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A Dual Hesitant Fuzzy Set Theoretic Approach in Fuzzy Reliability Analysis of a Fuzzy System

Laxmi Rathour[1], Vinay Singh[1], Harendra Yadav[2], M. K. Sharma[2], and Vishnu N. Mishra[3,]*

[1]Department of Mathematics, National Institute of Technology, Chaltlang, Aizawl, Mizoram 796 012, India
[2]Department of Mathematics, Chaudhary Charan Singh University, Meerut, U.P.-250004, India
[3]Department of Mathematics, Faculty of Science, Indira Gandhi National Tribal University, Lalpur, Amarkantak, Anuppur, Madhya Pradesh 484 887, India

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Abstract: In this research work we investigated a dual hesitant fuzzy set theoretic approach with inverse Weibull distribution. It is a very common problem that many times data/information used to produce a system/equipment may have uncertainties. Dual hesitant fuzzy set (DHFS) plays an important role in reducing the effectiveness of such uncertainties. DHFS is a helpful alternative to handle situations where experts fail to provide a single choice of their satisfaction or rejection. DHFS is the superset of hesitant fuzzy set or intuitionistic fuzzy set or fuzzy set. In this research work, we proposed a method using DHFS as well as inverse Weibull distribution (IWD). With the help of IWD, it is easy to model failure rate of systems at various levels, which commonly takes place in reliability cases. Fuzzy IWD is used to get the fuzzy reliability of system's suffering failure during their life. Based onα-cut, a DHFS method is introduced. DHFS overcomes the results obtained by traditional methods because it is superior to hesitant fuzzy set theory as it includes multi gradings/choice for a single case. The dominance and importance of this method is verified by giving numerical examples.

Keywords: Fuzzy set, Dual hesitant fuzzy set, Weibull distribution, Failure rate and Fuzzy reliability.

1 Introduction

Reliability is the need of every scientific article/object/product. It has been a topic of focus at the start of the nineteenth century. The failure of the engineering product is a fixed property, or it is associated with its production/birth. Engineering systems play an important part in making/inventing better reliable products. Reliability or Survival function is the most helpful function for the prediction lifetime and reliability analysis. This function has the ability to provide probability of successful operation of a product up to a certain time without single failure. Every time, it is not possible to measure the various parameters exactly of a system due to uncertainties or fuzziness present in these parameters. In lack of exactness, it is reasonable to use fuzzy parameters in place of exact parameters (values). It is quite common that the error in parameters is due to error in machine or experiment or personal decision. Weibull distribution is the most used distribution, which is used in the study of reliability engineering. It is frequently used to model the failure time of a system. IWD has the flexibility to encircle the different distributions. It is able to model various life applications such as failure characteristics, degradation of mechanical parts like pistons, crankshafts of diesel engine etc. The requirement of reliability came into light in day-to-day life during the second world war, after that reliability theories and concepts were developed by Lloyd and Lipow [1] in 1952. The start of reliability as a separate subject including multi- level components was studied by Bazovsky [2].

Estimation of reliability using mathematical models was discussed (Roberts [3]) in engineering discipline. Dhillon [4] observed the concept of human reliability using human behavior/judgement and Gnedenko and Ushakov [5] analyzed probabilistic reliability. Onisawa and Kacprzyk [6] discussed the reliability and safety of engineering equipment's using fuzzy set theory. Sharma and Bansal [7] analysed the reliability of a three-unit redundant system based on stochastic model with two different failures. Zhu et al. [8] proposed DHFS, which covers the drawbacks of earlier fuzzy set theory extensions, such as hesitant and intuitionistic fuzzy sets. Fuzzy set theory, according to Zadeh [9], is primarily based on the membership function that accepts values between zero and one and corresponds to the problem. Bisht and Kumar [10] presented a technique that uses an aggregation operator with further fuzzy set and hesitant fuzzy information to aggregate hesitant fuzzy elements. Bollinger and Salvia [11] employed network theory to examine the reliability of a system's successive failures. Cai et al. [12], firstly given the fuzzy reliability theory and he described the fuzzy reliability based on states. Chang [13] presented the idea of triangular fuzzy number, which was constructed by membership function. Chang et al. [14] obtained the reliability of a linear system when components used were not

*Corresponding author e-mail:vnm@igntu.ac.in,vishnunarayanmishra@gmail.com

identical. Kumar and Singh [15] analyzed intuitionistic fuzzy reliability using the Weibull distribution and went on to determine the fuzzy reliability of many systems. Chaube and Singh [16-17] examined a two-stage, k-out-of-n weighted system and used a membership function to assess the fuzzy reliability. A triangular fuzzy number-based evaluation of fuzzy reliability was done by Mahapatra and Roy [18]. Qian et al. [19] evaluated the special case of a fuzzy set and applications in decision making using HFS. Torra and Narukawa [20] is known as the father of HFS, which is nothing but generalization of fuzzy set theory. Xia and Xu [21] discussed basic operations of the HFS and decision making using hesitant fuzzy information. Xie et al. [22] discussed the systems in which one unit or component fails then whole system fails to work and such systems are called series systems. Yu [23] analyzed the triangular HFS as well as its application to aggregate the operators. The measurement of triangular HFEs and operated it with weighted averaging operators. Zhang [24] discussed the hesitant fuzzy power aggregation operators as well as their applications. The evaluation of reliability was done by Sharma [25], which was based on intuitionistic fuzzy sets. Yadav et al. [26] evaluated the posfust reliability of nonrepairable multistate system using posfust failure rate. Sharma et al. [27] discussed the fuzzy decomposable approach to fond the posfust reliability of repairable substation automation system. [28] Sharma et.al. also, applied Neutrosophic Monte Carlo Simulation Approach for Decision Making.

Present research work has been divided into six sections. In section 2, useful definitions like fuzzy set as well as dual hesitant fuzzy set and others are given. Inverse Weibull distribution is used to model the present work and discussion is given in section 3 while section 4 presents the process of working to get the desired result. The numerical interpretation of the results is given in section 5, while section 6 contains conclusion.

2 Definitions

The relevant basic definitions used in this current research work are given in this section 2.

Fuzzy set: The membership function (μ_A) of a fuzzy set A on a universal set$X(A \subset X)$ can be used to characterize. The values of μ_A span the range [0,1]. Mathematically, it can be represented as;

$$A = \{x, \mu_A(x) | x \in X\}$$

And $\mu_A(x) = A \rightarrow [0,1]$ for element x in A.

Hesitant fuzzy set: If there is a fixed set X. When applied to X, the hesitant fuzzy set H can be defined as a function that yields a fixed subset within the interval [0,1]. Mathematically, it can be written as;

$$H = \{x, \tilde{h}_A(x) | x \in X\}$$

where$\tilde{h}_A(x)$ is representing the set of few values in [0,1] that correspond to the element $x \in X$ ‘s possible membership degrees of for set A.

Dual hesitant fuzzy set: A dual hesitant fuzzy set (D_H) can define for a fixed set X as:

$$D_H = \{x, \tilde{h}_A(x), \tilde{g}_A(x) | x \in X\}$$

where $\tilde{h}_A(x)$and$\tilde{g}_A(x)$ are the sets having values in the range [0,1] and representing the degree of satisfaction and rejection of the element $x \in X$ to the set D_H as well as satisfying the conditions: $0 \leq p, q \leq 1$, where $p \in \tilde{h}_A(x)$and $q \in \tilde{g}_A(x)$ and also $0 \leq p^+ + q^+ \leq 1$, where $p^+ \in \tilde{h}_A{}^+(x)$, $q^+ \in \tilde{g}_A{}^+(x)$.

3 Formulation of the model

The random variable T(t) has an inverse Weibull distribution if its CDF has the form

$$G(t) = \exp\left[-\left(\frac{\delta}{t}\right)^{\omega}\right], t > 0 \;\&\; \delta, \omega > 0$$

And

PDF $\quad g(t) = \omega\delta^{\omega} t^{-(\omega+1)} \exp\left[-\left(\frac{\delta}{t}\right)^{\omega}\right].$

Where, $\delta$is representing the scale parameter and ω is representing the shape parameter.

The reliability function or survival function is denoted by the expression;

$$R(t) = 1 - \exp\left[-\left(\frac{\delta}{t}\right)^{\omega}\right], t > 0 \,\&\, \delta, \omega > 0$$

And failure rate is denoted by the expression;

$$\lambda(t) = \frac{g(t)}{R(t)}$$

$$= \omega\delta^{\omega} t^{-(\omega+1)} \exp\left[-\left(\frac{\delta}{t}\right)^{\omega}\right]\left\{1 - \exp\left[-\left(\frac{\delta}{t}\right)^{\omega}\right]\right\}^{-1}.$$

Let T be the fuzzy random variable and $t \approx \tilde{t} = \{\tilde{h}_A(x), \tilde{g}_A(x)\}t$, here t is the observed value and $\{\tilde{h}_A(x), \tilde{g}_A(x)\}$ is the hesitancy or vagueness coefficient. We have used dual hesitant fuzzy number to cover the hesitation present in the parameter $\tilde{t}$.

Now,

$$\lambda(t) = \omega\delta t^{-(\omega+1)} \exp\left[-\left(\frac{\delta}{\{\tilde{h}_A(x), \tilde{g}_A(x)\}t}\right)^{\omega}\right]\left\{1 - \exp\left[-\left(\frac{\delta}{\{\tilde{h}_A(x), \tilde{g}_A(x)\}t}\right)^{\omega}\right]\right\}^{-1} \text{ and}$$

If (a, b, c) and (d,e,f) are then triangular fuzzy number then

$$\tilde{h}_A(x) = \begin{cases} \dfrac{x-a}{b-a}, & a < x \le b \\ \dfrac{c-x}{c-b}, & b \le x < c \\ 0 & otherwise, \end{cases}$$

$$\tilde{g}_A(x) =$$

$$\begin{cases} \dfrac{f-x}{f-e}, & e < x \le f \\ \dfrac{x-d}{e-d}, & d \le x < e \\ 1 & otherwise \end{cases}$$

So, the failure rate (based on α, β −cut) becomes as;

$$\lambda(t) = \omega\delta^{\omega} t^{-(\omega+1)} \left(\exp\left[-\left(\frac{\delta}{\{(a+\alpha(b-a)), d+\beta(e-d)\}t}\right)^{\omega}\right]\left\{1 - \exp\left[-\left(\frac{\delta}{\{(a+\alpha(b-a)), d+\beta(e-d)\}t}\right)^{\omega}\right]\right\}^{-1}, \exp\left[-\left(\frac{\delta}{\{(c-\alpha(c-b)), f-\beta(f-e)\}t}\right)^{\omega}\right]\left\{1 - \exp\left[-\left(\frac{\delta}{\{(c-\alpha(c-b)), f-\beta(f-e)\}t}\right)^{\omega}\right]\right\}^{-1}\right).$$

And fuzzy reliability is given by

$$R(t) = 1 - \left\{\exp\left[-\left(\frac{\delta}{\{(a+\alpha(b-a)),\ d+\beta(e-d)\}t}\right)^{\omega}\right], \exp\left[-\left(\frac{\delta}{\{(c-\alpha(c-b)), f-\beta(f-e)\}t}\right)^{\omega}\right]\right\}, t > 0 \,\&\, \delta, \omega > 0$$

4 Algorithm

This research work focuses on failure rate of systems by using DHFS as well as IWD to determine the fuzzy reliability.

IWD is also a good alternative to handle the failure rates of systems. The algorithm developed for this this work is as follows:

Step 1: The data in form of DHFS is selected/chosen on the basis of expert's opinion.

Step 2: Then general union is taken for both membership as well as non-membership degrees offeror and rejection respectively.

Step 3: A middle value is calculated using the data obtained in step 2 using the formula;

$$middle\ value\ = \frac{sum\ of\ all\ choices}{no.of\ choices}$$

Step4: Using the middle values of step 3, triangular fuzzy numbers are created for membership and non-membership degrees respectively.

Step 5: Now, an interval (α, β −cut) is created for both membership and non-membership.

Step 6: Fuzzy reliability is obtained in the form of interval having the minimum and maximum ranges of the membership/non-membership using reliability expression.

5 Numerical Interpretation

Let us take some data for membership and non-membership based on three experts {0.3,0.4,0.5}, {0.2, 0.6}, {0.1, 0.4, 0.6} and {0.2, 0.3}, {0.1, 0.2, 0.3, 0.4}, {0.3, 0.4} respectively for an electronic equipment. Now, taking general union for both, we get {0.1, 0.2, 0.3, 0.4, 0.5, 0.6} and {0.1, 0.2, 0.3, 0.4}.

Now

$$\frac{0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6}{6} = 0.35$$

and

$$\frac{0.1 + 0.2 + 0.3 + 0.4}{4} = 0.25$$

We used these middle values to form a triangular fuzzy number and triangular fuzzy number corresponding to membership and non-membership middle values formed as; (0.25, 0.35, 0.45) and (0.20, 0.25 0.30). Then fuzzy reliability

$$R(t) = 1 - \left\{\exp\left[-\left(\frac{\delta}{\{(0.25 + \alpha(0.1)),\ 0.2 + \beta(0.05)\}t}\right)^{\omega}\right], \exp\left[-\left(\frac{\delta}{\{(0.45 - \alpha(0.1)), 0.3 - \beta(0.05)\}t}\right)^{\omega}\right]\right\}$$

Taking $\delta = 1, \omega = 0.5$ and $t = 2$.

For α, β =0.3

$$= 1 - \left\{\exp \begin{matrix} \left[-\left(\frac{1}{\{(0.25 + 0.3(0.1)),\ 0.2 + 03(0.05)\}2}\right)^{0.5}\right], \\ \exp\left[-\left(\frac{1}{\{(0.45 - 0.3(0.1)), 0.3 - 0.3(0.05)\}2}\right)^{0.5}\right] \end{matrix}\right\}$$

$$= 1 - \{\exp[-(1.3363, 105249\ )], \exp[-(1.0910, 1.3245)]\}$$

$$= [(0.7372, 0.7824), (0.6642, 0.7341)]$$

Similarly, we can calculate other values for different choice of α, β. Other computational results corresponding to membership as well as non-membership are given in the table 1. (M_u= upper range of membership, NM_u = upper range of non-membership,M_l = lower range of membership, NM_l = lower range of non − membership).

Table 1: Presenting the range of reliability according to the α, β −cut.

α, β −cut	M_u	NM_u	M_l	NM_l
0	0.7569	0.7943	0.6515	0.725
0.1	0.7501	0.7903	0.6557	0.728
0.2	0.7436	0.7863	0.6599	0.731
0.3	0.7372	0.7824	0.6642	0.7341
0.4	0.731	0.7786	0.6686	0.7372
0.5	0.725	0.7748	0.6731	0.7404
0.6	0.7192	0.7711	0.6777	0.7436
0.7	0.7135	0.7675	0.6825	0.7469
0.8	0.708	0.7639	0.6873	0.7502
0.9	0.7026	0.7604	0.6923	0.7535
1	0.6974	0.7569	0.6974	0.7569

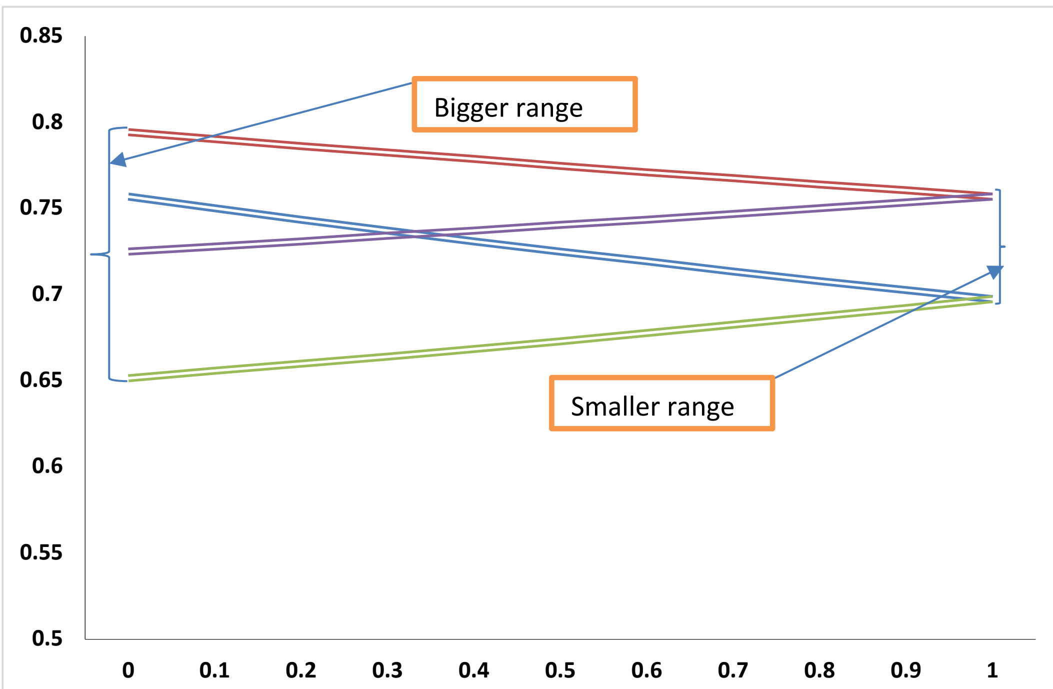

Fig. 1: Showing the change in range for different values of α, β.

As the time passes away, every object present on this earth is going to become weaker or we can say losing their efficiency or rate of performance. This is the reason of unreliability. Table 2 and Figure 2's graphical depiction of the fuzzy reliability of the suggested system demonstrate how it decreases over time. The fuzzy reliability of the suggested method is demonstrated by table 2 and figure 2, which shows how it decreases over time.

Table 2: Showing the difference between max and min range of fuzzy reliability.

Time (Hours)	Maximum Fuzzy Reliability	Minimum Fuzzy Reliability
0	1	1
2	0.725	0.6731
4	0.5987	0.5464
6	0.5255	0.4756
8	0.4756	0.4283
10	0.4386	0.3935

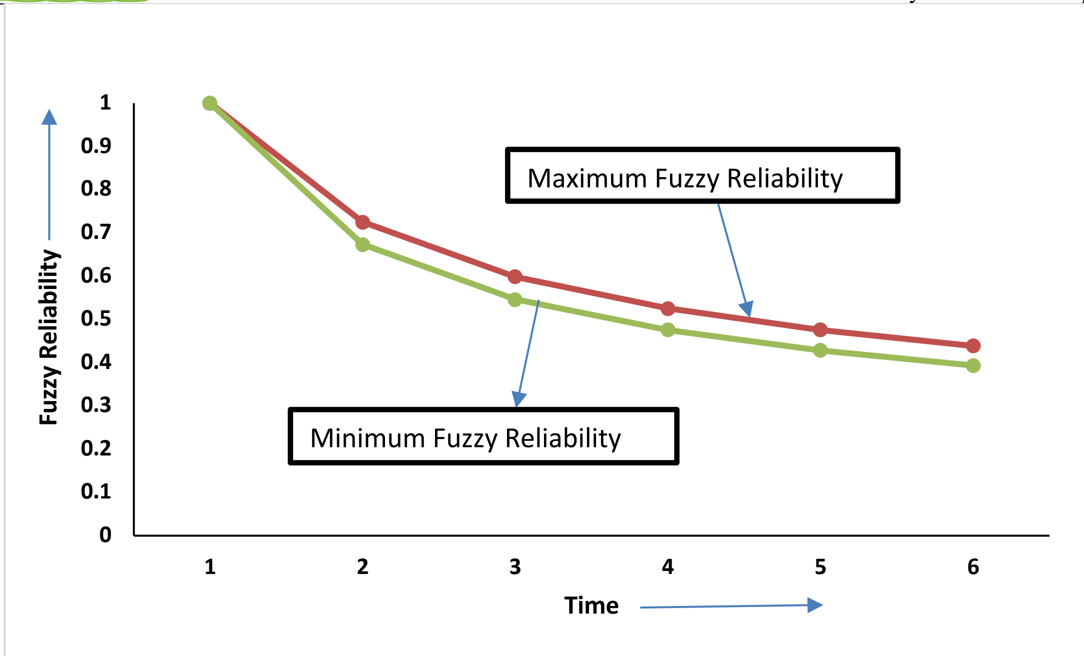

Fig. 2: Presenting the fuzzy reliability change with time

6 Conclusion

The importance of this method is increased because we took into account both the values of membership as well as non-membership to evaluate the fuzzy reliability of a system. So, DHFS becomes more realistic and practical as rejection also plays role in determination of reliability. Exactness is a difficult task in various systems due to the uncertainty in parameters at different stages/levels. But DHFS has more flexibility and accuracy than other methods as non-membership is also included to get nearest results. In the current research paper, inverse Weibull distribution is used with DHFS to get more preferable and realistic results. IWD is able to reflect closeness to reality in case of failure rates of systems. The results of this research work demonstrate the effectiveness of the proposed method, as the larger uncertainty range (Fig. 1) is reduced to a smaller range (0.6974, 0.7569), and figure 2 illustrates the reliability of the suggested system.

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Author's Contribution:

L.R.: Original Draft, Review, Methodology

V.S.: Supervision, Validation, review

HY: Original Draft, Developing methodology, Numerical Computations

M.K.S: Interpretation of the results, drafting, editing and critical revision of the manuscript.

V. N. M.: Review, Editing, Supervision

Conflicts of Interest Statement

The authors certify that they have NO affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript.

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