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An Optimal Control Problem for Dengue Fever Model Using Caputo Fractional Derivatives

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Abstract: In this paper, we study the optimal control of the dengue disease model with the vertical transmission in terms of the Caputo fractional derivatives. We apply the control parameters like larvicide, fogging, vaccination, and isolation to stop the spread of the dengue epidemic and explore the influence of the fractional order α $(0.6 \leq \alpha \leq 1)$ on the dengue transmission model. We apply a forward-backward sweep scheme using the Adams-type predictor-corrector approach for solving the proposed control problem. Finally, the effects of optimal controls considering three different cases in the given model are discussed.

Keywords: Optimal control, dengue fever, Caputo fractional derivative, forward-backward sweep method.

1 Introduction

Dengue fever is transmitted by Aedes mosquitoes and it is the most common and important vector-borne viral disease in humans. Mostly, it affects tropical and subtropical areas. Recently the World Health Organisation(WHO) mentioned that the dengue epidemic is one of the top 10 threats in the world. Dengue virus is a single-stranded, non-segmented RNA virus and it has four serotypes. During an epidemic period, several serotypes can be in circulation. Infection with one serotype will provide only lifelong immunity against that particular serotype. On the other hand, the subsequent infection with a different serotype will give serious illness (severe dengue(DHF) and dengue shock syndrome).

Mathematical modeling has played an important role in analyzing the transmission dynamics of several deadly diseases. The mathematical modeling of any disease can help us to observe the mechanism of influencing the spread of the epidemic, epidemiological patterns, and disease control. Recently, a number of mathematical models have been introduced to explore the dynamics of various diseases, like Covid-19 [1,2], malaria [3], cancer [4], cavity [5], HIV [6], mosaic (a plant disease) [7], etc. Several mathematical models have been introduced by the various researchers to explore the effects of possible factors in the transmission of dengue dynamics. In the last few years, several researchers have attracted to work on the dengue epidemic using optimal control theory due to its ability of decision making. In [8,9], the authors proposed a mathematical model to represent the relationship between human and dengue disease mosquito populations and used an optimal control approach to find the most effective technique to fight against the dengue infection. In [10], the authors considered the effects of Antibody-Dependent Enhancement(ADE) in the mathematical model and found that vaccination could decrease dengue incidence and provide population benefits even in the presence of ADE. *Roberto et. al* in [11] presented the dengue model with the effectiveness of the application of sterile insect technique and insecticide to the mosquito population using an optimal control approach. In [12], the authors applied a multi-objective method to determine an optimal control for a dengue disease model. In [13], the researchers formulated an optimization problem with infinite-time quadratic cost functional and used three control strategies to reduce the mosquito population and dengue infection. *Windarto et. al* in [14] proposed a method for estimating the parameters of the host-vector and SIR type dengue disease models using the particle swarm optimization approach in the sense of the Atangana-Baleanu fractional derivative.

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In [15], the authors investigated the impact of the combination of vaccination and sterile insect technique for reducing the number of infected individuals. In [16], the authors studied the impact of prey-predator dynamics in controlling the dengue disease in pre-adult mosquito populations. In [17], the authors used some effective computational techniques to derive the solution of the reaction-diffusion epidemic model of dengue. In [18], analyses of a fractional-order dengue model with control strategies have been given. *Xue et al.* recently investigated the optimal control of dengue virus mitigation measures in ref. [19]. For more information, one can refer to ref. [20,21,22,23,24,25,26,27,28,29,30].

Motivated by the above works, in this article, we consider a fractional-order dengue fever model with the vertical transmission in Taiwan given by *Defterli* in [31] and generalize that model to fractional optimal control problem (FOCP) using control strategies to minimize the spread of transmission of dengue. The necessary conditions for optimality of the FOCP are explored using the concept of Pontriagin's Minimum principle. The FOCP is given, in which the state and costate equations are expressed in terms of left fractional derivatives. Finally, the numerical results of the FOCP are obtained by using the forward-backward sweep method [32].

The paper is structured as follows: In section 2, we give some essential definitions as well as a mathematical model to represent dengue dynamics transmission. In section 3, we formulate an optimal control model to keep the spread of the disease under control, and then the provision of larvicide, fogging, vaccination, and isolation are used as control variables to obtain the optimal cost and reduce the number of the affected human population. The numerical results and discussion are shown in Section 4. A conclusion is given in Section 5.

2 The Mathematical Model

Firstly, we recall the definition of Caputo fractional derivative and an important lemma.

Definition 21*The left and right-sided Caputo fractional derivative of order $\alpha \in (m-1,m]$, $m \in N$ is given by*

$$^{C}D^{\alpha}_{\nu_0+}x(\nu) = \frac{1}{\Gamma(n-\alpha)}\int_{\nu_0}^{\nu}(\nu-\tau)^{n-\alpha-1}x^{(m)}(\tau)\mathrm{d}\tau$$

and

$$^{C}D^{\alpha}_{\nu_F-}x(\nu) = \frac{(-1)^m}{\Gamma(n-\alpha)}\int_{\nu}^{\nu_F}(\tau-\nu)^{n-\alpha-1}x^{(m)}(\tau)\mathrm{d}\tau$$

provided their existence almost everywhere on $[\nu_0,\nu_F]$.

Lemma 21*[32] The following given constraints are equivalent:*

$$^{C}_{\nu}D^{\alpha}_{\nu_F}\lambda(\nu) = \frac{\partial H}{\partial x}(\nu,x(\nu),u(\nu),\lambda(\nu))$$
$$^{C}_{\nu_0}D^{\alpha}_{\nu}\lambda(\nu_F-\nu) = \frac{\partial H}{\partial x}(\nu_F-\nu,x(\nu_F-\nu),u(\nu_F-\nu),\lambda(\nu_F-\nu))$$

where $0<\alpha\le 1$.

Now, we consider the fractional-order mathematical model with the vertical transmission dynamics of dengue fever given in [31]. The total population is classified into three main classes, namely human(host), pre-adult female mosquito(vector), and adult female mosquito(vector) population. In human(host) class, there are susceptible human H_s, infected/infectious human H_i and recovered/immune human H_r. The pre-adult female mosquito(vector) population consists of two classes; susceptible E_s and infected E_i. The adult female mosquito(vector) population is divided into three classes; susceptible M_s, infectious M_e and infected M_i. Thus, the dengue model is given by

$$\begin{aligned}
{}^{C}D^{\alpha}_{\nu_0+}E_s(\nu) &= e^{\alpha}_{v}\left(1-p\left(\frac{M_i(\nu)}{M_s(\nu)+M_e(\nu)+M_i(\nu)}\right)\right)-\eta^{\alpha}E_s(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}E_i(\nu) &= e^{\alpha}_{v}p\left(\frac{M_i(\nu)}{M_s(\nu)+M_e(\nu)+M_i(\nu)}\right)-\eta^{\alpha}E_i(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}M_s(\nu) &= \eta^{\alpha}E_s(\nu)-b^{\alpha}\frac{H_i(\nu)}{N_h}M_s(\nu)-\delta^{\alpha}M_s(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}M_e(\nu) &= b^{\alpha}\frac{H_i(\nu)}{N_h}M_s(\nu)-\gamma^{\alpha}M_e(\nu)-\delta^{\alpha}M_e(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}M_i(\nu) &= \gamma^{\alpha}M_e(\nu)+\eta^{\alpha}E_i(\nu)-\delta^{\alpha}M_i(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}H_s(\nu) &= R^{\alpha}_{hb}N_h-b^{\alpha}\frac{H_i(\nu)}{N_h}M_i(\nu)-R^{\alpha}_{hd}H_s(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}H_i(\nu) &= b^{\alpha}\frac{H_i(\nu)}{N_h}M_i(\nu)-\zeta^{\alpha}H_i(\nu)-R^{\alpha}_{hd}H_i(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}H_r(\nu) &= \zeta^{\alpha}H_i(\nu)-R^{\alpha}_{hd}H_r(\nu),
\end{aligned} \tag{1}$$

where $0<\alpha\leq 1$ and the corresponding model parameters are e_v; oviposition rate, p; proportion of eggs, η; pre-adult mosquito maturation rate, b; the biting rate, N_h; total size of human population, δ; adult mosquito mortality rate, γ; virus incubation rate in mosquito, R_{hb}; human birth rate, R_{hd}; human mortality rate, and ζ; human recovery rate.

3 Derivation of Optimal Control Problem

In this section, we formulate and solve an optimal control problem described as follows:

Find the optimal control u(t) for minimizing the cost functional

$$J(u)=\int_{\nu_0}^{\nu_F}\Phi\Big(\nu,x(\nu),u(\nu)\Big)\mathrm{d}\nu$$

subject to the dynamic constraint

$$\begin{aligned}
{}^{C}D^{\alpha}_{\nu_0+}x(\nu) &= F(\nu,x(\nu),u(\nu)),\\
x(0) &= x_0,
\end{aligned}$$

where $0<\alpha\leq 1$, the state and control variables $x(\nu)$ and $u(\nu)$ simultaneously, $\Phi(\cdot,\cdot,\cdot)$ and $F(\cdot,\cdot,\cdot)$ are differentiable functions.

In the given system, the state variable is defined by

$$x(\nu)=\Big(E_s(\nu),E_i(\nu),M_s(\nu),M_e(\nu),M_i(\nu),H_s(\nu),H_i(\nu),H_r(\nu)\Big)^T\in R^8.$$

Our task is to determine the optimal control $u^*(\nu)=\Big(u_1(\nu),u_2(\nu),u_3(\nu),u_4(\nu)\Big)^T\in R^4$ to reduce the number of infected humans and the expense of controlling infections through larvicide, fogging, vaccine, and isolation.

Let us take the cost functional

$$J(u_1,u_2,u_3,u_4)=\frac{1}{2}\int_0^{\nu_F}\Big(c_1H_i^2+c_2u_1^2+c_3u_2^2+c_4u_3^2+c_5u_4^2\Big)\mathrm{d}\nu$$

to be minimized with the weighting parameters $c_k>0$ for $k=1,2,3,4,5$. The OCP is constructed with time dependent optimal controls such as larvicide control $u_1(\nu)$, fogging control $u_2(\nu)$, vaccination control $u_3(\nu)$, and isolation control

$u_4(\nu)$ to minimize the infection of the dengue epidemic. Thus, the model (1) is given by

$$
\begin{aligned}
{}^{C}D^{\alpha}_{\nu_0+}E_s(\nu) &= e^{\alpha}_{\nu}\left(1-p\left(\frac{M_i(\nu)}{M_s(\nu)+M_e(\nu)+M_i(\nu)}\right)\right)-\eta^{\alpha}E_s(\nu)-u_1(\nu)E_s(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}E_i(\nu) &= e^{\alpha}_{\nu}p\left(\frac{M_i(\nu)}{M_s(\nu)+M_e(\nu)+M_i(\nu)}\right)-\eta^{\alpha}E_i(\nu)-u_1(\nu)E_i(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}M_s(\nu) &= \eta^{\alpha}E_s(\nu)-b^{\alpha}\frac{H_i(\nu)}{N_h}M_s(\nu)-\delta^{\alpha}M_s(\nu)-u_2(\nu)M_s(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}M_e(\nu) &= b^{\alpha}\frac{H_i(\nu)}{N_h}M_s(\nu)-\gamma^{\alpha}M_e(\nu)-\delta^{\alpha}M_e(\nu)-u_2(\nu)M_e(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}M_i(\nu) &= \gamma^{\alpha}M_e(\nu)+\eta^{\alpha}E_i(\nu)-\delta^{\alpha}M_i(\nu)-u_2(\nu)M_i(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}H_s(\nu) &= R^{\alpha}_{hb}N_h-b^{\alpha}\frac{H_i(\nu)}{N_h}M_i(\nu)-R^{\alpha}_{hd}H_s(\nu)-u_3(\nu)H_s(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}H_i(\nu) &= b^{\alpha}\frac{H_i(\nu)}{N_h}M_i(\nu)-\zeta^{\alpha}H_i(\nu)-R^{\alpha}_{hd}H_i(\nu)-u_4(\nu)H_i(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}H_r(\nu) &= \zeta^{\alpha}H_i(\nu)-R^{\alpha}_{hd}H_r(\nu)+u_3(\nu)H_s(\nu)+u_4(\nu)H_i(\nu),
\end{aligned}
$$

where $0<\alpha\leq 1$ and $0\leq u_k(\nu)\leq 1,\ k=1,2,3,4$. The theory of Pontryagin's minimum principle is used to determine the necessary optimality constraints of the OCP.

Consider the costate vector $\lambda(\nu)=\Big(\lambda_1(\nu),\lambda_2(\nu),\lambda_3(\nu),\lambda_4(\nu),\lambda_5(\nu),\lambda_6(\nu),\lambda_7(\nu),\lambda_8(\nu)\Big)^T\in R^8$ and the Hamiltonian of the given system

$$
\begin{aligned}
H =& \left(c_1H_i^2(\nu)+c_2u_1^2(\nu)+c_3u_2^2(\nu)+c_4u_3^2\right)\\
&+\lambda_1(\nu)\left(e^{\alpha}_{\nu}-e^{\alpha}_{\nu}p\left(\frac{M_i(\nu)}{M_s(\nu)+M_e(\nu)+M_i(\nu)}\right)-\eta^{\alpha}E_s(\nu)-u_1(\nu)E_s(\nu)\right)\\
&+\lambda_2(\nu)\left(e^{\alpha}_{\nu}p\left(\frac{M_i(\nu)}{M_s(\nu)+M_e(\nu)+M_i(\nu)}\right)-\eta^{\alpha}E_i(\nu)-u_1(\nu)E_i(\nu)\right)\\
&+\lambda_3(\nu)\left(\eta^{\alpha}E_s(\nu)-b^{\alpha}\frac{H_i(\nu)}{N_h}M_s(\nu)-\delta^{\alpha}M_s(\nu)-u_2(\nu)M_s(\nu)\right)\\
&+\lambda_4(\nu)\left(b^{\alpha}\frac{H_i(\nu)}{N_h}M_s(\nu)-\gamma^{\alpha}M_e(\nu)-\delta^{\alpha}M_s(\nu)-u_2(\nu)M_e(\nu)\right)\\
&+\lambda_5(\nu)\left(\gamma^{\alpha}M_e(\nu)+\eta^{\alpha}E_i(\nu)-\delta^{\alpha}M_i(\nu)-u_2(\nu)M_i(\nu)\right)\\
&+\lambda_6(\nu)\left(R^{\alpha}_{b}N_h-b^{\alpha}\frac{H_s(\nu)}{N_h}M_i(\nu)-R^{\alpha}_{d}H_s(\nu)-u_3(\nu)H_s(\nu)\right)\\
&+\lambda_7(\nu)\left(b^{\alpha}\frac{H_s(\nu)}{N_h}M_i(\nu)-\zeta^{\alpha}H_i(\nu)-R^{\alpha}_{d}H_i(\nu)-u_4(\nu)H_i(\nu)\right)\\
&+\lambda_8(\nu)\left(\zeta^{\alpha}H_i(\nu)-R^{\alpha}_{d}H_r(\nu)+u_3(\nu)H_s(\nu)+u_4(\nu)H_i(\nu)\right).
\end{aligned}
$$

Using Pontryagin minimum principle, we have

$$
\begin{aligned}
\frac{\partial H}{\partial u_1}=0 &\implies u_1^*(\nu)=\frac{1}{c_1}(E_s\lambda_1+E_i\lambda_2)\\
\frac{\partial H}{\partial u_2}=0 &\implies u_2^*(\nu)=\frac{1}{c_2}(M_s\lambda_3+M_e\lambda_4+M_i\lambda_5)\\
\frac{\partial H}{\partial u_3}=0 &\implies u_3^*(\nu)=\frac{1}{c_3}(H_s\lambda_6-H_s\lambda_8)\\
.\frac{\partial H}{\partial u_4}=0 &\implies u_4^*(\nu)=\frac{1}{c_4}(H_i\lambda_7-H_i\lambda_8),
\end{aligned}
$$

the state equations are

$$
\begin{aligned}
{}^{C}D^{\alpha}_{\nu_0+}E_s(\nu) &= \frac{\partial H}{\partial \lambda_1} = e^{\alpha}_{\nu} - e^{\alpha}_{\nu} p\left(\frac{M_i(\nu)}{M_s(\nu)+M_e(\nu)+M_i(\nu)}\right) - \eta^{\alpha}E_s(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}E_i(\nu) &= \frac{\partial H}{\partial \lambda_2} = e^{\alpha}_{\nu} p\left(\frac{M_i(\nu)}{M_s(\nu)+M_e(\nu)+M_i(\nu)}\right) - \eta^{\alpha}E_i(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}M_s(\nu) &= \frac{\partial H}{\partial \lambda_3} = \eta^{\alpha}E_s(\nu) - b^{\alpha}\frac{H_i(\nu)}{N_h}M_s(\nu) - \delta^{\alpha}M_s(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}M_e(\nu) &= \frac{\partial H}{\partial \lambda_4} = b^{\alpha}\frac{H_i(\nu)}{N_h}M_s(\nu) - \gamma^{\alpha}M_e(\nu) - \delta^{\alpha}M_s(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}M_i(\nu) &= \frac{\partial H}{\partial \lambda_5} = \gamma^{\alpha}M_e(\nu) + \eta^{\alpha}E_i(\nu) - \delta^{\alpha}M_i(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}H_s(\nu) &= \frac{\partial H}{\partial \lambda_6} = R^{\alpha}_{b}N_h - b^{\alpha}\frac{H_s(\nu)}{N_h}M_i(\nu) - R^{\alpha}_{d}H_s(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}H_i(\nu) &= \frac{\partial H}{\partial \lambda_7} = b^{\alpha}\frac{H_s(\nu)}{N_h}M_i(\nu) - \zeta^{\alpha}H_i(\nu) - R^{\alpha}_{d}H_i(\nu)\\
{}^{C}D^{\alpha}_{\nu_0+}H_r(\nu) &= \frac{\partial H}{\partial \lambda_8} = \zeta^{\alpha}H_i(\nu) - R^{\alpha}_{d}H_r(\nu)
\end{aligned}
\tag{2}
$$

and with the help of Lemma 21, the costate equations are

$$
\begin{aligned}
{}^{C}D^{\alpha}_{\nu_F-}\lambda_1(\nu_F-\nu) &= \frac{\partial H}{\partial E_s} = -\eta^{\alpha}\lambda_1(\nu_F-\nu) + eta^{\alpha}\lambda_3(\nu_F-\nu)\\
{}^{C}D^{\alpha}_{\nu_F-}\lambda_2(\nu_F-\nu) &= \frac{\partial H}{\partial E_i} = -\eta^{\alpha}\lambda_2(\nu_F-\nu) + eta^{\alpha}\lambda_5(\nu_F-\nu)\\
{}^{C}D^{\alpha}_{\nu_F-}\lambda_3(\nu_F-\nu) &= \frac{\partial H}{\partial M_s} = e^{\alpha}_{\nu} p\frac{M_i(\nu_F-\nu)\lambda_1(\nu_F-\nu)}{(M_s(\nu_F-\nu)+M_e(\nu_F-\nu)+M_i(\nu_F-\nu))^2} - \delta^{\alpha}\lambda_3(\nu_F-\nu)\\
&\quad - e^{\alpha}_{\nu} p\frac{M_i(\nu_F-\nu)\lambda_2(\nu_F-\nu)}{(M_s(\nu_F-\nu)+M_e(\nu_F-\nu)+M_i(\nu_F-\nu))^2}\\
&\quad + \frac{b^{\alpha}}{N_h}H_i(\nu_F-\nu)\lambda_4(\nu_F-\nu)\\
&\quad - \frac{b^{\alpha}}{N_h}H_i(\nu_F-\nu)\lambda_3(\nu_F-\nu) - u_3(\nu_F-\nu)\lambda_3(\nu_F-\nu)
\end{aligned}
$$

$$
\begin{aligned}
{}^{C}D^{\alpha}_{\nu_F-}\lambda_4(\nu_F-\nu) = \frac{\partial H}{\partial M_e} &= e_{\nu}^{\alpha} p \frac{M_i(\nu_F-\nu)\lambda_1(\nu_F-\nu)}{(M_s(\nu_F-\nu)+M_e(\nu_F-\nu)+M_i(\nu_F-\nu))^2} - \delta^{\alpha}\lambda_4(\nu_F-\nu) \\
&\quad - e_{\nu}^{\alpha} p \frac{M_i(\nu_F-\nu)\lambda_2(\nu_F-\nu)}{(M_s(\nu_F-\nu)+M_e(\nu_F-\nu)+M_i(\nu_F-\nu))^2} - \gamma^{\alpha}\lambda_5(\nu_F-\nu) \\
&\quad - \gamma^{\alpha}\lambda_4(\nu_F-\nu) - u_3(\nu_F-\nu)\lambda_4(\nu_F-\nu) \\
{}^{C}D^{\alpha}_{\nu_F-}\lambda_5(\nu_F-\nu) = \frac{\partial H}{\partial M_i} &= -e_{\nu}^{\alpha} p \frac{(M_s(\nu_F-\nu)+M_e(\nu_F-\nu))\,\lambda_1(\nu_F-\nu)}{(M_s(\nu_F-\nu)+M_e(\nu_F-\nu)+M_i(\nu_F-\nu))^2} + \delta^{\alpha}\lambda_5(\nu_F-\nu) \\
&\quad - e_{\nu}^{\alpha} p \frac{M_s(\nu_F-\nu)\lambda_1(\nu_F-\nu)+M_e(\nu_F-\nu)\lambda_1(\nu_F-\nu)}{(M_s(\nu_F-\nu)+M_e(\nu_F-\nu)+M_i(\nu_F-\nu))^2} \\
&\quad - \frac{b^{\alpha}}{N_h} H_s(\nu_F-\nu)\lambda_7(\nu_F-\nu) \\
&\quad - \frac{b^{\alpha}}{N_h} H_s(\nu_F-\nu)\lambda_6(\nu_F-\nu) - u_3(\nu_F-\nu)\lambda_5(\nu_F-\nu) \qquad (3) \\
{}^{C}D^{\alpha}_{\nu_F-}\lambda_6(\nu_F-\nu) = \frac{\partial H}{\partial H_s} &= -\frac{b^{\alpha}}{N_h} M_e(\nu_F-\nu)\lambda_6(\nu_F-\nu) + \frac{b^{\alpha}}{N_h} M_e(\nu_F-\nu)\lambda_7(\nu_F-\nu) \\
&\quad - R_d^{\alpha}\lambda_6(\nu_F-\nu) - u_2(\nu_F-\nu)\lambda_6(\nu_F-\nu) + u_2(\nu_F-\nu)\lambda_8(\nu_F-\nu) \\
{}^{C}D^{\alpha}_{\nu_F-}\lambda_7(\nu_F-\nu) = \frac{\partial H}{\partial H_i} &= H_i(\nu_F-\nu) - \frac{b^{\alpha}}{N_h} M_s(\nu_F-\nu)\lambda_3(\nu_F-\nu) - R_d^{\alpha}\lambda_7(\nu_F-\nu) \\
&\quad - \zeta^{\alpha}\lambda_7(\nu_F-\nu) + \frac{b^{\alpha}}{N_h} M_s(\nu_F-\nu)\lambda_4(\nu_F-\nu) + \zeta^{\alpha}\lambda_8(\nu_F-\nu) \\
&\quad - u_1(\nu_F-\nu)\lambda_7(\nu_F-\nu) + u_1(\nu_F-\nu)\lambda_8(\nu_F-\nu) \\
{}^{C}D^{\alpha}_{\nu_F-}\lambda_8(\nu_F-\nu) = \frac{\partial H}{\partial H_r} &= -R_d^{\alpha}\lambda_8(\nu_F-\nu)
\end{aligned}
$$

with the terminal conditions

$\lambda_l(\nu_F) = 0, \quad$ for all $l = 1,2,3,\cdots,8.$

By setting $c_k = 1$ for $k = 1,2,3,4,5$ and the time dependent optimal controls $u_1^*(\nu)$, $u_2^*(\nu)$, $u_3^*(\nu)$ and $u_4^*(\nu)$ are

$$u_1^* = min\Big(1, max\Big(0, (E_s\lambda_1 + E_i\lambda_2)\Big)\Big)$$

$$u_2^* = min\Big(1, max\Big(0, (M_s\lambda_3 + M_e\lambda_4 + M_i\lambda_5)\Big)\Big)$$

$$u_3^* = min\Big(1, max\Big(0, (H_s\lambda_6 - H_s\lambda_8)\Big)\Big)$$

$$u_4^* = min\Big(1, max\Big(0, (H_i\lambda_7 - H_i\lambda_8)\Big)\Big).$$

4 Experimental Simulations

The numerical results for the proposed FOCP are simulated in this section. To obtain the optimal states and controls, the forward-backward sweep scheme in MATLAB is used to solve the obtained state and costate equations, respectively. The initial values are $E_s(0) = 0$, $E_i(0) = 0$, $M_s(0) = 341120$, $M_e(0) = 0$, $M_i(0) = 0$, $H_s(0) = 341094$, $H_i(0) = 26$ and $H_r(0) = 0$. The values of the parameters are listed in the table below.

Table 1: Parameter values

Parameter	e_{ν}	p	η	b	N_h	δ	γ	R_{hb}	R_{hd}	ζ
Value	6.218	0.028	0.099	0.33	341120	0.0331	0.0607	0.00002	0.000016	1/7

Firstly, we explore the impact of the fractional order α $(0.6 \leq \alpha \leq 1)$ in the model and then add controls to explore the roles of the proposed control parameters in the given dynamics.

In Figures 1 - 8, the effects of the fractional orders $\alpha = 1, 0.9, 0.8, 0.7$ and 0.6 are shown on the given dengue model (1). Here, we observe that when α is reduced from 1, the dengue spreads slowly, and the number of dengue-infected people are rising for a long time. From this observation, one can say that the derivative of order α becomes an important factor to analyze the spread of dengue transmission and treatment of the dengue infection.

Now, consider the following strategies to investigate the numerical outcomes of the proposed control problem.

Simulation results of the fractional order dengue model for $\alpha = 1, 0.9, 0.8, 0.7$ and 0.6.

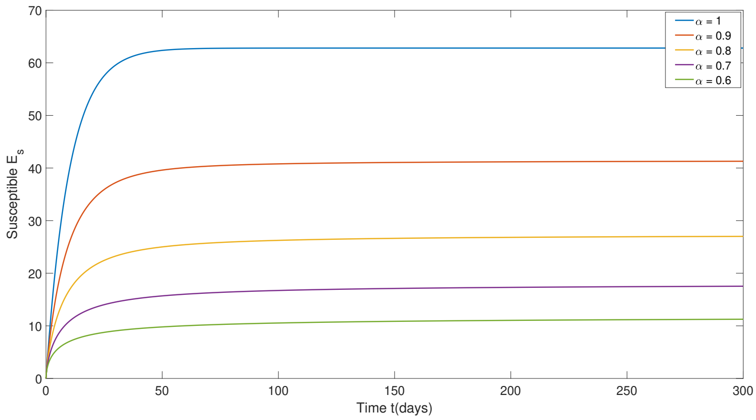

Fig. 1: Pre-adult female mosquito-Susceptible

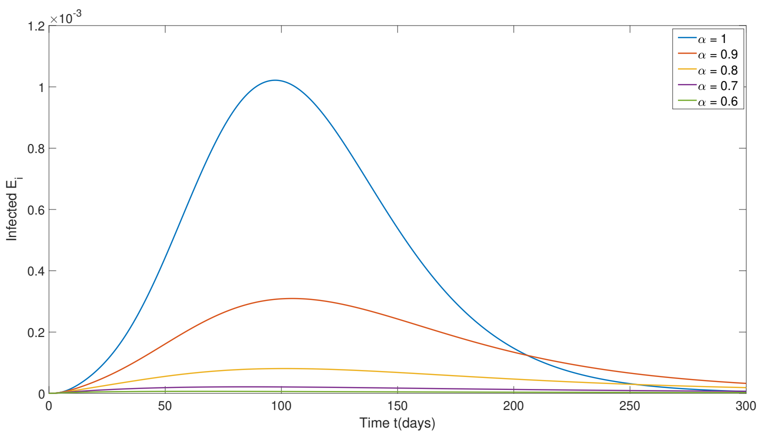

Fig. 2: Pre-adult female mosquito-Infected

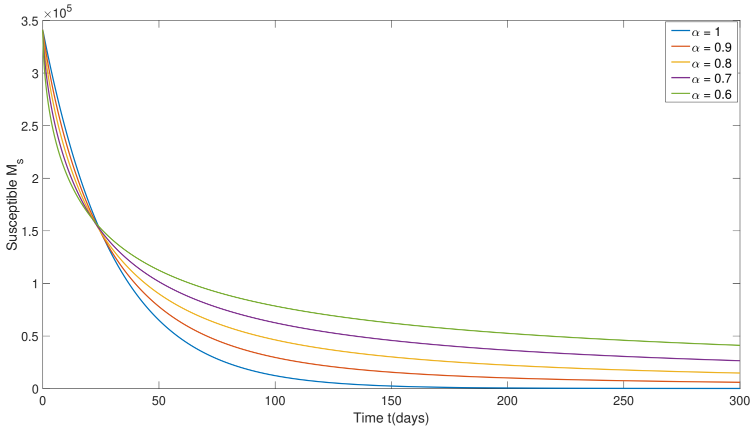

Fig. 3: Adult female mosquito-Susceptible

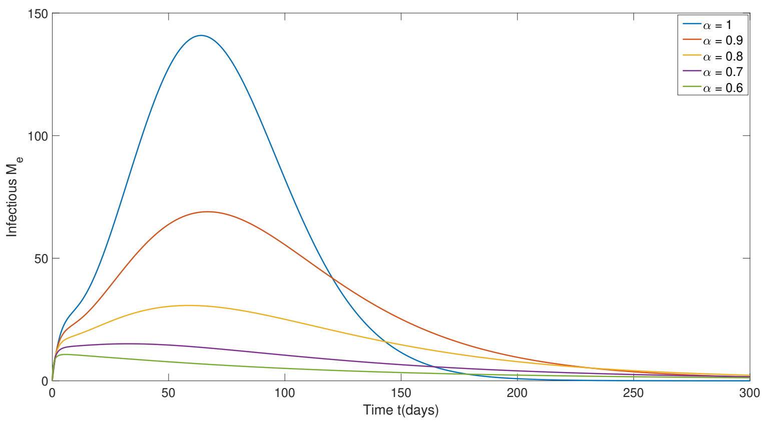

Fig. 4: Adult female mosquito-Infectious

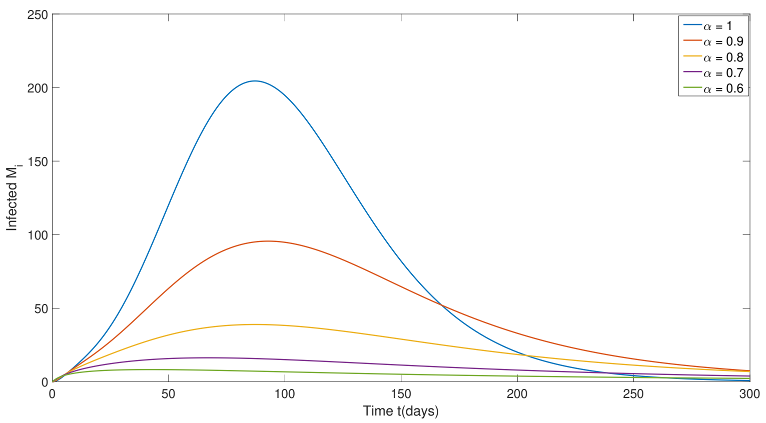

Fig. 5: Adult female mosquito-Infected

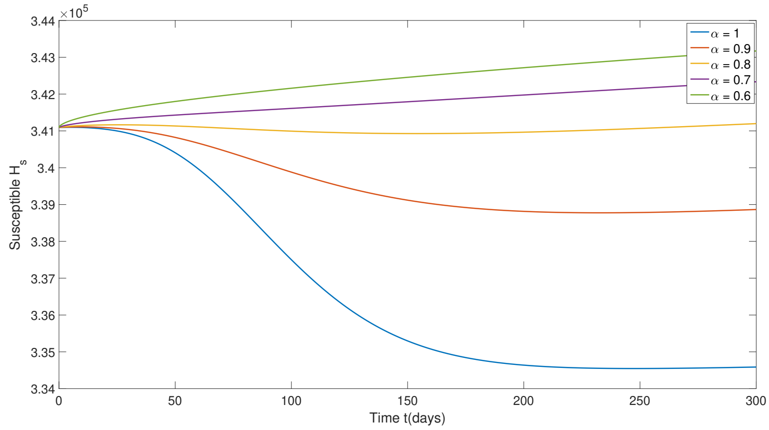

Fig. 6: Human population-Susceptible

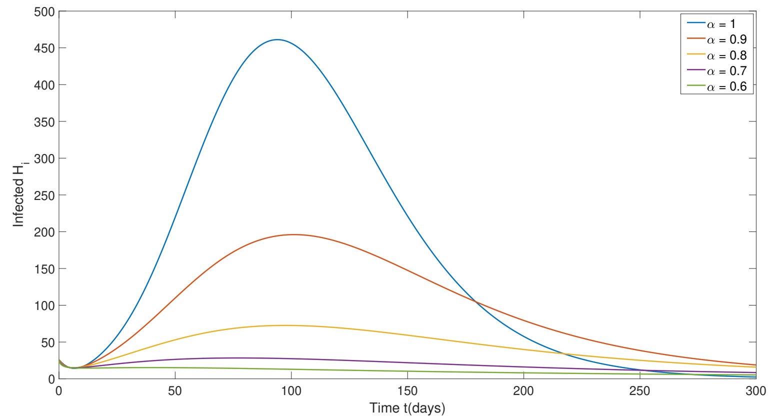

Fig. 7: Human population-Infected/Infectious

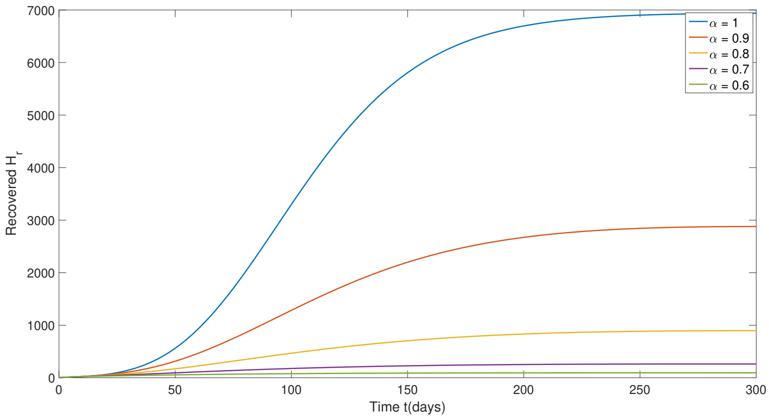

Fig. 8: Human population-Recovered/Immune

Strategy A: using larvicide, fogging and isolation controls (i.e. $u_1 \neq 0$, $u_2 \neq 0$, $u_3 = 0$ and $u_4 \neq 0$)

In the first case, the larvicide, fogging and isolation controls are used to minimize the cost function while vaccination control is assumed to be zero. Figures 9 - 12 show that larvicide, fogging and isolation controls decrease the number of infected human populations for a long time compared with the case without control. Figure 10 shows that strategy A and upcoming strategy B are similar to keep the spread of the dengue epidemic under control for $\alpha = 1$. When the vaccination is not available, then strategy A is effective to stop the spread of the dengue epidemic and reduce the number of the infected human population.

Strategy B: using vaccination and isolation controls (i.e. $u_1 = 0$, $u_2 = 0$, $u_3 \neq 0$ and $u_4 \neq 0$)

In this case, the isolation and vaccination controls are used to minimize the cost function while larvicide and fogging controls are assumed to be zero. We know that excessive usage of insecticide is harmful to humans and it may lead to the destruction of biodiversity. Fortunately, compared with all other cases strategy B is sufficient to reduce the transmission of dengue dynamics and decrease the number of infected humans as shown in Figures 9, 11 and 12. Since both isolation and vaccination controls are directly applied to the human population, it reduces the cost of larvicide and fogging control efforts.

Strategy C: using larvicide, fogging, vaccination and isolation controls (i.e. $u_1 \neq 0$, $u_2 \neq 0$, $u_3 \neq 0$ and $u_4 \neq 0$)

In this case, larvicide, fogging, vaccination, and isolation controls are used to minimize the cost function. The given Figures show that using four optimal controls in the dengue epidemic decreases the number of infected human populations and reduces the growth of spread of dengue transmission compared with the case without control. For $\alpha = 0.6$, it decreases the dengue infection rapidly but for $\alpha = 1$ and 0.8, it decreases the infection slowly compared to the $\alpha = 0.6$ case. Moreover, strategy C increases the cost of controlling efforts by comparison with strategies A and B.

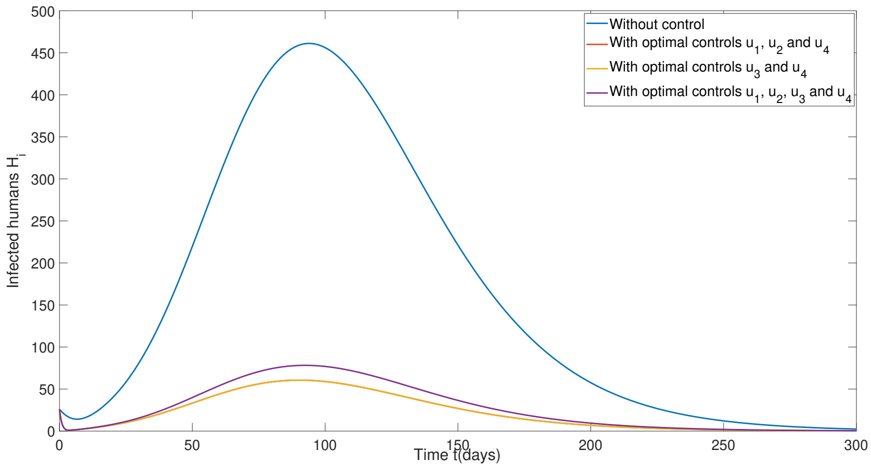

Fig. 9: The dynamic of infected humans H_i for $\alpha = 1$

5 Conclusion

The optimal control of larvicide, fogging, vaccine, and isolation in a dengue disease transmission model has been explored in this study. We have designed an optimal control problem to decrease the number of dengue infected humans along with

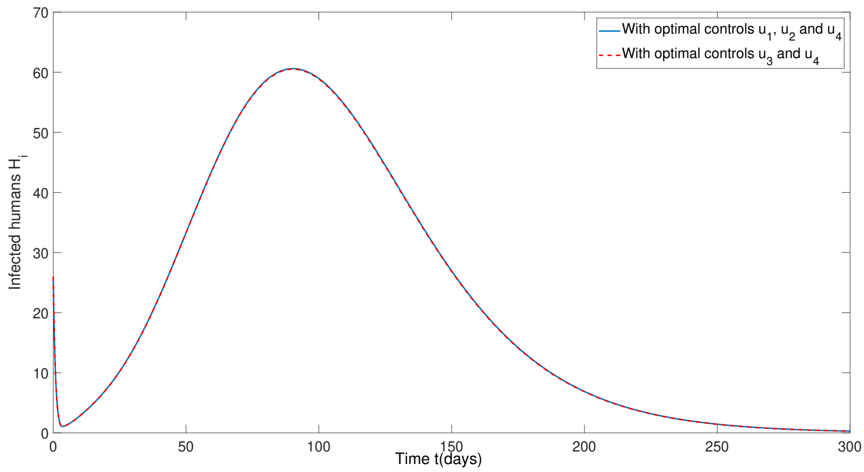

Fig. 10: Difference between the strategies A and B for $\alpha = 1$.

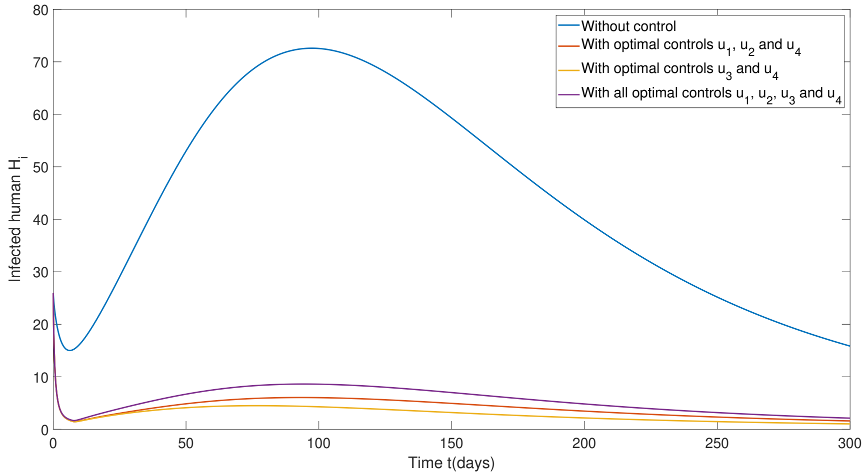

Fig. 11: The dynamic of infected humans H_i for $\alpha = 0.8$.

the control cost. We have solved the proposed control problem using the Pontryagin Minimum Principle. The numerical results of the provided situation are examined to demonstrate the impact of our optimal controls on the dengue outbreak. As a result of our findings, we can observe that the number of infected humans are reducing with time in the presence of possible controls. It suggests that maximizing the use of larvicide, fogging, vaccine, and isolation can have a significant impact on reducing the dengue fever outbreaks.

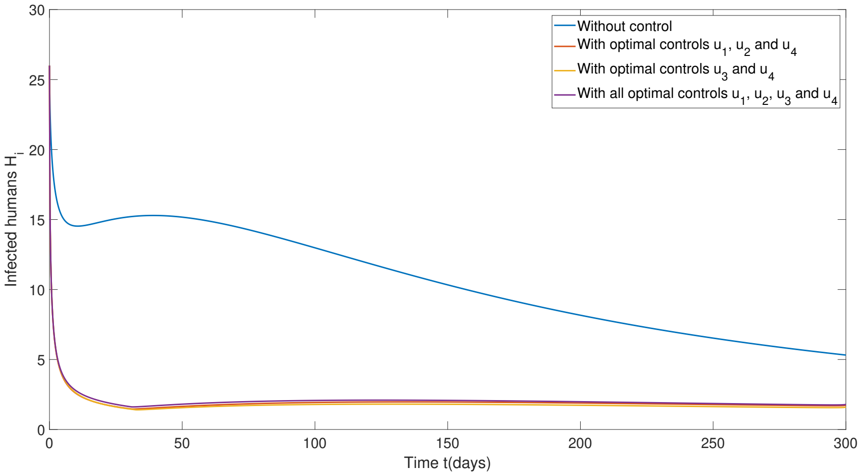

Fig. 12: The dynamic of the infected humans H_i for $\alpha = 0.6$.

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