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# Specific Properties of Fractional Kinetic Equations with the Advanced K-Bessel Function

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**Abstract:** A novel and comprehensive formulation of the fractional kinetic equation is devised by employing the advanced k-Bessel function. This work investigates the broad applicability of the advanced k-Bessel function in relation to solving the fractional kinetic equation. The findings presented in this study are of a general character and hold the potential to generate a substantial quantity of both established and original outcomes.

**Keywords:** Pochhammer symbol; Fractional calculus; advanced k-Bessel function; Laplace transform; Fractional derivative; Fractional integration.

## 1 An Overview and Setup

There is a branch of mathematics known as Fractional Order Calculus (FOC) that studies differentiating and integrating functions in any order [1,2,3]. This means that the order of the operation can be complex number or real. Despite the fact that Fractional Order Calculus is a problem that has been around for more than 300 years [4,5], its enormous implications for modern theoretical research and applications in the actual world have only recently become the subject of widespread discussion. The concept of a derivative that does not need an integer was possibly introduced for the first time in a letter written by Leibniz to L'Hospital in 1695. In later years, influential figures such as Riemann, Euler, Abel, Liouville, and Fourier contributed to the development of the FOC through their ground-breaking contributions. The reader eager to learn more about the FOC's history might look to, for example, [1], where they can get further information.

According to [4,6], FOC remained practically unexplored for engineering applications for such a long time because there are multiple definitions, the absence of a simple geometrical explanation, the absence of fractional order differential equation solution methods, and the apparent sufficiency of the Integer Order Calculus (IOC) for most complications. However, things are looking up as of late, as the FOC is proving to be a useful tool for addressing a wide range of concerns relating to fractal dimension, "infinite memory," chaotic behavior, and other related topics. As a result, the FOC has been successfully applied to a wide range of engineering disciplines [6], including signal processing, viscoelasticity, robotics, control theory, electronics, and bioengineering. There are a number of available control applications, some of which are [7,8,9]. During the twentieth century, a great deal more fractional calculus outcomes were presented. However, for the purpose of this discussion, we will only focus on one more result, which was given by M. Caputo and was first employed widely in [10]. In his definition of a fractional derivative, Caputo used the example of a function $f$ that had an absolute continuous derivative of $n-1$.

$$D^{\alpha}f(x)=\frac{1}{\Gamma(n-\alpha)}\int_0^t(t-s)^{n-\alpha-1}\left(\frac{d}{ds}\right)^n f(s)ds \tag{1.1}$$

Caputo fractional derivative [57] is the term given to this type of derivative today. The relationship between the derivative (1.1) and the Riemann-Liouville fractional derivative [11] is significant, and it holds considerable importance in various

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contemporary applications. The utilization of the Caputo derivative enables the representation of the initial conditions of fractional differential equations in a classical format., i.e.

$$y^{(k)}(0) = b_k, \quad k = 0, 1, 2, \ldots, n-1 \tag{1.2}$$

These differential equations, in contrast to differential equations that have a Riemann–Liouville differential operator [11], do not have that operator. Although we commonly refer to operator $D$ as the Caputo operator, Y. N. Rabotnov was the one who first brought this differential operator into the Russian viscoelastic research a year before Caputo's paper was printed. Rabotnov's work can be found in the Russian literature [12]. Today, we call operator $D$ the Caputo operator.

The branch of fractional calculus experienced such rapid expansion throughout the latter half of the 20th century that in 1974, New Haven played host to the very first conference that focused on the theory as well as the applications of fractional calculus [13]. The first book on fractional calculus was written by Oldham and Spanier [1] and released that same year. Since then, an enormous number of other volumes have been published, the most well-known of which were written by Miller and Ross [2], Samko et al. [14], and Podlubny [3]. The mathematics publication known as "Fractional calculus & applied analysis" released its very first issue in 1998. This magazine is solely devoted to discussing the theory behind fractional calculus as well as its applications in the real world.

In the year 2004, the city of Bordeaux played host to a sizable conference with the theme "Fractional Differentiation and Its Applications." During the course of this conference, 104 presentations on fractional calculus were given. Since its humble beginnings as a straightforward question posed by L'Hospital to Leibniz, fractional calculus has come a long way to become an indispensable tool in a wide variety of scientific disciplines. Even though it is almost as old as traditional calculus, it has grown in recent decades due to the fact that it can be applied to models depicting complicated problems that occur in the real world (see to current research [15,16,17,18,19] for more information). In addition, despite the fact that the term "fractional calculus" is technically inaccurate, we shall continue to make use of it for the entirety of this article. The theoretical and, more significantly, the numerical aspects of the issues coming up in this sector are the main focus of this text.

The importance of fractional differential equations in the area of applied research has expanded in order to build mathematical models of a wide variety of physical phenomena. This trend can be seen not only in mathematics and physics, but also in dynamical systems, control systems, and engineering. Kinetic equations, in particular, are used to characterize the unbroken flow of a substance's motion. In [20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,58,59,60,61,62], an extension and generalization of fractional kinetic equations that involve a large number of fractional operators is described in detail.

If a reaction, denoted by the time-dependent number $N = N(t)$, is considered, Haubold and Mathai's [23] fractional differential equation can be used to express the relationship between the reaction's rate of change, its rate of destruction, and its rate of $N$ production [58,59,60,61]. The equation is given as follows:

$$\frac{dN}{dt} = -d(N_t) + p(N_t) \tag{1.3}$$

Where $N = (N_t)$ the rate of reaction, $d = d(N)$ the rate of destruction, $p = p(N)$, the rate of production and $N_t = (t^*)N(t - t^*)$, $t^* > 0$.

When spatial fluctuations and inhomogeneities in the quantity $N(t)$ are disregarded, the following differential equation describes a special case of eqn. (1.3):

$$\frac{dN_i}{dt} = -c_i N_i(t) \tag{1.4}$$

such that $N_i(t = 0) = N_0$ is the numerical density of the species $i$ at time $t = 0$ and $c_i > 0$.

After taking out the index $i$ and integrating the usual kinetic equation (1.4), we get the following:

$$N(t) - N_0 = -c\,{}_0D_t^{-1}N(t) \tag{1.5}$$

In this context, let $c$ denote a constant and ${}_0D_t^{-1}$ denote a specific instance of the Riemann–Liouville fractional integral operator ${}_0D_t^{-\nu}$. The operator ${}_0D_t^{-\nu}$ is precisely defined as such.

$${}_0D_t^{-\nu} f(t) = \frac{1}{\Gamma(\nu)} \int_0^t (t-s)^{\nu-1} f(s)\,ds, \quad (t > 0, \mathrm{Re}(\nu) > 0). \tag{1.6}$$

Haubold and Mathai [23] provide the fractional generalisation of the standard kinetic equation (1.5) as follows:

$$N(t) - N_0 = -c^{\nu}\,{}_0D_t^{-\nu}N(t), \tag{1.7}$$

and obtained the solution of (1.7) as follows:

$$N(t) = N_0 \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma(\nu k+1)} (ct)^{\nu k} \tag{1.8}$$

Further, Saxena and Kalla [43] considered the following fractional kinetic equation:

$$N(t) - N_0 f(t) = -c^{\nu}\, {}_0D_t^{-\nu} N(t), \quad (\mathrm{Re}(\nu) > 0, c > 0) \tag{1.9}$$

Let $N(t)$ represent the number density of a certain species at a given time $t$. The initial number density of this species at time $t = 0$, is denoted by $N_0 = N(0)$. The constant $c$ and the function $f$, which belongs to the set $(0, \infty)$, are also involved in this context.

By utilizing the Laplace transform on equation (1.9) as referenced in [33],

$$L\{N(t);p\} = N_0 \frac{F(p)}{1+c^{\nu}p^{-\nu}} = N_0 \left( \sum_{n=0}^{\infty} (-c^{\nu})^n p^{-\nu n} \right) F(p) \quad (n \in \mathbb{N}_0, |c/p| < 1) \tag{1.10}$$

If $f(t)$ is a function of variable $t$ that takes real or complex values, and $p$ is a parameter that takes real or complex values, then the Laplace transform of $f(t)$ can be expressed as shown in reference [44].

$$F(p) = L\{f(t);p\} = \int_0^{\infty} e^{-pt} f(t)dt, \quad (\mathrm{Re}(p) > 0) \tag{1.11}$$

The kinetic equation holds significant value and demonstrates notable efficacy within several astrophysical contexts, the writers create a more extended version of the fractional kinetic equation that incorporates an improved k-Bessel function. The publication presents this version of the fractional kinetic equation.

The following words are crucial to our investigation in this work but will not be defined here. Lately, there has been an increasing curiosity among scholars in the study of unified integrals involving special functions due to their versatile applications in several domains (refer to references [27, **?**]). The k-gamma function and the k-Pochhammer symbol were presented by Diaz and Pariguan [46].

$$(\gamma)_{n,k} = \begin{cases} \frac{\Gamma_k(\gamma+nk)}{\Gamma_k(\gamma)} & (k \in \mathbb{R}; \gamma \in \mathbb{C} \setminus \{0\}) \\ \gamma(\gamma+k)\dots(\gamma+(n-1)k) & (n \in \mathbb{N}; \gamma \in \mathbb{C}) \end{cases} \tag{1.12}$$

They established a connection with the classical Euler's gamma function, as referenced in [34, **?**].:

$$\Gamma_k(\gamma) = k^{\gamma/k-1}\Gamma(\gamma/k) \tag{1.13}$$

It is evident that when $k$ is equal to 1, the expression (1.12) simplifies to the gamma function of Euler and the conventional Pochhammer symbol, as mentioned in [47].

In a recent study, Romero et al. [48] developed the First-kind k-Bessel function for $\lambda, \gamma$, and $\mu$ belonging to the complex number set, with the condition that $\mathrm{Re}(\lambda)$ and $\mathrm{Re}(\mu)$ are both greater than zero. This function is defined as follows, as previously mentioned in reference [49].

$$J_{k,\mu}^{(\gamma,\lambda)}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{n,k}}{\Gamma_k(\lambda n+\mu+1)} \frac{(-1)^n}{(n!)^2} \left(\frac{z}{2}\right)^n \tag{1.14}$$

The k-Pochhammer symbol, denoted by the notation $(\lambda)_{n,k}$ is defined by the equation (1.12).

Generalized k-Bessel function $\omega_{k,\mu,b,c}^{(\gamma,\lambda)}(z)$ is given by G. Singh et al. [55] as follows:

$$\omega_{k,\mu,b,c}^{(\gamma,\lambda)}(z) = \sum_{n=0}^{\infty} \frac{(-1)^n c^n (\gamma)_{n,k}}{\Gamma_k(\lambda n+\mu+(b+1)/2)} \frac{1}{(n!)^2} \left(\frac{z}{2}\right)^{\mu+2n} \tag{1.15}$$

where $b, c, \lambda, \gamma, \mu \in \mathbb{C}$ and $\mathrm{Re}(\lambda) > 0, \mathrm{Re}(\mu) > 0$.

We give the following description of the advanced variant of the k-Bessel function $F_{q,k,\mu,h,\lambda}^{(p,\gamma,c,()}z)$:

$$F_{q,k,\mu,h,\lambda}^{(p,\gamma,c,()}z) = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}(a_i)_n}{\prod_{j=1}^{q}(b_j)_n} \frac{(-1)^n c^n (\gamma)_{n,k}}{\Gamma_k(\lambda n+\mu+(h+1)/2)} \frac{1}{(n!)^2} \left(\frac{z}{2}\right)^{\mu+2n} \tag{1.16}$$

where $\lambda,\gamma,\mu,h,c \in \mathbb{C}$ and $\mathrm{Re}(\lambda) > 0, \mathrm{Re}(\mu) > 0$. Here, $p$ upper parameters $a_1, a_2, \ldots, a_p$ and $q$ lower parameters $b_1, b_2, \ldots, b_q$, $\gamma \in \mathbb{C}$, and $(a_i)_n, (b_j)_n$ are Pochhammer symbols. The series (1.16) is defined when none of the denominator parameters $b_j$'s, $j = 1, 2, \ldots, q$ is a negative integer or zero.

The equation (1.15) can be modified as equation (1.16) if no upper or lower parameters (i.e., $p = q = 0$).

The Fox–Wright function, denoted as ${}_p\psi_q\left[{}^{z}\;;\right.$ , *isamathematicalfunctionthatinvolvespnumeratorparametersandqdenominatorparameters.Andparametersa*$_i, b_j$ viz. $a_i, b_j \in \mathbb{C} (i = 1, \ldots, p; j = 1, \ldots, q)$ are defined according to the details provided in reference [50].

$$ {}_p\psi_q\left[\begin{matrix}(a_i,\alpha_i)_{1,p}\\(b_j,\beta_j)_{1,q}\end{matrix};z\right] = \sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma(a_i+\alpha_i n)}{\prod_{j=1}^{q}\Gamma(b_j+\beta_j n)}\frac{z^n}{n!} \tag{1.17} $$

under the condition

$$ \sum_{j=1}^{q}\beta_j - \sum_{i=1}^{p}\alpha_i > -1 \tag{1.18} $$

In particular, when $\alpha_i = \beta_j = 1 (i = 1, \ldots, p; j = 1, \ldots, q)$, with instantaneous reduction, ${}_p\psi_q\left[{}^{z}\;;\right.$ b *ecomesthegeneralizedhypergeometricfunction*${}_pF_q\left[{}^{z}_{(};p\right], q \in \mathbb{N}_0)$ to learn more, check [31]

$$ {}_p\psi_q\left[\begin{matrix}(a_i,1)_{1,p}\\(b_j,1)_{1,q}\end{matrix};z\right] = \frac{\prod_{i=1}^{p}\Gamma(a_i)}{\prod_{j=1}^{q}\Gamma(b_j)}\,{}_pF_q\left[\begin{matrix}a_1,\ldots,a_p\\b_1,\ldots,b_q\end{matrix};z\right] \tag{1.19} $$

The goal of this study is to find out how to solve the fractional kinetic equation that includes advanced k-Bessel function. The consequences that can be gotten from the M-series function are pretty general and can be used to make many different fractional kinetic equations, both old and new.

## 2 Kinetic equations with generalized fraction and their solution

This section of the study will examine how the advanced k-Bessel function can be used to solve the generalized fractional kinetic equations.

The generalized M-series with two parameters [56] is defined as follows, and it is used to express the solutions to the fractional kinetic equations.

$$ {}_pM_q^{(\alpha,\beta)}(z) = \sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}(a_i)_n}{\prod_{j=1}^{q}(b_j)_n}\frac{z^n}{\Gamma(\alpha n+\beta)}, \quad \mathrm{Re}(\alpha) > 0, \mathrm{Re}(\beta) > 0, \tag{2.1} $$

**Theorem 1.***If* $d > 0, v > 0, \lambda,\gamma,\mu,h,c \in \mathbb{C}, k \in \mathbb{N}, \mathrm{Re}(\mu) > 0$ *and* $\mathrm{Re}(\lambda) > 0$ *consequently, the equation's result*

$$ N(t) - N_0 F^{(p,\gamma,c,()}_{q,k,\mu,h,\lambda}t) = -d^v{}_0D_t^{-v}N(t) \tag{2.2} $$

*is defined by the equation*

$$ N(t) = N_0\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}(a_i)_n}{\prod_{j=1}^{q}(b_j)_n}\frac{(-1)^n c^n(\gamma)_{n,k}}{\Gamma_k(\lambda n+\mu+(h+1)/2)}\frac{\Gamma(\mu+2n+1)}{(n!)^2} \times\left(\frac{t}{2}\right)^{\mu+2n}{}_pM_q^{(v,\mu+2n+1)}(-d^v t^v) \tag{2.3} $$

*Proof.*According to Srivastava and Saxena [53], and Erdelyi et al. [52], the following expression describes the Laplace transform of the Riemann–Liouville fractional integral operator:

$$ L\{{}_0D_t^{-v}f(t);p\} = p^{-v}F(p) \tag{2.4} $$

Where $F(p)$ is defined in (1.11). When the Laplace transform is used on both sides of equation (2.2), we get the following:

$$ L\{N(t);p\} = N_0 L\{F^{(p,\gamma,c,()}_{q,k,\mu,h,\lambda}t);p\} - d^v L\{{}_0D_t^{-v}N(t);p\} \tag{2.5} $$

Let $N(p) = L\{N(t);p\}$. Using (2.4) and the definition (1.16):

$$N(p) = N_0 \int_0^\infty e^{-pt} \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}(a_i)_n}{\prod_{j=1}^{q}(b_j)_n} \frac{(-1)^n c^n (\gamma)_{n,k}}{\Gamma_k(\lambda n+\mu+(h+1)/2)} \frac{1}{(n!)^2} \left(\frac{t}{2}\right)^{\mu+2n} dt - d^v p^{-v} N(p) \quad (2.6)$$

Rearranging and evaluating the integral term-by-term (assuming valid interchange):

$$N(p)(1+d^v p^{-v}) = N_0 \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}(a_i)_n}{\prod_{j=1}^{q}(b_j)_n} \frac{(-1)^n c^n (\gamma)_{n,k}}{\Gamma_k(\lambda n+\mu+(h+1)/2)} \frac{1}{(n!)^2} \left(\frac{1}{2}\right)^{\mu+2n} \int_0^\infty e^{-pt} t^{\mu+2n} dt$$

$$= N_0 \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}(a_i)_n}{\prod_{j=1}^{q}(b_j)_n} \frac{(-1)^n c^n (\gamma)_{n,k}}{\Gamma_k(\lambda n+\mu+(h+1)/2)} \frac{1}{(n!)^2} \left(\frac{1}{2}\right)^{\mu+2n} \frac{\Gamma(\mu+2n+1)}{p^{\mu+2n+1}} \quad (2.7)$$

On solving for $N(p)$, using the geometric series expansion $(1+x)^{-1} = \sum_{r=0}^{\infty}(-x)^r$ for $x = d^v p^{-v}$ (assuming $|d^v p^{-v}| < 1$), we have:

$$N(p) = N_0 \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}(a_i)_n}{\prod_{j=1}^{q}(b_j)_n} \frac{(-1)^n c^n (\gamma)_{n,k}}{\Gamma_k(\lambda n+\mu+(h+1)/2)} \frac{\Gamma(\mu+2n+1)}{(n!)^2} \left(\frac{1}{2}\right)^{\mu+2n} \times p^{-(\mu+2n+1)} \sum_{r=0}^{\infty} (-d^v p^{-v})^r \quad (2.8)$$

$$N(p) = N_0 \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}(a_i)_n}{\prod_{j=1}^{q}(b_j)_n} \frac{(-1)^n c^n (\gamma)_{n,k}}{\Gamma_k(\lambda n+\mu+(h+1)/2)} \frac{\Gamma(\mu+2n+1)}{(n!)^2} \left(\frac{1}{2}\right)^{\mu+2n} \times \sum_{r=0}^{\infty} (-1)^r (d^v)^r p^{-(\mu+2n+1+vr)} \quad (2.9)$$

Applying the inverse Laplace transform $L^{-1}$ and taking into account

$$L^{-1}\{p^{-v};t\} = \frac{t^{v-1}}{\Gamma(v)}, \quad \mathrm{Re}(v) > 0 \quad (2.10)$$

we possess (assuming valid interchange of $L^{-1}$ and summation):

$$N(t) = L^{-1}\{N(p)\} = N_0 \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}(a_i)_n}{\prod_{j=1}^{q}(b_j)_n} \frac{(-1)^n c^n (\gamma)_{n,k}}{\Gamma_k(\lambda n+\mu+(h+1)/2)} \frac{\Gamma(\mu+2n+1)}{(n!)^2} \left(\frac{1}{2}\right)^{\mu+2n} \times L^{-1}\left[\sum_{r=0}^{\infty} (-1)^r (d^v)^r p^{-(\mu+2n+1+vr)}\right] \quad (2.11)$$

$$N(t) = N_0 \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}(a_i)_n}{\prod_{j=1}^{q}(b_j)_n} \frac{(-1)^n c^n (\gamma)_{n,k}}{\Gamma_k(\lambda n+\mu+(h+1)/2)} \frac{\Gamma(\mu+2n+1)}{(n!)^2} \left(\frac{1}{2}\right)^{\mu+2n} \times \sum_{r=0}^{\infty} (-1)^r (d^v)^r \frac{t^{\mu+2n+vr}}{\Gamma(\mu+2n+1+vr)} \quad (2.12)$$

$$N(t) = N_0 \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}(a_i)_n}{\prod_{j=1}^{q}(b_j)_n} \frac{(-1)^n c^n (\gamma)_{n,k}}{\Gamma_k(\lambda n+\mu+(h+1)/2)} \frac{\Gamma(\mu+2n+1)}{(n!)^2} \left(\frac{t}{2}\right)^{\mu+2n} \times \sum_{r=0}^{\infty} \frac{(-d^v t^v)^r}{\Gamma(vr+\mu+2n+1)} \quad (2.13)$$

By reinterpreting the inner sum in Eq. (2.13) using the definition (2.1), we arrive at the desired result (2.3).

**Theorem 2.***If* $d > 0, v > 0, \lambda, \gamma, \mu, h, c \in \mathbb{C}, k \in \mathbb{N}, \mathrm{Re}(\mu) > 0$ *and* $\mathrm{Re}(\lambda) > 0$ *consequently, the equation's result*

$$N(t) = N_0 F^{(p,\gamma,c,()}_{q,k,\mu,h,\lambda} d^v t^v) - d^v {}_0D_t^{-v} N(t) \tag{2.14}$$

*is defined by the equation*

$$N(t) = N_0 \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}(a_i)_n}{\prod_{j=1}^{q}(b_j)_n} \frac{(-1)^n c^n (\gamma)_{n,k}}{\Gamma_k(\lambda n + \mu + (h+1)/2)} \frac{\Gamma(v(\mu+2n)+1)}{(n!)^2} \times \left(\frac{(d^v t^v)}{2}\right)^{\mu+2n} {}_pM_q^{(v,v(\mu+2n)+1)}(-d^v t^v) \tag{2.15}$$

**Theorem 3.***If* $a > 0, d > 0, v > 0; a \neq d; \lambda, \gamma, \mu, h, c \in \mathbb{C}, k \in \mathbb{N}, \mathrm{Re}(\mu) > 0$ *and* $\mathrm{Re}(\lambda) > 0$ *consequently, the equation's result*

$$N(t) = N_0 F^{(p,\gamma,c,()}_{q,k,\mu,h,\lambda} d^v t^v) - a^v {}_0D_t^{-v} N(t) \tag{2.16}$$

*is defined by the equation*

$$N(t) = N_0 \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p}(a_i)_n}{\prod_{j=1}^{q}(b_j)_n} \frac{(-1)^n c^n (\gamma)_{n,k}}{\Gamma_k(\lambda n + \mu + (h+1)/2)} \frac{\Gamma(v(\mu+2n)+1)}{(n!)^2} \times \left(\frac{(d^v t^v)}{2}\right)^{\mu+2n} {}_pM_q^{(v,v(\mu+2n)+1)}(-a^v t^v) \tag{2.17}$$

*Proof.*Because the proofs of theorems 2 and 3 would be similar to the proof of theorem 1, we will skip over the specifics of those proofs.

## 3 Special cases

The advanced k-Bessel function can be reduced to the following form if we decide to use the parameters $h = 1, c = 1$ and neither the upper nor the lower parameter. :

$$F^{0,\gamma,1}_{0,k,\mu,1,\lambda}(z) = \left(\frac{z}{2}\right)^{\mu} \sum_{n=0}^{\infty} \frac{(-1)^n (\gamma)_{n,k}}{\Gamma_k(\lambda n + \mu + 1)} \frac{1}{(n!)^2} \left(\frac{z^2}{2}\right)^n = \left(\frac{z}{2}\right)^{\mu} J^{\gamma,\lambda}_{k,\mu}\left(\frac{z^2}{2}\right) \tag{3.1}$$

Where $\lambda, \gamma, \mu \in \mathbb{C}$, $\mathrm{Re}(\mu) > 0$, and $\mathrm{Re}(\lambda) > 0$

Theorems 1-3 can be condensed to the form that is presented below, and the conclusions presented here are in agreement with those given by G. Singh et al. [55].

**Corollary 1.***If* $d > 0, v > 0, \lambda, \gamma, \mu \in \mathbb{C}, k \in \mathbb{N}, \mathrm{Re}(\mu) > 0$ *and* $\mathrm{Re}(\lambda) > 0$, *consequently, the equation's result*

$$N(t) - N_0 \left(\frac{t}{2}\right)^{\mu} J^{(\gamma,\lambda)}_{k,\mu}\left(\frac{t^2}{2}\right) = -d^v {}_0D_t^{-v} N(t) \tag{3.2}$$

*is defined by the equation*

$$N(t) = N_0 \sum_{n=0}^{\infty} \frac{(-1)^n (\gamma)_{n,k}}{\Gamma_k(\lambda n + \mu + 1)} \frac{\Gamma(\mu+2n+1)}{(n!)^2} \left(\frac{t}{2}\right)^{\mu+2n} {}_0M_0^{(v,\mu+2n+1)}(-d^v t^v) \tag{3.3}$$

**Corollary 2.***If* $d > 0, v > 0, \lambda, \gamma, \mu \in \mathbb{C}, k \in \mathbb{N}, \mathrm{Re}(\mu) > 0$ *and* $\mathrm{Re}(\lambda) > 0$ *consequently, the equation's result*

$$N(t) = N_0 \left(\frac{d^v t^v}{2}\right)^{\mu} J^{(\gamma,\lambda)}_{k,\mu}\left(\frac{(d^v t^v)^2}{2}\right) - d^v {}_0D_t^{-v} N(t) \tag{3.4}$$

*is defined by the equation*

$$N(t) = N_0 \sum_{n=0}^{\infty} \frac{(-1)^n (\gamma)_{n,k}}{\Gamma_k(\lambda n + \mu + 1)} \frac{\Gamma(v(\mu+2n)+1)}{(n!)^2} \left(\frac{d^v t^v}{2}\right)^{\mu+2n} {}_0M_0^{(v,v(\mu+2n)+1)}(-d^v t^v) \tag{3.5}$$

**Corollary 3.** *If* $a>0, d>0, v>0; a\neq d; \lambda,\gamma,\mu\in\mathbb{C}, k\in\mathbb{N}, \mathrm{Re}(\mu)>0$ *and* $\mathrm{Re}(\lambda)>0$, *consequently, the equation's result*

$$N(t)=N_0\left(\frac{d^v t^v}{2}\right)^{\mu} J_{k,\mu}^{(\gamma,\lambda)}\left(\frac{(d^v t^v)^2}{2}\right)-a^v{}_0D_t^{-v}N(t) \tag{3.6}$$

*is defined by the equation*

$$N(t)=N_0\sum_{n=0}^{\infty}\frac{(-1)^n(\gamma)_{n,k}}{\Gamma_k(\lambda n+\mu+1)}\frac{\Gamma(v(\mu+2n)+1)}{(n!)^2}\left(\frac{d^v t^v}{2}\right)^{\mu+2n}{}_0M_0^{(v,v(\mu+2n)+1)}(-a^v t^v) \tag{3.7}$$

In the event that we decide to use the parameters $h=-1, c=1$, and neither an upper nor a lower parameter ($p=q=0$), the advanced k-Bessel function becomes related to the k-Wright function [54], which is connected with the following relation (following the text's version):

$$F_{0,k,\mu,-1,\lambda}^{0,\gamma,1}(z)=\left(\frac{z}{2}\right)^{\mu} W_{k,\lambda,\mu}^{(\gamma)}\left(-\frac{z^2}{2}\right) \tag{3.8}$$

Where $\lambda,\gamma,\mu\in\mathbb{C}$ and $\mathrm{Re}(\lambda)>0, \mathrm{Re}(\mu)>0$.

It is possible to rewrite Theorems 1-3 as shown below.

**Corollary 4.** *If* $d>0, v>0, \lambda,\gamma,\mu\in\mathbb{C}, k\in\mathbb{N}, \mathrm{Re}(\mu)>0$ *and* $\mathrm{Re}(\lambda)>0$ *consequently, the equation's result*

$$N(t)-N_0\left(\frac{t}{2}\right)^{\mu} W_{k,\lambda,\mu}^{(\gamma)}\left(-\frac{t^2}{2}\right)=-d^v{}_0D_t^{-v}N(t) \tag{3.9}$$

*is defined by the equation*

$$N(t)=N_0\sum_{n=0}^{\infty}\frac{(-1)^n(\gamma)_{n,k}}{\Gamma_k(\lambda n+\mu)}\frac{\Gamma(\mu+2n+1)}{(n!)^2}\left(\frac{t}{2}\right)^{\mu+2n}{}_0M_0^{(v,\mu+2n+1)}(-d^v t^v) \tag{3.10}$$

**Corollary 5.** *If* $d>0, v>0, \lambda,\gamma,\mu\in\mathbb{C}, k\in\mathbb{N}, \mathrm{Re}(\mu)>0$ *and* $\mathrm{Re}(\lambda)>0$, *consequently, the equation's result*

$$N(t)=N_0\left(\frac{d^v t^v}{2}\right)^{\mu} W_{k,\lambda,\mu}^{(\gamma)}\left(-\frac{(d^v t^v)^2}{2}\right)-d^v{}_0D_t^{-v}N(t) \tag{3.11}$$

*is defined by the equation*

$$N(t)=N_0\sum_{n=0}^{\infty}\frac{(-1)^n(\gamma)_{n,k}}{\Gamma_k(\lambda n+\mu)}\frac{\Gamma(v(\mu+2n)+1)}{(n!)^2}\left(\frac{d^v t^v}{2}\right)^{\mu+2n}{}_0M_0^{(v,v(\mu+2n)+1)}(-d^v t^v) \tag{3.12}$$

**Corollary 6.** *If* $a>0, d>0, v>0; a\neq d; \lambda,\gamma,\mu\in\mathbb{C}, k\in\mathbb{N}, \mathrm{Re}(\mu)>0$ *and* $\mathrm{Re}(\lambda)>0$, *consequently, the equation's result*

$$N(t)=N_0\left(\frac{d^v t^v}{2}\right)^{\mu} W_{k,\lambda,\mu}^{(\gamma)}\left(-\frac{(d^v t^v)^2}{2}\right)-a^v{}_0D_t^{-v}N(t) \tag{3.13}$$

*is defined by the equation*

$$N(t)=N_0\sum_{n=0}^{\infty}\frac{(-1)^n(\gamma)_{n,k}}{\Gamma_k(\lambda n+\mu)}\frac{\Gamma(v(\mu+2n)+1)}{(n!)^2}\left(\frac{d^v t^v}{2}\right)^{\mu+2n}{}_0M_0^{(v,v(\mu+2n)+1)}(-a^v t^v) \tag{3.14}$$

After making a few small reductions and using the results in equations (1.12) and (1.13), theorems 1-3 can be expressed as follows.

**Corollary 7.** *If* $d>0, v>0; \lambda,\gamma,\mu,b,c\in\mathbb{C}, k\in\mathbb{N}, \mathrm{Re}(\mu)>0$ *and* $\mathrm{Re}(\lambda)>0$, *consequently,*

$$N(t)-N_0k^{1-\mu/k-(b+1)/(2k)}\frac{1}{\Gamma(\gamma/k)} \times {}_1\psi_2\left[\begin{matrix}(\gamma/k,1)\\(\mu/k+(b+1)/(2k),\lambda/k),(1,1)\end{matrix};-ck^{\lambda/k-1}t\right]=-d^v{}_0D_t^{-v}N(t) \tag{3.15}$$

*is defined by the equation*

$$N(t) = N_0 k^{1-\mu/k-(b+1)/(2k)} \frac{1}{\Gamma(\gamma/k)} \sum_{n=0}^{\infty} \frac{(-ck^{\lambda/k-1})^n}{\Gamma(\mu/k+\lambda n/k+(b+1)/(2k))} \frac{\Gamma(\mu+2n+1)}{(n!)^2} \times \left(\frac{d^v t^v}{2}\right)^{\mu+2n} {}_0M_0^{(v,\mu+2n+1)}(-d^v t^v) \quad (3.16)$$

**Corollary 8.** *If* $d > 0, v > 0; \lambda, \gamma, \mu, b, c \in \mathbb{C}, k \in \mathbb{N}, \mathrm{Re}(\mu) > 0$ *and* $\mathrm{Re}(\lambda) > 0$*, consequently, the equation's result*

$$N(t) = N_0 k^{1-\mu/k-(b+1)/(2k)} \frac{1}{\Gamma(\gamma/k)} \times {}_1\psi_2 \left[ \begin{matrix} (\gamma/k, 1) \\ (\mu/k+(b+1)/(2k), \lambda/k), (1,1) \end{matrix} ; -ck^{\lambda/k-1}(d^v t^v) \right] - d^v {}_0D_t^{-v} N(t) \quad (3.17)$$

*is defined by the equation*

$$N(t) = N_0 k^{1-\mu/k-(b+1)/(2k)} \frac{1}{\Gamma(\gamma/k)} \sum_{n=0}^{\infty} \frac{(-ck^{\lambda/k-1})^n}{\Gamma(\mu/k+\lambda n/k+(b+1)/(2k))} \frac{\Gamma(v(\mu+2n)+1)}{(n!)^2} \times \left(\frac{d^v t^v}{2}\right)^{\mu+2n} {}_0M_0^{(v,v(\mu+2n)+1)}(-d^v t^v) \quad (3.18)$$

**Corollary 9.** *If* $a > 0, d > 0, v > 0; a \neq d; \lambda, \gamma, \mu, b, c \in \mathbb{C}, k \in \mathbb{N}, \mathrm{Re}(\mu) > 0$ *and* $\mathrm{Re}(\lambda) > 0$*,*

$$N(t) = N_0 k^{1-\mu/k-(b+1)/(2k)} \frac{1}{\Gamma(\gamma/k)} \times {}_1\psi_2 \left[ \begin{matrix} (\gamma/k, 1) \\ (\mu/k+(b+1)/(2k), \lambda/k), (1,1) \end{matrix} ; -ck^{\lambda/k-1}(d^v t^v) \right] - a^v {}_0D_t^{-v} N(t) \quad (3.19)$$

*is defined by the equation*

$$N(t) = N_0 k^{1-\mu/k-(b+1)/(2k)} \frac{1}{\Gamma(\gamma/k)} \sum_{n=0}^{\infty} \frac{(-ck^{\lambda/k-1})^n}{\Gamma(\mu/k+\lambda n/k+(b+1)/(2k))} \frac{\Gamma(v(\mu+2n)+1)}{(n!)^2} \times \left(\frac{d^v t^v}{2}\right)^{\mu+2n} {}_0M_0^{(v,v(\mu+2n)+1)}(-a^v t^v) \quad (3.20)$$

## 4 Concluding remarks

We present a fractional generalization of the standard kinetic equation as well as a derivation of the solution that corresponds to it in this piece of research that we have carried out. It is not difficult to obtain a great number of well-known and fractional kinetic equations as a result of the close relationship that exists between the advanced k-Bessel function and a big number of different special functions.

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