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Exploring the Role of Vacuum Fluctuations on Nuclear-Lepton Interactions in Atomic Systems

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Abstract: The nuclear-lepton wavefunction is the quantum mechanical description of the state of an atom. It describes the probability amplitude for finding leptons and nucleus at particular locations and times relative to each other. The Z/r nuclear potential is a commonly used model for describing the interaction between a point-like electron and nucleus. However, this model does not account for the finite-size charge distribution of both the electron and nucleus, which can lead to inaccurate predictions of their interactions. In this work, we developed a theoretical model for investigating the effects of vacuum fields on the interaction between a finite-sized electron and a nuclear system. The model provides wavefunctions for both the electron and nuclear system as a function of their separation, r, and is valid from r = 0 to r ≈ 1.8 r0, and from r ≈ 1.8 r0, to r ~ 0 ƛ where r0 and ƛ are the nuclear and lepton radius respectively. We found that our modified model showed a finite value of the electron wavefunction at the origin, which modifies the infinite value predicted by the Z/r model. Our theoretical framework provides a simplified description of the atomic system without considering nuclear structure. Our model can contribute to a better understanding of the interaction between electrons and nuclei, and the role of vacuum fields in atomic systems.

Keywords: *Lepton; Finite-size; Nuclear; Interaction; Quantum Vacuum; Wavefunction.*

1 Background

The interaction between a lepton and a nucleus in an atomic system is typically described by the point-charge *Z*/*r* nuclear potential, which assumes the lepton and nucleus are point-like particles [1]. However, this model fails to take into account the finite-size charge distribution of both particles, which arises due to their interaction with external fields, such as the fluctuating vacuum fields. To incorporate these effects, the *Z*/*r* potential must be modified, resulting in a perturbation on the Hamiltonian that describes the system [2-4]. To solve for this modified Hamiltonian, various approximation methods have been employed, with time-independent perturbation theory being the most commonly used [5-8]. The first-order perturbation theory is crucial in atomic and nuclear physics, as it allows for the calculation of small changes in lepton energy states due to various effects, including relativistic and spin-orbit effects, nuclear finite-size effects, and effects due to vacuum polarization and fluctuations in the vacuum fields [9-18]. These corrections to the energy levels have paved the way for more precise measurements of fundamental physical constants and the determination of nuclear radii [19-25].

In recent years, there has been increasing interest in the effects of vacuum fields on lepton-nuclear interactions [26,27]. These fields can affect the charge distribution and energy states of orbiting leptons, leading to further corrections to the energy levels and nuclear-lepton interaction models [28-32]. The finite-size nuclear models have provided valuable information about the nucleus, such as radii and quadrupole moments, but the effects of extended charged orbiting leptons and vacuum fields on the wavefunctions and quantum states of both the nucleus and its bound leptons remain unclear. This paper aims to determine the possible wavefunctions for two interacting finite-sized particles affected by vacuum fields and calculate the resulting changes in the quantum states of the leptons using the theoretical model developed in this study. This model provides a simplified description of the combined nuclear-lepton wavefunctions in terms of their separation distance and can help to further our understanding of lepton-nuclear interactions in atomic systems.

2 Methodology

The interaction of relativistic lepton with extended charge nucleus is affected by fluctuating electromagnetic fields in quantum vacuum. These

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fields perturbed the lepton-nuclear interaction by an amount $\vec{\varepsilon}$ as illustrated in Figure 1. The effective nuclear-lepton interaction can be obtained by adding small perturbation $\vec{\varepsilon}$ to the lepton position, $U_{eff} = U(\vec{r}+\vec{\varepsilon})$ and can be expanded using Taylor's series as

$$U_{eff}(\vec{r},\vec{\varepsilon}) = \frac{1}{V}\left[\int U(\vec{r})d^3\varepsilon + \int \vec{\varepsilon}\cdot\nabla U d^3\varepsilon + \frac{1}{2}\int \varepsilon_i\varepsilon_j \frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j} U d^3\varepsilon\right] \quad (1)$$

where for spherical symmetry the integrals,

$$\int U(\vec{r})d^3\varepsilon = U(\vec{r})\times V$$

$$\int \vec{\varepsilon}\cdot\nabla U d^3\varepsilon = \nabla U\cdot\int \vec{\varepsilon} d^3\varepsilon = 0$$

and

$$\int \varepsilon_i\varepsilon_j \frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j} U d^3\varepsilon = \lambda\!\!\!^{-2}\delta_{ij}\int \frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j} U d^3\varepsilon$$

$$= \frac{1}{3}\lambda\!\!\!^{-2}\nabla^2 U\delta_{ij}$$

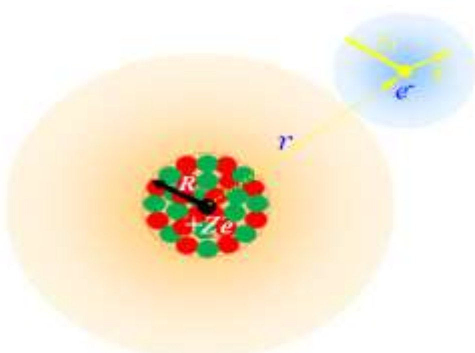

Fig.1: The sketch of nuclear-lepton charge distribution when interacts with fluctuation fields.

Therefore, equation 1 becomes,

$$U_{\text{eff}}(R,r_l) = U(R) + \zeta'(r_l) \quad (2)$$

where

$$\zeta'(r_l) = \frac{1}{6}\lambda\!\!\!^{-}{}_l^2\nabla^2 U(R)\delta_{ij} \quad (3)$$

and the potential for extended charge nucleus is given by

$$U(R) = -\frac{Z\gamma}{2R}\left[3-\left(\frac{r}{R}\right)^2\right] \quad (4)$$

with radius $R = r_0A^{1/3}$, A the nucleon number, $\lambda\!\!\!^{-}$ the Compton wavelength and $\gamma = ke^2$.

Thus, the effective interaction (1) can be simplified to give

$$U_{\text{eff}}(R,\lambda\!\!\!^{-}_l) = -\frac{Z\gamma}{2R}\left[3-\frac{1}{A^{1/3}}\left(\frac{r}{r_0}\right)^2-\frac{1}{3A^{1/3}}\left(\frac{\lambda\!\!\!^{-}_l}{r_0}\right)^2\right] \quad (5)$$

The solution to the effective interaction (5) can be sought from time independent perturbation theory,

$$\hat{E}'_n(\lambda) = \langle\psi_n|\hat{H}_0|\psi_n\rangle + \lambda\langle\psi_n|\hat{H}_{\text{pert.}}|\psi_n\rangle$$

$$+\lambda^2\sum_{m\neq n}\frac{\left|\langle\psi_n|\hat{H}_{\text{pert.}}|\psi_m\rangle\right|^2}{\hat{E}_n-\hat{E}_m} + \mathcal{O}(\lambda^3)$$

where $\hat{E}_n$ is the 0^{th} order correction to the n^{th} eigenvalue, and ψ_n is the 0^{th} order correction to the n^{th} eigenfunctions; $\hat{E}_n^{(1)}$ is the first order corrections; $\hat{E}_n^{(2)}$ is the second order corrections λ is taken to be dimensionless small number, $\lambda \ll 1$ and $\mathcal{O}(\lambda^3)$ is the Landau symbol [4,6,7,33]. The first-order correction to lepton energy states due to lepton extended wavefunctions is given by

$$\hat{E}_{nlm}^{(1)} = \int \psi_{nlm}^* U_{\text{eff}}(R,\lambda\!\!\!^{-}_l)\psi_{nlm}d\tau \quad (6)$$

where the ψ_{nlm} is the unperturbed wavefunction and its intensity corresponding to $n00$ states is given by:

$$|\psi_{n00}|^2 = \begin{cases}\frac{Z^3}{\pi n^3 a_0^3}; & l=0,\ m=0\\ 0 & l\geq 1\end{cases} \quad (7)$$

where $a_0 = \hbar^2/kme^2$, is the Bohr radius. The shift in $n00$ energy states are determined using (6) as,

$$\Delta E_{n00}^{(1)} = -\frac{Z\gamma}{2R}\int_0^R\left[3-\left(\frac{r}{R}\right)^2-\frac{1}{3}\left(\frac{\lambda\!\!\!^{-}_l}{R}\right)^2\right]|\psi_{n00}|^2 r^2 dr$$

$$= E_n\frac{Z^2}{n}\left(\frac{16A^{1/3}}{5}\rho_l - \frac{4}{9}\sigma_l\right) \quad (8)$$

where for an electron $\rho_e = (r_0/a_0)^2 = 5.14577921\times 10^{-11}$, $\sigma_e = (\lambda\!\!\!^{-}_l/a_0)^2 = 7.09401373\times10^{-10}$ and $\lambda\!\!\!^{-}_l = 1.4089698\times10^{-15}\ m$ [34].

3 Results

The interaction (8) is analyzed using Microsoft Excel package and Figure 2(*a*), (*b*), (*c*), (*a*), (*d*), (*e*) and (*f*) gives the information on variation of the nuclear-nuclear wavefunction *U*(*R*, ƛ) with distance *r* from the origin of the single-lepton atoms: *Li*, *Na*, *K*, *Rb*, *Sc* and *Fr*.

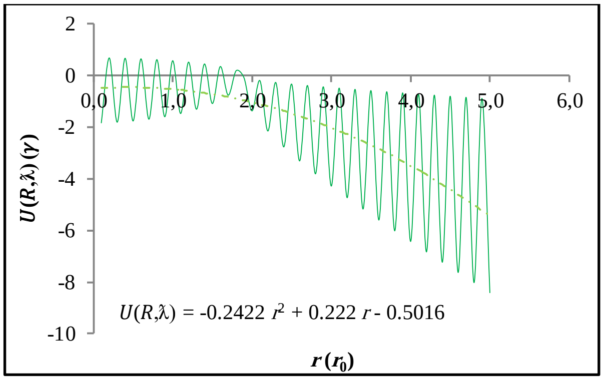

Fig.2(*a*): The lepton-nuclear wavefunctions for *Li* atom as a function of *r*.

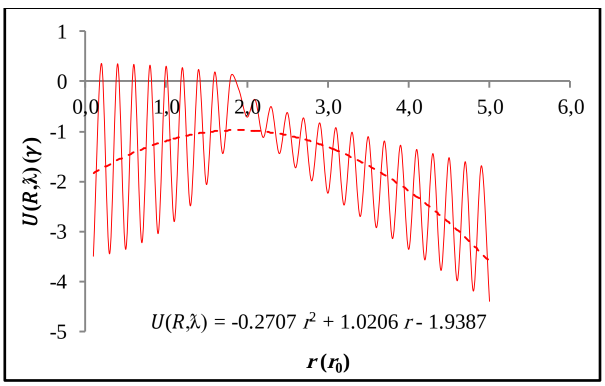

Fig.2(*b*): The lepton-nuclear wavefunctions for *Na* atom as a function of *r*.

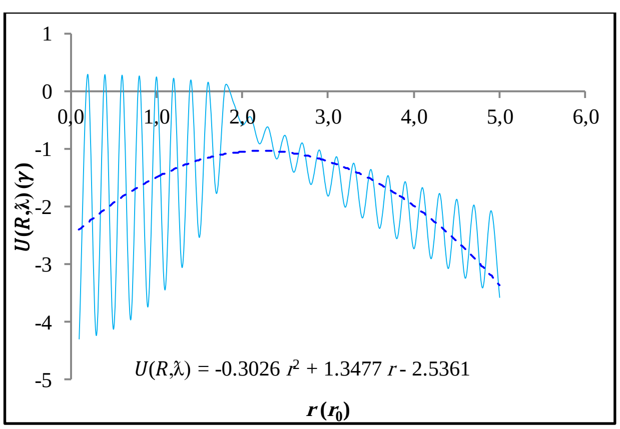

Fig. 2(*c*): The lepton-nuclear wavefunctions for *K* atom as a function of *r*.

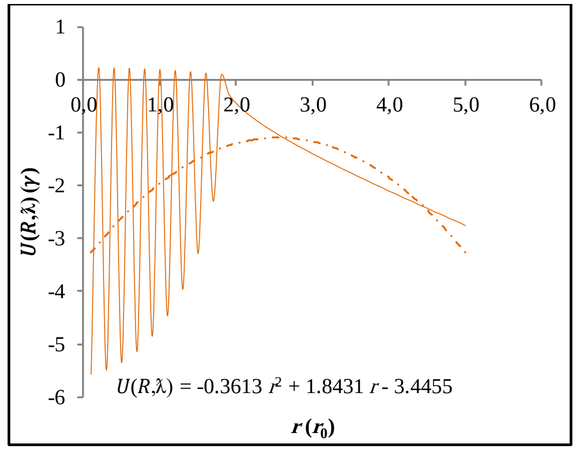

Fig.2(*d*): The lepton-nuclear wavefunctions for *Rb* atom as a function of *r*.

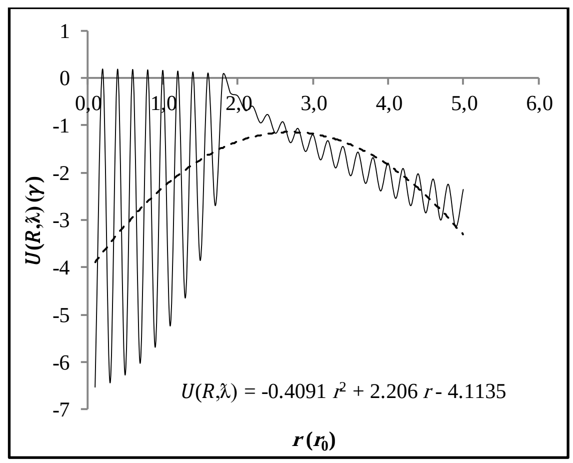

Fig. 2(*e*): The lepton-nuclear wavefunctions for *Sc* atom as a function of *r*.

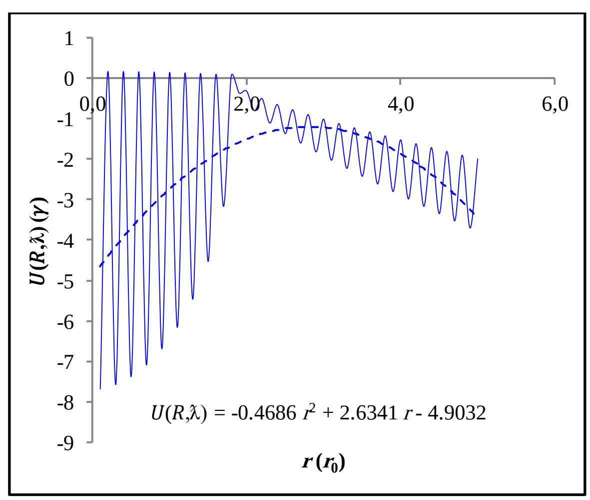

Fig. 2(*f*): The lepton-nuclear wavefunctions for *Fr* atom as a function of *r*.

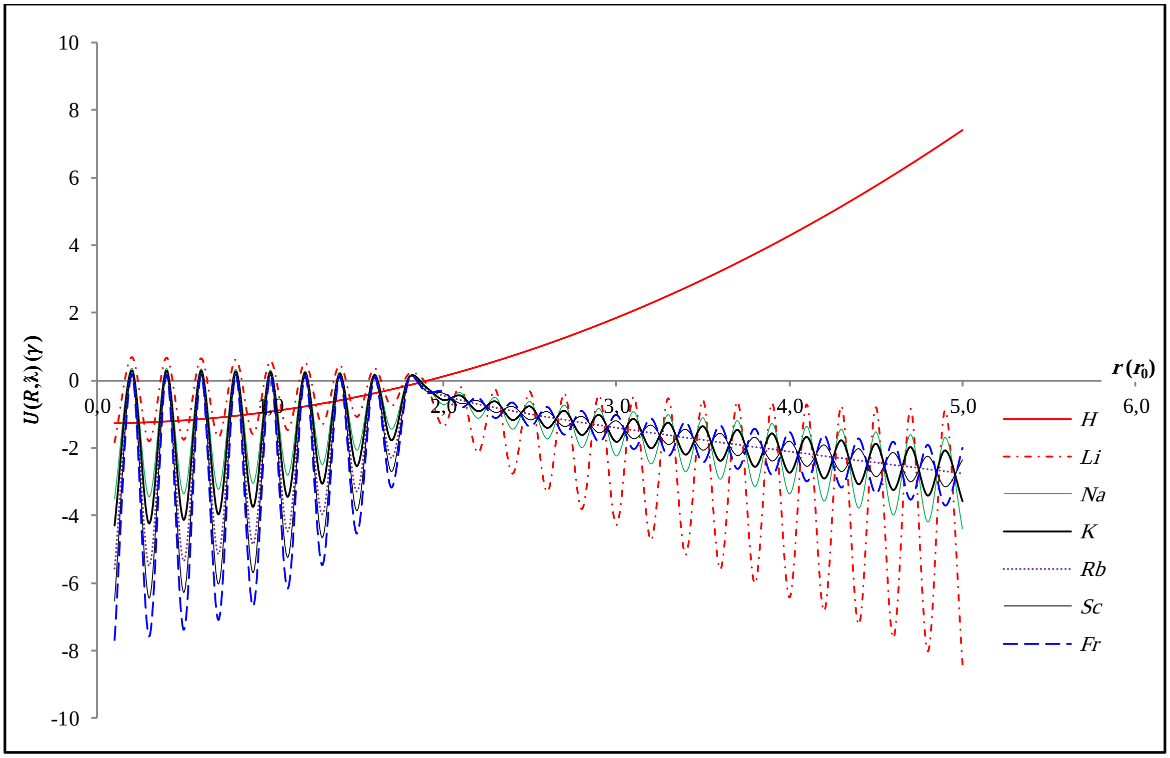

Fig. 3: The comparison of nuclear and lepton wavefunction as a function of distance *r* from the origin.

4 Discussion

Figure 2 (*a*) to (*f*) showed a sinusoidal plot of $U(R, ƛ)$ as a function of *r* visualizing the information of Equation 5 which was obtained from quantum electrodynamics' effects. The figures revealed different formation of wavefunction when extended charged lepton interacts with finite-size nucleus. There is an important characteristic of the potential at about $r = 1.8\, r_0$ exhibited by the entire nucleus (light, medium and heavy). This indicates the boundary between the nuclear and lepton wavefunctions. Figure 2 further interpreted the effective interaction (5) as having non-zero value at the origin (r ~ 0), this contradicts the coulomb point-charge interaction which is infinite at origin. This indicates that the nuclear potential is concentrated towards the origin. This modified the coulomb interaction by giving the correct analytical behavior of the nuclear wavefunctions at origin. It can also be observed that the extension of the wavefunctions towards leptons have different characteristics as it starts increasing towards the origin of lepton charge.

Figure 2 were studied using the regression analysis and developed a simple relationship between $U(R, ƛ)$ and the distance *r* that satisfied,

$$(R,ƛ) = -0.2422\, r^2 + 0.2220\, r - 0.5016$$

$$(R,ƛ) = -0.2707\, r^2 + 1.0206\, r - 1.9387$$

$$(R,ƛ) = -0.3026\, r^2 + 1.3477\, r - 2.5361$$

$$(R,ƛ) = -0.3613\, r^2 + 1.8431\, r - 3.4455$$

$$(R,ƛ) = -0.4091\, r^2 + 2.2060\, r - 4.1135$$

$$(R,ƛ) = -0.4686\, r^2 + 2.6341\, r - 4.9032$$

for *Li*, *Na*, *K*, *Rb*, *Sc* and *Fr* respectively. These equations show the potential-distance relationships that best suit the observed data from simulations. The r^2 component emphasizes a quadratic dependence on distance, whereas the coefficients show the direction and strength of these dependencies. The potential's finite, non-zero value at $r \approx 0$ suggests that the Coulomb interaction has changed, which gives nuclear wavefunctions at the origin a more realistic analytical behavior. The fitted functions reflect this change. The observed peak at $r = 1.8\, r_0$ in Figure 2 represents a transition or barrier between nuclear and lepton wavefunctions. The extension of nuclear wavefunctions towards leptons and the different properties detected as they approach the lepton's origin reveal the complex interplay between the charged lepton and the finite-size nucleus. The resulting equations are a valuable tool for understanding or predicting the behavior of the system beyond reported data points.

These results provide a quantitative explanation of the potential-distance connection within the context of quantum electrodynamics, and are important in understanding the behavior of wavefunctions and interactions between charged leptons and nuclei. The findings provide opportunities for further research, such as a closer look at the effects of the altered Coulomb interaction, a more thorough investigation of the physical meaning of the fitted coefficients, and the use of the model in different situations or systems.

Figure 3 gives the comparison of the variation of the effective interaction (5) with distance r from the origin of various atomic nuclei. The figure showed the finite behavior of the effective interaction at $r = 0$. For hydrogen atom, the wavefunction gives a curve stating form γ ($= ke^2$) to 0.8 at $r = 0$ and then starts to increase with the increase in distance, r from the nucleus. The curve exhibited by hydrogen atom could be attributed to interaction between a proton and single lepton having roughly spherical charge distributions with no shielding effect. Other atomic nuclei have the wavefunctions decreasing to the origin of lepton charge distribution. This suggested that the effective interaction (5) is more effective at a point very close to the atomic nucleus and when there is no electron-electron interaction. Figure 3 also showed the dependence of lepton-nuclear interaction described by the theoretical model (5) as a function of the both lepton and nuclear position r. The nuclear wavefunction starting from $r = 0$ to $r \approx 1.8\ r_0$, describes the nuclear potential field and lepton wavefunction starts to exist at a distance $r > 1.8\ r_0$, describes the lepton fields.

In this study, a new theoretical model of nuclear-electron interaction energy was formulated, which addresses the infinite value of Z/r potential at the origin ($r \sim 0$) that results from the point-charge nuclear model. The proposed model provides a remedy for this problem and modifies the undefined nature of the Z/r potential. The findings from this study suggest that the effective interaction (5) has a non-zero value at the origin ($r \sim 0$), unlike the coulomb point-charge interaction, which is infinite at the origin. This indicates that the nuclear potential is concentrated towards the origin, and it modifies the coulomb interaction by giving the correct analytical behavior of the nuclear wavefunctions at origin. The study also reveals different formations of wavefunction when an extended charged lepton interacts with a finite-size nucleus. The figures presented in this study show that there is an important characteristic of the potential at about $r = 1.8\ r_0$ exhibited by the entire nucleus (light, medium, and heavy), indicating the boundary between the nuclear and lepton wavefunctions. Moreover, the effective interaction (5) is more effective at a point very close to the atomic nucleus and when there is no electron-electron interaction. The proposed model presents a theoretical framework for describing atomic systems and understanding the pattern in which both lepton and nuclear fields interact. It could play a vital role in the quantum mechanical description of atomic systems. Additionally, this lepton-nuclear effective interaction, which takes into account the finite size of both particles together with vacuum field effects, could reflect on the nuclear coulomb energy, atomic quantum states or be used in determining the possible wavefunctions for two interacting particles (leptons and nucleus).

5 Conclusions

In conclusion, the modified electron-nucleus interaction proposed in this study would provide a clear understanding of the physical structure and details of atomic spectra that will remarkably agree with the measured values. It would also affect the energy levels of an electron and thus needs to be involved in the general understanding of electron energy level corrections such as the fine structure, hyperfine structure, Lamb shift, and isotope shift.

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