Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.18576/jsap/140203

Half Logistic Zeghdoudi Distribution: Statistical Properties, Estimation, Simulation and Applications

Mohammed Nasser Alshahrani

Department of Mathematics, College of Science and Humanities in Al-Kharj, Prince Sattam bin Abdulaziz University, Al-Kharj 11942, Saudi Arabia

Received: 5 Nov. 2024, Revised: 28 Dec. 2024, Accepted: 9 Feb. 2024
Published online: 1 Mar. 2025

Abstract: In this study, we introduce a new extension of the Zeghoudi distribution (ZD), which is an innovative modification of the ZD. The new proposed model is known as the half-logistic Zeghdoudi distribution (HLZD). This updated distribution simplifies the original ZD while increasing flexibility and accuracy when modeling data. The HLZD exhibits a variety of statistical properties, including right skewed, reduced, and unimodal probability density functions, skewness, kurtosis moments, incomplete moments, and order statistics. We use the maximum likelihood standard parameter estimation technique as well as a full simulation exercise to demonstrate the HLZD's efficacy and dependability. Furthermore, applying the HLZD to two real-world failure time/chemotherapy datasets demonstrates its utility. It is capable of outperforming well-known models such as generalized exponential, exponentiated generalized XLindley, power exponentiated Lindley, exponential, exponentiated generalized Lindley, XLindley, Weibull power Lindley, Weibull Lindley, half logistic new-Weibull Pareto, half logistic Weibull, half logistic exponential, half logistic Rayleigh, half logistic Pareto, and power Lindley distributions.

Keywords: Half logistic-G; Zeghdoudi distribution; Maximum likelihood; Simulation.

1 Introduction

Traditional distributions typically challenge correctly representing a large variety of data and processes in fields like engineering, biology, environmental science, and finance. This is because of the broad range of data and processes involved. The use of composite probability models is really necessary in order to triumph over these challenges. Due to the vast quantity of data and the practicality of distribution theory, there is a growing need for novel statistical distributions capable of correctly representing and modelling complicated events. A great number of researchers have made modifications to the statistical approaches that are already in use in order to improve distribution theory and its applications in modelling. As a consequence of this, several generalized families of distributions have been developed and expanded. Following are a few instances: a new Muth -G [1], type II general inverse exponential-G [2], a new truncated Muth-G [3], type II half logistic-G (HL-G) [4], sine exponentiated Weibull-G [5], alpha power weibull-G [6], a new sine-G [7], odd inverse power generalized Weibull-G [8], half-logistic odd Frćhet-G [9], truncated inverse Lomax-G [10], weighted exponentiated-G [11], ratio exponentiated general-G [12], generalized truncated Fréchet-G [13], compounded Bell-G [14], for more information see [15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32].
Ref. [4] established a novel family of distributions known as the HL-G family. The distribution function (CDF) and the probability density function (PDF) are provided via

$$F(y;\zeta,\eta)=\frac{2[G(y;\zeta)]^{\eta}}{1+[G(y;\zeta)]^{\eta}},\; y\in R\;,\;\; \eta>0, \tag{1}$$

and

$$f\left(y;\zeta,\eta\right)=\frac{2\eta g(y;\zeta)[G(y;\zeta)]^{\eta-1}}{\left[1+[G(y;\zeta)]^{\eta}\right]^{2}},\; y\in R,\;\; \eta>0, \tag{2}$$

* Corresponding author e-mail: mn.alshahrani@psau.edu.sa

where η is positive shape parameter. Also, we can write (2) as a linear combination of the exponentiated family

$$f(y;\zeta) = 2\eta g(y;\zeta)\sum_{i=0}^{\infty}(-1)^{i}(1+i)[G(y;\zeta)]^{i+\eta-1}. \tag{3}$$

Recentlt, many authors used the HL family to generate many distributions such as; half logistic modified Kies exponential model [33], half logistic inverted Nadarajah-Haghighi distribution [34], half logistic inverse Lomax distribution [35], half logistic inverted Kumaraswamy distribution [36], and half logistic Lindley distribution [37]. In recent years, many authors used the mixture method to create anew statistical distributions see [38,39,40,41,42,43,44,45]. Recently, Messaadia and Zeghdoudi [46] developed a one-parameter probability distribution, entitled the Zeghdoudi distribution (ZD), for modeling lifespan data in engineering and medical. Its PDF and CDF are as follows:

$$g(y;\alpha) = \left(\frac{\alpha^3 y(1+y)}{\alpha+2}\right)e^{-\alpha y}; \qquad y,\alpha>0, \tag{4}$$

and

$$G(y;\alpha) = 1-\left(1+\frac{\alpha^2y^2+(\alpha+2)\alpha y}{(\alpha+2)}\right)e^{-\alpha y}; \qquad \alpha,y>0. \tag{5}$$

Many authors studied and discussed the ZD such as; Aidi et al. [47] discussed the power ZD, Talhi et al. [48] proposed the truncated ZD, AlSultan and Al-Omari [49] proposed the acceptance sampling plans for the ZD, Bashiru et al. [50] introduced unit ZD, Ruidas [51] discussed the transmuted ZD and Elbatal et al. [52] proposed the inverse power ZD.

The primary purpose of this study is to explore and formulate the HLZD. Also, to introduce and investigate the statistical properties of the HLZD.

The substance of the article is organized as described in the following sections. The formulation of the HLZD and some reliability measures are presented in 2. In section 3, the mathematical properties of the HLZD are investigated in much greater detail. Section 4 discusses the statistical inferences that may be drawn from the HLZD based on the maximum likelihood estimation approach. Some simulation study are discussed in Section 5. Section 6 investigates two datasets taken from the actual world to demonstrate the applicability of the HLZD. A comprehensive summary of the study's most important findings and conclusions can be found in Section 7.

2 Formulation of The HLZD

In this section, we formulate the new suggested model which called the HLZD by inserting (4) and (5) in (1) and (2). The CDF and PDF of the HLZD are given by

$$F(y;\alpha,\eta) = \frac{2\left[1-\left(1+\frac{\alpha^2y^2+(\alpha+2)\alpha y}{(\alpha+2)}\right)e^{-\alpha y}\right]^{\eta}}{1+\left[1-\left(1+\frac{\alpha^2y^2+(\alpha+2)\alpha y}{(\alpha+2)}\right)e^{-\alpha y}\right]^{\eta}}, \; y>0\;,\;\alpha,\eta>0, \tag{6}$$

and

$$f(y;\alpha,\eta) = \frac{2\eta\alpha^3 y(1+y)e^{-\alpha y}\left[1-\left(1+\frac{\alpha^2y^2+(\alpha+2)\alpha y}{(\alpha+2)}\right)e^{-\alpha y}\right]^{\eta-1}}{(\alpha+2)\left[1+\left[1-\left(1+\frac{\alpha^2y^2+(\alpha+2)\alpha y}{(\alpha+2)}\right)e^{-\alpha y}\right]^{\eta}\right]^2}, \; y>0\;,\;\alpha,\eta>0. \tag{7}$$

The survival function (SF), HRF, reversed HRF and cumulative HRF of the HLZD are provided via

$$S(y;\alpha,\eta) = \frac{1-\left[1-\left(1+\frac{\alpha^2y^2+(\alpha+2)\alpha y}{(\alpha+2)}\right)e^{-\alpha y}\right]^{\eta}}{1+\left[1-\left(1+\frac{\alpha^2y^2+(\alpha+2)\alpha y}{(\alpha+2)}\right)e^{-\alpha y}\right]^{\eta}}, \; y>0\;,\;\alpha,\eta>0,$$

$$h(y;\alpha,\eta) = \frac{2\eta\alpha^3 y(1+y)e^{-\alpha y}\left[1-\left(1+\frac{\alpha^2y^2+(\alpha+2)\alpha y}{(\alpha+2)}\right)e^{-\alpha y}\right]^{\eta-1}}{(\alpha+2)\left[1-\left[1-\left(1+\frac{\alpha^2y^2+(\alpha+2)\alpha y}{(\alpha+2)}\right)e^{-\alpha y}\right]^{2\eta}\right]}, \; y>0\;,\;\alpha,\eta>0,$$

and

$$H(y;\alpha,\eta) = -\ln\left[\frac{1-\left[1-\left(1+\frac{\alpha^2y^2+(\alpha+2)\alpha y}{(\alpha+2)}\right)e^{-\alpha y}\right]^{\eta}}{1+\left[1-\left(1+\frac{\alpha^2y^2+(\alpha+2)\alpha y}{(\alpha+2)}\right)e^{-\alpha y}\right]^{\eta}}\right],\ y>0\ ,\ \alpha,\eta>0.$$

Figures 1 and 2 illustrate the curves of the PDF and HRF for the HLZD. From these figures we can note that the PDF can be decreasing or unimodal or right skewed but the HRF can be decreased or increased or constant.

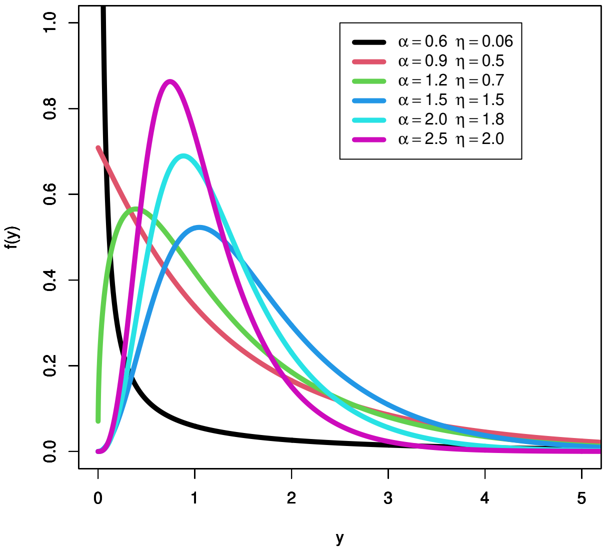

Fig. 1 Plots of PDF for the HLZD

3 Statistical Properties

In this section, we compute some important statistical features for the HLZD.

3.1 Quantile function

The finding below indicates that the quantile function (QF) of the HLZD, denoted as $Q(u)$ for $0 < u < 1$, is the solution to the equation $F(Q(u)) = u$.

$$u = \frac{2\left[1-\left(1+\frac{\alpha^2Q(u)^2+(\alpha+2)\alpha Q(u)}{(\alpha+2)}\right)e^{-\alpha Q(u)}\right]^{\eta}}{1+\left[1-\left(1+\frac{\alpha^2Q(u)^2+(\alpha+2)\alpha Q(u)}{(\alpha+2)}\right)e^{-\alpha Q(u)}\right]^{\eta}},$$

$$1-\left(\frac{u}{2-u}\right)^{\frac{1}{\eta}} = \left(1+\frac{\alpha^2Q(u)^2+(\alpha+2)\alpha Q(u)}{(\alpha+2)}\right)e^{-\alpha Q(u)}. \quad (8)$$

Equation (8) yields to the first, second and third QFs (Q1, Q2 and Q3) for (u=0.25, 0.5 and 0.75).

For a selection of u values and parameter values, Table 1 shows the possible quantile values, Q_1, Q_2 and Q_3, BSK, and MKUR of BSK, and MKUR of the HLZD. As can be observed from Table 1, as the value of α increases at fixed value of η, the quantiles values are increasing, while values of BSK and MKUR are decreasing.

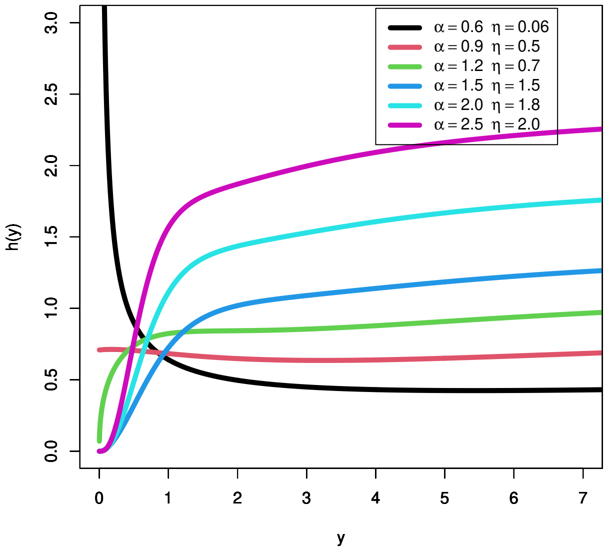

Fig. 2 Plots of HRF for the HLZD

3.2 Important Expansion

In this subsection, we derive the important expansion for the PDF of the HLZD by inserting (4) and (5) in (3), we have

$$f\left(y;\alpha,\eta\right)=2\eta\left(\frac{\alpha^{3}y(1+y)}{\alpha+2}\right)e^{-\alpha y}\sum_{i=0}^{\infty}\left(-1\right)^{i}\left(1+i\right)\left[1-\left(1+\frac{\alpha^{2}y^{2}+(\alpha+2)\alpha y}{(\alpha+2)}\right)e^{-\alpha y}\right]^{i+\eta-1}. \tag{9}$$

By applying the binomial expansion to the last term of Equation (9), then

$$\left[1-\left(1+\frac{\alpha^{2}y^{2}+(\alpha+2)\alpha y}{(\alpha+2)}\right)e^{-\alpha y}\right]^{i+\eta-1}=\sum_{j=0}^{\infty}(-1)^{j}\binom{i+\eta-1}{j}\left(1+\frac{\alpha^{2}y^{2}+(\alpha+2)\alpha y}{(\alpha+2)}\right)^{j}e^{-\alpha jy}. \tag{10}$$

By inserting (10) in (9), we have

$$f\left(y;\alpha,\eta\right)=2\eta\left(\frac{\alpha^{3}y(1+y)}{\alpha+2}\right)\sum_{i,j=0}^{\infty}\left(-1\right)^{i+j}\left(1+i\right)\binom{i+\eta-1}{j}\left(1+\alpha\left[1+\frac{\alpha}{\alpha+2}y\right]y\right)^{j}e^{-\alpha(j+1)y}. \tag{11}$$

$$\left(1+\alpha\left[1+\frac{\alpha}{\alpha+2}y\right]y\right)^{j}=\sum_{k=0}^{j}\sum_{l=0}^{k}\binom{j}{k}\binom{k}{l}\frac{\alpha^{k+l}}{(\alpha+2)^{l}}y^{k+l}. \tag{12}$$

By inserting (12) in (11), we get the PDF of the HLZD as below

$$f\left(y;\alpha,\eta\right)=\sum_{i,j=0}^{\infty}\sum_{k=0}^{j}\sum_{l=0}^{k}\Xi_{i,j,k,l}y^{k+l+1}\left(1+y\right)e^{-\alpha(j+1)y}, \tag{13}$$

where $\Xi_{i,j,k,l}=\binom{j}{k}\binom{k}{l}\binom{i+\eta-1}{j}\frac{2\eta(-1)^{i+j}(1+i)\alpha^{k+l+3}}{(\alpha+2)^{l+1}}$.

Table 1 Results of Q_1, Q_2, Q_3, BSK and MKUR associated with the HLZD distribution

η	α	Q_1	Q_2	Q_3	BSK	MKUR
0.6	1.1	0.4413	0.9619	1.8433	0.2573	1.3246
	1.3	0.3597	0.7916	1.5296	0.2618	1.3277
	1.5	0.3020	0.6695	1.3027	0.2656	1.3308
	1.7	0.2593	0.5782	1.1315	0.2688	1.3338
	1.9	0.2267	0.5076	0.9981	0.2717	1.3367
	2.1	0.2009	0.4516	0.8916	0.2741	1.3393
	2.3	0.1801	0.4061	0.8045	0.2762	1.3417
	2.5	0.1630	0.3685	0.7323	0.2780	1.3440
0.8	1.1	0.6826	1.2761	2.2006	0.2181	1.3085
	1.3	0.5592	1.0539	1.8300	0.2214	1.3100
	1.5	0.4714	0.8941	1.5614	0.2243	1.3117
	1.7	0.4060	0.7741	1.3583	0.2269	1.3134
	1.9	0.3556	0.6809	1.1998	0.2292	1.3151
	2.1	0.3158	0.6068	1.0729	0.2312	1.3167
	2.3	0.2835	0.5465	0.9691	0.2329	1.3183
	2.5	0.2570	0.4965	0.8829	0.2346	1.3197
1.2	1.1	1.0913	1.7546	2.7136	0.1823	1.2993
	1.3	0.8995	1.4551	2.2619	0.1844	1.3000
	1.5	0.7618	1.2386	1.9337	0.1863	1.3007
	1.7	0.6587	1.0754	1.6852	0.1881	1.3015
	1.9	0.5787	0.9482	1.4908	0.1898	1.3023
	2.1	0.5152	0.8467	1.3349	0.1913	1.3032
	2.3	0.4636	0.7638	1.2073	0.1926	1.3039
	2.5	0.4209	0.6951	1.1009	0.1938	1.3048
1.4	1.1	1.2624	1.9437	2.9098	0.1728	1.2978
	1.3	1.0425	1.6140	2.4272	0.1746	1.2983
	1.5	0.8843	1.3753	2.0764	0.1762	1.2988
	1.7	0.7655	1.1951	1.8106	0.1778	1.2995
	1.9	0.6734	1.0547	1.6025	0.1792	1.3002
	2.1	0.6000	0.9424	1.4356	0.1805	1.3008
	2.3	0.5403	0.8507	1.2988	0.1816	1.3014
	2.5	0.4909	0.7744	1.1848	0.1829	1.3021

3.3 Moments

Moments are measurable values that define the characteristics of a probability distribution. They illustrate a dataset's skewness, kurtosis, variability, and central tendency. The rth moments of the HLZD is derived via PDF (13) as follows:

$$\begin{aligned}
\mu'_r &= \int_0^\infty y^r f(y;\alpha,\eta)dy = \sum_{i,j=0}^{\infty}\sum_{k=0}^{j}\sum_{l=0}^{k} \Xi_{i,j,k,l} \int_0^\infty y^{r+k+l+1}(1+y)e^{-\alpha(j+1)y}dy \\
&= \sum_{i,j=0}^{\infty}\sum_{k=0}^{j}\sum_{l=0}^{k} \Xi_{i,j,k,l} \left[\frac{\Gamma(r+k+l+2)}{[\alpha(j+1)]^{r+k+l+2}} + \frac{\Gamma(r+k+l+3)}{[\alpha(j+1)]^{r+k+l+3}}\right].
\end{aligned} \tag{14}$$

The moment generating function is derived from Equation (14) of the HLZD and the rth moment as below

$$M(t) = E(e^{tY}) = \sum_{r=0}^{\infty} \frac{t^r \mu'_r}{r!} = \sum_{r=0}^{\infty}\sum_{i,j=0}^{\infty}\sum_{k=0}^{j}\sum_{l=0}^{k} \frac{t^r \Xi_{i,j,k,l_r}}{r!} \left[\frac{\Gamma(r+k+l+2)}{[\alpha(j+1)]^{r+k+l+2}} + \frac{\Gamma(r+k+l+3)}{[\alpha(j+1)]^{r+k+l+3}}\right]. \tag{15}$$

The lower sth incomplete moments for the HLZD is defined as below

$$\mu'_s(t) = \int_0^t y^s f(y;\alpha,\eta)dy = \sum_{i,j=0}^{\infty}\sum_{k=0}^{j}\sum_{l=0}^{k}\Xi_{i,j,k,l}\int_0^t y^{s+k+l+1}(1+y)e^{-\alpha(j+1)y}dy$$
$$= \sum_{i,j=0}^{\infty}\sum_{k=0}^{j}\sum_{l=0}^{k}\Xi_{i,j,k,l}\left[\frac{\gamma(s+k+l+2,\alpha(j+1)t)}{[\alpha(j+1)]^{s+k+l+2}} + \frac{\gamma(s+k+l+3,\alpha(j+1))}{[\alpha(j+1)]^{s+k+l+3}}\right]. \quad (16)$$

Table 2 displays the numerical values of μ_1', μ_2', μ_3', μ_4', variance (σ^2), standard deviation (σ), CS, CK, and coefficient of variation (CV). According to numerical results, we conclude that values of μ_1', μ_2', μ_3', μ_4', σ^2, and σ, are decreasing as the value of α increases and for fixed value of η, while values of CS, CK, and CV are increasing. These conclusions are also confirmed from Table 2.

Table 2 Results of μ_1', μ_2', μ_3', μ_4', σ^2, σ, CS, CK, and CV associated with the HLZD distribution

η	α	μ_1'	μ_2'	μ_3'	μ_4'	σ^2	σ	CS	CK	CV
0.6	1.1	1.3443	3.4172	12.7694	62.2585	1.6101	1.2689	1.8830	8.0428	0.9439
	1.3	1.1157	2.3784	7.4824	30.7632	1.1335	1.0647	1.9053	8.1606	0.9542
	1.5	0.9506	1.7416	4.7258	16.7830	0.8380	0.9154	1.9255	8.2697	0.9629
	1.7	0.8262	1.3252	3.1582	9.8646	0.6427	0.8017	1.9437	8.3704	0.9703
	1.9	0.7292	1.0391	2.2059	6.1456	0.5073	0.7123	1.9601	8.4632	0.9767
	2.1	0.6518	0.8346	1.5964	4.0117	0.4098	0.6402	1.9749	8.5486	0.9822
	2.3	0.5885	0.6838	1.1894	2.7212	0.3374	0.5809	1.9883	8.6274	0.9870
	2.5	0.5360	0.5696	0.9080	1.9059	0.2823	0.5313	2.0005	8.6999	0.9913
0.8	1.1	1.6312	4.4129	16.8395	82.6910	1.7520	1.3236	1.6926	7.1761	0.8114
	1.3	1.3554	3.0737	9.8708	40.8655	1.2365	1.1120	1.7111	7.2632	0.8204
	1.5	1.1560	2.2522	6.2361	22.2972	0.9160	0.9571	1.7281	7.3452	0.8279
	1.7	1.0054	1.7146	4.1686	13.1071	0.7038	0.8389	1.7436	7.4218	0.8344
	1.9	0.8880	1.3450	2.9123	8.1663	0.5564	0.7460	1.7577	7.4932	0.8400
	2.1	0.7941	1.0808	2.1079	5.3313	0.4502	0.6709	1.7706	7.5596	0.8449
	2.3	0.7174	0.8857	1.5708	3.6165	0.3711	0.6092	1.7823	7.6212	0.8492
	2.5	0.6536	0.7380	1.1994	2.5332	0.3108	0.5575	1.7930	7.6784	0.8529
1.2	1.1	2.0754	6.2007	24.6301	122.8535	1.8935	1.3760	1.4976	6.4083	0.6630
	1.3	1.7275	4.3247	14.4481	60.7347	1.3405	1.1578	1.5114	6.4652	0.6702
	1.5	1.4753	3.1724	9.1335	33.1479	0.9958	0.9979	1.5243	6.5200	0.6764
	1.7	1.2847	2.4175	6.1086	19.4903	0.7671	0.8758	1.5364	6.5723	0.6817
	1.9	1.1358	1.8978	4.2694	12.1459	0.6078	0.7796	1.5475	6.6218	0.6864
	2.1	1.0165	1.5260	3.0914	7.9308	0.4926	0.7019	1.5578	6.6684	0.6905
	2.3	0.9190	1.2514	2.3044	5.3807	0.4068	0.6378	1.5674	6.7123	0.6940
	2.5	0.8378	1.0432	1.7601	3.7694	0.3412	0.5842	1.5762	6.7534	0.6972
1.4	1.1	2.2535	7.0076	28.3527	142.5523	1.9295	1.3891	1.4422	6.2143	0.6164
	1.3	1.8769	4.8903	16.6376	70.4860	1.3674	1.1694	1.4544	6.2629	0.6230
	1.5	1.6038	3.5890	10.5208	38.4759	1.0168	1.0084	1.4660	6.3101	0.6287
	1.7	1.3972	2.7360	7.0382	22.6261	0.7839	0.8854	1.4769	6.3554	0.6337
	1.9	1.2358	2.1487	4.9202	14.1017	0.6216	0.7884	1.4870	6.3986	0.6380
	2.1	1.1064	1.7282	3.5633	9.2087	0.5042	0.7101	1.4965	6.4395	0.6418
	2.3	1.0005	1.4176	2.6567	6.2483	0.4166	0.6455	1.5052	6.4782	0.6452
	2.5	0.9123	1.1820	2.0293	4.3775	0.3497	0.5913	1.5133	6.5146	0.6481

3.4 Order Statistics

Assume that $Y_1, Y_2, \ldots, Y_n$ be a random sample from the HLZD with order statistics $Y_{(1)}, Y_{(2)}, \ldots, Y_{(n)}$. The PDF of $Y_{(s)}$ of order statistics is given by

$$f_{Y_{(s)}}(y) = \frac{n!}{(s-1)!(n-s)!}f(y;\alpha,\eta)[F(y;\alpha,\eta)]^{s-1}[1-F(y;\alpha,\eta)]^{n-s}, \quad (17)$$

The PDF of $y_{(s)}$ can be computed as below

$$f_{Y_{(s)}}(y)=\frac{n!2^s\eta\alpha^3 y(1+y)e^{-\alpha y}\Pi(y,\alpha)^{\eta(s+1)-s}}{(s-1)!(n-s)!\,(\alpha+2)\,[1+\Pi(y,\alpha)^{\eta}]^{n+1}}[1-\Pi(y,\alpha)^{\eta}]^{n-s}, \tag{18}$$

where $\Pi(y,\alpha)=1-\left(1+\frac{\alpha^2y^2+(\alpha+2)\alpha y}{(\alpha+2)}\right)e^{-\alpha y}$.

4 Maximum Likelihood Estimation

Assume that $Y_1, Y_2, ..., Y_n$ be n independent and identical random variables returning to the HLZD having parameters (α,η). The log-likelihood function for observed samples, $\underline{\mathrm{Y}}=(y_1, y_2, ...y_n)$, is

$$\log L(\alpha,\eta)=n\log(2\eta)+3n\log(\alpha)-n\log(\alpha+2)+\sum_{i=1}^{n}\log(y_i)+\sum_{i=1}^{n}\log(1+y_i)$$
$$-\alpha\sum_{i=1}^{n}y_i+(\eta-1)\sum_{i=1}^{n}\log\left(\Pi(y_i,\alpha)^{\eta-1}\right)-2\sum_{i=1}^{n}\log\left(1+\Pi(y_i,\alpha)^{\eta}\right).$$

Non-linear optimization techniques, such as the quasi-Newton technique, are frequently more effective for deriving HLZD maximum likelihood estimators (MLE). The explicit formula for the MLEs of the parameters is very complex to utilize.

5 Simulation

In this section, we conduct simulations to explore the behaviour of estimates for the parameters α and η of the $HLZD$ distribution. This involves generating random samples of varying sizes ($n=50,70,100,150,...,500$) from the $HLZD$ distribution and repeating the process 1000 times. Additionally, different sets of initial parameter values for α and η are selected. which are summarized in Table 3

Table 3 Initial parameter values for α and η

Table	α	η	Table	α	η	Table	α	η
4	0.1	0.5	5	1.2	0.8	6	0.5	0.5
7	0.7	1.5	8	0.7	0.2	9	1.7	1.2

The simulation outcomes provide several metrics, including the mean, Bias, mean square error (MSE), relative MSE (RMSE) average interval length (AIL), and coverage probability (CP). These results are presented in Tables 4-9.

– As the sample size increases, the estimates for α and η tend to approach their true values, evidenced by a decrease in Bias metric.
– With larger sample sizes, the MSE diminishes, indicating enhanced precision in parameter estimation.
– Similarly, the AIL decreases with increasing sample size, indicating tighter confidence intervals around the estimated parameters.
– Throughout various sample sizes, the CP remains consistently high, implying robust confidence intervals that encompass the true parameter values with high probability.
– In summary, augmenting sample sizes leads to more accurate parameter estimates and narrower confidence intervals, thereby bolstering the reliability of the estimation procedure.

6 Data analysis

This section uses two real datasets to demonstrate the efficacy of the HLZD in a data-fitting situation. The HLZD is contrasted with fourteen rival models, including generalized exponential (GED) [53], exponentiated generalized XLindley (EGXLD) [54], power exponentiated Lindley (PELD) [55], exponential (ED), exponentiated generalized

Table 4 simulation data with initial values $\alpha = 0.1$ and $\eta = 0.5$

n	Estimate	Mean	Bias	MSE	RMSE	AIL	CP
50	α	0.1306	0.0306	0.0020	0.0452	0.1304	96.00%
	η	0.6665	0.1665	0.0681	0.2610	0.7885	95.30%
70	α	0.1269	0.0269	0.0013	0.0366	0.0974	95.80%
	η	0.6464	0.1464	0.0410	0.2025	0.5486	96.30%
100	α	0.1229	0.0229	0.0009	0.0296	0.0736	96.40%
	η	0.6214	0.1214	0.0237	0.1541	0.3721	95.60%
150	α	0.1215	0.0215	0.0007	0.0262	0.0586	95.90%
	η	0.6122	0.1122	0.0181	0.1345	0.2907	95.80%
200	α	0.1208	0.0208	0.0005	0.0232	0.0404	96.60%
	η	0.6063	0.1063	0.0140	0.1182	0.2026	96.00%
250	α	0.1205	0.0205	0.0005	0.0222	0.0337	96.80%
	η	0.6040	0.1040	0.0128	0.1132	0.1757	96.40%
300	α	0.1204	0.0204	0.0005	0.0218	0.0306	96.90%
	η	0.6037	0.1037	0.0123	0.1108	0.1531	96.60%
350	α	0.1200	0.0200	0.0004	0.0209	0.0237	97.40%
	η	0.6030	0.1030	0.0120	0.1095	0.1461	96.90%
400	α	0.1200	0.0200	0.0004	0.0207	0.0215	96.80%
	η	0.6013	0.1013	0.0112	0.1058	0.1195	97.00%
450	α	0.1198	0.0198	0.0004	0.0205	0.0200	96.70%
	η	0.6000	0.1000	0.0106	0.1031	0.0986	97.30%
500	α	0.1197	0.0197	0.0004	0.0202	0.0171	96.40%
	η	0.5991	0.0991	0.0103	0.1015	0.0851	97.40%

Table 5 simulation data with initial values $\alpha = 1.2$ and $\eta = 0.8$

n	Estimate	Mean	Bias	MSE	RMSE	AIL	CP
50	α	1.5196	0.3196	0.2068	0.4547	1.2686	96.10%
	η	1.1246	0.3246	0.2295	0.4791	1.3819	95.70%
70	α	1.4660	0.2660	0.1320	0.3633	0.9705	95.80%
	η	1.0626	0.2626	0.1282	0.3580	0.9544	96.00%
100	α	1.4302	0.2302	0.0885	0.2974	0.7387	95.80%
	η	1.0128	0.2128	0.0720	0.2684	0.6415	95.00%
150	α	1.4180	0.2180	0.0696	0.2639	0.5835	95.90%
	η	1.0031	0.2031	0.0606	0.2463	0.5462	95.50%
200	α	1.4124	0.2124	0.0618	0.2485	0.5060	97.10%
	η	0.9916	0.1916	0.0495	0.2224	0.4431	95.90%
250	α	1.4121	0.2121	0.0528	0.2298	0.3476	96.50%
	η	0.9864	0.1864	0.0423	0.2056	0.3402	96.60%
300	α	1.4071	0.2071	0.0469	0.2166	0.2494	97.30%
	η	0.9851	0.1851	0.0393	0.1983	0.2794	96.80%
350	α	1.4062	0.2062	0.0461	0.2146	0.2342	97.40%
	η	0.9833	0.1833	0.0376	0.1940	0.2484	96.40%
400	α	1.4052	0.2052	0.0445	0.2109	0.1920	97.00%
	η	0.9806	0.1806	0.0355	0.1885	0.2113	97.30%
450	α	1.4046	0.2046	0.0437	0.2091	0.1689	97.10%
	η	0.9784	0.1784	0.0344	0.1854	0.1974	97.40%
500	α	1.4045	0.2045	0.0434	0.2084	0.1559	96.80%
	η	0.9756	0.1756	0.0328	0.1810	0.1728	97.20%

Table 6 simulation data with initial values $\alpha = 0.5$ and $\eta = 0.5$

n	Estimate	Mean	Bias	MSE	RMSE	AIL	CP
50	α	0.6609	0.1609	0.0560	0.2367	0.6806	95.50%
	η	0.6704	0.1704	0.0664	0.2576	0.7575	96.20%
70	α	0.6310	0.1310	0.0328	0.1812	0.4911	95.50%
	η	0.6378	0.1378	0.0388	0.1970	0.5524	96.50%
100	α	0.6196	0.1196	0.0226	0.1504	0.3577	96.60%
	η	0.6162	0.1162	0.0221	0.1488	0.3647	95.80%
150	α	0.6129	0.1129	0.0189	0.1373	0.3070	95.40%
	η	0.6142	0.1142	0.0195	0.1396	0.3146	96.00%
200	α	0.6078	0.1078	0.0157	0.1252	0.2498	96.30%
	η	0.6108	0.1108	0.0166	0.1287	0.2571	95.60%
250	α	0.6077	0.1077	0.0141	0.1188	0.1965	96.40%
	η	0.6087	0.1087	0.0145	0.1205	0.2044	96.20%
300	α	0.6021	0.1021	0.0123	0.1111	0.1711	97.40%
	η	0.6018	0.1018	0.0122	0.1106	0.1693	97.10%
350	α	0.6005	0.1005	0.0116	0.1078	0.1532	96.60%
	η	0.6010	0.1010	0.0118	0.1085	0.1548	96.40%
400	α	0.5998	0.0998	0.0112	0.1057	0.1363	96.90%
	η	0.6005	0.1005	0.0114	0.1067	0.1401	97.00%
450	α	0.5984	0.0984	0.0102	0.1008	0.0853	96.90%
	η	0.5983	0.0983	0.0106	0.1028	0.1179	96.60%
500	α	0.5969	0.0969	0.0098	0.0988	0.0764	97.10%
	η	0.5967	0.0967	0.0098	0.0988	0.0788	96.70%

Table 7 simulation data with initial values $\alpha = 0.7$ and $\eta = 1.5$

n	Estimate	Mean	Bias	MSE	RMSE	AIL	CP
50	α	0.8398	0.1398	0.0444	0.2107	0.6180	95.60%
	η	1.9975	0.4975	0.3736	0.6112	1.3926	95.80%
70	α	0.8352	0.1352	0.0336	0.1834	0.4858	95.90%
	η	1.9760	0.4760	0.3003	0.5480	1.0646	96.60%
100	α	0.8135	0.1135	0.0216	0.1469	0.3660	95.70%
	η	1.9456	0.4456	0.2423	0.4922	0.8204	96.50%
150	α	0.8119	0.1119	0.0185	0.1360	0.3034	97.10%
	η	1.9388	0.4388	0.2279	0.4774	0.7379	96.20%
200	α	0.8076	0.1076	0.0156	0.1251	0.2496	96.60%
	η	1.9212	0.4212	0.1947	0.4412	0.5155	96.10%
250	α	0.8052	0.1052	0.0136	0.1167	0.1985	96.60%
	η	1.9205	0.4205	0.2031	0.4507	0.6361	96.20%
300	α	0.8016	0.1016	0.0118	0.1088	0.1532	96.30%
	η	1.9198	0.4198	0.1892	0.4349	0.4466	96.70%
350	α	0.8015	0.1015	0.0113	0.1064	0.1255	96.60%
	η	1.9135	0.4135	0.1819	0.4265	0.4089	97.30%
400	α	0.8009	0.1009	0.0109	0.1045	0.1068	97.00%
	η	1.9127	0.4127	0.1780	0.4219	0.3519	96.50%
450	α	0.8005	0.1005	0.0108	0.1037	0.1012	96.90%
	η	1.9126	0.4126	0.1774	0.4212	0.3332	97.30%
500	α	0.7997	0.0997	0.0103	0.1017	0.0788	97.90%
	η	1.9122	0.4122	0.1770	0.4207	0.3206	96.80%

Table 8 simulation data with initial values $\alpha = 0.7$ and $\eta = 0.2$

n	Estimate	Mean	Bias	MSE	RMSE	AIL	CP
50	α	1.0772	0.3772	0.3525	0.5938	1.7983	95.69%
	η	0.2474	0.0474	0.0050	0.0707	0.2056	95.99%
70	α	0.9882	0.2882	0.1615	0.4019	1.0986	95.30%
	η	0.2386	0.0386	0.0032	0.0562	0.1601	95.50%
100	α	0.9613	0.2613	0.1114	0.3338	0.8150	95.90%
	η	0.2351	0.0351	0.0021	0.0463	0.1181	96.50%
150	α	0.9522	0.2522	0.0940	0.3066	0.6833	95.49%
	η	0.2338	0.0338	0.0018	0.0426	0.1015	95.89%
200	α	0.9294	0.2294	0.0693	0.2633	0.5067	96.00%
	η	0.2319	0.0319	0.0014	0.0377	0.0790	96.90%
250	α	0.9232	0.2232	0.0610	0.2470	0.4141	95.50%
	η	0.2310	0.0310	0.0013	0.0354	0.0664	95.80%
300	α	0.9215	0.2215	0.0576	0.2399	0.3622	96.40%
	η	0.2301	0.0301	0.0011	0.0333	0.0559	96.90%
350	α	0.9170	0.2170	0.0531	0.2304	0.3031	96.80%
	η	0.2296	0.0296	0.0010	0.0320	0.0476	96.10%
400	α	0.9124	0.2124	0.0495	0.2226	0.2603	96.00%
	η	0.2292	0.0292	0.0010	0.0309	0.0396	96.70%
450	α	0.9123	0.2123	0.0471	0.2170	0.1760	96.60%
	η	0.2292	0.0292	0.0009	0.0302	0.0303	96.90%
500	α	0.9113	0.2113	0.0465	0.2157	0.1691	97.00%
	η	0.2292	0.0292	0.0009	0.0299	0.0261	96.90%

Table 9 simulation data with initial values $\alpha = 1.7$ and $\eta = 1.2$

n	Estimate	Mean	Bias	MSE	RMSE	AIL	CP
50	α	2.1279	0.4279	0.3691	0.6076	1.6916	95.70%
	η	1.7620	0.5620	0.7885	0.8880	2.6961	96.20%
70	α	2.0437	0.3437	0.2244	0.4737	1.2787	96.50%
	η	1.6509	0.4509	0.3811	0.6173	1.6537	95.20%
100	α	2.0163	0.3163	0.1608	0.4010	0.9664	96.70%
	η	1.5844	0.3844	0.2431	0.4930	1.2109	96.40%
150	α	2.0009	0.3009	0.1332	0.3650	0.8097	96.70%
	η	1.5566	0.3566	0.1875	0.4330	0.9637	95.20%
200	α	1.9911	0.2911	0.1131	0.3363	0.6603	96.10%
	η	1.5387	0.3387	0.1504	0.3878	0.7407	96.30%
250	α	1.9793	0.2793	0.0918	0.3030	0.5266	96.80%
	η	1.5185	0.3185	0.1267	0.3560	0.6238	95.20%
300	α	1.9681	0.2681	0.0899	0.2999	0.4608	97.60%
	η	1.5062	0.3062	0.1081	0.3287	0.4691	96.50%
350	α	1.9636	0.2636	0.0806	0.2839	0.4139	96.80%
	η	1.5060	0.3060	0.0995	0.3155	0.3001	96.80%
400	α	1.9627	0.2627	0.0783	0.2798	0.3777	97.00%
	η	1.5050	0.3050	0.0977	0.3125	0.2670	96.70%
450	α	1.9592	0.2592	0.0723	0.2689	0.2811	97.70%
	η	1.5028	0.3028	0.0959	0.3096	0.2534	96.60%
500	α	1.9586	0.2586	0.0701	0.2647	0.2223	96.70%
	η	1.5000	0.3000	0.0941	0.3068	0.2527	96.80%

Lindley (EGLD) [56], XLindley (XLD) [57], Weibull power Lindley (WPLD) [58], Weibull Lindley (WLD) [59], half logistic new-Weibull Pareto (HLNWPD) [60], half logistic Weibull (HLWD) [60], half logistic exponential (HLED) [60], half logistic Rayleigh (HLRD) [61], half logistic Pareto (HLP) [60], and power Lindley (PLD) [62].

We compare the related models using eight well-referenced measures of goodness of fit, including the Kolmogorov-Smirnov (KS) statistic, p-value of KS (P_value), Anderson-Darling statistics (A), Hannan-Quinn information criterion (HQIC), Akaike IC (AIC), Bayesian IC (BIC), consistent AIC (CAIC), and Cramér von Mises statistic (W). The model with the lowest conceivable manner of KS, A, W, HQIC, AIC, CAIC, and BIC values and the greatest P_value value.

First Dataset The dataset shows 100 bank customers' wait times (in minutes) for service; it was examined in earlier research to assess the Lindley [63] distribution. These kinds of datasets are frequently used to comprehend patterns of consumer behaviour and the effectiveness of financial operations.The first publication of the data set was in [64]. 0.8, 0.8, 1.3, 1.5, 1.8, 1.9, 1.9, 2.1, 2.6, 2.7, 2.9, 3.1, 3.2, 3.3, 3.5, 3.6, 4, 4.1, 4.2, 4.2, 4.3, 4.3, 4.4, 4.4, 4.6, 4.7, 4.7, 4.8, 4.9, 4.9, 5, 5.3, 5.5, 5.7, 5.7, 6.1, 6.2, 6.2, 6.2, 6.3, 6.7, 6.9, 7.1, 7.1, 7.1, 7.1, 7.4, 7.6, 7.7, 8, 8.2, 8.6, 8.6, 8.6, 8.8, 8.8, 8.9, 8.9, 9.5, 9.6, 9.7, 9.8, 10.7, 10.9, 11, 11, 11.1, 11.2, 11.2, 11.5, 11.9, 12.4, 12.5, 12.9, 13, 13.1, 13.3, 13.6, 13.7, 13.9, 14.1, 15.4, 15.4, 17.3, 17.3, 18.1, 18.2, 18.4, 18.9, 19, 19.9, 20.6, 21.3, 21.4, 21.9, 23, 27, 31.6, 33.1, 38.5.

Second Dataset: This study uses a dataset that reflects the survival times (in years) of 46 individuals who received just chemotherapy. This dataset's previous reports were published by [65] and [66]. 0.047, 0.115, 0.121, 0.132, 0.164, 0.197, 0.203, 0.260, 0.282, 0.296, 0.334, 0.395, 0.458, 0.466, 0.501, 0.507, 0.529, 0.534, 0.540, 0.641, 0.644, 0.696, 0.841, 0.863, 1.099, 1.219, 1.271, 1.326, 1.447, 1.485, 1.553, 1.581, 1.589, 2.178, 2.343, 2.416, 2.444, 2.825, 2.830, 3.578, 3.658, 3.743, 3.978, 4.003, 4.033.

For the model parameters, the maximum likelihood estimators (MLEs) and associated standard errors (SEs) have been calculated. For the competing models under examination, the MLEs and SEs are given in Tables 10 and 12. Additionally, Tables 11-13 provide a summary of the goodness-of-fit metrics for the examined datasets, including KS, A, W, HQIC, AIC, CAIC, BIC, and P_value.

The findings show that the two-parameter HLZD distribution performs better than the rival models. The best goodness-of-fit among the models is demonstrated by the HLZD distribution, which obtained the highest P_value and the lowest values for KS, A, W, HQIC, AIC, CAIC, and BIC.

Additionally, the estimated pdf, cdf, and probability plots (PP) for the models applied to the datasets are visually compared in Figures 3-8. The robustness and efficacy of the HLZD distribution in fitting the data are further supported by these graphical representations.

The usefulness of the HLZD distribution in simulating datasets with comparable properties is demonstrated by this investigation, which provides statistical and visual evidence of its superiority over other distributions.

Table 10 MLEs and SEs for the first dataset.

Name	$\hat{\alpha}$	$\hat{\eta}$	$\hat{\delta}$	$\hat{\beta}$	$SE(\hat{\alpha})$	$SE(\hat{\eta})$	$SE(\hat{\delta})$	$SE(\hat{\beta})$
HLZD	0.2108	0.8371			0.0227	0.1092		
GED	2.1829	0.1591			0.3342	0.0175		
EGXLD	0.1696	0.9918	1.9278		0.0191	0.0034	0.2881	
PELD	1.3846	0.2674	0.1745		0.1937	0.3862	0.0473	
ED	0.1013				0.0101			
EGLD	0.9835	0.1718	1.7370		0.0056	0.0198	0.2538	
XLD	0.1744				0.0125			
WPLD	0.0146	1.0469	93.2170	65.4072	0.0113	0.2937	2703.5400	41.4436
WLD	1.5145	0.0465	0.1366		0.3550	0.0513	0.0941	
HLNWPD	0.8106	1.2339	0.0596		2.4502	0.0980	0.2215	
HLWD	1.2339	0.0772			0.0978	0.0216		
HLED	0.1445				0.0119			
HLRD	0.0086				0.0008			
HLPD	6.9188				0.5716			
PLD	1.0832	0.1529			0.0704	0.0282		

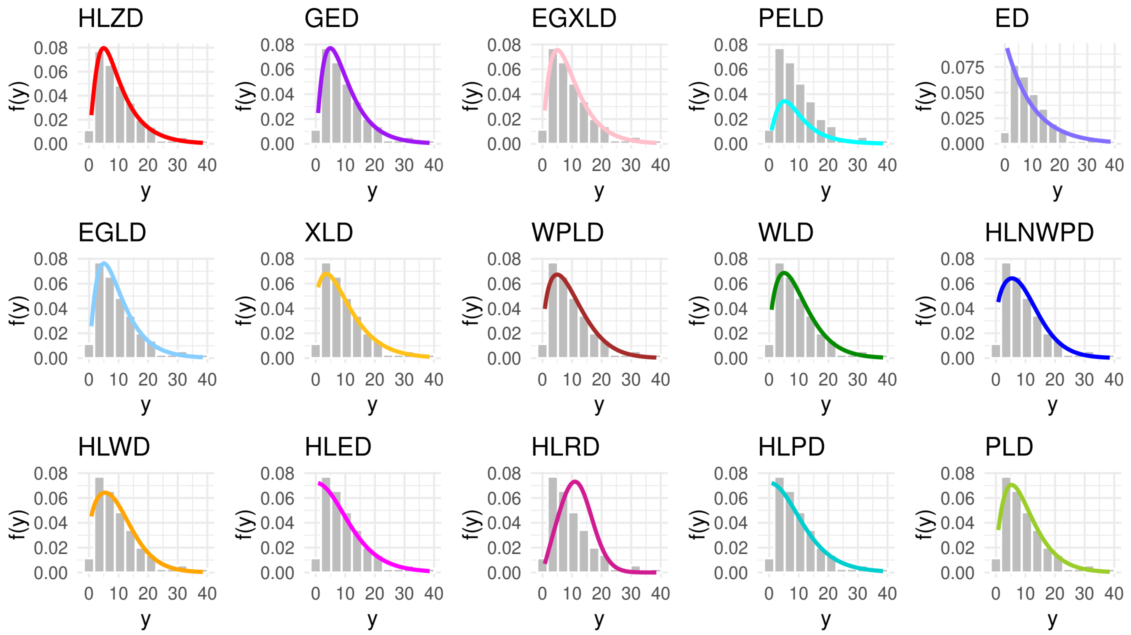

Fig. 3 Estimated pdf plots for the first dataset

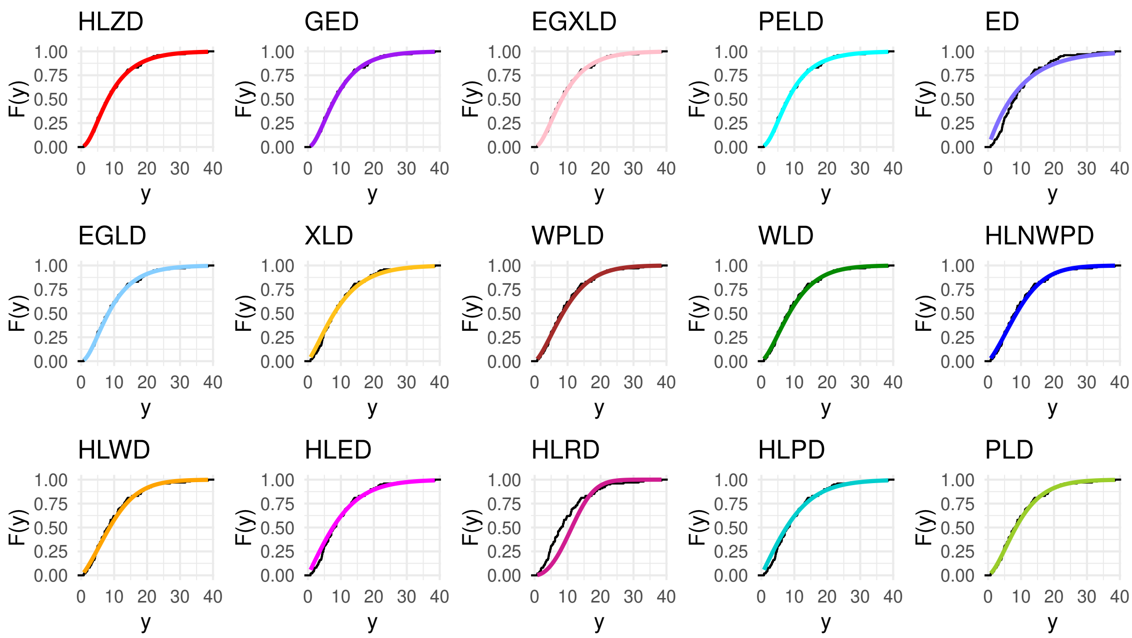

Fig. 4 Estimated cdf plots for the first dataset

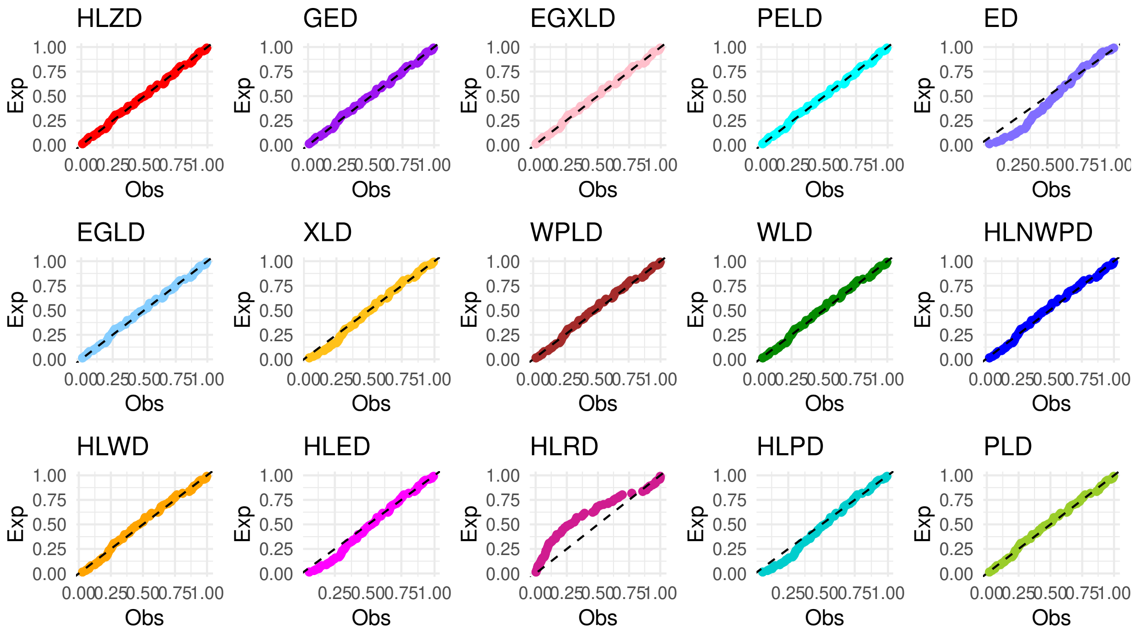

Fig. 5 Estimated PP plots for the first dataset

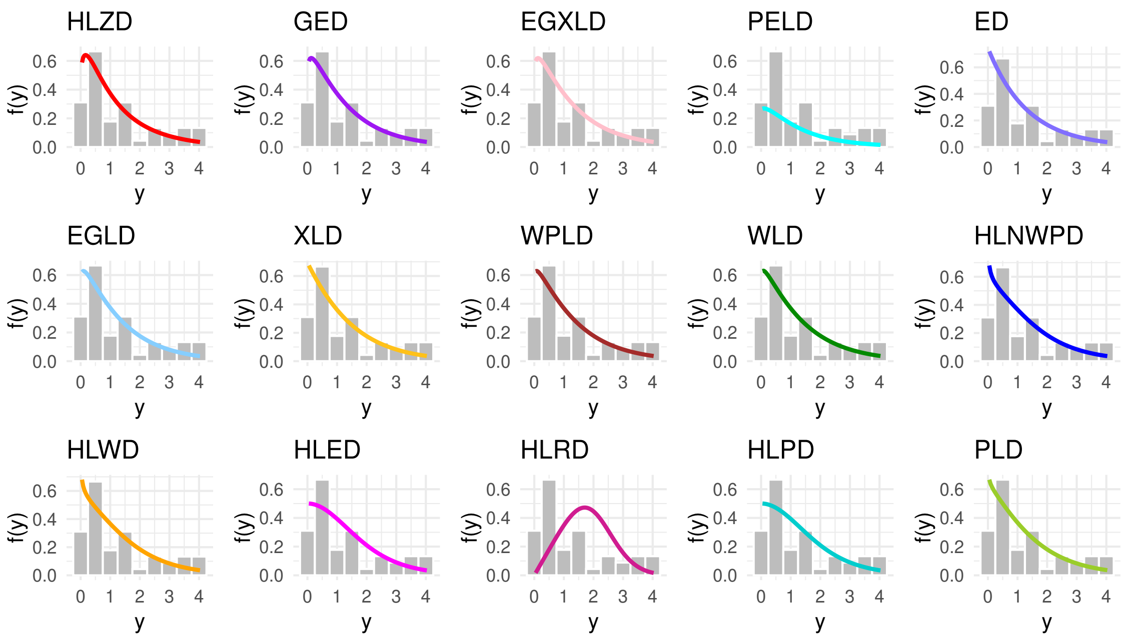

Fig. 6 Estimated pdf plots for the second dataset

Table 11 Measures of fitting for the first dataset.

Name	lnL	AIC	BIC	CAIC	HQIC	KS	P_value	W	A
HLZD	634.1439	638.1439	643.3542	638.2676	640.2526	0.0395	0.9977	0.0178	0.1268
GED	634.1906	638.1906	643.4009	638.3143	640.2993	0.0402	0.9970	0.0207	0.1428
EGXLD	634.0699	640.0699	647.8854	640.3199	643.2329	0.0420	0.9945	0.0242	0.1616
PELD	801.3233	807.3233	815.1388	807.5733	810.4864	0.0437	0.9910	0.0209	0.1413
ED	658.0418	660.0418	662.6469	660.0826	661.0961	0.1730	0.0050	0.0270	0.1790
EGLD	634.3046	640.3046	648.1201	640.5546	643.4676	0.0428	0.9931	0.0232	0.1560
XLD	641.5234	643.5234	646.1286	643.5642	644.5778	0.0905	0.3856	0.0468	0.2970
WPLD	637.6373	645.6373	656.0580	646.0584	649.8548	0.0585	0.8840	0.0648	0.4076
WLD	637.3022	643.3022	651.1177	643.5522	646.4653	0.0518	0.9516	0.0557	0.3527
HLNWPD	639.7300	645.7300	653.5455	645.9800	648.8930	0.0627	0.8263	0.0935	0.5742
HLWD	639.7300	643.7300	648.9403	643.8537	645.8387	0.0628	0.8254	0.0935	0.5742
HLED	646.0759	648.0759	650.6811	648.1167	649.1303	0.1213	0.1056	0.0583	0.3624
HLRD	686.6693	688.6693	691.2745	688.7102	689.7237	0.2507	0.0000	0.2387	1.4314
HLPD	646.0759	648.0759	650.6811	648.1167	649.1303	0.1213	0.1055	0.0583	0.3624
PLD	636.6372	640.6372	645.8475	640.7609	642.7459	0.0521	0.9493	0.0492	0.3130

Table 12 MLEs and SEs for the second dataset.

Name	$\hat{\alpha}$	$\hat{\eta}$	$\hat{\delta}$	$\hat{\beta}$	$SE(\hat{\alpha})$	$SE(\hat{\eta})$	$SE(\hat{\delta})$	$SE(\hat{\beta})$
HLZD	1.0881	0.5706			0.1955	0.0986		
GED	1.1050	0.7945			0.2196	0.1511		
EGXLD	0.1356	5.9568	1.1026		0.0369	1.1280	0.2191	
PELD	1.0391	0.4105	0.9483		0.2184	0.5384	0.2549	
ED	0.7458				0.1112			
EGLD	8.7303	0.0952	1.0639		0.0105	0.0182	0.2094	
XLD	0.9433				0.1116			
WPLD	0.0072	1.0872	95.5839	95.7929	0.0020	0.0039	194.0285	24.0535
WLD	1.0485	0.7187	0.0856		0.1416	0.2066	0.7238	
HLNWPD	0.4228	0.8913	0.5087		12.8567	0.1110	13.7860	
HLWD	0.8912	1.0958			0.1110	0.1688		
HLED	1.0012				0.1274			
HLRD	0.3598				0.0490			
HLPD	0.9988				0.1271			
PLD	0.9465	1.1350			0.1076	0.1465		

Table 13 Measures of fitting for the second dataset.

Name	lnL	AIC	BIC	CAIC	HQIC	KS	P_value	W	A
HLZD	115.7734	119.7734	123.3867	120.0591	121.1204	0.0994	0.7280	0.0726	0.4902
GED	116.1897	120.1897	123.8030	120.4754	121.5367	0.1099	0.6095	0.0785	0.5269
EGXLD	116.1882	122.1882	127.6082	122.7736	124.2087	0.1101	0.6069	0.0787	0.5280
PELD	191.5246	197.5246	202.9445	198.1099	199.5451	0.1079	0.6323	0.0821	0.5482
ED	119.6437	121.6437	123.4504	121.7367	122.3172	0.0907	0.8203	0.0788	0.5284
EGLD	116.2586	122.2586	127.6786	122.8440	124.2792	0.1076	0.6354	0.0812	0.5430
XLD	119.9593	121.9593	123.7660	122.0523	122.6328	0.1041	0.6750	0.0837	0.5580
WPLD	116.2718	124.2718	131.4985	125.2718	126.9658	0.1095	0.6137	0.0819	0.5473
WLD	116.2401	122.2401	127.6601	122.8255	124.2607	0.1094	0.6155	0.0816	0.5457
HLNWPD	117.5185	123.5185	128.9385	124.1039	125.5390	0.1160	0.5411	0.0976	0.6434
HLWD	117.5185	121.5185	125.1318	121.8042	122.8655	0.1160	0.5415	0.0976	0.6434
HLED	119.6103	121.6103	123.4170	121.7033	122.2838	0.1583	0.1882	0.1061	0.6958
HLRD	175.0309	177.0309	178.8375	177.1239	177.7044	0.4020	0.0000	0.1727	1.1033
HLPD	119.9610	121.9610	123.7677	122.0541	122.6345	0.1830	0.2816	0.0961	0.5804
PLD	116.8056	120.8056	124.4189	121.0913	122.1526	0.1105	0.6031	0.0900	0.5967

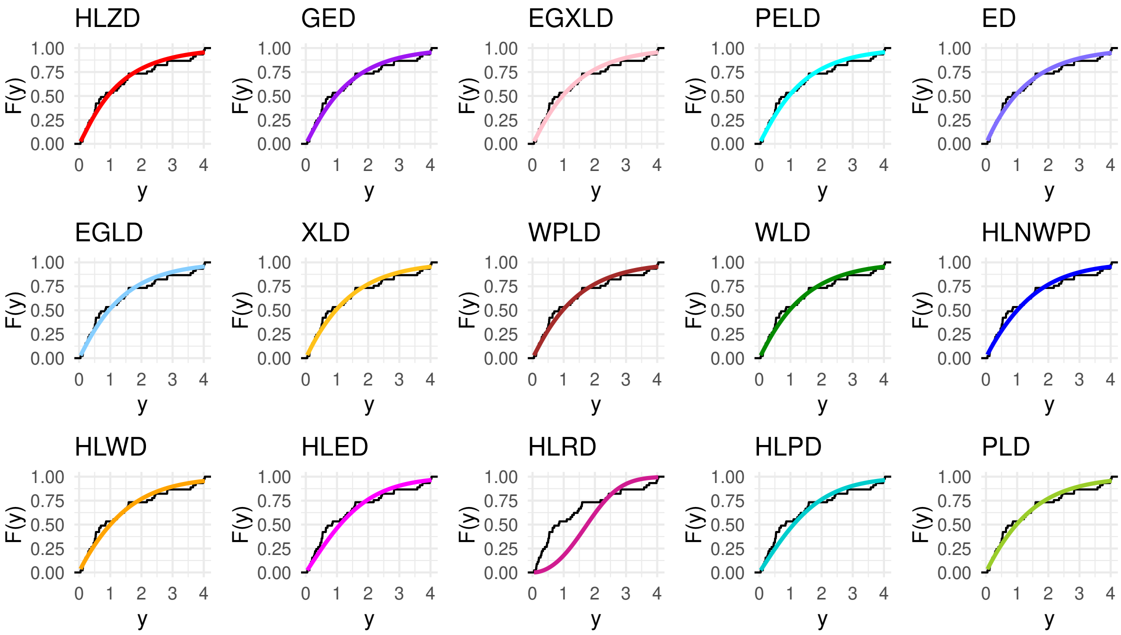

Fig. 7 Estimated cdf plots for the second dataset

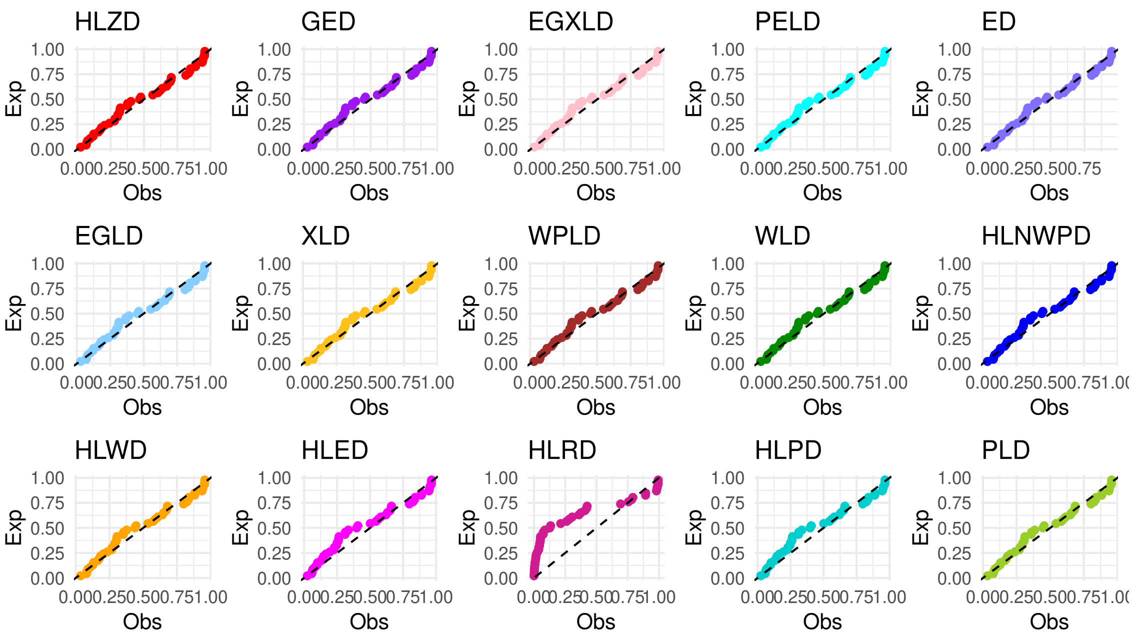

Fig. 8 Estimated PP plots for the second dataset

7 Concluding Remarks

This work introduces a novel extension of the Zeghoudi distribution (ZD), a unique variation of the ZD. The proposed model is termed the half-logistic Zeghdoudi distribution (HLZD). This enhanced distribution remains the original ZD's simplicity while enhancing flexibility and precision in data modeling. The HLZD possesses several statistical attributes, including right skewness, reduced and unimodal probability density functions, skewness, kurtosis moments, incomplete moments, and order statistics. We demonstrate the efficacy and reliability of the HLZD by the maximum likelihood standard parameter estimation method and a comprehensive simulation exercise. Moreover, applying the HLZD to two practical failure times and chemotherapy datasets demonstrates its efficacy.

References

[1] Almarashi, A. M., & Elgarhy, M. (2018). A new Muth generated family of distributions with applications. J. Nonlinear Sci. Appl, 11, 1171-1184.
[2] Jamal, F., Chesneau, C., & Elgarhy, M. (2020). Type II general inverse exponential family of distributions. Journal of Statistics and Management Systems, 23(3), 617-641.
[3] Almarashi, A. M., Jamal, F., Chesneau, C., & Elgarhy, M. (2021). A new truncated Muth generated family of distributions with applications. Complexity, 2021(1), 1211526.
[4] Hassan, A. S., Elgarhy, M. A. E., & Shakil, M. (2017). Type II Half Logistic Family of Distributions with Applications. Pakistan Journal of Statistics and Operation Research, 13(2), 245-264.
[5] Alyami, S. A., Elbatal, I., Alotaibi, N., Almetwally, E. M., & Elgarhy, M. (2022). Modeling to factor productivity of the United Kingdom food chain: Using a new lifetime-generated family of distributions. Sustainability, 14(14), 8942.
[6] Elbatal, I., Elgarhy, M., & Kibria, B. G. (2021). Alpha power transformed weibull-g family of distributions: Theory and applications. Journal of Statistical Theory and Applications, 20(2), 340-354.
[7] Benchiha, S., Sapkota, L. P., Al Mutairi, A., Kumar, V., Khashab, R. H., Gemeay, A. M., ... & Nassr, S. G. (2023). A new sine family of generalized distributions: Statistical inference with applications. Mathematical and Computational Applications, 28(4), 83.
[8] Al-Moisheer, A. S., Elbatal, I., Almutiry, W., & Elgarhy, M. (2021). Odd inverse power generalized Weibull generated family of distributions: Properties and applications. Mathematical Problems in Engineering, 2021(1), 5082192.
[9] Alyami, S. A., Babu, M. G., Elbatal, I., Alotaibi, N., & Elgarhy, M. (2022). Type II half-logistic odd Frćhet class of distributions: Statistical theory and applications. Symmetry, 14(6), 1222.
[10] Algarni, A., Almarashi, A. M., Jamal, F., Chesneau, C., & Elgarhy, M. (2021). Truncated inverse Lomax generated family of distributions with applications to biomedical data. Journal of Medical Imaging and Health Informatics, 11(9), 2425-2439.
[11] Ahmad, Z., Hamedani, G., & Elgarhy, M. (2020). The weighted exponentiated family of distributions: Properties, applications and characterizations. Journal of the Iranian Statistical Society, 19(1), 209-228.
[12] Bantan, R. A., Jamal, F., Chesneau, C., & Elgarhy, M. (2020). On a new result on the ratio exponentiated general family of distributions with applications. Mathematics, 8(4), 598.
[13] ZeinEldin, R. A., Chesneau, C., Jamal, F., Elgarhy, M., Almarashi, A. M., & Al-Marzouki, S. (2021). Generalized Truncated Fréchet Generated Family Distributions and Their Applications. Computer Modeling in Engineering & Sciences, 126(2), 791-819.
[14] Alsadat, N., Imran, M., Tahir, M. H., Jamal, F., Ahmad, H., & Elgarhy, M. (2023). Compounded Bell-G class of statistical models with applications to COVID-19 and actuarial data. Open Physics, 21(1), 20220242.
[15] Aldahlan, M., Jamal, F., Chesneau, C., Elbatal, I. & Elgarhy, M. Exponentiated power generalized Weibull power series family of distributions: Properties, estimation andapplications. PLOS ONE. 20 pp. 1-25 (2020).
[16] Al-Marzouki, S., Jamal, F., Chesneau, C. & Elgarhy, M. Topp-Leone Odd Frechet Generated Family of Distributions with Applications to COVID-19 Data Sets. Computer Modeling In Engineering & Sciences. 125, 437-458 (2020)
[17] Al-Babtain, A., Elbatal, I., Chesneau, C. & Elgarhy, M. Sine Topp-Leone-G family of models: Theory and applications. Open Physics. 18 pp. 74-593 (2020)
[18] Elbatal, I., Alotaibi, N., Almetwally, E., Alyami, S. & Elgarhy, M. On Odd Perks-G Class of Models: Properties, Regression Model, Discretization. Bayesian and Non-Bayesian Estimation, And Applications. Symmetry. 14, pp. 883 (2022)
[19] Kumar, D., Singh, U. & Singh, S. A method of proposing new model and its application to bladder cancer patients data. J. Statist. Appl. Probab. Lett. 2 pp. 235-245 (2015)
[20] Alotaibi, N., Elbatal, I., Almetwally, E., Alyami, S., Al-Moisheer, A. & Elgarhy, M. Truncated Cauchy Power Weibull-G Class of Models: Bayesian and Non-Bayesian Inference Modelling for COVID-19 and Carbon Fiber Data. Mathematics. 10 pp. 1565(2022)
[21] Haq, M., Elgarhy, M. \& Hashmi, S. The generalized odd Burr III family of distributions: properties, and applications. em Journal Of Taibah University For Science. 13, 961-971 (2019)
[22] Maurya, S., Kaushik, A., Singh, S. & Singh, U. A new class of exponential transformed Lindley model and its application to Yarn data. Int. J. Statist. Econo. 18 pp. 135-151(2017)
[23] Afify, A. & Alizadeh, M. The odd Dagum family of models: Properties and applications. J. Appl. Probab. 15 pp. 45-72(2020)
[24] Nascimento, A., Silva, K., Cordeiro, M., Alizadeh, M., Yousof, H. & Hamedani, G. The odd Nadarajah-Haghighi family of models. Prop. Appl. Stud. Sci. Math. Hung. 56 pp. 1-2(2019)

[25] Al-Marzouki, S., Jamal, F., Chesneau, C. & Elgarhy, M. Topp-Leone Odd Fréchet Generated Family of Distributions with Applications to COVID-19 Data Sets. Computer Modeling In Engineering & Sciences. 125, 437-458(2020)
[26] Bantan, R.A.R., Chesneau, C., Jamal, F., Elbatal, I. & Elgarhy, M. The Truncated Burr X-G Family of Distributions: Properties and Applications to Actuarial and Financial Data. Entropy. 23, 1088(2021)
[27] Afify, A. Z., Cordeiro, G. M., Ibrahim, N. A., Jamal, F., Elgarhy, M., & Nasir, M. A. The Marshall-Olkin Odd Burr III-G Family: Theory, Estimation, and Engineering Applications. IEEE Access. 9, 4376-4387(2021)
[28] Al-Mofleh, H., Elgarhy, M., Afify, A. & Zannon, M. Type II Exponentiated Half Logistic Generated Family of Distributions with Applications. Electronic Journal of Applied Statistical Analysis. 13, 36-561(2020)
[29] Jamal, F., Nasir, M., Ozel, G., M. & Khan, N. Generalized inverted Kumaraswamy generated family of distributions. Journal Of Applied Statistics. 46, 2927-2944(2019)
[30] Muhammad, M., Bantan, R., Liu, L., Chesneau, C., Tahir, M., Jamal, F. & Elgarhy, M. A New Extended Cosine-G Distributions for Lifetime Studies. Mathematics. 9 pp. 2758(2021)
[31] Ahmad, Z., Elgarhy, M., Hamedani, G. & Butt, N. Odd Generalized N-H Generated Family of Distributions with Application to Exponential Model. Pakistan Journal of Statistics and Operation Research. 16, 53-71 (2020)
[32] Mahdavi, A., and Kundu, D. (2017). A new method for generating distributions with an application to exponential distribution. Communications in Statistics-Theory and Methods, 46 (13), 6543-6557.
[33] Alghamdi, S.M.; Shrahili, M.; Hassan, A.S.; Gemeay, A.M.; Elbatal, I.; Elgarhy, M. (2023). Statistical Inference of the Half Logistic Modified Kies Exponential Model with Modeling to Engineering Data. Symmetry 2023, 15, 586.
[34] Alotaibi, N.; Al-Moisheer, A.S.; Elbatal, I.; Shrahili, M.; Elgarhy, M.; Almetwally, E.M. (2023). Half Logistic Inverted Nadarajah-Haghighi Distribution under Ranked Set Sampling with Applications. Mathematics, 11, 1693.
[35] Al-Marzouki, S., Jamal, F., Chesneau, Ch. and Elgarhy, M. (2021). Half Logistic Inverse Lomax Distribution with Applications, Symmetry, 13 (2), 1-26.
[36] ZeinEldin, A. R., Chesneau, Ch., Jamal, F. and Elgarhy, M. (2019). Statistical properties and different methods of estimation for Type I half logistic inverted Kumaraswamy distribution. Mathematics, 7, 1-22.
[37] Elgarhy, M., Hassan, A. S. and Fayomi, A. (2018). Maximum Likelihood and Bayesian Estimation for two-Parameter Type I Half Logistic Lindley Distribution. Journal of Computational and Theoretical Nanoscience, 15, 1-9.
[38] Elgarhy, M., Haq, M., Elbatal, I. and Hassan, A.S. (2018). Transmuted Kumaraswamy Quasi Lindley Distribution with Applications. Annals of Data Science, DOI: 10.1007/s40745-018-0153-4
[39] Elgarhy, M., Sharma, V. K. and Elbatal, I. (2018). Transmuted Kumaraswamy Lindley distribution with application. Journal of Statistics & Management Systems, 21 (6), No. 6 1083-1104.
[40] Elgarhy, M., Elbatal, I., Diab, L. S., Hwas, H. K. and Shawki, A. W. (2017).Transmuted Generalized Quasi Lindley Distribution. International Journal of Scientific Engineering and Science, 1(7), 1-8.
[41] Elbatal, I., Diab, L.S. and Elgarhy, M. (2016). Exponentiated Quasi Lindley Distribution. International Journal of Reliability and Applications, 17(1), 1-19.
[42] Elgarhy, M., Rashed, M. and Shawki, A. W. (2016). Transmuted Generalized Lindley Distribution. International Journal of Mathematics Trends and Technology, 29(2), 145-154.
[43] Elbatal, I. and Elgarhy, M. (2013a). Transmuted quasi Lindley distribution: a Generalization of the quasi Lindley distribution. International Journal of Pure and Applied Sciences and Technology, 18(2), 59-69.
[44] Elbatal, I. and Elgarhy, M. (2013b). Kumaraswamy Quasi Lindley Distribution. International Journal of Mathematics Trends and Technology, 4(10), 237-246.
[45] Elbatal, I., Merovci, F. and Elgarhy, M. (2013). A new generalized Lindley distribution. Mathematical Theory and Modeling, 3(13), 30-47.
[46] Messaadia, H., & Zeghdoudi, H. (2018). Zeghdoudi distribution and its applications. International Journal of Computing Science and Mathematics, 9(1), 58-65.
[47] Aidi, K., Al-Omari, A. I., & Alsultan, R. (2022). The Power Zeghdoudi Distribution: Properties, Estimation, and Applications to Real Right-Censored Data. Applied Sciences, 12(23), 12081.
[48] Talhi, H., & Aiachi, H. (2021). On Truncated Zeghdoudi Distribution : Posterior Analysis under Different Loss Functions for Type II Censored Data. Pakistan Journal of Statistics and Operation Research, 17(2), 497-508.
[49] AlSultan, R., & Al-Omari, A. (2023). Zeghdoudi distribution in acceptance sampling plans based on truncated life tests with real data application. Decision Making Applications in Management and Engineering, 6(1), 432-448.
[50] S. O. Bashiru, M. Kayid, R. Mahmoud, O. S. Balogun, M. M. Abd El-Raouf, A. M. Gemeay. (2025). Introducing the unit Zeghdoudi distribution as a novel statistical model for analyzing proportional data, Journal of Radiation Research and Applied Sciences, 18(1), 101204.
[51] Ruidas, M. K. (2020). Transmuted Zeghdoudi Distribution: Theory and Applications. Int. J. Sci. Res. in Mathematical and Statistical Sciences Vol, 7(2), 95-101.
[52] Elbatal, I., Hassan, A. S., Gemeay, A. M., Diab, L. S., Ghorbal, A. B., & Elgarhy, M. (2024). Statistical analysis of the inverse power Zeghdoudi model: Estimation, simulation and modeling to engineering and environmental data. Physica Scripta, 99(6), 065231.
[53] Gupta, R. D., and Kundu, D. (1999). Generalized exponential distributions. Australian and New Zealand Journal of Statistics, 41(2), 173-188.

[54] MusEGPLDa, R. R., & Makubate, B. (2024). A flexible generalized XLindley distribution with application to engineering. Scientific African, 24, e02192.
[55] Rajitha, C. S., and Akhilnath, A. (2022). Generalization of the Lindley distribution with application to COVID-19 data. International Journal of Data Science and Analytics, 1-21.
[56] MirMostafaee, S. M. T. K., Alizadeh, M., Altun, E., & Nadarajah, S. (2019). The exponentiated generalized power Lindley distribution: Properties and applications. Applied Mathematics-A Journal of Chinese Universities, 34, 127-148.
[57] Chouia, S., & Zeghdoudi, H. (2021). The xlindley distribution: Properties and application. Journal of Statistical Theory and Applications, 20(2), 318-327.
[58] M Bourguignon, R B Silva, G M Cordeiro. The Weibull-G family of probability distributions, J Data Sci, 2014, 12(1): 53-68.
[59] Asgharzadeh, A. K. B. A. R., Nadarajah, S., & Sharafi, F. (2018). Weibull-Linley distribution. Revstat-Statistical Journal, 16(1), 87-113.
[60] Sasai, S. (2023). The Half Logistic New Weibull Pareto Distribution: Model, Properties and Applications (Master's thesis, Indiana University of Pennsylvania).
[61] Al-Babtain, A. A. (2020). A new extended Rayleigh distribution. Journal of King Saud University-Science, 32(5), 2576-2581.
[62] Ghitany, M. E., Al-Mutairi, D. K., Balakrishnan, N., & Al-Enezi, L. J. (2013). Power Lindley distribution and associated inference. Computational Statistics and Data Analysis, 64, 20-33.
[63] Lindley, D. V. (1958). Fiducial distributions and Bayes' theorem. Journal of the Royal Statistical Society. Series B (Methodological), 102-107.
[64] Ghitany, M. E., Atieh, B., & Nadarajah, S. (2008). Lindley distribution and its application. Mathematics and computers in simulation, 78(4), 493-506.
[65] Bekker, A., Roux, J. J. J., & Mosteit, P. J. (2000). A generalization of the compound Rayleigh distribution: using a Bayesian method on cancer survival times. Communications in Statistics-Theory and Methods, 29(7), 1419-1433.
[66] Fulment, A. K., Gadde, S. R., & Peter, J. K. (2022). The odd log-logistic generalized exponential distribution: Application on survival times of chemotherapy patients data. F1000research, 11. 3

Mohammed Nasser Alshahrani Assistant Professor in the Department of Mathematics at Prince Sattam Bin Abdulaziz University. Earned a Master of Science in Statistics and Mathematics from the College of Science at the University of Arkansas, Fayetteville, USA. Earned a Ph.D. in Biostatistics and Complex Data Analysis from the School of Mathematics, College of Science, University of Leeds, UK.
Research Interests: Advanced Statistical Modeling and Methodology Development, Genomic Big Data Analysis, Biostatistics and its Applications in Health and Biological Sciences, High-Dimensional and Complex Data Analysis, Official Statistics and National Statistical Systems Design, Econometric and Statistical Economic Models.
Professional Experience: Currently serving as an Assistant Professor at the university.