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Divided Square Divisor Cordial and Fibonacci Prime Labeling of Theta Graphs in Python

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Abstract: Let $\Omega = (W(\Omega), F(\Omega))$ be a graph and let h from the node set $W(\Omega)$ to set of 1, 2, ... up to total count of nodes be a one–one correspondence function. For each arc $f = ab$, give it a label of 1 if the absolute value of the square of $h(a)$ minus the square of $h(b)$, divided by the difference between $h(a)$ and $h(b)$ is uneven and label 0 when it is even. If the discrepancy between arcs classified 0 and 1 is no more than 1, the function h is called divided square *DC* labeling. Divided square *DC* graph has the divided square *DC* labeling. In this paper, we explore divided square *DC* labeling in the theta graph and its variations formed by merging nodes within its cycle and altering its central node and analyze its application in small social networks. Also, Fibonacci prime labeling of a graph $\Omega = (W(\Omega), F(\Omega))$ with $|W(\Omega)| = n$ is a one-to-one function $h(\Omega) \to \{f_2, f_3, ..., f_{n+1}\}$ with f_n representing the n^{th} Fibonacci number. This labeling induces a function $h*(\Omega) \to N$ defined as $h*(cd)$= maximum common divisorof $h(c)$ and $h(d)$, for all $cd \in F(\Omega)$. A graph Ω that admits a Fibonacci prime labeling is referred to as a Fibonacci prime graph. Additionally, we examine Fibonacci prime labeling in theta related graphs supported by a Python implementation.

Keywords: Divided square divisor cordial labeling, Fibonacci prime labeling, tiny social networks, Python programming, Drug.

1 Introduction

Graph theory is a rapidly expanding field with applications spanning various domains of mathematics, science, and technology. It has been extensively employed in biological chemistry, chemical studies, communication setups, encoding principles, informatics (particularly in algorithms and computing), and operational inquiry (such as timetabling). Moreover, chart philosophy is employed in encoding principles, X-ray diffraction analysis, radar, celestial studies, electronic layout, communication grid allocation, and data administration amidst myriad other domains [1,2]. It is relevant in computer science and intersects with various engineering disciplines and geographic analysis. It encompasses the study of distance and time within the framework of road network optimization, employing relational strategies for thorough evaluation [3,4,5,6].

In biological networks, nodes represent biomolecular entities such as chromosomes, proteins, or metabolites, while a denrcsote the interactive, physical, or chemical connections between these biomolecules. Graph theory finds application in transcriptional regulatory and metabolic networks, facilitating analysis and understanding. Additionally, graph theory is instrumental in studying protein-protein interaction (PPI) networks, allowing for the characterization of drug-target interactions and drug-goal partnerships [7,8,9,10]. The

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branch of mathematics known as graph theory investigates networks composed of vertices connected by edges. In "Graph Theory in Network Analysis," Barnes and Harary [11] examine the application of graph theory to network analysis. They explore how graph theoretical concepts help model and understand complex networks, focusing on using graph structures to analyze relationships and interactions. The paper highlights various graph models and their practical relevance to real-world network problems [11,12,13]. Sadavare and Kulkarni [14] discuss the critical role of efficient route planning in modern networks, emphasizing the use of graph theory to address increasing complexity and costs.

They review various shortest-path algorithms, such as Dijkstra's and Bellman-Ford's, and their applications in different network systems. The paper also suggests areas for further research on these topics. Researchers can utilize graph theory to model and analyze the configuration of a network [15,16,17]. Graph theory is quickly becoming a central area of mathematics, primarily due to its applications across various disciplines such as Biology, Electronic Engineering, Communication Networks, and Mathematics. The extensive range of these and other applications is well-established. In particular, research areas in computer science, including knowledge discovery in data, image region, clustering, image capturing, and networking, rely heavily on graph theory. For instance, a data structure can be organized as a tree, which inherently uses vertices and edges [18]. Graph theory is ideally suited for representing numerous concepts in computer science. In this regard, we introduce a project where we develop a computational technique for solving electric circuits using graph-theoretic principles. This interdisciplinary approach integrates abstract mathematics, linear algebra, circuit physics, and computer programming to accomplish the ambitious objective of automated circuit solving [19,20,21,22].

Due to its extensive range of applications, graph labeling is becoming an increasingly appealing field. Labeled graphs have played a crucial role in various areas of graph theory. Some notable fields that benefit from graph labeling include coding theory, missile guidance codes, the design of efficient radar codes, astronomy, circuit design, X-ray crystallography, and database management [23,24]. Additionally, the conditions for labeling can be verified using Python programming. Python programming [25,26] has generated over a thousand patterns of Graceful and Odd Even Graceful labeling patterns for the Grotzsch graph and Peterson Graph. Dürr and Vie [27] have demonstrated a Python program code capable of producing the vertex labels for the union of a subdivided star graph and a bistar graph. Mohammad et al. [28] and Maheswari & Purnalakshimi [29] have illustrated that combining Graph theory with Chaldean numerology and arithmetic number labeling can encrypt text. Mohammad et al. [30] and Aanisha & Manoharan [31] used Python to investigate edge sum divisor cordial labeling of circular ladder CLs, where s is even, subdivisions of the star S(K1,s), bistar graphs, Bs,s graph where s is oddand the star graph where s is even which are proven to be ESDC graphs. Additionally, graph theory is often used in social networks. A social network is a structure where the nodes are a group of social actors connected through various kinds of relationships [32,33,34].

A weighted, labeled, directed graph is always used to depict social networks due to the intricacy of the actors and the relationships among them. Before the Internet, researchers in sociology studied small group structures, as analyzing larger groups was challenging. Traditional networks identified importance based on a person's connections, with centrality indicating influence. To address limitations, modern social networks, like online communities, have emerged [35,36]. Graph labeling encompasses various types such as cordial, prime, graceful, and harmonious labeling. Among these, Fibonacci labeling is a notable category. Numerous other labeling methods are in graph theory like Sum Divisor Cordial Labeling or SDC labeling [31], Edge Sum Divisor Cordial Labeling [37], Fibonacci Prime Anti-magic Vertex Labeling [22,38,39], Odd Fibonacci Mean Labeling [40,41], Kth Fibonacci Prime Labeling [42], Radio Mean labeling [43,44,45].

2 Literature Review

Fibonacci prime labeling of a graph $\Omega = (W(\Omega), F(\Omega))$ with $|W(\Omega) = n|$ is a one–to-one function $h(\Omega) \rightarrow \{f_2, f_3, \ldots f_{n+1}\}$ with fn representing the nth Fibonacci number. This labeling induces a function $h*(\Omega) \rightarrow N$ defined as $h*(cd)$= maximum common divisor of $h(c)$ and $h(d)$, for all $cd \in F(\Omega)$. A graph Ω that admits a Fibonacci prime labeling is referred to as a Fibonacci prime graph [46].

Let $\Omega = (W(\Omega), F(\Omega))$ be a graph that is neither loop nor multiple lines(arcs), where $W(\Omega)$ represents the node set and $F(\Omega)$ represents the arc set and let h from the node-set $W(\Omega)$ to set of 1, 2, ... up to total count of nodes is a one-one correspondence function. For each arc $f = ab$, give it a label of 1 if the absolute value of the square of $h(a)$ minus the square of $h(b)$, divided by the difference between $h(a)$ and $h(b)$ is uneven and label 0 when the absolute value of the square of $h(a)$ minus the square of $h(b)$, divided by the difference between $h(a)$ and $h(b)$ is even. If the discrepancy between arcs classified 0 and 1 is no more than 1, the function h is called divided square divisor cordial labeling or divided square DC labeling. A divided square divisor cordial graph or divided square DC graph has the divided square DC labeling [47].

A theta graph is characterized by a block featuring two non-adjacent vertices of degree 3, with all other vertices possessing a degree of 2 [48].

Given two distinct nodes, u and v, within a graph Ω, a new graph Ω_1 is constructed by merging these nodes into a single node x in Ω_1, where all edges incident with either u or v in Ω are now incident with x in Ω_1 [48].

The node switching operation in a graph Ω results in a new graph Ω_v, achieved by removing all edges incident with a chosen node v in Ω and introducing edges connecting v to every non-adjacent node in Ω [48].

3 Methodology

The methodology is to prove the theta-related graphsare divided square DC by checking if the number of edges labeled differently does not exceed one and analyze their applicability to small social networks. Additionally, we investigate Fibonacci prime labeling in similar graphs and verify with python implementation.

4 Data Analysis

4.1 Divided Square Divisor Cordial Labeling in Theta–Related Graphs

In this section, we will prove that Theta graph and cycle combining any two nodes in the graph T_a are divided square divisor cordial labeling and also investigate its applications in tiny social networks. The application of divided square DC labeling in a tiny social network depends on the specific network structure and the assignment of labels to nodes.

4.1.1 Theorem

The theta graph T_a is divided square DC graph.

Proof
Let $\Omega = T_a$ be a theta graph with center a_0. Let $W(\Omega) = \{a_0, a_1, a_2, a_3, a_4, a_5, a_6\}$ be the node set and $F(\Omega) = \{a_k \quad a_{k+1} : 1 \le k \le 5\} \cup \{a_0 \quad a_1, a_0 \quad a_4, a_1 \quad a_6\}$ be the arc set. Then cordinality of the node set is 7 and cordinality of the arc set is 8. Define $h : W(\Omega) \to \{1, 2, \dots |W(\Omega)|\}$ as follows:
$h(a_0) = 7; h(a_{4k-3}) = 3k : 1 \le k \le 2 : h(a_{2k}) = 4k-3 : 1 \le k \le 2; h(a_{3k}) = 2k : 1 \le k \le 2$

Then the induced arc labels are
$h*(a_0 \quad a_{3k-2}) = 0 : 1 \le k \le 2; h*(a_{4k-3} \quad a_{4k-2}) = 0 : 1 \le k \le 2; h*(a_1 \quad a_6) = 1; h*(a_{k+1} \quad a_{k+2}) = 1 : 1 \le k \le 3$

Therefore $f_h(0) = f_h(1) = 4$, where f represents arbitrary arc. Here, $f_h(0)$ represents the count of arcs categorized with 0 and $f_h(1)$ represents the count of arcs categorized with 1. So,
$|f_h(0) - f_h(1)| = |4-4| = 0 \le 1$

Hence, the theta graph T_a is divided square DC graph.

Example
The divided square DC labeling of theta graph T_a is shown in Figure 1.

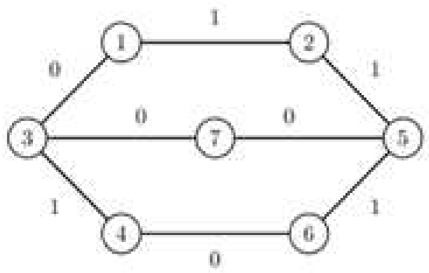

Fig. 1: Divided square DC labeling of theta graph T_a

From Figure 1,
$|f_h(0) - f_h(1)| = |4-4| = 0 \le 1.$
So, we conclude that theta graph T_a is divided square DC graph.

4.1.2 Theorem

The merging of couple of nodes within the cycle of theta graph T_a is a divided square DC graph.

Proof
Let $\Omega = T_a$ be a theta graph with center a_0. Let $W(\Omega) = \{a_0, a_1, a_2, a_3, a_4, a_5, a_6\}$ be the node set and $F(\Omega) = \{a_k \quad a_{k+1} : 1 \le k \le 5\} \cup \{a_0 \quad a_1, a_0 \quad a_4, a_1 \quad a_6\}$ be the arc set of T_a. Then cordinality of the node set of Ta is 7 and cordinality of the arc set of Ta is 8. Let Ω_1 be a graph obtained by fusion of two nodes a_2 and a_3 in the cycle of Ta and we call it as node a_2. Then cordinality of the node set of Ω_1 is 6 and cordinality of the arc set Ω_1 is 7.
Define $h : \quad W(\Omega) \to \{1, 2, \dots |W(\Omega)|\}$ as follows:
$h(a_0) = 5; h(a_1) = 1; h(a_{3k-1}) = 2k; 1 \le k \le 2; h(a_{2k+2}) = 12/2i; 1 \le k \le 2$

Then the induced arc labels are
$h*(a_0 \quad a_4) = 1; h*(a_{4k-3} \quad a_{4k-2}) = 1; 1 \le k \le 2; h*(a_4 \quad a_{3k-1}) = 0; 1 \le k \le 2; h*(a_1 \quad a_{6k}) = 0; 0 \le k \le 1$

Therefore $f_h(0) = 4$ and $f_h(1) = 3$, where f represents an arbitrary arc. Here, $f_h(0)$ represents the count of arcs categorized with 0 and $f_h(1)$ represents the count of arcs categorized with 1. So,
$|f_h(0) - f_h(1)| = |4-3| = 1 \le 1$
Hence, the merging of couple of nodes within the cycle of theta graph T_a is a divided square DC graph.

Example

The merging of a couple of nodes within the cycle of theta graph T_a is shown in Figure 2.

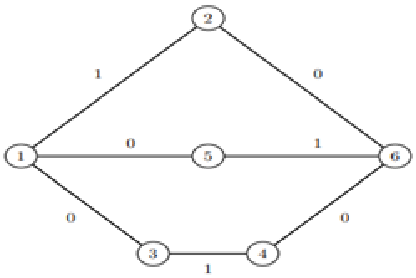

Fig. 2: Divided square DC labeling of the merging of couple of nodes within the cycle of theta graph T_a

From Figure 2,

$|f_h(0) - f_h(1)| = |4-3| = 0 \leq 1$

So, we conclude that the merging of a couple of nodes within the cycle of theta graph T_a is divided square DC graph.

4.2 Application of Divided square DC labeling in Tiny Social Networks

The application of divided square DC labeling in a tiny social network depends on the specific network structure and the assignment of labels to nodes. A divided square DC labeling in a tiny social network can be viewed as follows. Each person in a tiny social network is represented by a node, and the connections between them are represented by arcs. Now, assign labels to each person(node) such that the difference between the square of the adjacent nodes (connected individuals) follows the rules of divided square divisor cordial labeling [49].

–Assigning labels: Assign each person in the network a label from the set $\{1,2,..n\}$, where n is total number of individuals in the network. This assignment should be one-to–one correspondence function, meaning each person gets a unique label.

–Edge Labeling: Now consider each arc between individuals. Label the arc with 1 if the difference between the square of the adjacent nodes is divisible by difference between those adjacent nodes and give it the label 0 if it is.

–Main Condition: The discrepancy between arcs classified 0 and 1 is no more than 1, following the properties of divided square divisor cordial labeling.

–The following graph (Figure 3) is an example of tiny social network with divided square difference cordial labeling [49].

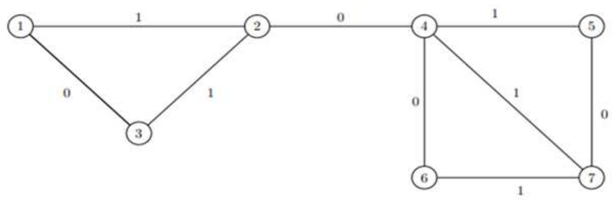

Fig. 3: Divided square divisor cordial labeling in example of Tiny Social Network

Not all the tiny social network figure will necessarily satisfy the condition for divided square divisor cordial labeling. Whether a particular social network graph satisfies this condition depends on the arrangement of nodes and arc within the graph and how labels are assigned to the nodes. To determine if a given social network satisfies the divided square divisor cordial labeling condition the following conditions could need to

–Assign labels to each node in the graph in a way that forms a one–to–one correspondence function h from the node-set $W(\Omega)$ to set of 1, 2, . . .up to the total count of nodes.

–Label the arc with 1 if the difference between the square of the adjacent nodes is divisible by difference between those adjacent nodes and give it the label 0 if it is.

–Make sure that the discrepancy between arcs classified 0 and 1 is no more than 1.

–If these conditions are met, then that social network graph can be said to have a divided square DC labeling.

4.3 Fibonacci Prime Labeling in Theta–Related Graphs

In this section, we will prove that Theta graph, cycle combining any two nodes in the graph Ta and the alteration of a central node in Taare Fibonacci Prime labeling and, we explore the Fibonacci prime graphs of these structures using Python programming.

4.3.1 Theorem

T_a graph is a Fibonacci prime graph

Proof

Let $\Omega = T_a$ be a theta graph with centre v_0. Let $W(T_a) = \{z_0, z_1, z_2, z_3, z_4, z_5, z_6\}$ be the node set and $F(T_a) = \{f_k = z_k \quad z_{k+1} : 1 \leq k \leq 5; f_{k+6} = z_0 \quad z_{3k} : 1 \leq k \leq 2; f_6 = z_1 \quad z_6\}$ be the line set of theta graph T_a.

Define $h : W(\Omega) \to \{f_2, f_3, ...f_{n+1} = f_8\}$ by

$h(z_1) = f_3; h(z_2) = f_4; h(z_3) = f_6; h(z_4) = f_2; h(z_5) = f_8; h(z_6) = f_7; h(z_0) = f_5$

Then, the induced maximum common divisor of $h(c)$ and

$h(d)$.
Nowmaximum common divisor of $h(z_1)$ and $h(z_2)$=maximum common divisor of f_3 and $f_4 = 1$
maximum common divisor of $h(z_2)$, $h(z_3)$=maximum common divisor of f_4 and f_6=1
maximum common divisor of $h(z_3)$ and $h(z_4)$=maximum common divisor of f_6 and $f_2 = 1$
maximum common divisor of $h(z_4)$ and $h(z_5)$=maximum common divisor of f_2 and $f_8 = 1$
maximum common divisor of $h(z_5)$ and $h(z_6)$=maximum common divisor f_8 and $f_7 = 1$
maximum common divisor of $h(z_6)$ and $h(z_1)$=maximum common divisor of f_7 and $f_3 = 1$
maximum common divisor of $h(z_0)$ and $h(z_6)$=maximum common divisor of f_5 and $f_7 = 1$
maximum common divisor of $h(z_0)$ and $h(z_3)$=maximum common divisor of f_5 and $f_6 = 1$
Thus, $h*(cd)$= maximum common divisor of $g(c)$ and $g(d) = 1$, for all $cd \in F(\Omega)$.
Hence, theta graph T_a is Fibonacci prime graph.

Example

Fibonacci prime labeling of T_a is shown in Figure 4

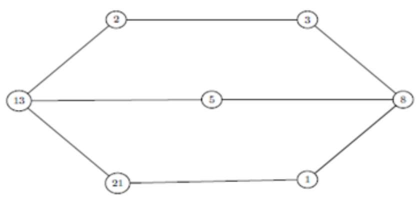

Fig. 4: Fibonacci Prime Labeling of theta graph T_a

From Figure 4, maximum common divisor of all the adjacent vertices is one.

Hence theta graph T_a is a Fibonacci Prime Labeling.

4.3.2 Theorem

The merging any two nodes in T_a graph cycle is Fibonacci prime graph.

Proof

Let T_a be the theta graph with centre z_0. Let $W(T_a) = \{z_0, z_1, z_2, z_3, z_4, z_5, z_6\}$ be the node set and $\{f_k = z_k \quad z_{k+1} : 1 \leq k \leq 5; f_{k+6} = z_0 z_{3k} : 1 \leq k \leq 2; f_6 = z_1 \quad z_6\}$ be the line set of theta graph T_a.
Here, $|W(T_a)| = 7$ and $|F(T_a)| = 8$
Let Ω be a graph obtained by fusion of two nodes z_1 and z_2 in the cycle of T_a and we call it as vertex z_1.
Then, $|W(T_a)| = 6$ and $|F(T_a)| = 7$
Define $h : W(\Omega) \to \{f_2, f_3, ... f_{n+1} = f_7\}$ by
$h(z_1) = f_2; h(z_3) = f_6; h(z_4) = f_7; h(z_5) = f_5; h(z_6) = f_3; h(z_0) = f_4$
Then, the induced function $h* : F(\Omega) \to N$ is defined such that for each edge $cd \in F(\Omega)$, $h*(cd)$= maximum common divisor of $h(c)$ and $h(d)$.
Now, maximum common divisor of $h(z_1)$ and $h(z_3)$=maximum common divisor of f_2 and $f_6 = 1$
maximum common divisor of $h(z_3)$ and $h(z_4)$=maximum common divisor of f_6 and $f_7 = 1$
maximum common divisor of $h(z_4)$ and $h(z_5)$=maximum common divisor of f_7 and $f_5 = 1$
maximum common divisor of $h(z_5)$ and $h(z_6)$=maximum common divisor of f_5 and $f_3 = 1$
maximum common divisor of $h(z_6)$ and $h(z_1)$=maximum common divisor of f_3 and $f_2 = 1$
maximum common divisor of $h(z_6)$ and $h(z_0)$=maximum common divisor of f_3 and $f_4 = 1$
maximum common divisor of $h(z_0)$ and $h(z_3)$=maximum common divisor of f_4 and $f_6 = 1$
Thus, $h*(cd)$ = maximum common divisor of $g(c)$ and $g(d) = 1$, for all $cd \in F(\Omega)$.
Hence, the merging any two nodes in T_a graph cycle is a Fibonacci prime graph.

Example

Fibonacci prime labeling of the merging any two nodes in T_a graph cycle is shown in Figure 5

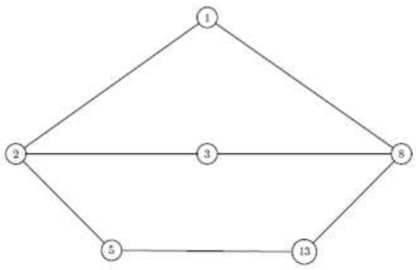

Fig. 5: Fibonacci Prime labeling of the merging of couple of nodes within the cycle of theta graph T_a

From Figure 5, the maximum common divisor of all the adjacent vertices is one.

Hence, we conclude that merging of couple of nodes within the cycle of theta graph T_a is Fibonacci Prime Labeling.

4.3.3 Theorem

The alteration of a central node in T_a graph is Fibonacci prime graph.

Proof

Let T_a be the theta graph with centre z_0.
Let $W(T_a) = \{z_0, z_1, z_2, z_3, z_4, z_5, z_6\}$ be the node set and $\{f_k = z_k \quad z_{k+1} : 1 \leq k \leq 5; f_{k+6} = z_0 \quad z_{3k} : 1 \leq k \leq$

$2; f_6 = z_1 \quad z_6\}$ be the line set of theta graph T_a.
Here, $|W(Ta)| = 7$ and $|F(Ta)| = 8$
Let Ω be a graph obtained from T_a after switching the central node z_0 of T_a.
Define $h : W(\Omega) \to \{f_2, f_3, ...f_{n+1} = f_8\}$ by
$h(z_1) = f_2; h(z_2) = f_4; h(z_3) = f_6; h(z_4) = f_7; h(z_5) = f_5; h(z_6) = f_8; h(z_0) = f_3$
Then, the induced function $h* : F(\Omega) \to N$ is defined such that for each edge $cd \in F(\Omega)$, $h*(cd) =$ of $h(c)$ and $h(d)$
Now, maximum common divisor of $h(z_1)$ and $h(z_2)$=maximum common divisor of f_2 and $f_4 = 1$
maximum common divisor of $h(z_2)$ and $h(z_3)$=maximum common divisor of f_4 and $_6 = 1$
maximum common divisor of $h(z_3)$ and $h(z_4)$=maximum common divisor of f_6 and $f_7 = 1$
maximum common divisor of $h(z_4)$ and $h(z_5)$=maximum common divisor of f_7 and $f_5 = 1$
maximum common divisor of $h(z_5)$ and $h(z_6)$=maximum common divisor of f_5 and $f_8 = 1$
maximum common divisor of $h(z_6)$ and $h(z_1)$=maximum common divisor of f_8 and $f_2 = 1$
maximum common divisor of $h(z_0)$ and $h(z_1)$=maximum common divisor of f_3 and $f_2 = 1$
maximum common divisor of $h(z_0)$ and $h(z_2)$=maximum common divisor of f_3 and $f_4 = 1$
maximum common divisor of $h(z_0)$ and $h(z_4)$=maximum common divisor of f_3 and $f_7 = 1$
maximum common divisor of $h(z_0)$ and $h(z_5)$=maximum common divisor of f_3 and $f_5 = 1$
Thus, $h*(cd) =$ maximum common divisor of $g(c)$ and $g(d) = 1$, for all $cd \in F(\Omega)$.
Hence, the alteration of a central node in T_a graph is a Fibonacci prime graph.

Example

Fibonacci prime labeling of the alteration of a central node in T_a graph is shown in Figure 6.

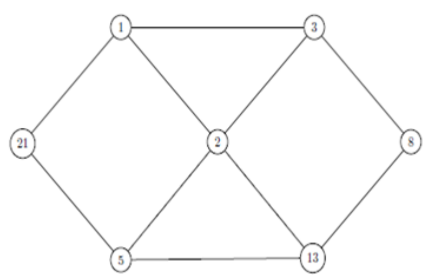

Fig. 6: Fibonacci Prime labeling of alteration of a central node in T_a

From Figure 6, the maximum common divisor of all the adjacent vertices is one.

Hence, we conclude that the alteration of a central node in T_a is Fibonacci Prime Labeling.

4.4 Exploring edge sum divisor cordial labeling in graphs using Python

Python Implementation to Verify Fibonacci prime labeling of the merging of couple of nodes within the cycle of theta graph T_a:6.1

```
import math
from functools import lru_cache
# Step 1: Generate Fibonacci numbers up to the required index
@lru_cache(None)
def fibonacci(n):
if n == 0:
return 0
elif n == 1:
return 1
else:
return fibonacci(n - 1) + fibonacci(n - 2)
# Generate Fibonacci sequence up to f8 (since we need up to f(n+1) with n = 7)
fib_sequence = [fibonacci(i) for i in range(9)]
# Step 2: Define the vertex labels according to h
vertex_labels = {
'z1': fib_sequence[3],
'z2': fib_sequence[4],
'z3': fib_sequence[6],
'z4': fib_sequence[2],
'z5': fib_sequence[8],
'z6': fib_sequence[7],
'z0': fib_sequence[5],
}
# Step 3: Define the edges of the theta graph Ta
edges = [
('z1', 'z2'),
('z2', 'z3'),
('z3', 'z4'),
('z4', 'z5'),
('z5', 'z6'),
('z6', 'z1'),
('z0', 'z6'),
('z0', 'z3'),
]
# Step 4: Check the GCD for each edge
def gcd(a, b):
return math.gcd(a, b)
is_fibonacci_prime_graph = True
for (u, v) in edges:
hu = vertex_labels[u]
hv = vertex_labels[v]
if gcd(hu, hv) != 1:
is_fibonacci_prime_graph = False
break
# Print the results
```

print("Vertex Labels:", vertex_labels)
print("Edge GCDs:")
for (u, v) in edges:
hu = vertex_labels[u]
hv = vertex_labels[v]
print(f'GCD of u (hu) and v (hv): gcd(hu, hv)")
print("\ nIs the theta graph a Fibonacci prime graph?", is_fibonacci_prime_graph)

Output
Vertex Labels: {'z1': 2, 'z2': 3, 'z3': 8, 'z4': 1, 'z5': 21, 'z6': 13, 'z0': 5}
Edge maximum common divisors:
maximum common divisor of z1 (2) and z2 (3)= 1
maximum common divisor ofz2 (3) and z3 (8) = 1
maximum common divisor of z3 (8) and z4 (1) = 1
maximum common divisor of z4 (1) and z5 (21 = 1
maximum common divisor of z5 (21) and z6 (13) = 1
maximum common divisor of z6 (13) and z1 (2) = 1
maximum common divisor of z0 (5) and z6 (13) = 1
maximum common divisor ofz0 (5) and z3 (8) = 1
Is the theta graph a Fibonacci prime graph? True

Python Implementation to Verify Fibonacci prime labeling of the merging of couple of nodes within the cycle of theta graph T_a: 6.2
import math
from functools import lru_cache
# Step 1: Generate Fibonacci numbers up to the required index
@lru_cache(None)
def fibonacci(n):
if n == 0:
return 0
elif n == 1:
return 1
else:
return fibonacci(n - 1) + fibonacci(n - 2)
# Generate Fibonacci sequence up to f7 (since we need up to f(n+1) with n = 6)
fib_sequence = [fibonacci(i) for i in range(8)]
# Step 2: Define the vertex labels according to h
vertex_labels = {
'z1': fib_sequence[2],
'z3': fib_sequence[6],
'z4': fib_sequence[7],
'z5': fib_sequence[5],
'z6': fib_sequence[3],
'z0': fib_sequence[4],
}
# Step 3: Define the edges of the graph Ω after fusion
edges = [
('z1', 'z3'),
('z3', 'z4'),
('z4', 'z5'),
('z5', 'z6'),
('z6', 'z1'),
('z6', 'z0'),
('z0', 'z3'),
]
# Step 4: Check the GCD for each edge
def gcd(a, b):
return math.gcd(a, b)
is_fibonacci_prime_graph = True
edge_gcds = []
for (u, v) in edges:
hu = vertex_labels[u]
hv = vertex_labels[v]
edge_gcds.append((u, v, gcd(hu, hv)))
if gcd(hu, hv) != 1:
is_fibonacci_prime_graph = False
# Print the results
print("Vertex Labels:", vertex_labels)
print("Edge GCDs:")
for (u, v, g) in edge_gcds:
print(f'GCD of u ({vertex_labels[u]}) and v ({vertex_labels[v]}): {g}")
print("\ nIs the graph a Fibonacci prime graph?", is_fibonacci_prime_graph)
Output
Vertex Labels: {'z1': 1, 'z3': 8, 'z4': 13, 'z5': 5, 'z6': 2, 'z0': 3}
Edge maximum common divisors:
maximum common divisor of z1 (1) and z3 (8) = 1
maximum common divisor of z3 (8) and z4 (13) = 1
maximum common divisor of z4 (13) and z5 (5) = 1
maximum common divisor ofz5 (5) and z6 (2) = 1
maximum common divisor of z6 (2) and z1 (1) = 1
maximum common divisor of z6 (2) and z0 (3) = 1
maximum common divisor ofz0 (3) and z3 (8) = 1
Is the graph a Fibonacci prime graph? True

Python Implementation to Verify Fibonacci prime labeling of the merging of couple of nodes within the cycle of theta graph T_a
import math
from functools import lru_cache
# Step 1: Generate Fibonacci numbers up to the required index
@lru_cache(None)
def fibonacci(n):
if n == 0:
return 0
elif n == 1:
return 1
else:
return fibonacci(n - 1) + fibonacci(n - 2)
# Generate Fibonacci sequence up to f8 (since we need up to f(n+1) with n = 7)
fib_sequence = [fibonacci(i) for i in range(9)]
# Step 2: Define the vertex labels according to h
vertex_labels = {
'z1': fib_sequence[2],
'z2': fib_sequence[4],
'z3': fib_sequence[6],
'z4': fib_sequence[7],

```
'z5': fib_sequence[5],
'z6': fib_sequence[8],
'z0': fib_sequence[3],
}
# Step 3: Define the edges of the graph Ω after switching the central node
edges = [
('z1', 'z2'),
('z2', 'z3'),
('z3', 'z4'),
('z4', 'z5'),
('z5', 'z6'),
('z6', 'z1'),
('z0', 'z1'),
('z0', 'z2'),
('z0', 'z4'),
('z0', 'z5'),
]
# Step 4: Check the GCD for each edge
def gcd(a, b):
return math.gcd(a, b)
is_fibonacci_prime_graph = True
edge_gcds = []
for (u, v) in edges:
hu = vertex_labels[u]
hv = vertex_labels[v]
edge_gcds.append((u, v, gcd(hu, hv)))
if gcd(hu, hv) != 1:
is_fibonacci_prime_graph = False
# Print the results
print("Vertex Labels:", vertex_labels)
print("Edge GCDs:")
for (u, v, g) in edge_gcds:
print(f"GCD of u ({vertex_labels[u]}) and {v} ({vertex_labels[v]}): {g}")
print("\nIs the graph a Fibonacci prime graph?", is_fibonacci_prime_graph)
```

Output

Vertex Labels: {'z1': 1, 'z2': 3, 'z3': 8, 'z4': 13, 'z5': 5, 'z6': 21, 'z0': 2}
Edge maximum common divisors:
maximum common divisor of z1 (1) and z2 (3) = 1
maximum common divisor of z2 (3) and z3 (8) = 1
maximum common divisor of z3 (8) and z4 (13) =1
maximum common divisor of z4 (13) and z5 (5) = 1
maximum common divisor of z5 (5) and z6 (21) = 1
maximum common divisor of z6 (21) and z1 (1) = 1
maximum common divisor of z0 (2) and z1 (1) = 1
maximum common divisor of f z0 (2) and z2 (3) = 1
maximum common divisor of z0 (2) and z4 (13) = 1
maximum common divisor of z0 (2) and z5 (5) = 1
Is the graph a Fibonacci prime graph? True

5 Conclusion

In conclusion, this study highlights that the theta graph, along with its variations formed by merging nodes within its cycle and altering its central node, are consistently demonstrated as Fibonacci prime graphs. Additionally, the application of divided square DC labeling in these graphs, particularly within the context of tiny social networks, underscores their mathematical properties and structural uniqueness. The successful implementation and validation of these labeling techniques using Python programming further emphasize the practical relevance and computational feasibility of these approaches in analyzing complex graph structures within social network contexts.

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Conflict of Interest

The authors have no conflict of interest to declare.

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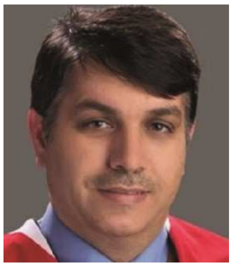

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