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A Novel Algorithm with Orthogonal Arrays for the Global Optimization of Design of Experiments

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Abstract: In this paper, we develop a novel algorithm that determines the overall best parameter setting in the design of experiments. The algorithm begins with successive orthogonal array experiments and ends with a full factorial experiment. The setup for the next orthogonal array experiment is obtained from the previous experiment by either fixing a factor at a given level or by reducing the number of levels considered for all currently non-fixed factors. We illustrate this method using a light-gauge steel wall sound isolation system with four parameters, each with three levels. In a previous study, the full factorial of 81 experiments was conducted, and the optimal parameter settings were determined. With the proposed algorithm, we found the same result using 15 experiments. As a further comparison, we obtained the optimal settings using a traditional Taguchi method and found that they correspond to the 23rd experiment out of the 81 experiments when sorted by the objective (or quality) function. We conclude that the proposed algorithm can provide an accurate, fast, and economic tool for the global optimization of design of experiments.

Keywords: Design of experiments, orthogonal array, factorial, Taguchi method.

1. Introduction

The introduction of a new product or process design is an effective way for a corporation to handle competition from its industry rivals. When a new design is introduced, several parameters (or factors) influence its objective and quality characteristics. Each parameter may have several optional levels for costs, safety, reliability, performance, and so on. Thus, the parameters and their levels form many optional settings, from which the ideal settings are expected to be selected for the design.

Within the levels specified for the factors of a design, most organizations aim to determine the optimal or a workable setting through the design of experiment. Typically, two approaches are used in the design of experiments, a full factorial experiment (Full-FFE) and a fractional factorial one (FFE).

A full factorial experiment tests all possible combinations of the factor levels and can therefore find the overall optimum setting. However, this process becomes overwhelming as the number of design parameters or levels increases. For example, if a new design involves 12 three-level parameters, the experiment must include 531,441 ($= 3^{12}$) settings. Therefore, the approach is only practical for a limited number of parameters and levels.

To overcome the limitations of a full factorial experiment, a fractional factorial approach was developed by the British statistician R. A. Fisher in the 1920s [1]. Fisher showed that the full factorial array used in a full factorial experiment could be reduced to a smaller but still statistically meaningful array, which is typically referred to as an orthogonal array.

An orthogonal array has two major requirements. The first is that the levels of any factor must occur with the same frequency. The second is that for any two factors, each possible combination of levels must occur with the same frequency. If all factors have q levels, an orthogonal array is typically expressed as $L_M(q^m)$, where m is the number of factors and M represents the number of rows in the array (a multiple of q^2).

In a study of orthogonal arrays, Taguchi [2] explored the entire design space with a few experiments and suggested several standard orthogonal arrays. He classified them into three types: 2-level arrays (L_4, L_8, L_{16}, L_{32}, L_{64}), 3-level arrays (L_9, L_{27}, L_{81}), and mixed 2-and-3-level arrays (L_{18}, L_{36}, L_{54}). For example, if a design has 12 three-level parameters, the Taguchi method

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would select the L_{27} design and would test only 27 specified settings. In this manner, for a design containing many parameters and levels, a fractional factorial approach could determine a feasible parameter setting with significantly less effort and time through the use of an orthogonal array.

Taguchi further defined a loss function to evaluate the difference between an experimental value and a desired value. The loss function is a way to measure the performance characteristic deviation from the desired value. The value of a loss function was further redefined as a signal-to-noise (S/N) ratio. In general, a larger S/N ratio corresponds to a better design; that is, the design parameter setting is the one with the highest S/N ratio in the orthogonal array.

The Taguchi method has been widely used to design high-quality products at low cost in the engineering industry because the approach effectively provides an acceptable solution. Ghani *et al.* [3] applied the Taguchi approach to optimize end milling parameters. Their results showed that the optimal end milling parameters were determined with a minimum number of trials rather than with the significant number of trials required for a full factorial design. Chen *et al.* [4] used finite-element techniques and the Taguchi approach to optimize the package design of plastic quad flat packages (PQFPs) with and without an assembled heat slug. Lin *et al.* [5] combined grey relational analysis based on an orthogonal array and the fuzzy-based Taguchi approach to optimize the electrical discharge machining (EDM) process with multiple process responses.

However, the design parameter setting determined by the Taguchi process is essentially a local optimum [6, 7]. The difference between this local optimum and a global optimum must be known to establish the quality of the local optimum. Unfortunately, finding the global optimum can only be guaranteed via a full factorial experiment, which is prohibitive when there is a high number of factors and levels. Consequently, only a limited number of studies have compared the best two parameter settings obtained from the different approaches.

In this study, a novel approach is developed that combines both the full and fractional factorial approaches. The developed algorithm begins with several successive fractional factorial experiments and ends with a full factorial approach. In the algorithm, we use analysis of variance (ANOVA) to rank the involved factors according to their influence on the designs objective characteristics.

After the1999 Jiji Earthquake, architectural requirements for earthquake resistance in Taiwan became more demanding. According to National Cheng Kung University (NCKU) professor Sheu [8] in his article The Jiji EarthquakeVDisastrous Earthquake Effects on Civil Architecture, partition walls enhance and assume a portion of the earthquake resistance in low-story buildings. Currently, light-gauge steel walls are the main partition wall systems used during architectural repair, construction, and expansion. A research report by Huang *et al.* [9] from the Architecture and Building Research Institute of the Ministry of the Interior in December 1996 (before the earthquake) indicated that light-gauge steel walls have been the most popular type of partition wall system since 1994. Although the popularity of light-gauge steel walls continues to increase, the lack of suitable sound isolation for these walls remains an unsolved problem. To verify the proposed approach, we apply it to the design of a light-gauge steel wall sound isolation system. The details of the application of the proposed approach are described in the following sections.

2. Algorithm of the proposed approach

The proposed algorithm begins with a series of fractional factorial experiments. The number of the series depends on the number of parameters (or factors) in the new design. The parameters are gradually optimized for each fractional factorial step. Finally, a full factorial experiment is conducted for the remaining level-varying parameters to reach the globally optimal parameter settings in a design of experiments. The algorithm is formulated below in ten steps.

Step 1:Define a design and identify its objective characteristics (or quality characteristics).
Step 2:Identify the factors affecting the designs objective characteristics
Step 3:Identify each factors levels of design.
Step 4:Select a suitable orthogonal array based the number of factors to conduct a fractional factorial experiment.
Step 5:Determine the designs objective value for each setting in the orthogonal array and weigh its influence by signal-to-noise (S/N) analysis.
Step 6:Calculate the average characteristic values of all settings in the array (denoted as AV_{all}) and obtain the average characteristic value of the settings for each factor with a given level (denoted as AV_{lvl}). Denote the setting formed by the best level of each factor as the orthogonal array point (*OAP*).
Step 7:If $OAP(S/N) > AV_{all}$, then proceed to Step 8; if not, eliminate the level with the poorest AV_{lvl} value for each factor that has not yet been fixed to a single level and proceed to Step 4.
Step 8:Determine the most influential factor in this experiment by analysis of variance and fix its level at the factors optimal level. Because the factor determined to be the most influential has its level fixed as the best one, we will call it a fixed factor in the successive tests and refer to the other factors as non-fixed.
Step 9:Examine whether the number of non-fixed factors has been sufficiently reduced to conduct a full factorial experiment. If yes, then proceed to Step 10; otherwise, proceed to Step 4. Step10: Review

the function values of all of the settings employed in the previous experiments and then achieve the optimal parameter settings for the design.

Step 10: Review the function values of all of the settings employed in the previous experiments and then achieve the optimal parameter settings for the design. It is worth mentioning that if we end at step 6 in the first fractional factorial experiment, our approach would obtain the same solution as the Taguchi approach.

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3. Design optimization of a light-gauge steel wall sound isolation system

Light-gauge partition systems are indoor partition walls that are non-load bearing. A wide variety of light-gauge steel wall systems are used, and their functions include space separation, fire prevention, sound isolation, and humidity resistance. The assembly and disassembly of light-gauge steel partition walls requires little effort or labor. In addition, such walls can be dismantled without damaging the materials; therefore, they can be used repeatedly at other locations. Whether they are used for repairs, new construction, or expanded architecture, light-gauge steel walls are the most common type of partition wall system [10].

The experimental model used in this experiment design was a light-gauge steel wall system, and the study measured the sound isolation capability of the system in the anechoic and reverberation chambers of the acoustic laboratory at National Taiwan Ocean University. The width and height of both openings of the anechoic and reverberation chambers were 1485 mm and 1240 mm, respectively.

An illustration (a) and picture (b) of the light-gauge steel wall system are shown in Figure 1.

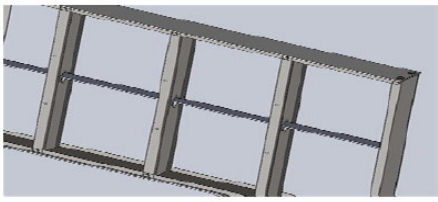
The cross-sectional component diagram of the light-gauge steel wall system design shown in Figure 2 was used to investigate the performances of the proposed method and the Taguchi method.

The parameter components for the design of the system include the stud width (W), stud spacing (S), screw spacing (D), and frequency (F).

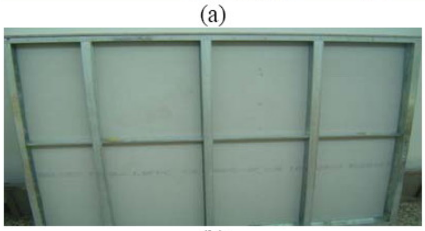
The noise reduction capability of a material is defined as its sound transmission loss (TL) effectiveness. The TL value, given in decibels (dB), is defined as

$$TL = 10\log(\frac{1}{\tau}) = 10\log(\frac{I_i}{I_t})$$

The acoustic transmission coefficient τ is the ratio of transmitted to incident sound energy (I_t/I_i) at an interface in a sound medium. Using the design of experiment



(a)



(b)

Figure 1 Illustration (a) and picture (b) of the inner structure of the light-gauge steel wall system (the distance between the light-gauge steel studs was 400 mm).

approach to optimize the parameter settings of the light-gauge steel wall sound isolation system, the sound transmission loss (TL) was considered as the quality characteristic. In signal-to-noise analysis, the TL value denoted as y is related to the S/N ratio in the form of

$$S/N = -10\log(\frac{1}{y^2})$$

A larger S/N value corresponds to a better quality, representing a larger is better characteristic. Using the design of experiment method, we selected four factors with three levels each, as shown in Table 1, for the system design, and the orthogonal array $L_9(3^4)$ was applied.

Table 1 The given data for four factors, each with three levels.

Parameter(Unit)	No. of levels	Level		
		1	2	3
Stud width (mm)	3	65	92	125
Stud spacing (mm)	3	200	400	600
Screw spacing (mm)	3	200	400	600
Frequency (Hz)	3	500	800	1250

3.1. The first fractional factorial experiment

We adopted an orthogonal array with nine settings; the initial S/N ratio values are displayed in Table 2. The average value of the nine settings was then calculated to be AV_{all} = 34.94, and the average value of each factor within a given level was calculated; these values are

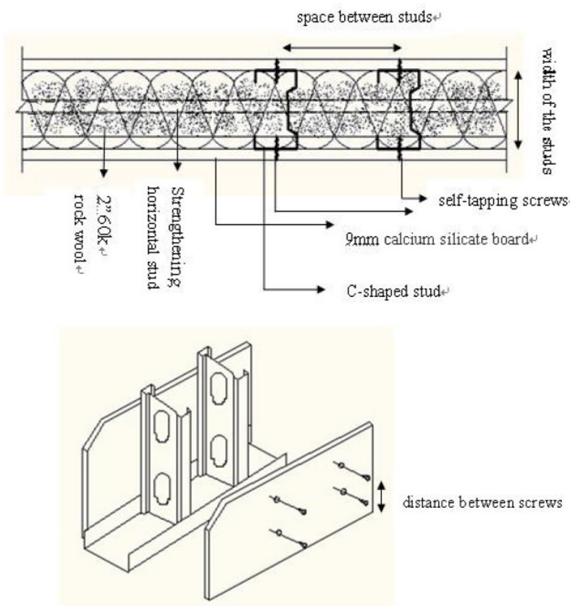


Figure 2 A cross-sectional component diagram of the light-gauge steel wall system.

presented in Table 3, and the influence of the level for each factor is shown by a line graph in Fig. 3.

Table 2 orthogonal array with nine trials

Trial	W	S	D	F	S/N
1	65	200	200	500	33.64
2	65	400	400	800	34.57
3	65	600	600	1250	35.86
4	92	200	400	1250	35.13
5	92	400	600	500	35.03
6	92	600	200	800	35.03
7	125	200	600	800	35.04
8	125	400	200	1250	34.87
9	125	600	400	500	34.93
OAP	92	600	600	1250	35.34

In this work, an analysis of variance was used to determine the most influential factor in the experiment. To weigh a factors effect on the characteristic value of the design problem, we defined a parameter SSF function of the problems factor as the sum of the squares of the difference between the AV_{lvl} of each factor and AV_{all}. The SSF function was expressed as follows:

$$SSF(factor) = \sum_{i=1}^{nl} ns(AV_{lvl}^{i} - AV_{all})^2$$

Table 3 Average function value, AV_{lvl} , for each factor and each given level.

Level	W	S	D	F
1	34.69	34.61	34.62	34.53
2	35.17	34.82	34.88	34.98
3	34.95	35.38	35.31	35.29
SSF	0.35	0.95	0.73	0.88
Rank	4	1	3	2

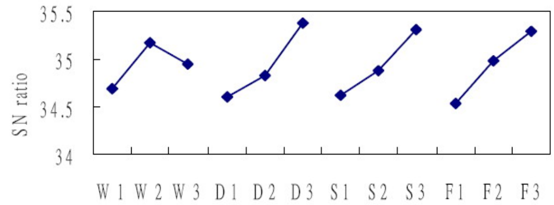


Figure 3 A line graph for three-level versus four factors.

Here, nl represents the number of a factors level, and represents the number of settings within a given level. The SSF values were calculated as $SSF(W) = 0.35$, $SSF(S) = 0.95$, $SSF(D) = 0.73$, and $SSF(F) = 0.88$, respectively. Based on these SSF values, the factor S was obtained as the most influential one among the four factors in the experiment.

3.2. The second fractional factorial experiment

The remaining three non-fixed factors were maintained using the $L_9(3^4)$ orthogonal array for the subsequent orthogonal array experiment. The nine settings and corresponding S/N values are displayed in Table 4, and the influence of the level for each factor is shown by a line graph in Fig.4. After comparing the function values of the three settings, we justified level 3 (1,250) of factor F as the best level for the next experiment.

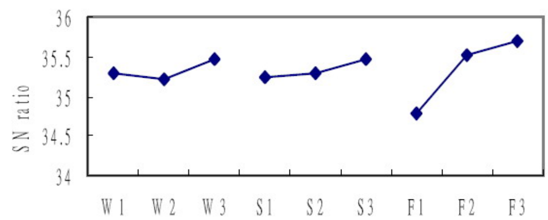


Figure 4 A line graph for three-level versus three factors.

Table 4 orthogonal array with nine trials and varied levels specified for three factors with the fixed-level factor D

Trial	W	D	S	F	S/N
1	65	600	200	500	34.52
2	65	600	400	800	35.53
3	65	600	600	1250	35.86
4	92	600	400	1250	35.42
5	92	600	600	500	34.90
6	92	600	200	800	35.34
7	125	600	600	800	35.68
8	125	600	200	1250	34.83
9	125	600	400	500	34.93

3.3. The full factorial experiment

At this step in the algorithm, the S/N values of the nine settings in the full factorial experiment were calculated; they are presented in Table 5. The best parameter settings were determined from among the 81(=3^4) setting-trial experiments [11] for the two different methods. The optimal settings, the corresponding S/N ratios of the sound transmission loss (TL) values obtained, and their ranks are presented in Table 6. The results shown in Table 6 indicate that the proposed method successfully achieved the globally optimal parameter settings with a much lower percentage of 18.5

Table 5 Nine settings with varied levels specified for two non-fixed factors, W and S, and two fixed factors, D and F.

Trial	W	D	S	F	S/N
1	65	600	200	1250	34.52
2	65	600	400	1250	35.53
3	65	600	600	1250	35.86
4	92	600	400	1250	35.42
5	92	600	600	1250	34.90
6	92	600	200	1250	35.34
7	125	600	600	1250	35.68
8	125	600	200	1250	34.83
9	125	600	400	1250	34.93

Table 6 The optimal settings and the corresponding S/N values obtained from the proposed method and the conventional Taguchi method (TM).

	Optimal settings	Optimal value	Rank
TM (The 1st FFE)	92,600,600,1250	35.34	23
Present(The 2nd FFE)	65,600,600,1250	35.86	1
Present(The Full FFE)	65,600,600,1250	35.86	1

4. Conclusion

We have developed a novel, effective and efficient approach for the design of experiments. The performance of the proposed approach was verified through the design of a new light-gauge steel system design. From the results, we can make the following conclusions:

(1) Although the Taguchi method can offer a feasible solution, it does not provide a global optimum. The approach presented herein can successfully find the globally optimal parameter settings of the design of experiments with a significantly lower level of effort.

(2) The proposed algorithm combines the benefits of both the fractional and full factorial approaches, is effective and efficient, and can serve as a complete solution tool for the global optimization of design of experiments.

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