p}{L_e}\frac{\partial\bar{\theta}/\partial z}{\partial\bar{q}/\partial z} = \frac{c_p\Delta\bar{\theta}}{L_e\Delta\bar{q}} \quad (42)$$

The differences in potential temperature and specific humidity between the two heights, denoted as $\Delta\bar{\theta}$ and $\Delta\bar{q}$, respectively, can be deduced from the temperature measurements of the dry and wet bulbs at the two heights in the surface layer.

Height of Mixing on the PBL Depth

In addition to defining the upper bound on the vertical diffusion of plumes or material puffs released in the PBL, the mixing height, or PBL depth "h" is the most significant parameter because it influences several other parameters and scales related to turbulence, difference, and diffusion, such as $w_*, h/w_*, L_x, L_y$ *and* L_z.

In middle- and high-latitude regions, the neutral PBL's depth can be determined using the following fundamental theoretical parameters:

$$h \cong 0.3\frac{u_*}{|f|} \tag{43}$$

Eq. (43) is appropriate only in steady-state near-neutral conditions where $\lfloor h/L \rfloor < 1$ and no law-level high inversions are present. employing, respectively, the Coriolis parameter = 10^{-4} s^{-1} and u***** = 0.4 (u / ln [Z/Zo]).

The mixing height for the continually stratified nocturnal boundary layer with moderate to high winds is determined by:

$$h \cong 0.4\left(u_*\frac{L}{|f|}\right)^{0.5} \qquad for\ \frac{h}{L} > 0 \tag{44}$$

This is absolutely accurate for the conditions of steady-state equilibrium. u_* = 0.4 (u / (ln [Z/Z_o] + 5 Z/L))

For the unstable and convective boundary layer throughout the day, a prediction rate equation for the mixing height is determined to be appropriate.

$$\frac{\partial h}{\partial t} = \overline{w}_h + w_e \tag{45}$$

is thought to work better with PBL that change during the day. We are the entrainment velocity in this instance, and the mean vertical velocity at the top of the PBL is denoted by $\overline{w}_h$. The parameters utilized to parameterize the entrainment velocity are typically the friction velocity u_*, the convective velocity w_*, the environmental lapse rate immediately above the PBL, and maybe other ones[11]. In contrast, large- or meso-scale divergence must be used to compute $\overline{w}_h$.

Z_i = [12-(σ_θ u / u*)3 L] / 0.5 for Z/L < 0

where,

u_* = 0.4 [Z / (1-15(Z/L))]$^{-1/4}$Δu/Δz

Average wind profile

As one ascends, the mean wind speed likewise increases in parallel with the shift in mean wind direction, at least in the lower portion of the PBL. According to Monin-Obukhov similarity relative (17), the wind speed profile in the surface layer can be expressed as follows:

$$\frac{\overline{u}(z)}{\overline{u}(z_r)} = \frac{\ln\left(\frac{z}{z_0}\right) - \psi_m\left(\frac{z}{L}\right)}{\ln\left(\frac{z_r}{z_0}\right) - \psi_m\left(\frac{z_r}{L}\right)} \tag{46}$$

where measurements are taken at the height designated as reference, z_r. Eq. (28) can only be used to determine ψ_m in the lowest 10 to 15 percent of the PBL, where there is little variation in wind direction with height.

On the basis of measured wind profiles under different stability circumstances, one can assume that mean wind in unstable and convective situations, regardless of height, are comparatively uniform. Citing, the mean mixed-layer wind speed can be determined generally as follows[12]:

$$\frac{\overline{V}_m}{u_*} = \frac{1}{k}\left[ln\frac{h}{z_0} - \frac{1}{2}ln\left|\frac{h}{L}\right| - 2.3\right] \qquad ,for\ \frac{h}{L} < 0 \tag{47}$$

However, under generally stable conditions, a substantial body of observational evidence supports a nearly linear wind speed profile up to the height of the low-level jet, which frequently aligns with the top of the PBL, which is:

$$\overline{V} = \overline{V}_s + \frac{z-h_s}{h-h_s}(\overline{V}_h - \overline{V}_s) \qquad ,for\ \frac{h}{L} > 0 \tag{48}$$

Where the subscripts "s" and "h" denote the PBL and the top of the surface layer, respectively, and V_s can be calculated using the surface layer profile relations.

The power-law profile is an alternate empirical depiction of the mean wind distribution that is widely employed in applications involving the dispersion of air pollution.

$$\frac{\overline{V}}{\overline{V}_r} = \left(\frac{z}{z_r}\right)^m \tag{49}$$

Given a comparison of measured wind-speed profiles from various sites and stability conditions with Eq. (40), where $\overline{V}_r$ is the wind speed at the reference height "z_r" and "m" increases with increasing surface roughness and, for the same site, increases with increasing stability. where the exponent m is either equal to unity or smaller.

In more straightforward dispersion models that do not require direct turbulence input, stability is typically utilized as a stand-in for turbulence, and diffusion is defined as a function of stability. The properties that depend on the potential temperature or vertical temperature gradient are displayed in Table (1) and include $\partial\overline{\theta}/\partial z$ and s.

Table 1: General characterizations of static stability

Lapse Rate	$\frac{\partial\overline{\theta}_v}{\partial z}$ *or s*	$\partial\overline{T}_v/\partial z$	Static stability
Sub-adiabatic	> 0	$> -\Gamma$	Stable
Adiabatic	=0	$= -\Gamma$	Neutral
Super adiabatic	< 0	$< -\Gamma$	Unstable

The different Richardson number variations in this category—such as the ones below-are significant features.

$$Gradients:\ Ri = \frac{g}{T_0}\frac{\partial\overline{\theta}/\partial z}{|\partial\overline{V}/\partial z|^2}$$

$$Bulk: Ri_b = \frac{g}{T_0}\frac{\Delta\overline{\theta}z_r}{\overline{V}_r^2} \tag{50}$$

$$Mixed: Ri_B = \frac{g}{T_0}\frac{(\partial\overline{\theta}/\partial z)z_r^2}{\overline{V}_r^2}$$

And here is the Obukhov length (L), also referred to as the

Monin-Obukhov stability parameter:

$$\frac{z}{L} = -\frac{zk}{u_*^3}\frac{g}{T_0}\frac{H_0}{\rho c_p} = \frac{zk}{u_*^2}\frac{g\theta_*}{T_0} \tag{51}$$

The previously listed parameters are all local because they depend on either wind speed at a reference height z_r , local gradient, or altitude above the surface. As the vertical gradients of potential temperature and velocity are most pronounced in the surface layer, it is thought that they are accurate indicators of turbulence. Monin-Obukhov similarity theory in this layer accurately associates the different Richardson numbers with z/L (Ri_b and Ri_B also depend on z/z_0).

The most accurate ratio to use when assessing the PBL's overall stability is this one:

$$\frac{h}{L} = -\frac{hk}{u_*^3}\frac{g}{T_0}\frac{H_0}{\rho c_p} = \frac{hk}{u_*^2}\frac{g\theta_*}{T_0} \tag{52}$$

The PBL similarity hypothesis is used to obtain this, and meteorologists who research boundary layers and air pollution highly suggest it. In the absence of information about the heat and momentum surface fluxes (or, equivalently, u_* and θ_*), the PBL's bulk Richardson number is unknown.

$$Ri_h = \frac{g}{T_0}\frac{h\Delta\bar{\theta}}{\bar{V}_h^2} \tag{53}$$

where h/L can be replaced by the potential-temperature difference over the PBL depth, $\Delta\bar{\theta}$. It is important to highlight that Ri_h is well defined for stable, unstable, or convective boundary layers and is not restricted by the local bulk or the gradient Richardson number. It is evident that in unstable and convective conditions, the ratio of the friction and convective velocity scales is specifically related to h/L, that is:

$$\frac{w_*}{u_*} = k^{1/3}\left(-\frac{h}{L}\right)^{1/3} \tag{54}$$

that h/L estimation can also be made using.

When using dispersion modeling for the PBL as a whole, the following empirical similarity relations should be applied:

$$\frac{\sigma_u}{u_*} = 2.5\left(1-\frac{z}{h}\right)^{\alpha}$$
$$\frac{\sigma_v}{u_*} = 1.9\left(1-\frac{z}{h}\right)^{\alpha} \tag{55}$$
$$\frac{\sigma_w}{u_*} = 1.3\left(1-\frac{z}{h}\right)^{\alpha}$$

In the presence of neutral and stable conditions, use the PBL turbulence data [13,14-17] to determine α=0.5 to 1.

$$\frac{\sigma_u}{w_*} = \frac{\sigma_v}{w_*} = \frac{\sigma_w}{w_*} = 0.60 \tag{56}$$

for unstable and convective conditions.

Analytically, the diffusion equation that will be addressed later can be solved in simple terms by either providing the eddy diffusivities as functions of height using the following sorts of power law relations, or by considering them as constants.

$$\frac{K}{K_r} = \left(\frac{z}{z_r}\right)^n \tag{57}$$

Depending on the stability and surface roughness of the exponent "n," this could change.

Though the rigorous conjugate connection "n=1-m" proposed in the literature may not always hold true, there is a roughly inverse relationship between "n" and "m." By imposing the constraint that the momentum flux in the surface layer must be independent of height, the conjugate connection, $K\frac{\partial u}{\partial z} = constant$, is produced. Only a power law profile with a finite exponent m>0 can adequately characterize the logarithmic velocity profile. On the other hand, the eddy viscosity in the neutral surface layer with continuous flow varies in a linear fashion with height (n=1). The conjugate relationship between the two exponents is therefore not satisfied (m+n>1). As a result, (m+n>1) holds true in both extra ordinarily unstable or free convective scenarios (n=4/3) and exceptionally stable conditions (m≅1)

Technique applied

The following formula can be used to express three-dimensional advection-diffusion:

$$u\frac{\partial C}{\partial x} = \frac{\partial}{\partial y}\left(k_y\frac{\partial C}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial C}{\partial z}\right) \tag{58}$$

The concentration in three dimensions is represented by C (Bq/m^3), and the turbulent diffusivities in the y and z directions are indicated by k_y and k_z.

In order to solve Equation (1), take into consideration the following scenario:

$$C = Q\, C_y C_z \tag{59}$$

Q stands for the emission rate (Bq).

It is possible to substitute equations (59) and (58) to obtain the following two equations:

$$u(z)\frac{\partial\, C_y(x,y)}{\partial x} = \frac{\partial}{\partial y}\left(k_y\frac{\partial\, C_y(x,y)}{\partial y}\right) \tag{60.1}$$

$$u(z)\frac{\partial\, C_z(x,y)}{\partial x} = \frac{\partial}{\partial z}\left(k_z\frac{\partial\, C_z(x,y)}{\partial z}\right) \tag{60.2}$$

Two equations, (60.1) and (60.2), are estimated under the subsequent boundary conditions:

(a) At the mixing height and surface, there is no vertical flow, i.e.

$$k_z\frac{\partial C}{\partial z} = 0, \qquad \text{at } z = 0, h \tag{61a}$$

(b) At y=0 and L_y, there is no y-direction flux, i.e.

$$k_y\frac{\partial C}{\partial y} = 0, \qquad at\ \ y = 0, L_y \tag{61b}$$

© Examples of mass continuity are as follows:

$$uC_z = Q\delta(z - h_s) \quad \text{at } x = 0 \tag{61c1}$$

$$uC_y = Q\delta(y - y_o) \quad \text{at } x = 0 \tag{61c2}$$

where, y_0 is a small distance in y-direction.

(d) At a great distance, no concentration is present in the following ways:

$$C \to 0 \quad \text{as } y \to \pm\infty \quad and\ z \to \infty \tag{61d}$$

where h is the height of the Planetary Boundary Layer (PBL) (m), L_y is a large distance in the y direction, and δ is a Dirac delta function.

Using the following average values and the assumption that "h" is discretizing stepwise into N sub-intervals where k (z) and u (z):

$$k_n = \frac{1}{z_{n+1}-z_n}\int_{z_n}^{z_{n+1}} k_n(z)dz \tag{62}$$

$$u_n = \frac{1}{z_{n+1}-z_n}\int_{z_n}^{z_{n+1}} u(z)dz \tag{63}$$

Assuming the crosswind turbulence parameters have the following form:

$$k_y(x,z) = \beta x u \tag{64}$$

i- First Model

Assuming the crosswind turbulence parameters have the following form:

$$\frac{\partial\, C_y}{\partial x} = \beta\, x\frac{\partial^2\, C_y}{\partial y^2} \tag{65}$$

The separation method is used to compute Eq. (65) as follows:

$$C_y = \chi_l(x)\eta_l(y) \tag{66}$$

Then, Eq. (65) becomes:

$$\frac{1}{x}\frac{1}{\chi_l(x)}\frac{\partial\chi_l(x)}{\partial x} = \frac{\beta}{\eta_l(y)}\frac{\partial^2\eta_l(y)}{\partial y^2} = -\lambda_l^2 \tag{67}$$

here the constant is denoted by λ_l. It then results in the following two differential equations:

$$\frac{\partial\chi_l(x)}{\partial x} = -\lambda_l^2\; x\; \chi_l(x) \tag{68a}$$

$$\frac{\partial^2\eta_l(y)}{\partial y^2} = \frac{-\lambda_l^2}{\beta}\eta_l(y) \tag{68b}$$

Equations (68 a, b) appear as follows after evaluation:

$$\chi_l(x) = a_1 e^{\frac{-\lambda_l^2 x^2}{2}} \tag{69a}$$

$$\eta_l(y) = a_2 cos\left(\frac{\lambda_l}{\sqrt{\beta}}y\right) + a_3 sin\left(\frac{\lambda_l}{\sqrt{\beta}}y\right) \tag{69b}$$

where the boundary condition (4b) is used to estimate the constant values a_1, a_2, and a_3,and $a_3 = 0$ and

$$\lambda_l = \frac{l\pi\sqrt{\beta}}{L_y}\ , l = 0,1,2,\ldots$$

For Eq. (66), the following is the response:

$$C_y = \sum_{l=0}^{\infty} B_l e^{\frac{-\lambda_m^2 x^2}{2}} cos\left(\frac{l\pi}{L_y}y\right) \tag{70}$$

Consequently, using Equations (61c) and (67), one obtains: $B_0 = \frac{1}{L_y},\ B_l = \frac{2}{L_y},\quad l = 1,2,3, \ldots$ where, $B_l = a_1 a_2$.

ii- Second Model

Equations (62) and (63) will provide the following value for Equation (60.2).

$$\frac{k_n(z)}{u_n(z)}\frac{\partial^2 C_{zn}(x,z)}{\partial z^2} = \frac{\partial C_{zn}(x,z)}{\partial x}\ , \qquad n = 1{:}\,N \tag{71}$$

Using the proper boundary conditions and the Laplace transform on "x"

$$C_{zn}(0,z_n) = \frac{Q}{u_n}\delta(z_n - h_s) \tag{i}$$

$$k_n(z)\frac{\partial c_{zn}(x,z)}{\partial z} = 0 \qquad \text{at} \quad z_n = 0, \mathrm{h} \tag{ii}$$

Equation (71) transforms into:

$$\int_0^{\infty} u\frac{\partial c_{z_n}}{\partial x}e^{-sx}dx = k_n(z)\int_0^{\infty}\frac{\partial^2 c_{z_n}}{\partial z^2{}_n}e^{-sx}dx \tag{72}$$

The formula for Equation (71) is as follows:

$$-uc_{z_n}(0,\mathrm{z}) + \mathrm{su}\ c_{z_n}\tilde{c}(\mathrm{s},\mathrm{z}) = k_n(z)\frac{\partial^2\tilde{c}_{z_n}(\mathrm{s},\mathrm{z})}{\partial z^2{}_n} \tag{73}$$

By using the boundary condition (i), Eq. (73) becomes:

$$\frac{\partial^2\tilde{c}_{z_n}(\mathrm{s},\mathrm{z})}{\partial z^2{}_n} - \frac{su}{k_n}\tilde{c}_{z_n}(\mathrm{s},\mathrm{z}) = -\frac{Q}{k_n}\delta(z_n - h_s) \tag{74}$$

After that, let's apply the Laplace transform to z:

$$p^2\tilde{\tilde{c}}_{z_n}(s,p) - pc_{y_n}(s,0) - \frac{\partial\tilde{c}_{z_n}(s,0)}{\partial z} - \frac{us}{k_n}\tilde{\tilde{c}}_{z_n}(s,p) = -\frac{Q}{k_n}e^{-ph_s} \tag{75}$$

Eq. (75) becomes: when the condition (ii) is substituted

$$\tilde{\tilde{c}}_{z_n}(s,p) = \frac{c_{z_n}(s,0)p}{(p^2-\frac{us}{k_n})} - \frac{Qe^{-ph_s}}{k_n(p^2-\frac{us}{k_n})} \tag{76}$$

$$\tilde{\tilde{c}}_{z_n}(s,p) = c_{z_n}(s,0)F(s,p) - \frac{Q}{k_n}e^{-ph_s}G(s,p) \tag{77}$$

where, $\quad \mathrm{F}\,(\mathrm{s},\mathrm{p}) = \frac{p}{(p^2-\frac{us}{k_n})}$

and $\quad G(s,p) = \frac{1}{(p^2-\frac{us}{k})}$

After taking Eq. (77) and flipping it, one gets:

$$\tilde{c}_{z_n}(s,z) = \frac{c_{z_n}(s,0)}{2}\left[e^{\sqrt{\frac{su}{k_n}}z} + e^{-\sqrt{\frac{su}{k_n}}z}\right] -$$

$$\frac{Q}{2k_n}\sqrt{\frac{k_n}{su}}\left[e^{\sqrt{\frac{su}{k_n}}(z-h_s)} - e^{-\sqrt{\frac{su}{k_n}}(z-h_s)}\right]H(z-h_s)$$
(78)

Let $R_n = \sqrt{\frac{su}{k_n}}$ and $R_a = \sqrt{suk_n}$

$$\tilde{c}_{z_n}(s,z) = \frac{c_{z_n}(s,0)}{2}\left[e^{R_n z} + e^{-R_n z}\right] - \frac{Q}{2R_a}\left[e^{R_n(z-h_s)} - e^{-R_n(z-h_s)}\right]H(z-h_s) \quad (79)$$

$$\tilde{c}_{z_n}(s,z) = c_{z_n}(s,0)\cosh R_n z - \frac{Q}{R_a}\sinh R_n(z-h_s) * H(z-h_s) \quad (80)$$

By employing boundary condition (ii), one obtains:

$$\frac{\partial}{\partial z}\tilde{c}_{z_n}(s,z) = R_n c_{z_n}(s,0)\sinh R_n z - \frac{Q}{R_a}R_n\cosh R_n(z-h_s)H(z-h_s) - \frac{Q}{R_a}\sinh R_n(z-h_s)\frac{\partial}{\partial z}H(z-h_s) \quad (81)$$

$$c_{z_n}(s,0)\sinh(R_n \mathrm{h}) = \frac{Q}{R_a}\cosh\left(R_n(\mathrm{h}-h_s)\right)H(\mathrm{h}-h_s) \quad (82)$$

$$c_{z_n}(s,0) = \frac{Q}{R_a}\frac{\cosh R_n(\mathrm{h}-h_s)}{\sinh(R_n \mathrm{h})}$$

$$c_{z_n}(s,0) = \frac{Q}{\sqrt{suk_n}}\frac{\cosh\sqrt{\frac{su}{k_n}}(\mathrm{h}-h_s)}{\sinh\sqrt{\frac{su}{k_n}}\mathrm{h}} \quad (83)$$

Upon substituting equation (81) for equation (83), the following results are obtained:

$$\tilde{c}_{z_n}(s,z) = \frac{Q}{\sqrt{suk_n}}\frac{\cosh\sqrt{\frac{su}{k_n}}(\mathrm{h}-h_s)}{\sinh\sqrt{\frac{su}{k_n}}\mathrm{h}}\cosh R_n z - \frac{Q}{R_a}\sinh R_n(z-h_s) * H(z-h_s) \quad (84)$$

Using the Gaussian quadrature formula method, the following results can be obtained:

$$\frac{c_{z_n}(x,z)}{Q} = \sum_{i=1}^{8} a_i\left(\frac{p_i}{x}\right)\frac{1}{\sqrt{\frac{uk_n(z)p_i}{x}}}\frac{\cosh\sqrt{\frac{p_i u}{xk_n}}(z_i-h_s)Cosh\,(R_n z)}{\sinh\sqrt{\frac{p_i u}{xk_n}}z_i} \quad (85)$$

Equations (70) and (85) are used to calculate Eq. (58) as follows:

$$C(x,y,z) = \sum_{i=1}^{8} a_i\left(\frac{p_i}{x}\right)\frac{Q}{\sqrt{\frac{uk_n(z)p_i}{x}}}\frac{\cosh\sqrt{\frac{p_i u}{xk_n}}(z_i-h_s)Cosh(R_n z)}{\sinh\sqrt{\frac{p_i u}{xk_n}}z_i}$$

$$\sum_{l=0}^{\infty} B_l e^{\frac{-\lambda_l^2 x^2}{2}-\frac{vx}{u}}\cos\left(\frac{l\pi}{L_y}y\right) \quad (86)$$

where the isotope's radioactive decay is represented by $e^{-vx/u}$.

Unstable Condition

In the unstable state, the wind speed and vertical turbulence take the following forms:

$$u(z) = \frac{u_*}{0.4}\left(\mathrm{Ln}\,\frac{z}{z_o} - 2\ln\left[0.5\left(1+\frac{1}{\emptyset_m}\right)\right] - \mathrm{Ln}\left[0.5\left(1+\frac{1}{\emptyset_m{}^2}\right)\right] + 2Tan^{-1}\frac{1}{\emptyset_m} - \frac{\pi}{2}\right) \quad (87)$$

$$k_z(x,z) = 0.4\,u_*\frac{z}{\emptyset_m(\frac{z}{L})}, \quad (88)$$

where,

$$\emptyset_m\left(\frac{z}{L}\right) = \left(1 - 15\frac{z}{L}\right)^{-1/4} \quad (89)$$

Findings and Discussions

Air samples with the reported I^{135} unstable isotope concentrations were collected at the Egyptian Atomic Energy Authority's, First Research Reactor at Inshas[18]. At 43 meters in height and 0.6 meters in roughness length, the experiments were conducted inside the stack. Table 1 displays the I^{135} meteorological data that are taken into consideration[19] Essa and Maha's (2008). Table (2) contains the concentrations that Eq. (55) should be used below the plume centerline. As illustrated in Figs. (1) and (2), it is evident that the suggested model fits the observed concentrations and the proposed data, when unstable circumstances are met, extremely well, within a factor of two.

Table 2: displays meteorological information from the nine convective test runs conducted between March and May 2006 at the Inshas site.

Run no.	Working hours of the source	Release rate (Bq)	Wind speed U10 (m s^{-1})	Wind direction (deg)	U_{uns} (m s^{-1})	W_* (ms^{-1})	P-G stability class	h (m)	Vertical distance (m)
1	48	1028571	4	301.1	0.27257	2.27	A	600.85	5
2	49	1050000	4	278.7	0.0748057	3.05	A	801.13	10
3	1.5	42857.14	6	190.2	2.41774	1.61	B	973	5
4	22	471428.6	4	197.9	1.43425	1.23	C	888	5
5	23	492857.1	4	181.5	0.98887	0.958	A	921	2
6	24	514285.7	4	347.3	1.27652	1.3	D	443	8.0
7	28	1007143	4	330.8	1.55442	1.51	C	1271	7.5
8	48.7	1043571	4	187.6	1.61173	1.64	C	1842	7.5
9	48.25	1033929	4	141.7	0.987627	2.1	A	1642	5.0

Table 3: Meteorological characteristics and concentrations measured during the Inshas experiment under unstable conditions.

Run	*Stability class*	*Distance (m)*	*Q (Bq)*	*Observed Conc. C(Bq/m^3)*	Gaussian Conc. C(Bq/m^3) (Bq/m^3)	*Predicted Conc. C(Bq/m^3)*
1	A	100	1028571	0.025	0.018	0.03699
2	A	98	1050000	0.037	0.02831	0.04511
3	B	115	42857.14	0.091	0.06962	0.11957
4	C	135	471428.6	0.197	0.146	0.218
5	A	99	492857.1	0.272	0.239	0.251
6	D	184	514285.7	0.188	0.159	0.201
7	C	165	1007143	0.447	0.411	0.459
8	C	134	1043571	0.123	0.112	0.129
9	A	96	1033929	0.032	0.043	0.039

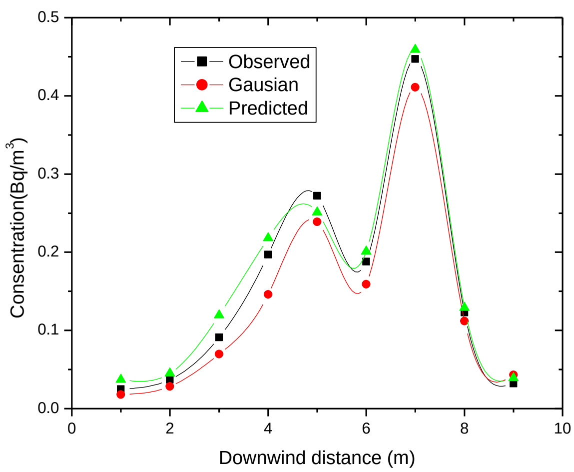

Fig. 1: shows how the downwind distance (m) varies under unstable conditions for the Gaussian, projected, and observed concentration models.

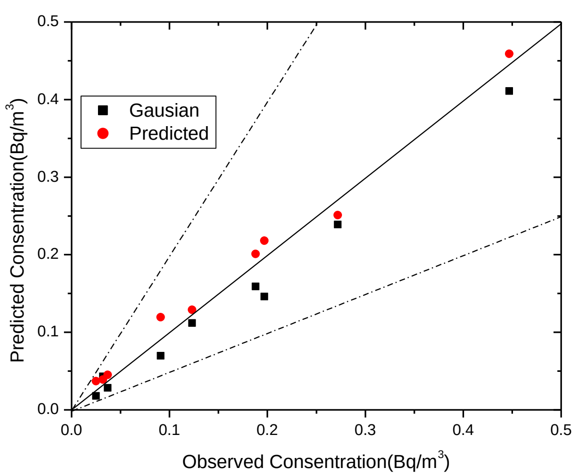

Fig. 2: Gaussian and proposed model fluctuations based on measured concentrations under unstable circumstances.

Methods of Statistics

Gaussian, predicted, and observed concentrations were compared by [20]. Factor of two is represented by FAC2, fraction bias by FB, In this case, the normalized mean square error (NMSE) and correlation coefficient (COR) are used.

Table 4: Gaussian, predicted, and observed concentrations in unstable situations are compared.

	NMSE	FB	COR	FAC2
Gaussian	0.027	0.14	0.995	0.87
Predicted	0.009	-0.06	0.995	1

In combination with the observed concentration data for I^{135}, Table (4) unequivocally demonstrates that the entire suggested model and the Gaussian model under unstable state fell within a factor of two.

Conclusions

The first work covers the intensities of turbulence as well as its formation, maintenance, and variations. Covariance and turbulent fluxes, gradient-transport theories, the mixing length hypothesis, the Monin-Obukhov similarity theory, and its values in unstable circumstances are all discussed. Estimating surface stress and heat flux, mean wind profile, wind speed in unstable conditions, boundary-layer parameterization for dispersion applications, mixed-layer and PBL similarity theory, and local free-convection similarity theory are all covered in the second article.

Under unstable situations, the proposed model and the Gaussian model agreed on the measured concentration values of I^{135} within a factor of two. Compared to Gaussian concentrations, the recommended concentrations under unstable circumstances are substantially closer to the reported I^{135} concentration values.

Declarations

PBL: Planetary boundary layer.

I^{135}: Iodine-135.

Ri: Richardson number.

$\overline{\acute{u}^2}, \overline{\acute{v}^2}, \overline{\acute{w}^2} and\ \overline{\acute{\theta}^2}$**:** the variance or mean square fluctuations.

i_u; i_v; i_w **:** The level of turbulence.

σ_θ; σ_ϕ**:** Turbulence intensities.

$\overline{\acute{u}\acute{w}}, \overline{\acute{v}\acute{w}}, \overline{\acute{\theta}\acute{w}}$ **:** Covariances.

r_{uw}, r_{qw}**:** Normalized Covariance.

τ_{zx}, τ_{zy}: Turbulence flux of momentum.

F, H, and E: momentum, heat, and water vapor vertical fluxes.

k_m, k_h, and k_z: Eddy diffusivities of mass, heat and momentum.

$\overline{\theta w}$**:** Heat flux.

L$_h$ and **L$_m$** : The average mixing lengths for momentum heat transmission.

K$_x$, k$_y$ and k$_z$: turbulent diffusivities of passive.

$\frac{\partial \bar{u}}{\partial z}$**:** Mean velocity gradient.

$\frac{\sigma_u}{u_*}$ **:** Normalized standard deviation.

τ_0/ρ : Surface Stress.

M-O: The Monin-Obukhov.

$H_0/\rho c_p$: Kinematic heat flux.

g/T_0 : The buoyancy velocity.

u_*: friction velocity.

θ_*: friction temperature.

L: Buoyancy length.

z/L : fluctuate linearly.

h: height of the planetary boundary layer.

CBL: Convective Boundary Layer.

$B = H_0/L_e E_0$ Brown Ratio.

f: Corioils Parameter.

“s" and "h" denote the PBL and the top of the surface layer.

Ri_b**:** Bulk Richardson number.

Ri_B**:** Mixed Richardson number.

$\frac{\sigma_u}{u_*}, \frac{\sigma_v}{u_*}$ **and** $\frac{\sigma_w}{u_*}$: Empirical similarity relations**.**

V$_r$: Wind speed at the reference height Z_r.

C **(Bq/m^3):** Concentration in three dimensions (Bq/m^3)

k_y **and** k_z: Turbulent diffusivities in the y and z directions.

Q : Stands for the emission rate (Bq).

L$_y$: is a large distance in the y direction.

$e^{-vx/u}$: Isotope radioactive decay.

P-G stability class.

W$_*$: Convective vertical velocity.

FAC2: factor of Two.

Z: Vertical distance (m)

FB: Friction Bias.

NMSE: Normalized Mean Square Error.

COR: Correlation Coefficient.

<u>The role of Khaled Essa</u>

Contribute to conducting this research

Contribute to collecting scientific material

Contribute to writing this research

Conduct correspondence for publication

<u>The role of Hanaa Taha</u>

Contribute to collecting scientific material

Contribute to writing this research

Make drawings

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