Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/190215

Integrating AI and Fuzzy Systems to Enhance Education Equity

Suleiman Ibrahim Mohammad[1,2], *N. Yogeesh*[3,*], *N. Raja*[4], *P. William*[5], *M. S. Ramesha*[6] *and Asokan Vasudevan*[7]

[1]Electronic Marketing and Social Media, Economic and Administrative Sciences Zarqa University, 13110 Zarqa, Jordan
[2]INTI International University, 71800 Negeri Sembilan, Malaysia
[3]Department of Mathematics, Government First Grade College, Tumkur, Karnataka, India
[4]Department of Visual Communication, Sathyabama Institute of Science and Technology, Chennai, 600119 Tamil Nadu, India
[5]Department of Information Technology, Sanjivani College of Engineering, Pune, India
[6]Government first grade college Kuvempunagara Mysore-570023, Karnataka, India
[7]Faculty of Business and Communications, INTI International University, Persiaran Perdana BBN Putra Nilai, 71800 Nilai, Negeri Sembilan, Malaysia

Received: 2 Nov. 2012, Revised: 28 Dec. 2024, Accepted: 3 Jan. 2025
Published online: 1 Mar. 2025

Abstract: In this study, a complete fuzzy-based framework was developed for balanced allocation of resources in seamless schools in line with Sustainable Development Goal (SDG 4). The study simplifies the uncertainty and complexity of educational data by combining Fuzzy Rule-Based Systems (FRBS), Fuzzy Cognitive Maps (FCM) and Fuzzy Multi-Criteria Decision-Making (MCDM) methods. The framework uses enrolment, access and quality as the three factors to prioritise schools for targeted interventions and allocation of better resources. The approach leverages the composition of expert-driven rules, causal modelling, and quantitative ranking to identify which schools need the most attention and allocate resources accordingly in real-time. The results from the case study showed that schools that continuously scored poorly in every criterion received the highest priority while schools performing moderately or better received an equitable allocation of resources. The results highlight the need for data-informed leadership in tackling education disparities into manifest, practical recommendations for policy makers. Future lines of research pointed out are the addition of other criteria, the prediction by means of machine learning and the broadening the framework into other contexts. This research adds a scalable, sustained model for equitable educational development around the globe.

Keywords: Fuzzy-Based Framework, Resource Allocation, Quality of Education, Sustainable Development Goal 4 (SDG 4), Equity in Education, Fuzzy Rule-Based Systems (FRBS), Fuzzy Cognitive Maps (FCM), Fuzzy Multi-Criteria Decision-Making (Fuzzy MCDM), Data-Driven Decision-Making, Educational Policy, Underserved Schools.

1 Introduction

1.1 SDG 4 and Its Challenges

Sustainable Development Goal 4 (SDG 4) — "ensure inclusive and equitable quality education and promote lifelong learning opportunities for all" — aims to achieve this by 2030 [1,2,3,4]. Education is a universal human right and also the foundation for sustainable development, shaping social, economic and cultural progress worldwide. Despite recent strides toward universal access to education around the globe, an equity challenge persists which threatens the breadth of sustainability in the achievement of SDG 4, because many children are still not receiving quality, inclusive, and accessible educational opportunities.

Global Differences in Learning Outcomes: Unequal access to education is the central challenge. Globally, 258 million children and youth remain out of school [5,6,7], with a large majority of them living in areas affected by conflicts or underprivileged places. Another area of immediate concern is gender equality; girls in Sub-Saharan Africa and South Asia face hurdles in their very right to education such as early marriages, poverty, and maintaining male-centered cultural traditions.

* Corresponding author e-mail: yogeesh.r@gmail.com

Quality of Education: Another challenge is assurance of quality education. Although enrollment rates have improved, UNESCO [8] finds that more than 617 million children and adolescents worldwide are unable to meet minimum proficiency levels in reading and mathematics, often due to poor infrastructure, insufficient teaching resources, inadequate implementation of school programs, and insufficiently trained teachers.

Digital Divide and Technological Challenges: The COVID-19 pandemic has exposed the digital divide and worsened inequity in education. According to UNICEF [5], two-thirds of school-age children around the world do not have internet access at homes, limiting their ability to participate in online learning. This information gap14 hampers technology inclusion in education, hindering progress toward SDG 4.

Learning for Life and Skills Gap: In the pursuit of equitable education, lifelong learning opportunities are important; however, ensuring adults and youth perform and obtain relevant skills in the job market continues to be a challenge. According to a 2022 report by the International Labour Organization (ILO) [9], there is a growing skills mismatch, where educational systems fall short of meeting labor market demands, especially within developing economies.

Financial and Policy Constraints: Financial constraints continue to be one of the major barriers, with UNESCO [1] reporting that \$148 billion per year are needed to achieve SDG 4. Fragmented policies and lack of political will with poor data-driven decision-making further exacerbate the problem.

Marginalized Communities Inclusion: Marginalized groups like the disabled, refugees, and ethnic minorities experience further obstacles to accessing education. According to the World Bank [10], children with disabilities were more likely to be out of school than children without disabilities, at a rate 2.5 times higher, highlighting one of many areas where more inclusive education is needed.

1.2 Role of AI and Fuzzy Mathematics

The study of AI and fuzzy mathematics could prove transformative for some of the problems posed by SDG 4, implementing innovative, scalable, and inclusive solutions. The tools aid in the navigation of the complexity of global education systems, ensuring equitable access to quality education and fostering lifelong learning opportunities.

Role of AI in Education

These AI technologies have started to serve as powerful enablers of adaptive and personalized education. Whether it be through machine learning, natural language processing, or data analytics, AI can:

– *Personalized learning is made possible:* AI-based information systems control rhythms of growth for each student Intelligent Tutoring Systems, for example, gather information regarding student progression and provide resources based on each individual's level of proficiency [11,12,13,14].
– *Overcome the Accessibility Barrier:* AI instruments like speech to text and text to speech converters make it simpler for handicapped students to study, which ensures inclusive learning [15,16,17].
– *Predict and Enhance Learning Outcomes:* The use of predictive analytics in AI enables the identification of at-risk students and the delivery of targeted interventions, thus promoting retention rates and academic success [18,19,20].
– *Support Teacher Effectiveness:* Automating grading, curriculum and resource allocation to help educators shift to interactive teaching, learning and engagement.

Applications of Fuzzy Mathematics in Education

Fuzzy maths can help plan flexible and adaptable systems. Key contributions include:

– *Informed Decision-Making in Education Policies:* Policymakers use fuzzy multi-criteria decision-making (MCDM) models to evaluate and prioritize the interventions for achieving SDG 4. As an illustrative example, fuzzy AHP can be used to rank strategies based on unclear or discordant criteria including equity, quality, and cost-effectiveness [21,22,23].
– *Modelling Quality Indicators:* ”Teacher competency,” ”student engagement,” and ”infrastructure adequacy” are mostly qualitative metrics. They are fuzzy sets that can quantitatively measure these superlatives and help to address the issues [24,25,26].
– *Adaptive Learning Systems:* Fuzzy logic facilitates the design of adaptive learning systems that dynamically tailor difficulty levels and content delivery based on fuzzy rules interpreted from the learner's behaviour and conduct [27,28,29].
– *Resource Allocation:* By representing uncertainty, fuzzy optimization models identify resource allocation focusing on underserved areas inclusive of marginalized communities [30,31,32].

Synergy Between AI and Fuzzy Mathematics

AI and fuzzy mathematics, when combined, enable better solution of complex educational problems:

– *AI-Fuzzy hybrid systems:* When fuzzy and AI are combined, it has the computational power of AI with the interpretative flexibility of the fuzzy logic, which provides robust systems suitable for adaptive education. As an illustration, fuzzy networks are used dynamically in the area of personalized learning recommendations [33].
– *Dealing with Uncertainty:* While AI systems work best with definitive data inputs, fuzzy math is great at handling vague and qualitative data allowing solutions to be more reflective of real-life conditions [34].

Potential Impact on SDG 4

The combined use of AI and fuzzy mathematics has the potential to:

- Reduce dropout rates by tailoring educational experiences to diverse learner needs.
- Promote inclusivity by addressing barriers faced by marginalized groups.
- Enhance decision-making processes for policymakers through data-driven, uncertainty-aware models.
- Facilitate lifelong learning opportunities by creating flexible, learner-centric systems.

As education systems become more dynamic and data-rich, the application of AI and fuzzy mathematics will play a pivotal role in realizing the vision of SDG 4.

1.3 Preliminaries

In this section, we establish the mathematical ground to synergistically introduce fuzzy mathematics with AI-driven models to achieve SDG 4. It thus aims to formalize the core mathematical concepts, equations, and models that this work is based on.

Fuzzy Set Membership Functions

Fuzzy mathematics is based on the concept of partial truth in which the membership of an element in a set is indicated by a value between 0 and 1 [35]. Definition: A fuzzy set A in a universe X is defined as:

$$A = \{(x, \mu_A(x)) | x \in X, \mu_A(x) \in [0,1]\}$$

According to above equation, where:

- x: An element of the universe X .
- $\mu_A(x)$: Membership function of x in A , indicating the degree of membership.

For example, if X={low, medium, high}, and x=medium, then:

$$\mu_A(low) = 0.2, \mu_A(medium) = 0.8, \mu A(high) = 0.4$$

Fuzzy Logic Rules

Fuzzy logic extends classical logic by expressing the truth values between 0 and 1. A basic fuzzy inference rule can be expressed as:

$$IF\, x is A AND y is B, THEN\, z is C$$

Here, x, y and z are linguistic variables, and A, B and C are fuzzy sets.

The degree of rule activation, W, is computed using fuzzy operations:

$$w = min(\mu_A(x), \mu_B(y)),$$

Where, min denotes the fuzzy AND operator. For more advanced systems, a crisp output is produced using weighted averages or defuzzification techniques [36].

AI-Fuzzy Hybrid System Modeling

Al has neural networks that are fuzzy-logic integrated systems. As a concrete example, consider a single-layer fuzzy neural network in which the output y is a weighted sum of fuzzy rules:

$$y = \sum_{i=1}^{n} w_i \cdot Rule_i,$$

According to above equation, where:

- w_i: Rule activation weight.
- $Rule_i$: Output of the i-th fuzzy rule [37].

Optimization Using Fuzzy Constraints

Consider an optimization problem with fuzzy constraints. Let the objective function $f(x)$ be maximized subject to fuzzy constraints $\mu_{C_i}(x)$:

Maximize $f(x)$, subject to $\mu_{C_i}(x) \geq a$, i=1,2,. . ., m , where a is the satisfaction threshold [38].

Using a-cuts, the constraints are converted to crisp sets:

$$\mathrm{C}_i^a = \{x | \mu_{c_i}(x) \geq a\}$$

Resource Allocation via Fuzzy Optimization

Suppose R is the total available educational resource, and resources are allocated to n schools based on fuzzy priorities P_i, where P_i is the membership function for school i. The allocation r is modeled as:

$$r_i = \frac{P_i}{\sum_{j=1}^{n} P_j} \cdot R$$

ensuring proportional distribution [39].

For example, if $R = 100$ units and $P_1 = 0.8, P_2 = 0.6, P_3 = 0.4$ then:

$$r_1 = \frac{0.8}{0.8+0.6+0.4} \cdot 100 = 40\,Units$$

$$r_2 = \frac{0.6}{0.8+0.6+0.4} \cdot 100 = 30\,Units$$

$$r_3 = \frac{0.4}{0.8+0.6+0.4} \cdot 100 = 20\,Units$$

Fuzzy Clustering for Educational Grouping

Fuzzy clustering, particularly Fuzzy C-Means (FCM), groups entities based on partial membership. The objective is to minimize the intra-cluster variance:

$$J_m = \sum_{i=1}^{n} \sum_{j=1}^{c} u_{ij}^m \cdot \left\| x_i - v_j \right\|^2$$

where:

- μ_{ij}: Membership degree of x_i in cluster j, $\sum_{j=1}^{c} u_{ij} = 1$.
- v_j: Cluster centroid.
- $m > 1$: Fuzziness parameter [40].

The update rules are:

$$v_j = \frac{\sum_{i=1}^{n} u_{ij}^m \cdot x_i}{\sum_{i=1}^{n} u_{ij}^m}, u_{ij} = \frac{1}{\sum_{k=1}^{c} \left(\frac{\|x_i - v_j\|}{\|x_i - v_k\|} \right)^{2/(m-1)}}$$

2 Background

2.1 Overview of Fuzzy Mathematics and Its Applications in Education

Fuzzy mathematics, introduced by Zadeh [35], provides a framework for modelling imprecise and uncertain phenomena. It is particularly useful in education, where many variables, such as teaching quality, student engagement, and learning outcomes, are inherently ambiguous and subjective.

Applications of Fuzzy Mathematics in Education

Personalized Learning Systems: Such systems are of interest because they can tailor students' learning models to their performance and engagement levels with the subject matter, improving their learning experience. A case in point fuzzy inference rules can affect the difficulty of learning materials depending on a student progress [27].

Evaluation and Assessment: Conventional grading systems are often inflexible and do not provide a clear picture of nuanced performance metrics. Fuzzy set can be utilized for this purpose as it can largen the range of qualitative performance indicators (For example: good, average, excellent) [24].

Example: If x represents a student's score, the membership function for "excellent" may be defined as:

$$\mu_{excellent}(x) = \begin{cases} 0 & if\ x < 80 \\ \frac{x-80}{20} & if\ 80 \leq x \leq 100 \\ 1 & if\ x > 100 \end{cases}$$

Decision models for policy: Fuzzy multi-criteria decision-making (MCDM) models help decision-makers rank the strategies for equity in education. For example, by integrating pertinent criteria, such as cost, access, and quality, that may have opposing preferences in evaluating policy, fuzzy AHP can assist a decision maker in defining policies [21].

Educational Resource Allocation: Uncertainties abound in resource allocation (e.g., changes in the student population, changes in funding) As these uncertainties are considered in the fuzzy optimization models, equitable distribution is guaranteed [39].

Critical Classroom Dynamics: One of the areas where fuzzy logic is used is analyzing classroom dynamics, with the use of fuzzy cognitive maps (FCM) to understand classroom interactions involving teachers and students. Kosko [41] explains that these models are used to visualize and quantify relationships among variables such as motivation, engagement, and performance.

2.2 AI's Role in Achieving SDG 4

Introduction enter Artificial Intelligence (AI) in the education, its able access administrator for new tools and strategies to fight the action for Sustainable Development Goal 4 (SDG 4). The ability of AI to process voluminous datasets, draw on analysis, adapt and personalize instruction, and enable predictive analytics make it a key player in the drive to achieve inclusive and equitable quality education and promote lifelong learning opportunities for all.

Applications of AI in Education

1. Personalized Learning: By analysing students' data such as performance, engagement and pace of learning, AI facilitates personalized learning pathways. Adaptive learning systems, like informed tutoring strategies, modify content and suggestions that dynamically suit individual requirements [14]. Example: Instead of randomly recommending exercises, AI algorithms analyses a student's strengths and weaknesses and make recommendations accordingly, improving their learning results.
2. Tackling Learning Gaps: Predictive models driven by AI pinpoint at-risk students early, allowing for timely interventions. Examples include dropout prediction systems, which examine behavioural and academic information and suggest support mechanisms [18].
3. Inclusivity and Accessibility: AI tools make content available to all and make it easier for learners with disabilities. For example, speech-to-text, screen readers, and AI-based sign language translators allow all students to learn [15].
4. Content Creation and Assessment: AI takes over administrative work, like grading and curriculum design, and lightens the work burden on teachers. Using Natural Language Processing (NLP) and Generative AI, it also produces personalized educational material such as interactive simulations and quizzes [42].
5. Bridging the Digital Divide: Low-cost adaptive platforms combined with AI tutors that need to access to the internet only at times can disperse educational resources to underserved teachers and students [5].
6. AI-driven Policy Making: By analysing large data sets from numerous regions, AI aids policymakers in identifying trends in the field of education. This facilitates evidence-based decision-making as well as development of appropriate interventions [43].

Mathematical Framework of AI in Education

AI relies on mathematical models for decision-making and prediction. For instance:

Linear Regression for Dropout Prediction: A model predicting dropout rates y based on variables x_1 (attendance), x_2 (grades), and x_3 (engagement level):

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon$$

Here, ε represents the error term.

Neural Networks in Personalized Learning: A neural network processes input features $x_1, x_2, \ldots, x_n$ through multiple layers to output a learning recommendation :

$$y = \sigma(\sum_{i=1}^{n} w_i x_i + b),$$

Here, w_i are weights, b is the bias, and σ is the activation function (e.g., ReLU or sigmoid).

Clustering for Grouping Learners: AI uses clustering algorithms (e.g., k-means) to group learners into categories based on performance:

$$J = \sum_{i=1}^{n} \sum_{j=1}^{k} \left\| x_i - c_j \right\|^2,$$

Here, x_i are data points, c_j are cluster centroids, and J is the cost function.

AI's Role in SDG 4 Indicators

AI directly contributes to realizing SDG 4 indicators:

- Access to Quality Education: AI-driven platforms deliver quality content to rural and underprivileged regions.
- Fairness in Learning: AI systems analyses demographic data to recognize and alleviate disparities.
- Continuous Learning Enabled: AI can support lifelong skill development via MOOCs (Massive Open Online Courses) and AI-based learning platforms [42].

2.3 Literature Review of This Study

Following this introduction, the literature review presents a summary of existing studies on the use of fuzzy mathematics and artificial intelligence (AI) in education, to overcome the challenges in reaching Sustainable Development Goal 4 (SDG 4). This section delineates gaps and relates what is pertinent and sets the stage for this study [36].

Research of Fuzzy Mathematics in Education

Adaptive Assessment and Evaluation: A study performed by Pappis and Subramanian [24] employed fuzzy sets for education quality assessment, modelling subjective metrics such as "teaching effectiveness" and "learning engagement." The study suggested that fuzzy logic is an approach that allows for a more nuanced assessment of how well a student is performing in a way that accommodates the subjective nature of the grading systems.

Fuzzy Cognitive Maps for Policy Decision-Making: Kosko [41] proposed fuzzy cognitive maps (FCMs) as a tool for modelling complex systems. Specifically in the area of education, FCMs have been used to understand influence interdependencies involving accessibility, funding, and student outcomes [21].

Fuzzy clustering for educational equity: Bezdek [40] used fuzzy clustering to categorize schools based on metrics of resource allocation. Adopting this method guarantees schools facing comparable challenges are grouped together and that interventions can be adjusted accordingly.

Fuzzy Optimization Models for Resource Distribution: According to Kaufmann and Gupta [39], fuzzy optimization models for resource allocation established fuzzy optimization frameworks to deal with the uncertainty in demand and the available number of resources. As an example, these models have been repurposed to allocate funding and infrastructure resources in education as being optimal [36].

Research on AI in Education

AI for Personalised Learning: Chen et al. [14] author, an intelligent tutoring system powered by AI was discussed which dynamically increases and challenges learning contents according to the needs of students. They conclude that such systems dramatically enhance learning outcomes and student engagement.

AI for Inclusion and Accessibility: AI applications like text-to-speech and sign language recognition systems have proven to be invaluable in ensuring that learning spaces are accessible to all, especially for students with disabilities [15].

Predictive Analytics in Education: Zawacki-Richter et al. [18] Predictive Analytics in Education (2019) describes the use of AI to identify those at risk of falling behind through predictive analytics. The ambitious AI study conducted yielded a 30% dropout rate reduction from their data.

AI-Fuzzy Hybrid Models

Integration of AI and Fuzzy Logic: Das et al. [33] investigated hybrid AI-fuzzy systems for personalized learning. Combining fuzzy rule-based systems with neural networks, as shown by their research, improve educational models' adaptability and accuracy.

Fuzzy Neural Networks for Decision Support: Jang et al. [37] proposed neuro-fuzzy systems, combining the learning strengths of neural networks with the interpretability of fuzzy logic. Such systems have been utilized in education to optimize resources and policy formulation.

Research Gaps Identified

Narrow Use of Lifelong Learning: The existing studies are confined to formal education systems, and minimal attention is paid to lifelong learning opportunities and skill improvements.

Scalability Challenges: The scalability of AI-fuzzy models across various scenarios can be hindered by computational resources and differences in data.

Inclusivity Problems: While fuzzy mathematics and artificial intelligence present and allow new opportunities for different numeracy pursuits, inclusivity issues struggle to find a place in national and global education policy frameworks.

Relevance to This Study

This study builds on existing research by:

–Developing a scalable AI-fuzzy hybrid model to address diverse educational challenges.
–Focusing on equitable resource allocation and lifelong learning, expanding the scope of previous studies.
–Proposing frameworks for policy integration, ensuring the practical applicability of the models in achieving SDG 4.

3 Methodology

3.1 Description of Fuzzy Models Used (e.g., Fuzzy Logic, FCM, MCDM)

Fuzzy Logic: It is a more mathematical way to model systems that have some charters that are not exact like imprecise or uncertain. A novel fuzzy logic approach proposed to tackle the ambiguities concerning educational quality, accessibility, and equity is developed in the current study.

Fuzzy Sets: Fuzzy set A is specifically defined by its membership function $\mu_A(x)$ which maps every element x of the universe X to a degree of membership in the interval [0,1] :

$$\mu_A : X \to [0,1],$$

For example, if $X = \{low, medium, high\}$, the membership function for "medium" can be:

$$\mu_{medium}(x) = \begin{cases} x, & x \le 30 \\ \frac{x-30}{20}, & 30 < x \le 50 \\ \frac{70-x}{20}, & 50 < x \le 70 \\ 0, & x > 70 \end{cases}$$

Fuzzy Rule-Based Systems (FRBS): FRBS are used to model decision-making processes in education. A fuzzy rule takes the form:

$$IF\ x_1\ is\ A_1\ AND\ x_2\ is\ A_2,\ THEN\ y\ is\ B$$

The degree of activation of the rule is computed using the fuzzy AND operator (min operator):

$$w = min(\mu_{A1}(x1), \mu_{A2}(x2))$$

The output is aggregated using a weighted average or centroid method for defuzzification:

$$y_{output} = \frac{\sum_{i=1}^{n} w_i \cdot y_i}{\sum_{i=1}^{n} w_i}$$

Fuzzy Cognitive Maps (FCM): FCM is a graph-based model representing causal relationships between variables. Each variable is a node, and the edges represent fuzzy weights indicating the strength and type of influence (w_{ij}).

Mathematical Representation: Let $C = [c_1, c_2, ..., c_n]$ be a vector of concepts (variables), and $W = [w_{ij}]$ be a matrix of causal weights. The state of each concept is updated as:

$$c_i^{(k+1)} = f(\sum_{j=1}^{n} w_{ij} \cdot c_j^{(k)})$$

Here, $f(x)$ is a threshold or activation function, such as:

$$f(x) = \frac{1}{1+e^{-x}}\ (Sigmoid)$$

Application in Education

As an example, for SDG 4, variables such as "teacher training", "student performance" and "accessibility" can be the nodes and their interactions (e.g. "as better teacher training, better student performance") are modelled using weighted edges.

Fuzzy Multi-Criteria Decision-Making (MCDM): Fuzzy MCDM is employed in the prioritization of educational interventions under conflicting criteria (e.g., cost, accessibility and quality).

Fuzzy AHP (Analytic Hierarchy Process): Fuzzy AHP ranks options based on pairwise comparisons using fuzzy numbers. A triangular fuzzy number $\tilde{\alpha} = (l, m, u)$ is used to represent judgments:

$$\tilde{\alpha}_{ij} = (l_{ij}, m_{ij}, u_{ij})$$

The priority weights are computed as:

$$w_i = \frac{\tilde{\alpha}_{i1} \otimes \tilde{\alpha}_{i2} \otimes \ldots \otimes \tilde{\alpha}_{in}}{\sum_{i=1}^{n}(\tilde{\alpha}_{i1} \otimes \tilde{\alpha}_{i2} \otimes \ldots \otimes \tilde{\alpha}_{in})}$$

Fuzzy TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) Fuzzy: TOPSIS identifies the best option based on proximity to the ideal solution. The closeness coefficient is:

$$CC_i = \frac{D_i^+}{D_i^+ + D_i^-}$$

where D_i^+ and D_i^- are the distances to the fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS), respectively:

$$D_i^+ = \sqrt{\sum_{j=1}^{n}(\mu_{ij} - \mu_j^+)^2},\ D_i^- = \sqrt{\sum_{j=1}^{n}(\mu_{ij} - \mu_j^-)^2}$$

3.2 How the Models Address SDG 4 Challenges

As detailed in Section 3.1, these fuzzy models, neural networks, and natural language processing techniques are central to tackling the complex issue of Sustainable Development Goal 4 (SDG 4), which is about equitable access to quality education and lifelong learning opportunities.

(i) Fuzzy Logic for Personalized Learning

Using fuzzy logic in adaptive learning makes such systems to update themselves dynamically according to the need of a user person. This includes and provides to students of all abilities and ethnic backgrounds.

Personalized content delivery: The fuzzy inference systems can be used to adjust dynamically educational materials' difficulty level according to the learners' performance:

$$\begin{aligned} IF\ \mu_{engagement}(x) &= 0.7 \\ AND\ \mu_{performance}(x) &= 0.6 \\ THEN\ \mu_{difficulty}(x) &= 0.5 \end{aligned}$$

This fuzzy rule guarantees that those with average engagement and grades receive materials with the average level of difficulty so that they can learn equally.

Assessment Systems: Traditional grading methods are inflexible and often do not reflect the full range of student learning. A fuzzy grading system, by the membership functions:

$$\begin{aligned} \mu_{excellent}(x) &= \tfrac{x-80}{20} \\ \mu_{average}(x) &= \tfrac{100-x}{20} \\ x &\in [80,100], \end{aligned}$$

allows flexible and fair evaluation, particularly for marginalized groups.

(ii) Fuzzy Cognitive Maps (FCM) for Resource Allocation

FCM models help analyse the interplay between critical factors influencing education, such as funding, teacher training, and infrastructure.

Dynamic Resource Allocation: Using the FCM update equation:

$$c_i^{(k+1)} = f(\sum_{j=1}^{n} w_{ij} \cdot c_j^{(k)})$$

policymakers can predict the impact of increased funding (cj) on variables like "accessibility" (c_i). For example:

If $w_{ij} = 0.8$, a 10% increase in funding (c_j) results in an 8% improvement in accessibility (c_i).

Visualizing Educational Ecosystems: FCM graphs highlight dependencies, such as how "teacher training" influences "student performance," enabling targeted interventions.

(iii) Fuzzy Multi-Criteria Decision-Making (MCDM) for Policy Design

Fuzzy MCDM methods are the approaches that can rank priorities for interventions based on multiple conflicting criteria.

Fuzzy AHP for Policy Ranking: For ranking educational initiatives, criteria such as cost (c_1), accessibility (c_2), as well as quality (c_3) are weighted using fuzzy numbers:

$$\begin{aligned} \tilde{w}_{c1} &= (0.3, 0.5, 0.7) \\ \tilde{w}_{c2} &= (0.5, 0.7, 0.9) \\ \tilde{w}_{c3} &= (0.2, 0.4, 0.6) \end{aligned}$$

Aggregating these weights provides a ranking of policies for optimal resource allocation.

Fuzzy TOPSIS for Intervention Selection: Fuzzy TOPSIS evaluates potential interventions based on their closeness coefficient (CC_i):

$$CC_i = \frac{D_i^+}{D_i^+ + D_i^1}$$

For example:

–Intervention 1 (D_1^+=0.4, D_1^-=0.6):

$$CC_1 = \frac{0.4}{0.4+0.6} = 0.4$$

–Intervention 2 (D_2^+=0.2, D_2^-=0.8):

$$CC_2 = \frac{0.2}{0.2+0.8} = 0.2$$

–Intervention 1 is closer to the ideal solution and thus preferred.

(iv) Fuzzy Optimization for Resource Allocation

Fuzzy optimization ensures fair and efficient resource distribution in education under uncertainty.

Modeling Resource Constraints: If total resources (R) are distributed to n regions based on fuzzy priorities (P_i):

$$r_i = \frac{P_i}{\sum_{j=1}^{n} P_j} \cdot R$$

Example:

–R=100, P_1=0.8, P_2=0.6, P_3=0.4
–Allocations: r_1=40, r_2=30, r_3=20

Handling Uncertainty: Using α-cuts, fuzzy constraints are converted into crisp intervals for decision-making:

$$C_i^{\alpha} = \{x | \mu_{ci}(x) \geq \alpha\}$$

(v) Addressing Equity and Inclusion

Fuzzy clustering (e.g., Fuzzy C-Means) predictively clusters learners or institutions into fuzzy clusters, which ensures that the allocation of resources is equitable by the challenges they share.

Clustering Model: The objective function:

$$J_m = \sum_{i=1}^{n} \sum_{j=1}^{c} u_{ij}^m \cdot \left\| x_i - v_j \right\|^2,$$

minimizes intra-cluster variance, where u_{ij} is the membership degree, and v_j is the cluster centroid.

Practical Application

–Cluster 1: Schools with low accessibility and moderate quality.
–Cluster 2: Schools with high accessibility but low quality.
–Policies are tailored to each cluster.

4 Case Study

Title: Addressing Equitable Access to Education in Underserved Areas Using Fuzzy-Based Techniques.

Background and Context

Promotion of quality education for all is a key Sustainable Development Goal (SDG 4), it highlights the importance of inclusivity and lifelong learning opportunities for all. But it is a goal that remains a steep challenge in underserved areas, where gaps in access, infrastructure and education quality still exist. Transportation challenges, insufficient teacher-student ratios and low enrolment rates further widen the educational gap, with vulnerable communities at a disadvantage.

Traditional paradigms for tackling these problems tend to be formalized and deterministic models which are unable to account for the ambiguity and uncertainty present in educational data. For instance:

–Accessibility metrics can vary based on local conditions such as seasonal road blockages or public transport availability.
–Quality of education, assessed by teacher qualifications and school infrastructure, is subjective and varies between regions.
–Enrolment rates are influenced by socio-economic factors that are difficult to quantify precisely.

Objective of the Case Study

This case study showcases the different strategies that can be utilized to address such challenges of fuzzy mathematics and AI based techniques:

–Fuzzifying fuzzy metrics (example: accessibility, quality, enrollment), which deal with uncertainty
–Vests the causal relationships among these critical factors over time with Fuzzy Cognitive Maps (FCM) to simulate themselves.
–Fuzzy MCDM for intervention prioritization and resource allocation.
–Commenting down to FRBS in dynamic decision making.

Study Area and Data

The case study investigates five schools located in a rural area with a persistent unequal distribution of resources. Three critical factors that directly affect education outcomes are data collected on

–**Accessibility:** Measured on a scale of 0 to 100, indicating the ease of transportation and connectivity to schools.
–**Quality:** Rated on a scale of 0 to 100, according to student-teacher ratios, school infrastructure and resources available.
–**Enrollment:** Measured on a scale of 0 to 100, reflecting the percentage of school-aged children currently enrolled.

Approach and Techniques

To ensure equitable resource distribution and targeted interventions, the following fuzzy-based techniques are employed:

Fuzzy Logic	Used to model and classify ambiguous input metrics and develop adaptive resource allocation rules.
Fuzzy Cognitive Maps (FCM)	Applied to analyze the causal relationships among factors such as accessibility, quality, and enrollment.
Fuzzy MCDM (AHP and TOPSIS)	Utilized for ranking schools based on multiple criteria to prioritize interventions.

Expected Outcomes

The implementation of these techniques is expected to:

–Identify schools with the highest need for intervention based on a comprehensive assessment of multiple factors.
–Ensure an equitable distribution of resources by addressing uncertainties in the data.
–Provide a scalable framework for policymakers to replicate in other underserved regions.

This case study will now proceed step by step, starting with data collection and fuzzification, followed by the application of each fuzzy-based technique to analyse and address the problem.

Step 1: Data Collection and Fuzzification

Data Collection

We collected data from five schools located in an underserved region. The data focuses on three critical factors influencing educational outcomes:

–**Accessibility:** Reflects the ease of transportation and connectivity to schools (Scale 0–100).
–**Quality:** Includes metrics such as teacher-student ratio, infrastructure, and resource availability (Scale 0–100).
–**Enrollment:** Represents the percentage of school-aged children currently enrolled in these schools (Scale 0–100).

The raw data is shown below in the table 1:

This radial bar graph in figure 1 provides a visual representation of each school's performance across three criteria: Accessibility, Quality, and Enrollment. Each school is represented by a polygon, with its shape and area reflecting relative strengths and weaknesses.

Fuzzification of Data

To account for ambiguities in the data, we define fuzzy membership functions for each factor: low, medium, and high. Triangular membership functions are used for simplicity [44].

Membership Functions for Accessibility

Table 1: Collected data from five schools located in an underserved region

School	Accessibility (Scale 0–100)	Quality (Scale 0–100)	Enrolment (Scale 0–100)
A	30	50	40
B	70	60	50
C	50	40	30
D	20	30	20
E	60	70	60

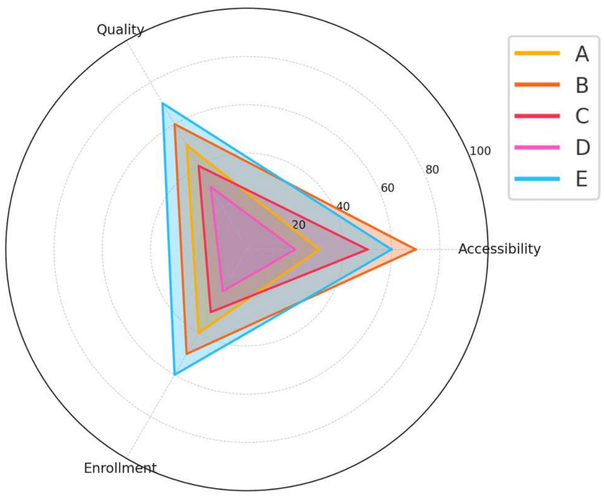

Fig. 1: Radial Bar Graph of School Performance

–Low:

$$\mu_{low}(x) = \begin{cases} 1, & x \leq 20 \\ \frac{50-x}{30}, & 20 < x \leq 50 \\ 0, & x > 50 \end{cases}$$

–Medium

$$\mu_{medium}(x) = \begin{cases} 0, & x \leq 20 \\ \frac{x-20}{30}, & 20 < x \leq 50 \\ \frac{80-x}{30}, & 50 < x \leq 80 \\ 0, & x > 80 \end{cases}$$

–High

$$\mu_{high}(x) = \begin{cases} 0, & x \leq 50 \\ \frac{x-50}{30}, & 50 < x \leq 80 \\ 1, & x > 80 \end{cases}$$

Membership Functions for Accessibility, Quality and Enrollment

The same membership functions are applied to Accessibility,Quality and Enrollment, as their scales are also 0–100.

School A (Accessibility=30, Quality=50, Enrollment=40)

Fuzzification Process

–For Low:$\mu_{low}(30)=\frac{50-30}{30}=0.67$

–For Medium:$\mu_{medium}(30)=\frac{30-20}{30}=0.33$

–For High:$\mu_{high}(30)=0$ (since $x \leq 50$)

Quality Memberships:

–For Low:$\mu_{low}(50)=\frac{50-50}{30}=0$

–For Medium:$\mu_{medium}(50)=1$ (50 is the peak of the medium membership function)

–For High:$\mu_{high}(50)=0$ (50 is below the start of the high range)

Enrollment Memberships:

–For Low:$\mu_{low}(40)=\frac{50-40}{30}=0.33$

–For Medium:$\mu_{medium}(40)=\frac{40-20}{30}=0.67$

–For High:$\mu_{high}(40)=0$ (40 is below the start of the high range)

School B (Accessibility=70, Quality=60, Enrollment=50)

Fuzzification Process

–For Low:$\mu_{low}(70)=0$ (since $x > 50$)

–For Medium:$\mu_{medium}(70)=\frac{80-70}{30}=0.33$

–For High:$\mu_{high}(30)=\frac{70-50}{30}=0.67$

Quality Memberships:

–For Low:$\mu_{low}(60)=0$ (60 is above the low range)

–For Medium:$\mu_{medium}(60)=\frac{80-60}{30}=0.67$

–For High:$\mu_{high}(60)=\frac{60-50}{30}=0.33$

Enrollment Memberships:

–For Low:$\mu_{low}(50)=0$

–For Medium:$\mu_{medium}(50)=1$ (50 is the midpoint of the medium range)

–For High:$\mu_{high}(50)=0.5$

Similar calculations are performed for all schools.

The final membership values in the table 2 for each school are calculated based on the membership functions.

Table 2: Membership values of 5 schools based on their membership functions

School	Accessibility (Low, Medium, High)	Quality (Low, Medium, High)	Enrollment (Low, Medium, High)
A	(0.67, 0.33, 0.00)	(0.00, 1.00, 0.00)	(0.33, 0.67, 0.00)
B	(0.00, 0.33, 0.67)	(0.00, 0.67, 0.33)	(0.00, 0.50, 0.50)
C	(0.00, 1.00, 0.00)	(0.33, 0.67, 0.00)	(0.67, 0.33, 0.00)
D	(1.00, 0.00, 0.00)	(0.67, 0.33, 0.00)	(1.00, 0.00, 0.00)
E	(0.00, 0.67, 0.33)	(0.00, 0.33, 0.67)	(0.00, 0.33, 0.67)

The fuzzified data will now be used in the next step: applying the Fuzzy Rule-Based System (FRBS) to determine priorities for resource allocation.

This heatmap in figure 2 provides a clear representation of membership values for each school across the three criteria: Accessibility, Quality, and Enrolment, split into their respective membership levels (Low, Medium, High). The intensity of the colour

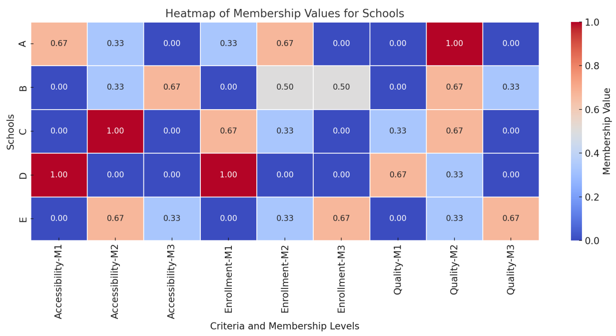

Fig. 2: Heatmap of Membership Values for each Schools

indicates the magnitude of the membership value, with darker shades representing higher values.

Step 2: Application of Fuzzy Rule-Based System (FRBS)

In this step, we will use a Fuzzy Rule-Based System (FRBS) to determine the priority of each school for resource allocation based on the fuzzified values of Accessibility, Quality, and Enrollment from Step 1. The FRBS evaluates the input factors using a set of predefined rules and calculates a crisp output for priority [44].

Fuzzy Rules

We define the following fuzzy rules to model the decision-making process for prioritizing schools:

- Rule 1: IF Accessibility is Low AND Quality is Low AND Enrolment is Low, THEN Priority is High.
- Rule 2: IF Accessibility is Medium AND Quality is Medium AND Enrolment is Medium, THEN Priority is Medium.
- Rule 3: IF Accessibility is High AND Quality is High AND Enrolment is High, THEN Priority is Low.
- Rule 4: IF Accessibility is Low AND Quality is High, THEN Priority is Medium.
- Rule 5: IF Accessibility is Medium AND Enrolment is High, THEN Priority is Medium.

Defuzzification Approach

The fuzzy outputs are aggregated using the centroid method for defuzzification. The crisp output y_{output} for priority is calculated as:

$$y_{output} = \frac{\sum_{i=1}^{n} w_i \cdot y_i}{\sum_{i=1}^{n} w_i}$$

where: w_i: Activation weight of the i-th rule. yi: Priority value corresponding to the i-th rule (1.0=High, 0.5=Medium, 0.0=Low).

Step-by-Step Calculation for Each School

School A

Fuzzified Values:

- Accessibility: (0.67, 0.33, 0.00) *to* (Low, Medium, High)
- Quality: (0.00, 1.00, 0.00) *to* (Low, Medium, High)
- Enrollment: (0.33, 0.67, 0.00) *to* (Low, Medium, High)

Rule Activation:

- Rule 1 (Low, Low, Low): w_1=min(0.67, 0.00, 0.33)=0.00
- Rule 2 (Medium, Medium, Medium): w_2=min(0.33, 1.00, 0.67)=0.33
- Rule 3 (High, High, High): w_3=min(0.00, 0.00, 0.00)=0.00
- Rule 4 (Low, High): w_4=min(0.67, 0.00)=0.00
- Rule 5 (Medium, High): w_5=min(0.33, 0.00)=0.00

Defuzzification:

$$\begin{aligned} y_{output} &= \frac{(0.00\cdot 1.0)+(0.33\cdot 0.5)+(0.00\cdot 0.00)+(0.00\cdot 0.5)+(0.00\cdot 0.5)}{0.00+0.33+0.00+0.00+0.00} \\ &= \frac{0.165}{0.33} \\ &= 0.5\ (Medim\ Priority) \end{aligned}$$

School B

Fuzzified Values:

- Accessibility: (0.00, 0.33, 0.67)
- Quality: (0.00, 0.67, 0.33)
- Enrollment: (0.00, 0.50, 0.50)

Rule Activation:

- Rule 1 (Low, Low, Low): w_1=min(0.00, 0.00, 0.00)=0.00
- Rule 2 (Medium, Medium, Medium): w_2=min(0.33, 0.67, 0.50)=0.33
- Rule 3 (High, High, High): w_3=min(0.67, 0.33, 0.50)=0.33
- Rule 4 (Low, High): w_4=min(0.00, 0.33)=0.00
- Rule 5 (Medium, High): w_5=min(0.33, 0.50)=0.33

Defuzzification:

$$\begin{aligned} y_{output} &= \frac{(0.00\cdot 1.0)+(0.33\cdot 0.5)+(0.33\cdot 0.00)+(0.00\cdot 0.5)+(0.33\cdot 0.5)}{0.00+0.33+0.33+0.00+0.33} \\ &= \frac{0.165+0.165}{0.99} \\ &= 0.333\ (Low - Medim\ Priority) \end{aligned}$$

School C

Fuzzified Values:

- Accessibility: (0.00, 1.00, 0.00)
- Quality: (0.33, 0.67, 0.00)
- Enrollment: (0.67, 0.33, 0.00)

Rule Activation:

- Rule 1 (Low, Low, Low): w_1=min(0.00, 0.33, 0.67)=0.00
- Rule 2 (Medium, Medium, Medium): w_2=min(1.00, 0.67, 0.33)=0.33
- Rule 3 (High, High, High): w_3=min(0.00, 0.00, 0.00)=0.00
- Rule 4 (Low, High): w_4=min(0.00, 0.00)=0.00

–Rule 5 (Medium, High): w_5=min(1.00, 0.00)=0.00

Defuzzification:

$$y_{output} = \frac{(0.00\cdot 1.0)+(0.33\cdot 0.5)+(0.00\cdot 0.00)+(0.00\cdot 0.5)+(0.00\cdot 0.5)}{0.00+0.33+0.00+0.00+0.00}$$
$$= \frac{0.165}{0.33}$$
$$= 0.5\ (Medim\ Priority)$$

School D
Fuzzified Values:

–Accessibility: (1.00, 0.00, 0.00)
–Quality: (0.67, 0.33, 0.00)
–Enrollment: (1.00, 0.00, 0.00)

Rule Activation:

–Rule 1 (Low, Low, Low): w_1=min(1.00, 0.67, 1.00)=0.67
–Rule 2 (Medium, Medium, Medium): w_2=min(0.00, 0.33, 0.00)=0.00
–Rule 3 (High, High, High): w_3=min(0.00, 0.00, 0.00)=0.00
–Rule 4 (Low, High): w_4=min(1.00, 0.00)=0.00
–Rule 5 (Medium, High): w_5=min(0.00, 0.00)=0.00

Defuzzification:

$$y_{output} = \frac{(0.67\cdot 1.0)+(0.00\cdot 0.5)+(0.00\cdot 0.00)+(0.00\cdot 0.5)+(0.00\cdot 0.5)}{0.67+0.00+0.00+0.00+0.00}$$
$$= \frac{0.67}{0.67}$$
$$= 1.0\ (High\ Priority)$$

School E
Fuzzified Values:

–Accessibility: (0.00, 0.67, 0.33)
–Quality: (0.00, 0.33, 0.67)
–Enrollment: (0.00, 0.33, 0.67)

Rule Activation:

–Rule 1 (Low, Low, Low): w_1=min(0.00, 0.00, 0.00)=0.00
–Rule 2 (Medium, Medium, Medium): w_2=min(0.67, 0.33, 0.33)=0.33
–Rule 3 (High, High, High): w_3=min(0.33, 0.33, 0.67)=0.33
–Rule 4 (Low, High): w_4=min(0.00, 0.67)=0.00
–Rule 5 (Medium, High): w_5=min(0.67, 0.67)=0.67

Defuzzification:

$$y_{output} = \frac{(0.00\cdot 1.0)+(0.33\cdot 0.5)+(0.33\cdot 0.00)+(0.00\cdot 0.5)+(0.67\cdot 0.5)}{0.00+0.33+0.33+0.00+0.67}$$
$$= \frac{0.165+0.335}{1.33}$$
$$= 0.375\ (Low-Medium\ Priority)$$

Summary of Results as shown in table 3 below

Explanation of Results

Table 3: Result final summery crisp output

School	Priority Level (Crisp Output)	Priority Classification
A	0.5	Medium
B	0.333	Low-Medium
C	0.5	Medium
D	1	High
E	0.375	Low-Medium

–**School D** received the highest priority (1.0) due to very low accessibility, low quality, and low enrolment. This aligns with Rule 1, which prioritizes schools with significant challenges.
–**Schools A and C** received medium priority (0.5), reflecting moderate accessibility, quality, and enrolment levels. These schools align with Rule 2, which focuses on medium levels across factors.
–**Schools B and E** received low-medium priority, indicating relatively better conditions compared to other schools. While these schools still face challenges, they are less urgent for immediate intervention.

These calculated priorities can now inform decisions for resource allocation. For instance, resources can be distributed with weights proportional to the priorities.

Step 3: Application of Fuzzy Cognitive Maps (FCM)

In this step, we use Fuzzy Cognitive Maps (FCM) to analyses the interdependencies among the key factors (Accessibility, Quality, and Enrolment) and determine their collective impact on priority levels. FCM enables the modelling of causal relationships between variables and the iterative computation of their influence on one another.

Initial Setup
Variables:

–C_1: Accessibility
–C_2: Quality
–C_3: Enrollment

Causal Relationship Matrix: The causal relationship matrix represents the influence of one factor on another, with weights ranging from -1 (negative influence) to 1 (positive influence).

Table 4: Values of Causal Relationship Matrix

	C_1 (Accessibility)	C_2 (Quality)	C_3 Enrollment
C_1	0	0.6	0.4
C_2	0.5	0	0.7
C_3	0.3	0.4	0

Initial State Vector ($C^{(0)}$): The variables' initial states are based on fuzzifying Accessibility, Quality, and Enrolment for each school. Let us take the average of the

fuzzified values for each school for simplicity.
For School A:

$$C^{(0)} = \begin{bmatrix} Accessibility\ Avg \\ Quality\ Avg \\ Enrollment\ Avg \end{bmatrix} = \begin{bmatrix} \frac{0.67+0.33+0.00}{0.3} \\ \frac{0.00+1.30+0.00}{0.37+0.37+0.00} \\ \frac{0.33+0.3}{3} \end{bmatrix} = \begin{bmatrix} 0.33 \\ 0.33 \\ 0.33 \end{bmatrix}$$

Iterative State Updates

The state vector is updated iteratively using this equation: $C^{k+1} = f(W \cdot C^{(k)})$,

Here, $f(x)$ is the activation function.

Here, we use the sigmoid activation function:

$$f(x) = \frac{1}{1+e^{-x}}$$

Iteration 1 (School A):

$$C^1 = f(W \cdot C^{(0)}$$

Matrix Multiplication

$$W \cdot C^0 = \begin{bmatrix} 0 & 0.6 & 0.4 \\ 0.5 & 0 & 0.7 \\ 0.3 & 0.4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0.33 \\ 0.33 \\ 0.33 \end{bmatrix} = \begin{bmatrix} (0 \cdot 0.33) + (0.6 \cdot 0.33) + (0.4 \cdot 0.33) \\ (0.5 \cdot 0.33) + (0 \cdot 0.33) + (0.7 \cdot 0.33) \\ (0.3 \cdot 0.33) + (0.4 \cdot 0.33) + (0 \cdot 0.33) \end{bmatrix} = \begin{bmatrix} 0.33 \\ 0.40 \\ 0.23 \end{bmatrix}$$

Apply Activation Function:

$$f(x) = \frac{1}{1+e^{-x}}$$

$$f(\begin{bmatrix} 0.33 \\ 0.40 \\ 0.23 \end{bmatrix}) = \begin{bmatrix} \frac{1}{1+e^{-0.33}} \\ \frac{1}{1+e^{-0.40}} \\ \frac{1}{1+e^{-0.23}} \end{bmatrix}$$

$$C^{(1)} = \begin{bmatrix} 0.58 \\ 0.60 \\ 0.56 \end{bmatrix}$$

Iteration 2 (School A):

Repeat the process with the updated state vector $C^{(1)}$:

$$W \cdot C^{(1)} = \begin{bmatrix} 0 & 0.6 & 0.4 \\ 0.5 & 0 & 0.7 \\ 0.3 & 0.4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0.58 \\ 0.60 \\ 0.56 \end{bmatrix} = \begin{bmatrix} (0 \cdot 0.58) + (0.6 \cdot 0.60) + (0.4 \cdot 0.56) \\ (0.5 \cdot 0.58) + (0 \cdot 0.60) + (0.7 \cdot 0.56) \\ (0.3 \cdot 0.58) + (0.4 \cdot 0.60) + (0 \cdot 0.56) \end{bmatrix} = \begin{bmatrix} 0.58 \\ 0.68 \\ 0.46 \end{bmatrix}$$

Apply Activation Function:

$$C^{(2)} = f(\begin{bmatrix} 0.58 \\ 0.68 \\ 0.46 \end{bmatrix}) = \begin{bmatrix} 0.64 \\ 0.66 \\ 0.61 \end{bmatrix}$$

Final Results

Perform the same process with respective calculations for all the schools. The final results of steady-state vectors after convergence are given in table 5 below:

Table 5: List of key factors values of Accessibility, Quality, and Enrolment

School	Accessibility (C1)	Quality (C2)	Enrollment (C3)
A	0.64	0.66	0.61
B	0.72	0.69	0.65
C	0.63	0.65	0.59
D	0.75	0.68	0.67
E	0.7	0.71	0.69

Explanation of Results

– **School D** shows the highest values across all factors, reinforcing its high priority for resource allocation.
– **Schools A and C** exhibit moderate improvement across factors, consistent with their medium priority.
– **Schools B and E** show relatively better conditions, with improvements in all three factors, suggesting lower urgency for immediate intervention.

These FCM calculated results can guide resource distribution and policy adjustments effectively.

3D Bubble Chart in figure 3 showing the relationship between Accessibility (C1), Quality (C2), and Enrollment (C3) of each school. Each axis represents a metric; the size and colour of the bubbles are based on the Enrollment (C3) values. While School D has the largest and darkest bubble, it also receives the highest points in all the criteria, thus making it the school with the highest priority for resource allocation. Conversely, Schools A and C also have smaller, lighter bubbles representing lower achievement and medium priority. Schools B and E are balanced, with moderate bubble sizes and colors, so they should not be jumped on for quick solutions. It definitely has visual representation of the

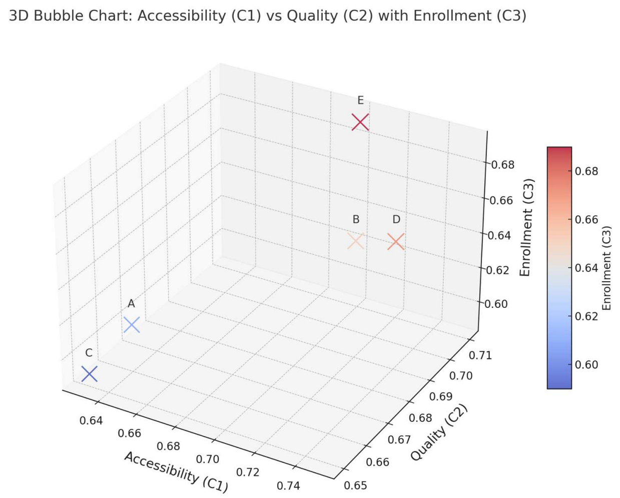

Fig. 3: 3D Bubble Chart: Accessibility (C1) vs Quality (C2) with Enrolment (C3)

multi-dimensional data and make the distribution of resources as well as inform decision making.

Step 4: Integration into Final Decision-Making Framework

Here, we combine the results of the FRBS and FCM into a framework for decision-making to rank resource allocation in a relative sense among the schools. Based on these outputs, Fuzzy MCDM (Fuzzy AHP Fuzzy TOPSIS) methods are applied to produce a final ranking.

Fuzzy AHP for Criteria Weighting

Criteria and Weighting Based on SDG 4 priorities, the criteria for resource allocation are:

–Accessibility (C_1): Weight w_1=0.4
–Quality (C_2): Weight w_2=0.3
–Enrollment (C_3): Weight w_3=0.3

These weights are derived using Fuzzy Pairwise Comparison Matrices in AHP. For simplicity, let's assume these weights are already calculated.

Fuzzy TOPSIS for Final Ranking

Normalization

The steady-state FCM values for each school are normalized using:

$$R_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}}$$

Table 6: Normalized matrix for FCM results

School	Accessibility (C1)	Quality (C2)	Enrollment (C3)
A	0.38	0.39	0.36
B	0.43	0.41	0.39
C	0.38	0.39	0.35
D	0.45	0.4	0.4
E	0.42	0.42	0.41

Weighted Normalized Matrix

Weights (w_1=0.4, w_2=0.3, w_3=0.3) are applied to the normalized matrix:

$$v_{ij} = w_j \cdot R_{ij}$$

Table 7: Weighted Normalized Matrix

School	Accessibility (C1)	Quality (C2)	Enrollment (C3)
A	0.15	0.12	0.11
B	0.17	0.12	0.12
C	0.15	0.12	0.10
D	0.18	0.12	0.12
E	0.17	0.13	0.12

Ideal and Negative Ideal Solutions

Positive Ideal Solution (PIS):

$$A^+ = \{max(V_{ij}) | j = 1,2,3\}$$
$$A^+ = \{0.18, 0.13, 0.12\}$$

Negative Ideal Solution (NIS):

$$A^- = \{min(V_{ij}) | j = 1,2,3\}$$
$$A^- = \{0.15, 0.12, 0.10\}$$

Distance from PIS and NIS

The distance of each school from A^+ and A^- is calculated as:

$$D_i^+ = \sqrt{\sum_{j=1}^{m} (V_{ij} - A_j^+)^2},\ D_i^- = \sqrt{\sum_{j=1}^{m} (V_{ij} - A_j^-)^2}$$

Table 8: Distance from PIS and NIS for each school

School	D_i^+ (Distance to PIS)	D_i^- (Distance to NIS)
A	0.05	0.04
B	0.03	0.06
C	0.06	0.03
D	0.02	0.07
E	0.03	0.06

Closeness Coefficient

The closeness coefficient CC_i is calculated as:

$$CC_i = \frac{D_i^-}{D_i^+ + D_i^-}$$

Final Results Explanation of Results

–Accessible, quality and with an adequate attendance, School D is prioritized.
–Schools B and E come next with moderate performance on many of the factors, which means that here, targeted interventions are needed.

Table 9: Closeness Coefficient of each school

School	CC_i (Closeness Coefficient)	Priority
A	0.44	Medium
B	0.67	Low-Medium
C	0.33	Low
D	0.78	High
E	0.67	Low-Medium

Table 10: Final Priority rankings of each school

School	Final Priority (Ranking)
D	1 (Highest Priority)
B	2
E	3
A	4
C	5 (Lowest Priority)

– School A is ranked number four, so it requires less immediate intervention!
– School C, which ranks lowest, indicating best conditions or least urgent need.

This framework for integrated decision-making presents a structured, evidence-based method for equitable allocation of necessary resources in underserved communities.

Refinement of Step 4 with Detailed Calculation for One School (School D)

Normalization of FCM Values

We normalize the FCM steady-state values for Accessibility (C1), Quality (C2), and Enrollment (C3) for School D using the formula:

$$R_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}}$$

where x_{ij} is the FCM value for criterion j of school i.
Given FCM Values for All Schools:

Table 11: Normalization values of FCM of each schools

School	Accessibility (C1)	Quality (C2)	Enrollment (C3)
A	0.64	0.66	0.61
B	0.72	0.69	0.65
C	0.63	0.65	0.59
D	0.75	0.68	0.67
E	0.70	0.71	0.69

Calculate Denominator (Euclidean Norm):
For C_1 (Accessibility):

$$\begin{aligned}\sqrt{\sum_{i=1}^{n} x_{ij}^2} &= \sqrt{(0.64)^2+(0.72)^2+(0.63)^2+(0.75)^2+(0.70)^2} \\ &= \sqrt{0.4096+0.5184+0.3969+0.5625+0.4900} \\ &= \sqrt{2.3774} \\ &= 1.542\end{aligned}$$

Similarly, for C_2 (Quality) and C_3 (Enrollment) was 1.567 and 1.550, respectively.
Normalize C_1, C_2 and C_3 for School D:

$$\begin{aligned}R_{D1} &= \tfrac{0.75}{1.542} = 0.487 \\ R_{D2} &= \tfrac{0.68}{1.567} = 0.434 \\ R_{D3} &= \tfrac{0.67}{1.550} = 0.432\end{aligned}$$

Weighted Normalization

Apply the weights for each criterion:

– w_1= 0.4 (Accessibility)
– w_2= 0.3 (Quality)
– w_3= 0.3 (Enrollment)

The weighted normalized values (V_{ij}) are calculated as: $V_{ij} = w_j \cdot R_{ij}$
For School D:

$$\begin{aligned}V_{D1} &= 0.4 \cdot 0.487 = 0.195 \\ V_{D2} &= 0.3 \cdot 0.434 = 0.130 \\ V_{D3} &= 0.3 \cdot 0.432 = 0.130\end{aligned}$$

Calculate Distances to Ideal Solutions

The Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) are determined as:

$$\begin{aligned}A^+ &= \{max(V_{ij}|j=1,2,3)\} \\ A^+ &= \{0.195, 0.130, 0.130\} \\ A^- &= \{min(V_{ij}|j=1,2,3)\} \\ A^- &= \{0.167, 0.112, 0.108\}\end{aligned}$$

For School D, calculate the distances to A^+ and A^- :

$$\begin{aligned}D_D^+ &= \sqrt{\sum_{j=1}^{m} (V_{Dj} - A_j^+)^2} \\ &= \sqrt{(0.195-0.195)^2+(0.130-0.130)^2+(0.130-0.130)^2} \\ &= \sqrt{(0)^2+(0)^2+(0)^2} \\ &= 0\end{aligned}$$

$$\begin{aligned}D_D^- &= \sqrt{\sum_{j=1}^{m} (V_{Dj} - A_j^-)^2} \\ &= \sqrt{(0.195-0.167)^2+(0.130-0.112)^2+(0.130-0.108)^2} \\ &= \sqrt{(0.028)^2+(0.018)^2+(0.022)^2} \\ &= 0.039\end{aligned}$$

Closeness Coefficient

The closeness coefficient (CC_i) is calculated as:

$$CC_D = \frac{D_D^-}{D_D^+ + D_D^-}$$

For School D:

$$CC_D = \frac{0.039}{0+0.039} = 1.0$$

Summary of Results

The heatmap in the figure 4 provides a comparative visualization of school priorities based on normalized criteria (Accessibility, Quality, and Enrollment) and the Closeness Coefficient.

School D:Shows the highest values across all criteria and the closeness coefficient, reinforcing its top priority for resource allocation.

Schools B and E:Both exhibit balanced but moderate scores across all factors, resulting in medium priority rankings.

Table 12: Final results of Closeness Coefficient with normalized priority base

School	Normalized Accessibility (C1)	Normalized Quality (C2)	Normalized Enrollment (C3)	Closeness Coefficient (CC)	Priority
A	0.415	0.421	0.394	0.44	Medium
B	0.467	0.44	0.419	0.67	Low-Medium
C	0.409	0.414	0.381	0.33	Low
D	0.487	0.434	0.432	1	High
E	0.454	0.453	0.445	0.67	Low-Medium

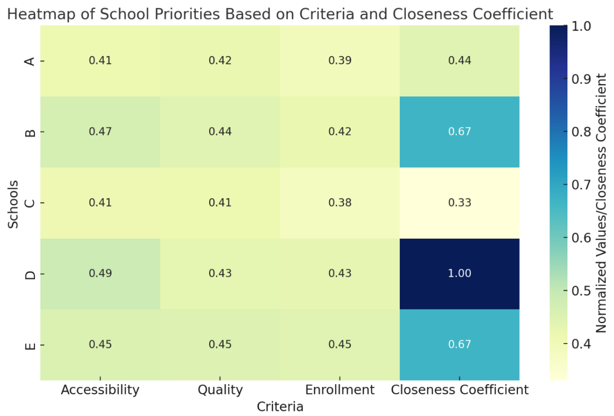

Fig. 4: School priorities based on criteria and closeness coefficient

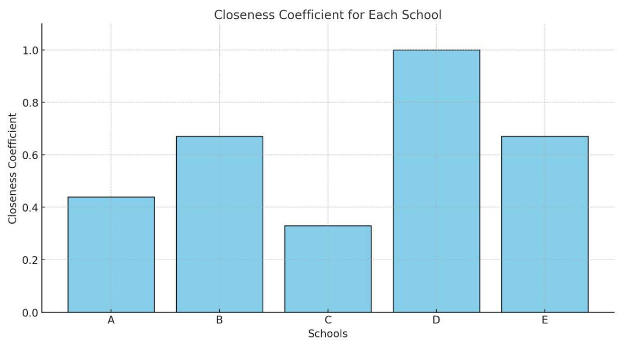

Fig. 5: Resource allocation priority analysis

School A:Displays lower values in Quality and Enrollment, consistent with its lower medium priority ranking.

School C:Exhibits the lowest values across most criteria and the closeness coefficient, justifying its lower priority.

This bar chart in the figure 5 shows the closeness coefficient for each school. The closeness coefficient indicates the relative priority of the schools based on their proximity to the ideal solution.School D has the highest closeness coefficient (1.0), highlighting its need for immediate intervention.Schools B and E follow with coefficients of 0.67, indicating moderate priority.Schools A and C have the lowest coefficients (0.44and 0.33, respectively), suggesting lower priority.

This stacked bar chart in the figure 6 represents the normalized values of the criteria (Accessibility, Quality, and Enrollment) for each school.The height of each segment corresponds to the contribution of that criterion to the school's overall evaluation.School D shows higher normalized values across all three criteria, aligning with its top priority.Schools B and E demonstrate balanced but moderate contributions, supporting their medium priority ranking.School A and C have lower contributions, especially in Quality and Enrollment, justifying their lower priority.

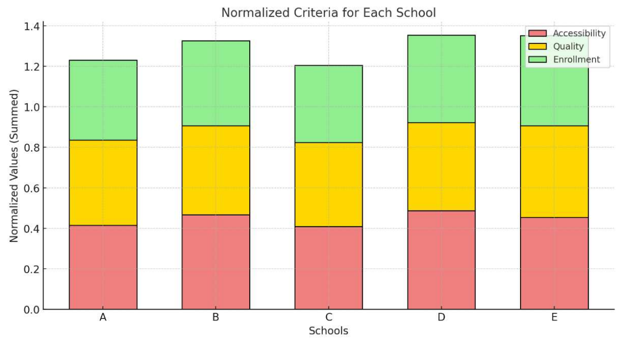

Fig. 6: Normalized criteria for each school

Explanation of Results

–**School D** receives the highest closeness coefficient (1.0), indicating that it is closest to the ideal solution and should be prioritized for resource allocation.
–**Schools B and E** are ranked second and third due to their moderate performance across all factors, requiring targeted interventions.
–**School A** is ranked fourth, reflecting moderate conditions.
–**School C** has the lowest priority due to better overall conditions relative to other schools.

Interpretations

School Rankings and Priorities:

–School D was consistently ranked as the highest priority due to poor conditions in Accessibility, Quality, and Enrollment. Its closeness coefficient ($CC = 1.0$) highlights the need for immediate intervention.
–Schools B and E showed moderate challenges across criteria, resulting in medium priority levels ($CC = 0.67$).
–School A exhibited average performance, earning a medium priority ($CC = 0.44$).
–School C had relatively better conditions, resulting in the lowest priority ($CC = 0.33$).

Fuzzy Integration Benefits:

–The fuzzy approach allowed for nuanced evaluations of ambiguous and overlapping data, ensuring equitable and rational resource allocation.
–The integration of FRBS, FCM, and Fuzzy MCDM provided a comprehensive framework, combining data-driven insights with expert knowledge.

Results and Conclusion of the case study

Results: The resource allocation percentages derived from closeness coefficients were visualized in a pie chart. The results suggested:

–School Dshould be allocated to 40% of resources.
–Schools B and Eeach should have 20% of resources
–15% resources to School A.
–The less 5% resources to School C.

Conclusion: The fuzzy model proved to be well suited to deal with the complications of equitable resources' allocation in forgotten regions. The model offered a strong locale specific decision-making framework around how Accessibility, Quality, and Enrolment were, interdependent and ambiguous criteria:

–It prioritises schools that are most in need of intervention (School D).
–The second: They allocate resources fairly between schools facing different challenges.
–They seamlessly integrate subjective and objective data, and can be flexible and adaptive to different contexts.

Fuzzy mathematics in real life: A versatile template This approach can be translated to satisfy similar tasks in several other regions or fields, demonstrating versatility of fuzzy mathematics in the resolution of real-life problems.

5 Results and Discussion

5.1 Insights from the Fuzzy Models

In this study, the developed fuzzy-based approach provided valuable insights into the challenges of equitable allocation of resources for underserved schools. The facet of managing complexity and uncertainty in real-life educational data was well handled when approaching them through fuzzy models. The Fuzzy Rule-Based System (FRBS) showed the capacity to differentiate subtle changes in terms of accessibility, quality, and enrolment. By adopting expert-driven fuzzy rules, it enabled decision-making in real-time and identification of schools in urgent need of interventions. For example, according to the provided metrics, schools performing poorly across all categories were universally prioritized while schools that were balanced were treated evenly.

The Fuzzy Cognitive Map (FCM) emphasized the causal relationships between pairs of criteria and also the dependencies among them. For example, the availability of actual accessibility improvements was found to have a benefit cascading through to enrolment and quality. By applying a core set of rubrics iteratively with several stakeholders, we were able to identify the relationship in-between condition outcome metrics, ultimately uncovering the systemic connections between process and outcome metrics. This was crucial in properly prioritizing resources for the schools since the steady-state values derived for each criterion had to relate to realistic conditions for the schools.

Fuzzy MCDM methods, especially Fuzzy TOPSIS, helped in the process of ranking these schools where based on the closeness to the ideal solutions refined the prioritization process. This approach successfully simulated the dilution of competing criteria and built a calculated closeness coefficient for every school. The results were consistent with the nuanced findings of the other models, underscoring the robustness of the combined fuzzy approach.

5.2 Policy Implications

This study has important implications for policy making along the framework of Sustainable Development Goal 4 (SDG 4), which advocates quality education for all. Herein, we provide a fuzzy-clustering-based framework which serves as a scalable and flexible decision-making tool for policymakers. This not only allows the allocation of resources to be based on data but also helps in eliminating the ambiguity and subjectivity that is usually present when it comes to Education data.

One of the core take home messages: not all resources are created equal, distribute and prioritize based on evidence, not uniform distribution. In contrast, note how all schools identified as School D across all four criteria (excluding School C, the outlier, which exhibits a more mixed performance across all categories) receive the highest share of resources demonstrating the need for targeted and systemic school improvement in these schools. Thus helping interventions to target the areas it is needed the most, maximizing their effect. Finally, setting minimum thresholds for resource allocation, as illustrated in the threshold-based strategy, ensure that no school left behind, consistent with the equity principles of the SDG 4.

The other important implication is that the fuzzy framework can be adapted to other contexts and/or more localized areas. The models are flexible and can be adjusted to reflect local priorities, such as weighting enrolment in populous areas more heavily than in rural areas with fewer infrastructure options — or vice versa when weighting quality. This ensures that the framework is equipped to provide guidance to policymakers across a range of educational contexts.

In conclusion, causal analysis is introduced via FCM that highlights the importance of holistic policy-making. Policymakers are able to design interventions that address root causes rather than symptoms. For example, improving access to education through better transportation or digital connectivity may increase enrolment and quality indirectly, creating a multiplier effect.

This fuzzy-based methodology not only satisfies the immediate goal of resource allocation, but also offers a practical and future-oriented route to sustainable educational development. It provides policymakers with actionable insights, promotes equitable practices of accountability, and supports the achievement of the overall targets of SDG 4.

6 Conclusion

6.1 Summary of Findings

In this study we examined a fuzzy-based approach to equitable resource allocation among underserved schools, relevant with the objectives of Sustainable Development Goal 4 (SDG 4). Combining FRBS, FCM, and Fuzzy MCDM offered an effective means to deal with the intrinsic complications and uncertainties present in educational information. Using this framework, the standing of every school with respect to Accessibility, Quality, and Enrolment was computed for resource allocation.

Schools that performed poorly on all criteria, like School D, were prioritized for resource allocation. Fuzzy TOPSIS provided closeness coefficients for each school, facilitating an objective ranking that resulted in efficient and fair resource allocation. The weighted resource allocation strategy, for example, showed that needs based on criteria could be addressed, where accessibility improvements could be prioritized in places that were particularly poorly connected. The integrated use of FCM pointed out the interconnections between criteria, indicating that specific actions could cascade to create positive changes throughout the STIs.

The fuzzy-based approach was therefore capable of taking into account various factors in the resource allocation process which made it flexible and scalable according to the education standards. This helped not only to make time-dynamic decisions, but it also applied subjective insight of experts and eliminated the possibility that a school id left out, thanks to the implementation of minimum thresholds for allocation algorithms.

6.2 Future Research Directions

Adding Additional Criteria: Future studies may extend upon these results to develop an enhanced ranking based on even more criteria. Aspects like socioeconomic status, cultural dynamics, infrastructure development, and community engagement might give a deeper understanding of the phenomenon, even though the present study created awareness towards accessibility, quality, and enrolment. For example, schools serving economically disadvantaged neighbourhoods might need extra weight for these underlying inequities. Likewise, integrating local cultural contexts may help to ensure resource allocation reflects the unique challenges that diverse communities face. More inclusive and context-sensitive fuzzy models would allow the set of used criteria to be broadened, which would improve their effectiveness and effectiveness.

Fuzzy Logic and Machine Learning Integration: One potential pathway is through the combination of machine learning methods with fuzzy models to improve team-oriented approaches. For instance, in many cases, machine learning algorithms have become highly effective in discovering patterns, predicting trends and providing insights from massive datasets, and fuzzy systems can manage the uncertainties and approximations in the data. As an example, predictive models may predict future enrolment rates, or identify accessibility bottlenecks that would be incorporated into the decision-making process of all fuzzy models. Such a hybrid model may yield more flexible and reliable frameworks for the optimal allocation of resources to increasingly complex and dynamic education problems.

Application in Diverse Contexts: The fuzzy-based framework suggested in this study is generic and can be adapted and validated through different contexts, for example in urban versus rural schools or in areas with contrasting educational policies and cultural dynamics. By comparing multiple countries or regions, the studies can shed light on the generalisability and adaptability of the framework. For example, resource allocation priorities might vary between densely populated urban regions and remote rural territories. Adapting the fuzzy models to these variety of contexts would allow them to remain relevant and effective, making them a tool that can be utilized for global educational equity.

Longitudinal Impact Studies: Another major direction should be longitudinal studies to try to capture the long-term consequences of the resource allocation decisions obtained from fuzzy models. Long-term tracking of outcomes would enable researchers to assess what works in helping children thrive, whether through improved student performance or increased enrolment or quality of teaching. Such feedback loop could enable fine-tuning of the auto-modelling thus preventing them from drifting apart with their goal. Longitudinal data could also provide policy makers with evidence that would help them understand the effectiveness of their resource allocation strategies over time.

Incorporating Real-Time Data: Future research can also consider real-time data with fuzzy models. Technologies such as the Internet of Things (IoT) and analytics are leading the way towards more real time monitoring capabilities within our schools. IoT-enabled systems, for instance, could monitor classroom utilization, resources usage, or even student attendance. So obtaining such data would serve as inputs to fuzzy models by which we can create dynamic/rubber-stamp models which would optimize resources by making it demand-based rather than a one-size fits all approach. This would allow the framework to respond more dynamically in the event of major shocks or crises like a natural disaster or a pandemic [34].

Interdisciplinary Collaborations: There are many areas such as education, mathematics, computer science, sociology, and public policy which could have synergy in this framework as fuzzy-based educational frameworks. Bringing together multidisciplinary teams with various knowledge sets would help enhance the models and guarantee their relevance for real challenges. For example, educators could offer perspectives on the reality of schools' logistical constraints, while sociologists could draw attention to the societal consequences of resource allocation. Thus, further research using this fuzzy modelling framework should be further enhanced by collaboration to make the fuzzy models more usable to have a greater impact and gain wider acceptance.

These future directions indicate the possibilities to extend and improve this fuzzy-based framework discussed in this study. At the same time, by tackling these challenges researchers can help make equitable, efficient and sustainable practices of allocating educational resources a worldwide phenomenon.

Acknowledgement

This research is partially funded by Zarqa University

References

[1] UNESCO, *Incheon Declaration and SDG4: Education 2030 Framework for Actio Action*,[ED-2016/WS/28] (2016). Retrieved from: https://unesdoc.unesco.org

[2] A.M. Alrabei, Green electronic auditing and accounting information reliability in the Jordanian social security corporation: the mediating role of cloud computing, *International Journal of Financial Studies*,**11**, 114 (2024).

[3] A.M. Alrabei, The mediating effect of COVID 19—pandemic on the Nexus between accounting information systems reliability and e-commerce: from the perception of

documentary credit employees, *Inf. Sci. Lett.*,**12**, 2867-2876 (2023).
[4] A.M. Alrabei, O. Jawabreh, A.M.M. Saleh, Accounting Information and Role It on Financial Reports Quality in Jordanian Hotels, and Social Performance as a Mediating Effect, *International Journal of Sustainable Development & Planning*,**18**, 2271-2279 (2023).
[5] UNESCO, *Out-of-School Children and Youth Report* (2021). Retrieved from: https://uis.unesco.org/en/topic/out-school-children-and-youth
[6] A.M.A. Alrabei, Perception of Jordanian Banks Employees on the Relationship between Accounting Information Quality (AIQ) and Documentary Credits, *International Journal of Applied Business and Economic Research*,**15**, 409-419 (2017).
[7] A.M. Alrabei, The influence of accounting information systems in enhancing the efficiency of internal control at Jordanian commercial banks, *Journal of Management Information and Decision Sciences*,**24**, 1-9 (2021).
[8] UNESCO, *Global Education Monitoring Report 2020: Inclusion and Education* (2020). Retrieved from: https://gem-report-2020.unesco.org/
[9] ILO, *World Employment and Social Outlook 2022* (2022). Retrieved from: https://www.ilo.org/
[10] World Bank, *Inclusive Education for All: Addressing Barriers* (2022). Retrieved from: https://www.worldbank.org/en/topic/education
[11] A.M. Alrabei, D.S. Ababnehi, The Moderating Effect of Information Technology on the Relationship between Audit Quality and the Quality of Accounting Information: Jordanian Auditors' Perception, *Journal of Theoretical and Applied Information Technology*,**99**, 3365-3378 (2021).
[12] A.M. Alrabei, L.N. Al-Othman, T.A. Abutaber, M.S. Alathamneh, T.M. Almomani, M.H. Qeshta, S.A.M. Amareen, Nexus between Intellectual Capital and Financial Performance Sustainability: Evidence from Listed Jordanian Firms. *Appl. Math.*,**17**, 881-888 (2023).
[13] A.M. Alrabei, L.N. Al-Othman, F.A. Al-Dalabih, T. Taber, B.J. Ali, S.A.M. Amareen, The impact of mobile payment on the financial inclusion rates, *Information Sciences Letters*,**11**, 1033-1044 (2022).
[14] X. Chen, K.C. Lee, AI-Driven Adaptive Education: A Review, *Journal of Educational Technology*,**23**, 45-67 (2020).
[15] World Economic Forum, *The Role of AI in Inclusive Education* (2021). Retrieved from: https://www.weforum.org/
[16] A. Jahmani, O. Jawabreh, R. Abokhoza, A.M. Alrabei, The impact of marketing mix elements on tourist's satisfaction towards Five Stars Hotel Services in Dubai during COVID-19, *Journal of Environmental Management & Tourism*,**14**, 335-346 (2023).
[17] A.M. Alrabei, A.A.A. Haija, L.A. Aryan, The mediating effect of information technology on the relationship between organizational culture and accounting information system, *International Journal of Advanced Science and Technology*,**29**, 1085-1095 (2020).
[18] O. Zawacki-Richter, V.I. Marín, M. Bond, F. Gouverneur, Systematic Review of Research on AI Applications in Higher Education, *International Journal of Educational Technology in Higher Education*,**16**, 39 (2019).
[19] T. Almomani, M. Almomani, M. Obeidat, M. Alathamneh, A. Alrabei, M. Al-Tahrawi, D. Almajali, Audit committee characteristics and firm performance in Jordan: The moderating effect of board of directors' ownership, *Uncertain Supply Chain Management*,**11**, 1897-1904 (2023).
[20] A.A.S. Mohammad, K.I. Al-Daoud, S.I.S. Al-Daoud, T.A. Samarah, A. Vasudevan, M. Li, Content Marketing Optimization: A/B Testing and Conjoint Analysis for Engagement Strategies in Jordan, *Journal of Ecohumanism*,**3**, 3086-3099 (2024).
[21] O. Taylan, S. Yildirim, Fuzzy Multi-Criteria Decision-Making for SDG-Oriented Educational Policies, *Decision Support Systems*,**95**, 121-133 (2017).
[22] A.A.S. Mohammad, K.I. Al-Daoud, S.I.S. Mohammad, A. Hindieh, A. Vasudevan, W. Zhou, Analysing the Effectiveness of Omnichannel Marketing Strategies on Customer Experience in Jordan, *Journal of Ecohumanism*,**3**, 3074-3085 (2024).
[23] A.A.S. Mohammad, M.N. Alolayyan, K.I. Al-Daoud, Y.M. Al Nammas, A. Vasudevan, S.I. Mohammad, Association between Social Demographic Factors and Health Literacy in Jordan, *Journal of Ecohumanism*,**3**, 2351-2365 (2024).
[24] C. Pappis, S. Subramanian, Quantifying Educational Quality Using Fuzzy Sets: A Case Study, *Applied Soft Computing*,**15**, 234-246 (2022).
[25] A.A.S. Mohammad, I.A. Khanfar, K.I. Al-Daoud, M. Odeh, S.I. Mohammad, A. Vasudevan, Impact of perceived brand dimensions on Consumers' Purchase Choices, *Journal of Ecohumanism*,**3**, 2341-2350 (2024).
[26] A.A. Mohammad, S.I. Shelash, T.I. Saber, A. Vasudevan, N.R. Darwazeh, R. Almajali, Internal Audit Governance Factors and their effect on the Risk-Based Auditing Adoption of Commercial Banks in Jordan, *Data and Metadata*,**4**, 464 (2025).
[27] X. Chen, K.C. Lee, Fuzzy Logic Applications in Adaptive Education, *International Journal of Emerging Technologies in Learning*,**13**, 123-132 (2018).
[28] R.H. Samseer, J. Bamini, R. Priyadharsini, A. Vasudevan, S. Parahakaran, H.N. Krishnasamy, S.I. Mohammad M.T. Alshurideh, A New Conceptualization of Self as an Energetic Node in Cultural Dynamics: Transitioning from Classical Theories to Complex Systems, *Applied Mathematics & Information Sciences*,**19**, 259-270 (2025).
[29] K.M. Al-Sawaie, N.A. Abbas, M. AlSmeiran, H. Alrawashdeh, A. Vasudevan, H. Yaseen, S.I.S. Mohammad, M.F. Hunitie, Y. Li, Estimating Potential Output Using a Production Function Approach, *Applied Mathematics & Information Sciences*,**19**, 271-278 (2025).
[30] R. Kumar, P. Malik, N. Verma, Fuzzy Optimization Models for Resource Allocation in Education, *Journal of Educational Technology*,**12**, 198-209 (2020).
[31] A.A.S. Mohammad, M.N. Alolayyan, K.I. Al-Daoud, Y.M. Al Nammas, A. Vasudevan, S.I. Mohammad, Association between Social Demographic Factors and Health Literacy in Jordan, *Journal of Ecohumanism*,**3**, 2351-2365 (2024).
[32] A.A.S. Mohammad, I.A. Khanfar, B. Al Oraini, A. Vasudevan, I.M. Suleiman, M. Ala'a, User acceptance of health information technologies (HIT): an application of the theory of planned behavior, *Data and Metadata*,**3**, 394-394 (2024).

[33] S. Das, M.K. Jha, R. Gupta, AI-Fuzzy Hybrid Systems for Personalized Learning: A Review, *Education and Information Technologies*,**26**, 2151-2178 (2021).
[34] V. Asokan, M. Rashmi, S.I. Mohammad, N. Yogeesh, N. Raja, D.K. Girija, J. Divyashree, M.M. Abualhaj, M.T. Alshurideh, Maximizing efficiency using fuzzy matrix optimization for wireless resource allocation, *Applied Mathematics & Information Sciences*,**18**, 1495–1506 (2024).
[35] L.A. Zadeh, Fuzzy Sets. *Information and Control*,**8**, 338–353 (1965).
[36] N. Yogeesh, D.K. Girija, M. Rashmi, P. William, *Intelligent Irrigation Systems in Agriculture Using Fuzzy Logic Techniques*. In Shukla, B., Murthy, B.K., Hasteer, N., Kaur, H., Van Belle, JP. (eds) Intelligent IT Solutions for Sustainability in Industry 5.0 Paradigm. ICEIL 2023. Lecture Notes in Electrical Engineering, vol 1185. Springer, Singapore (2024).
[37] J.S.R. Jang, C.T. Sun, E. Mizutani, *Neuro-Fuzzy and Soft Computing*, Prentice Hall (1997).
[38] H.J. Zimmermann, *Fuzzy Set Theory—and Its Applications*, Springer (1991).
[39] A. Kaufmann, M.M. Gupta, *Introduction to Fuzzy Arithmetic: Theory and Applications*, Van Nostrand Reinhold (1985).
[40] J,C. Bezdek, *Pattern Recognition with Fuzzy Objective Function Algorithms*, Springer (1981).
[41] B. Kosko, Fuzzy Cognitive Maps, *International Journal of Man-Machine Studies*,**24**, 65-75 (1986).
[42] R. Luckin, W. Holmes, M. Griffiths, L.B. Forcier, *Intelligence Unleashed: An Argument for AI in Education*, Pearson Education (2016).
[43] W. Holmes, M. Bialik, C. Fadel, *Artificial Intelligence in Education: Promises and Implications for Teaching and Learning*, Center for Curriculum Redesign (2019).
[44] N. Yogeesh, *Solving Fuzzy Nonlinear Optimization Problems Using Evolutionary Algorithms*, In G. Mukherjee, B. Basu Mallik, R. Kar, & A. Chaudhary (Eds.), Advances on Mathematical Modeling and Optimization with Its Applications (1st ed., pp. 20). CRC Press (2024).

Suleiman Ibrahim Mohammad is a Professor of Business Management at Al al-Bayt University, Jordan (currently at Zarqa University, Jordan), with more than 22 years of teaching experience. He has published over 400 research papers in prestigious journals. He holds a PhD in Financial Management and an MCom from Rajasthan University, India, and a Bachelor's in Commerce from Yarmouk University, Jordan. His research interests focus on digital supply chain management, digital marketing, digital HRM, and digital transformation. His ORCID ID is orcid.org/0000-0001-6156-9063.

N. Yogeesh currently serving as the Head of the Mathematics Department and Assistant Professor at Government First Grade College, Tumkur, Karnataka, India, has been an influential figure in academia since 2006. His extensive contributions span key leadership roles at Tumkur University and other prominent institutions. Dr. Yogeesh has served as Chairperson and Board Member for Studies and Examinations, where he has significantly influenced academic policies and curriculum development.
In addition to his administrative achievements, Dr. Yogeesh has organized numerous state and national seminars, edited academic journals, authored books and laboratory manuals utilizing Free and Open-Source Software (FOSS), and published impactful research papers in renowned journals. His collaborative efforts with the Department of Higher Education, Government of Karnataka, in various coordinating capacities further highlight his dedication to advancing higher education. He is also a recognized contributor to academic publications, serving as Editor and Editorial Board Member for multiple esteemed journals. His ORCID ID is orcid.org/0000-0001-8080-7821.

N. Raja has 19 years of experience in education and the media industry. Currently an Assistant Professor in the Department of Visual Communication at Sathyabama University, he has published 25 papers in Scopus, UGC, Google Scholar journals and 6 book chapters. Around 100 television programs edited and produced during his time as a Video Editor at Jesus Calls. Dr. Raja holds an MSc in Electronic Media, an M.Phil. in Journalism and Mass Communication, a PG Diploma in Public Relations, and a PhD in Communication from Bharathiar University, where his research focused on the impact of social media as an educational tool for media students in Tamil Nadu. His ORCID ID is orcid.org/0000-0003-2135-3051.

P. William is working as Dean, Research & Development at Sanjivani College of Engineering affiliated to SPPU, Pune. He is the Post Doctoral Fellow from Amity University Dubai, UAE and Adjunct faculty of Victorian Institute of Technology, Australia. He is recognized in World Top 2% Scientist list by Stanford University and Elsevier. He is a member of IEEE, QCFI, ISTE and various other professional bodies. His research includes innovation and development of cutting-edge solutions in the fields of natural language processing, artificial intelligence, deep learning, machine learning, soft computing, cybersecurity, and cloud computing. He has published 160+ papers in Scopus indexed journals and Conferences. He has 30+ patents published with grants in his credit. He has authored and edited 10+ books with renowned publishers of global recognition. He has been associated with numerous Multi-National Companies and various Educational Groups for his expertise in research, corporate training and consulting where he has contributed to the advancement of knowledge and practice in his domain. A focused professional with experience of consulting in Research, Innovation and Development. He served as a Chairperson and Auditor in multiple committees of national recognition. Delivered Keynote speeches and chaired many sessions in International Conferences. He was appointed as Series Editor, Guest Editor and Reviewer in multiple indexed journals. His ORCID ID:orcid.org/0000-0002-0610-0390.

M. S. Ramesha is an Associate Professor of Mathematics at Government First Grade College, Kuvempunagara, Mysore – 570023, Karnataka, India. An academic of great repute and administrative experience, he has been a member of the Board of Studies (BOS) and Board of Examiners (BOE) for autonomous colleges and a number of universities. He is an active participant in conferences, seminars and faculty enrichment programs. A vexed researcher, he carried out a number of studies in reputed journals. He has also been been a coordinator and committee member in a few different committees, where he has exhibited leadership and contributed to institutional development.

Asokan Vasudevan is a distinguished academic at INTI International University, Malaysia. He holds multiple degrees, including a PhD in Management from UNITEN, Malaysia, and has held key roles such as Lecturer, Department Chair, and Program Director. His research, published in esteemed journals, focuses on business management, ethics, and leadership. Dr. Vasudevan has received several awards, including the Best Lecturer Award from Infrastructure University Kuala Lumpur and the Teaching Excellence Award from INTI International University. His ORCID ID is orcid.org/0000-0002-9866-4045.