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Exploring the New Analytical Wave Solutions for M-Fractional Stochastic Nizhnik-Novikov-Veselov System by an Efficient Analytical Technique

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Abstract: In this paper, we derive new analytical wave solutions to the truncated M-fractional stochastic Nizhnik-Novikov-Veselov (SNNV) system by utilizing the modified simplest equation technique along the multiplicative noise effect. This system is an extension of the KdV equation and has many applications, including plasma, crystal networks, and shallow water waves. Achieved solutions are verified with the use of Mathematica software. Some of the obtained solutions are also described graphically by 2-dimensional, 3-dimensional and contour plots. The derived solutions are helpful in the further research of underlying model. Finally, this technique is simple and reliable to tackle the nonlinear PDEs.

Keywords: Stochastic Nizhnik-Novikov-Veselov system, truncated M-fractional derivative, modified simplest equation technique, multiplicative noise, new analytical wave solutions.

1 Introduction

The nonlinear partial differential equations (NLPDEs) arise in various types of physical problems such as fluid dynamics, plasma physics, quantum field theory etc. The system of nonlinear partial differential equations has been observed in chemical, biological, engineering and other areas of applied sciences. A lot of research have been done in these areas to find the numerical and analytical results of NLPDEs. Various schemes have been developed for this purpose. For example, the auxiliary rational method [1], new Kudryashov technique [2], two variable $(G'/G, 1/G)$-expansion technique [3], mapping technique [4], generalized auxiliary equation method [5], modified F-expansion technique [6], unified method [7], $\exp_a$ function scheme [8], [9] etc.

Aside from these schemes, there is another simple, useful, and significant technique: The modified simplest equation technique. In the literature, there are many uses of this technique. Instantly, some solitary wave solutions of BBM and Chan-Hilliard equations by utilizing modified simplest equation method [10], exact wave solutions of Boussinesq and coupled Boussinesq equations have been obtained by using this technique in [11], some new types of exact wave solutions of Hamiltonian amplitude equation have been attained in [12], different forms of exact wave solutions of Heisenberg ferromagnetic spin chain model have been derived in [13], five new solitary wave solutions for the Gardner and Whitham-Broer-Kaup equations have been achieved in [14].

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In our research, the study model is a truncated M-fractional non-linear (2+1)-dimensional stochastic Nizhnik-Novikov-Veselov system given as by [15].

$$u_t + a_1 D^{3\alpha,\Upsilon}_{M,3x} u + a_2 D^{3\alpha,\Upsilon}_{M,3y} u + a_3 D^{\alpha,\Upsilon}_{M,x} u + a_4 D^{\alpha,\Upsilon}_{M,y} u - 3a_1 D^{\alpha,\Upsilon}_{M,x}(uv) - 3a_2 D^{\alpha,\Upsilon}_{M,y}(uw) + \nu u \beta_t = 0. \quad (1)$$

$$D^{\alpha,\Upsilon}_{M,x} u = D^{\alpha,\Upsilon}_{M,y} v, \qquad D^{\alpha,\Upsilon}_{M,x} u = D^{\alpha,\Upsilon}_{M,x} w. \quad (2)$$

$$D^{\alpha,\Upsilon}_{M,x} u(x) = \lim_{\tau \to 0} \frac{u\left(x\, E_\Upsilon(\tau x^{1-\alpha})\right) - u(x)}{\tau}, \quad 0 < \alpha \leq 1, \ \Upsilon > 0, \quad (3)$$

where $E_\Upsilon(.)$ represents the truncated Mittag-Leffler function of one parameter as given in [16], [17]. Also where$u = u(x,y,t), v = v(x,y,t)$ and $w = w(x,y,t)$. The Symbols $a_i (i = 1,2,3,4)$ are arbitrary constants while ν denotes the control parameter. Symbol β_t represents the time noise or the Wiener Process. This model has many applications in different fields, such as shallow-water waves, sound-waves on the crystal networks, nonlinear geometrical optics, ionic acoustic waves in plasma, and long internal waves in density stratified oceans. Many researchers have been working on this model. For example, Mohammed W, obtained exact wave solutions by utilizing the sine-cosine and tanh-coth schemes in [18], some solitary wave solutions for this model have been achieved through the modified generalized rational function technique in [19], double periodic waves, shock wave solutions or kink-shaped soliton solutions, solitary waves or bell-shape solitons, and periodic wave solutions have been obtained by using the Jacobi elliptic function technique in [20].

The main purpose of this research is to search for new wave solutions of the nonlinear (2+1)-dimensional stochastic Nizhnik-Novikov-Veselov system based on the modified simplest equation method .

The motivation of this paper is to investigate the effect of the M-fractional derivative on the solutions of space-time fractional stochastic Nizhnik-Novikov-Veselov system, which were obtained by using the modified simplest equation method; an approach that has not been investigated before. The significance of the M-fractional derivative is that it fulfills both the properties of integer and fractional order derivatives. The derived solutions in this work are distinct from any other solutions present in the literature.

This research article is presented as follows: In Section 2, we present the mathematical analysis of the model. In Section 3, the modified simplest equation method is detailed. In Section 4, we utilize the method to derive a solution for the underlying model. Graphical illustrations of the solutions with discussions are given in section 5 followed by concluding remarks in section 6.

2 Mathematical Analysis

Assume that wave transformations is given by [21]:

$$u(x,y,t) = U(\zeta) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t), \quad v(x,y,t) = V(\zeta) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t);$$
$$w(x,y,t) = W(\zeta) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t);$$
$$\zeta = (\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^\alpha + \Omega y^\alpha) - \lambda t) \quad (4)$$

where U, V and W are the wave functions, the parameters τ and Ω are wave constants, λ denotes the speed of the light, and ν represents the noise strength. By inserting Eq.(4) into Eq.(1), considering Eq.(2), we obtain the following system of ODEs

$$(a_1\tau^3 + a_2\Omega^3)U''' + (a_3\tau + a_4\Omega - \lambda)U' - 3a_1\tau(UV)'\exp(-\nu\beta(t) - \frac{\nu^2}{2}t)$$
$$- 3a_2\Omega(UW)'\exp(-\nu\beta(t) - \frac{\nu^2}{2}t) = 0. \quad (5)$$

$$\tau U' = \Omega V', \qquad \Omega U' = \tau W' \tag{6}$$

Integrating Eq.(6) and taking integration constant equal to zero, we get:

$$\tau U = \Omega V, \qquad \Omega U = \tau W. \tag{7}$$

Therefore, we achieve from Eq.(7),

$$V = \frac{\tau}{\Omega} U, \qquad W = \frac{\Omega}{\tau} U \tag{8}$$

Putting Eq.(8) into Eq.(5), we gain'

$$(a_1\tau^3 + a_2\Omega^3)U''' + (a_3\tau + a_4\Omega - \lambda)U' - 3(\frac{a_1\tau^2}{\Omega} + \frac{a_2\Omega^2}{\tau})(U^2)' \exp(-\nu\beta(t) - \frac{\nu^2}{2}t) = 0. \tag{9}$$

Taking expectation on both side of Eq.(9), such as $E[\exp(-\nu\beta(t) - \frac{\nu^2}{2}t)] = 1$, we have

$$\tau\Omega U''' \left(a_1\tau^3 + a_2\Omega^3\right) - 3(U^2)' \left(a_1\tau^3 + a_2\Omega^3\right) + \tau U'\Omega \left(a_3\tau + a_4\Omega - \lambda\right) = 0 \tag{10}$$

Integrating Eq.(10) and taking the integration constant equal to zero, we obtain:

$$\tau\Omega U'' \left(a_1\tau^3 + a_2\Omega^3\right) - 3U^2 \left(a_1\tau^3 + a_2\Omega^3\right) + \tau U\Omega \left(a_3\tau + a_4\Omega - \lambda\right) = 0 \tag{11}$$

3 The Modified Simplest Equation Method (MSEM)

The main steps of this scheme are summarized as follows:
Step 1:
Consider a nonlinear PDE:

$$V(h, h^2, h^2h_x, h_y, h_{yy}, h_{xx}, h_{xy}, h_{xt}, \ldots) = 0, \tag{12}$$

where $h = h(x,y,t)$ denotes the wave function. Consider the wave transformation:

$$h(x,y,t) = H(\xi), \;\; \xi = x - \nu y + \kappa t. \tag{13}$$

Inserting Eq. (13) into Eq. (12) leads to the nonlinear ODE:

$$V(H(\xi), H^2(\xi)H'(\xi), H''(\xi), \ldots) = 0. \tag{14}$$

Step 2: Assume that Eq.(14) has the following solution form:

$$G(\xi) = \sum_{i=1}^{m} b_i \psi^i(\xi), \tag{15}$$

where $b_i (i = 1, 2, \ldots, m)$ are unknown and $b_m \neq 0$.
A function $\psi(\xi)$ satisfies the following ODE:

$$\psi'(\xi) = \psi^2(\xi) + \omega \tag{16}$$

where ω is a constant.
Eq.(16) has solutions for the following different cases of ω:
If $\omega < 0$,

$$\psi(\xi) = -\sqrt{-\omega}\tanh(\sqrt{-\omega}\xi) \tag{17}$$

$$\psi(\xi) = -\sqrt{-\omega}\coth(\sqrt{-\omega}\xi) \tag{18}$$

$$\psi(\xi) = \sqrt{-\omega}\left(-\tanh\left(2\sqrt{-\omega}\,\xi\right) \pm isech\left(2\sqrt{-\omega}\,\xi\right)\right), \tag{19}$$

$$\psi(\xi)=\sqrt{-\omega}\left(-\coth\left(2\sqrt{-\omega}\,\xi\right)\pm csch\left(2\sqrt{-\omega}\,\xi\right)\right), \tag{20}$$

$$\psi(\xi)=-\frac{\sqrt{-\omega}}{2}\left(\tanh\left(\frac{\sqrt{-\omega}}{2}\,\xi\right)+\coth\left(\frac{\sqrt{-\omega}}{2}\,\xi\right)\right). \tag{21}$$

If $\omega>0$,

$$\psi(\xi)=\sqrt{\omega}\tan(\sqrt{\omega}\xi) \tag{22}$$

$$\psi(\xi)=-\sqrt{\omega}\cot(\sqrt{\omega}\xi) \tag{23}$$

$$\psi(\xi)=\sqrt{\omega}\left(\tan\left(2\sqrt{\omega}\,\xi\right)\pm\sec\left(2\sqrt{\omega}\,\xi\right)\right), \tag{24}$$

$$\psi(\xi)=\sqrt{\omega}\left(-\cot\left(2\sqrt{\omega}\,\xi\right)\pm\csc\left(2\sqrt{\omega}\,\xi\right)\right), \tag{25}$$

$$\psi(\xi)=\frac{\sqrt{\omega}}{2}\left(\tan\left(\frac{\sqrt{\omega}}{2}\,\xi\right)-\cot\left(\frac{\sqrt{\omega}}{2}\,\xi\right)\right). \tag{26}$$

If $\omega=0$,

$$\psi(\xi)=-\frac{1}{\xi} \tag{27}$$

Step 3: Substitute Eq.(15) into Eq.(14) with Eq.(16) and collect the coefficients of each power of ψ^i. Put the coefficients of equal powers to 0, we obtain a system of algebraic equations in b_i, λ, and μ. **Step 4:** Inserting Eq.(15) whose parameters b_i, λ, μ were found into Eq.(14), we arrive at the analytical wave solitons of Eq.(12).

4 Exact Solutions of Eq.(11) via MSEM:

For $m=1$, Eq.(15) turns into:

$$U(\xi)=b_0+b_1\psi(\xi)+b_2\psi^2(\xi) \tag{28}$$

Using Eq.(28) into Eq.(10) with Eq.(16), and collecting coefficients of every power of $\psi(\xi)$ and taking them equal to 0, we obtain a system of algebraic equations. By solving the achieved system using a computer algebra software (Mathematica, in our work), we obtain the solutions.

Set 1:

$$\left\{b_0=\frac{2\tau\omega\Omega}{3},b_1=0,b_2=2\tau\Omega,\lambda=4a_1\tau^3\omega+a_3\tau+4a_2\omega\Omega^3+a_4\Omega\right\} \tag{29}$$

Case 1: If $\omega<0$,

$$\begin{aligned}u(x,y,t)=(&\frac{2\tau\omega\Omega}{3}+2\tau\Omega(-\sqrt{-\omega}\tanh(\sqrt{-\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^\alpha+\Omega y^\alpha)\\&-(4a_1\tau^3\omega+a_3\tau+4a_2\omega\Omega^3+a_4\Omega)t)))^2)\times\exp(-\nu\beta(t)-\frac{\nu^2}{2}t).\end{aligned} \tag{30}$$

$$\begin{aligned}v(x,y,t)=(&\frac{2\tau\omega\Omega}{3}+2\tau\Omega(-\sqrt{-\omega}\tanh(\sqrt{-\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^\alpha+\Omega y^\alpha)\\&-(4a_1\tau^3\omega+a_3\tau+4a_2\omega\Omega^3+a_4\Omega)t)))^2)\times\exp(-\nu\beta(t)-\frac{\nu^2}{2}t).\end{aligned} \tag{31}$$

$$\begin{aligned}w(x,y,t)=(&\frac{2\tau\omega\Omega}{3}+2\tau\Omega(-\sqrt{-\omega}\tanh(\sqrt{-\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^\alpha+\Omega y^\alpha)\\&-(4a_1\tau^3\omega+a_3\tau+4a_2\omega\Omega^3+a_4\Omega)t)))^2)\times\exp(-\nu\beta(t)-\frac{\nu^2}{2}t).\end{aligned} \tag{32}$$

$$u(x,y,t)=(\frac{2\tau\omega\Omega}{3}+2\tau\Omega(-\sqrt{-\omega}\coth(\sqrt{-\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) \\ -(4a_1\tau^3\omega+a_3\tau+4a_2\omega\Omega^3+a_4\Omega)t)))^2)\times\exp(-\nu\beta(t)-\frac{\nu^2}{2}t). \quad (33)$$

$$v(x,y,t)=(\frac{2\tau\omega\Omega}{3}+2\tau\Omega(-\sqrt{-\omega}\coth(\sqrt{-\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) \\ -(4a_1\tau^3\omega+a_3\tau+4a_2\omega\Omega^3+a_4\Omega)t)))^2)\times\exp(-\nu\beta(t)-\frac{\nu^2}{2}t). \quad (34)$$

$$w(x,y,t)=(\frac{2\tau\omega\Omega}{3}+2\tau\Omega(-\sqrt{-\omega}\coth(\sqrt{-\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) \\ -(4a_1\tau^3\omega+a_3\tau+4a_2\omega\Omega^3+a_4\Omega)t)))^2)\times\exp(-\nu\beta(t)-\frac{\nu^2}{2}t). \quad (35)$$

$$u(x,y,t)=(\frac{2\tau\omega\Omega}{3}+2\tau\Omega(\sqrt{-\omega}(-\tanh(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) \\ -(4a_1\tau^3\omega+a_3\tau+4a_2\omega\Omega^3+a_4\Omega)t)) \\ \pm isech(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) \\ -(4a_1\tau^3\omega+a_3\tau+4a_2\omega\Omega^3+a_4\Omega)t))))^2)\times\exp(-\nu\beta(t)-\frac{\nu^2}{2}t). \quad (36)$$

$$v(x,y,t)=(\frac{2\tau\omega\Omega}{3}+2\tau\Omega(\sqrt{-\omega}(-\tanh(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) \\ -(4a_1\tau^3\omega+a_3\tau+4a_2\omega\Omega^3+a_4\Omega)t))\pm isech(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) \\ -(4a_1\tau^3\omega+a_3\tau+4a_2\omega\Omega^3+a_4\Omega)t))))^2)\times\exp(-\nu\beta(t)-\frac{\nu^2}{2}t). \quad (37)$$

$$w(x,y,t)=(\frac{2\tau\omega\Omega}{3}+2\tau\Omega(\sqrt{-\omega}(-\tanh(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha})-(4a_1\tau^3\omega+a_3\tau+4a_2\omega\Omega^3+a_4\Omega)t)) \\ \pm isech(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha})-(4a_1\tau^3\omega+a_3\tau+4a_2\omega\Omega^3+a_4\Omega)t))))^2)\times\exp(-\nu\beta(t)-\frac{\nu^2}{2}t). \quad (38)$$

$$u(x,y,t)=(\frac{2\tau\omega\Omega}{3}+2\tau\Omega(\sqrt{-\omega}(-\coth(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha})-(4a_1\tau^3\omega+a_3\tau+4a_2\omega\Omega^3+a_4\Omega)t)) \\ \pm csch(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha})-(4a_1\tau^3\omega+a_3\tau+4a_2\omega\Omega^3+a_4\Omega)t))))^2)\times\exp(-\nu\beta(t)-\frac{\nu^2}{2}t). \quad (39)$$

$$v(x,y,t)=(\frac{2\tau\omega\Omega}{3}+2\tau\Omega(\sqrt{-\omega}(-\coth(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha})-(4a_1\tau^3\omega+a_3\tau+4a_2\omega\Omega^3+a_4\Omega)t)) \\ \pm csch(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha})-(4a_1\tau^3\omega+a_3\tau+4a_2\omega\Omega^3+a_4\Omega)t))))^2)\times\exp(-\nu\beta(t)-\frac{\nu^2}{2}t). \quad (40)$$

$$w(x,y,t)=(\frac{2\tau\omega\Omega}{3}+2\tau\Omega(\sqrt{-\omega}(-\coth(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha})-(4a_1\tau^3\omega+a_3\tau+4a_2\omega\Omega^3+a_4\Omega)t)) \\ \pm csch(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha})-(4a_1\tau^3\omega+a_3\tau+4a_2\omega\Omega^3+a_4\Omega)t))))^2)\times\exp(-\nu\beta(t)-\frac{\nu^2}{2}t). \quad (41)$$

$$u(x,y,t) = \left(\frac{2\tau\omega\Omega}{3} + 2\tau\Omega\left(-\frac{\sqrt{-\omega}}{2}\left(\tanh\left(\frac{\sqrt{-\omega}}{2}\left(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t\right)\right) + \coth\left(\frac{\sqrt{-\omega}}{2}\left(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t\right)\right)\right)\right)^2\right) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{42}$$

$$v(x,y,t) = \left(\frac{2\tau\omega\Omega}{3} + 2\tau\Omega\left(-\frac{\sqrt{-\omega}}{2}\left(\tanh\left(\frac{\sqrt{-\omega}}{2}\left(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t\right)\right) + \coth\left(\frac{\sqrt{-\omega}}{2}\left(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t\right)\right)\right)\right)^2\right) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{43}$$

$$w(x,y,t) = \left(\frac{2\tau\omega\Omega}{3} + 2\tau\Omega\left(-\frac{\sqrt{-\omega}}{2}\left(\tanh\left(\frac{\sqrt{-\omega}}{2}\left(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t\right)\right) + \coth\left(\frac{\sqrt{-\omega}}{2}\left(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t\right)\right)\right)\right)^2\right) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{44}$$

Case 2: $\omega > 0$,

$$u(x,y,t) = \left(\frac{2\tau\omega\Omega}{3} + 2\tau\Omega\left(\sqrt{\omega}\tan\left(\sqrt{\omega}\left(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t\right)\right)\right)^2\right) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{45}$$

$$v(x,y,t) = \left(\frac{2\tau\omega\Omega}{3} + 2\tau\Omega\left(\sqrt{\omega}\tan\left(\sqrt{\omega}\left(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t\right)\right)\right)^2\right) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{46}$$

$$w(x,y,t) = \left(\frac{2\tau\omega\Omega}{3} + 2\tau\Omega\left(\sqrt{\omega}\tan\left(\sqrt{\omega}\left(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t\right)\right)\right)^2\right) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{47}$$

$$u(x,y,t) = \left(\frac{2\tau\omega\Omega}{3} + 2\tau\Omega\left(-\sqrt{\omega}\cot\left(\sqrt{\omega}\left(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t\right)\right)\right)^2\right) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{48}$$

$$v(x,y,t) = \left(\frac{2\tau\omega\Omega}{3} + 2\tau\Omega\left(-\sqrt{\omega}\cot\left(\sqrt{\omega}\left(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t\right)\right)\right)^2\right) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{49}$$

$$w(x,y,t) = \left(\frac{2\tau\omega\Omega}{3} + 2\tau\Omega\left(-\sqrt{\omega}\cot\left(\sqrt{\omega}\left(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t\right)\right)\right)^2\right) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{50}$$

$$u(x,y,t) = \left(\frac{2\tau\omega\Omega}{3} + 2\tau\Omega\left(\sqrt{\omega}\left(\tan\left(2\sqrt{\omega}\left(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t\right)\right) \pm \sec\left(2\sqrt{\omega}\left(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t\right)\right)\right)\right)^2\right) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{51}$$

$$v(x,y,t) = \left(\frac{2\tau\omega\Omega}{3} + 2\tau\Omega\left(\sqrt{\omega}\left(\tan\left(2\sqrt{\omega}\left(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t\right)\right) \pm \sec\left(2\sqrt{\omega}\left(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t\right)\right)\right)\right)^2\right) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{52}$$

$$w(x,y,t) = (\frac{2\tau\omega\Omega}{3} + 2\tau\Omega(\sqrt{\omega}(\tan(2\sqrt{\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t)) \pm \sec(2\sqrt{\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \quad (53)$$

$$u(x,y,t) = (\frac{2\tau\omega\Omega}{3} + 2\tau\Omega(\sqrt{\omega}(-\cot(2\sqrt{\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t)) \pm \csc(2\sqrt{\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \quad (54)$$

$$v(x,y,t) = (\frac{2\tau\omega\Omega}{3} + 2\tau\Omega(\sqrt{\omega}(-\cot(2\sqrt{\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t)) \pm \csc(2\sqrt{\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \quad (55)$$

$$w(x,y,t) = (\frac{2\tau\omega\Omega}{3} + 2\tau\Omega(\sqrt{\omega}(-\cot(2\sqrt{\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t)) \pm \csc(2\sqrt{\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \quad (56)$$

$$u(x,y,t) = (\frac{2\tau\omega\Omega}{3} + 2\tau\Omega(\frac{\sqrt{\omega}}{2}(\tan(\frac{\sqrt{\omega}}{2}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t)) - \cot(\frac{\sqrt{\omega}}{2}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \quad (57)$$

$$v(x,y,t) = (\frac{2\tau\omega\Omega}{3} + 2\tau\Omega(\frac{\sqrt{\omega}}{2}(\tan(\frac{\sqrt{\omega}}{2}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t)) - \cot(\frac{\sqrt{\omega}}{2}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \quad (58)$$

$$w(x,y,t) = (\frac{2\tau\omega\Omega}{3} + 2\tau\Omega(\frac{\sqrt{\omega}}{2}(\tan(\frac{\sqrt{\omega}}{2}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t)) - \cot(\frac{\sqrt{\omega}}{2}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (4a_1\tau^3\omega + a_3\tau + 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \quad (59)$$

Case 3: $\omega = 0$,

$$u(x,y,t) = (2\tau\Omega(-\frac{1}{(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (a_3\tau + a_4\Omega)t)})^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t) \quad (60)$$

$$v(x,y,t) = (2\tau\Omega(-\frac{1}{(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (a_3\tau + a_4\Omega)t)})^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t) \quad (61)$$

$$w(x,y,t) = (2\tau\Omega(-\frac{1}{(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (a_3\tau + a_4\Omega)t)})^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t) \quad (62)$$

Set 2:

$$\{b_0 = 2\tau\omega\Omega, b_1 = 0, b_2 = 2\tau\Omega, \lambda = -4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega\} \quad (63)$$

Case 1: If $\omega < 0$,

$$u(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(-\sqrt{-\omega}\tanh(\sqrt{-\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha}+\Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \quad (64)$$

$$v(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(-\sqrt{-\omega}\tanh(\sqrt{-\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{65}$$

$$w(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(-\sqrt{-\omega}\tanh(\sqrt{-\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{66}$$

$$u(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(-\sqrt{-\omega}\coth(\sqrt{-\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{67}$$

$$v(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(-\sqrt{-\omega}\coth(\sqrt{-\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{68}$$

$$w(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(-\sqrt{-\omega}\coth(\sqrt{-\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{69}$$

$$u(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(\sqrt{-\omega}(-\tanh(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (a_3\tau - 4a_1\tau^3\omega - 4a_2\omega\Omega^3 + a_4\Omega)t)) \pm i sech(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{70}$$

$$v(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(\sqrt{-\omega}(-\tanh(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (a_3\tau - 4a_1\tau^3\omega - 4a_2\omega\Omega^3 + a_4\Omega)t)) \pm i sech(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{71}$$

$$w(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(\sqrt{-\omega}(-\tanh(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (a_3\tau - 4a_1\tau^3\omega - 4a_2\omega\Omega^3 + a_4\Omega)t)) \pm i sech(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{72}$$

$$u(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(\sqrt{-\omega}(-\coth(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (a_3\tau - 4a_1\tau^3\omega - 4a_2\omega\Omega^3 + a_4\Omega)t)) \pm csch(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{73}$$

$$v(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(\sqrt{-\omega}(-\coth(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (a_3\tau - 4a_1\tau^3\omega - 4a_2\omega\Omega^3 + a_4\Omega)t)) \pm csch(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{74}$$

$$w(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(\sqrt{-\omega}(-\coth(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (a_3\tau - 4a_1\tau^3\omega - 4a_2\omega\Omega^3 + a_4\Omega)t)) \pm csch(2\sqrt{-\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{75}$$

$$u(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(-\frac{\sqrt{-\omega}}{2}(\tanh(\frac{\sqrt{-\omega}}{2}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (a_3\tau - 4a_1\tau^3\omega - 4a_2\omega\Omega^3 + a_4\Omega)t)) + \coth(\frac{\sqrt{-\omega}}{2}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{76}$$

$$v(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(-\frac{\sqrt{-\omega}}{2}(\tanh(\frac{\sqrt{-\omega}}{2}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (a_3\tau - 4a_1\tau^3\omega - 4a_2\omega\Omega^3 + a_4\Omega)t)) + \coth(\frac{\sqrt{-\omega}}{2}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{77}$$

$$w(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(-\frac{\sqrt{-\omega}}{2}(\tanh(\frac{\sqrt{-\omega}}{2}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (a_3\tau - 4a_1\tau^3\omega - 4a_2\omega\Omega^3 + a_4\Omega)t)) + \coth(\frac{\sqrt{-\omega}}{2}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{78}$$

If $\omega > 0$,

$$u(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(\sqrt{\omega}\tan(\sqrt{\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{79}$$

$$v(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(\sqrt{\omega}\tan(\sqrt{\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{80}$$

$$w(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(\sqrt{\omega}\tan(\sqrt{\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{81}$$

$$u(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(-\sqrt{\omega}\cot(\sqrt{\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{82}$$

$$v(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(-\sqrt{\omega}\cot(\sqrt{\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{83}$$

$$w(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(-\sqrt{\omega}\cot(\sqrt{\omega}(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{84}$$

$$u(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(\sqrt{\omega}(\tan(2\sqrt{\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)) \pm \sec(2\sqrt{\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{85}$$

$$v(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(\sqrt{\omega}(\tan(2\sqrt{\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)) \pm \sec(2\sqrt{\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{86}$$

$$w(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(\sqrt{\omega}(\tan(2\sqrt{\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)) \pm \sec(2\sqrt{\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{87}$$

$$u(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(\sqrt{\omega}(-\cot(2\sqrt{\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)) \pm \csc(2\sqrt{\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{88}$$

$$v(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(\sqrt{\omega}(-\cot(2\sqrt{\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)) \pm \csc(2\sqrt{\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{89}$$

$$w(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(\sqrt{\omega}(-\cot(2\sqrt{\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)) \pm \csc(2\sqrt{\omega}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{90}$$

$$u(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(\frac{\sqrt{\omega}}{2}(\tan(\frac{\sqrt{\omega}}{2}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)) - \cot(\frac{\sqrt{\omega}}{2}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{91}$$

$$v(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(\frac{\sqrt{\omega}}{2}(\tan(\frac{\sqrt{\omega}}{2}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)) - \cot(\frac{\sqrt{\omega}}{2}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{92}$$

$$w(x,y,t) = (2\tau\omega\Omega + 2\tau\Omega(\frac{\sqrt{\omega}}{2}(\tan(\frac{\sqrt{\omega}}{2}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t)) - \cot(\frac{\sqrt{\omega}}{2}\,(\frac{\Gamma(1+\Upsilon)}{\alpha}(\tau x^{\alpha} + \Omega y^{\alpha}) - (-4a_1\tau^3\omega + a_3\tau - 4a_2\omega\Omega^3 + a_4\Omega)t))))^2) \times \exp(-\nu\beta(t) - \frac{\nu^2}{2}t). \tag{93}$$

5 Graphical illustrations and Discussion

In this section, we present graphs as visual representations to depict data, showcasing various solutions, with some of our solutions being presented in both two and three dimensions.
In Fig 1, We demonstrate the graphs of (30) at $a_1 = 0.01, a_2 = 0.01, a_3 = 0.001, a_4 = 0.001, \Upsilon = 0.4, \nu = 0.01, \tau = 0.3, \omega = -0.01, \Omega = 0.01, \beta(t) = 2t$. Utilizing the method, we present the graphs of (45) at $a_1 = 0.01, a_2 = 0.01, a_3 = 0.001, a_4 = 0.001, \Upsilon = 0.4, \nu = 0.01, \tau = 0.3, \omega = 0.01, \Omega = 0.01, \beta(t) = 2t$ in Fig 2. Fig. 3 exhibit the graph of (64) employing the method at $a_1 = 0.01, a_2 = 0.01, a_3 = 0.001, a_4 = 0.001, \Upsilon = 0.4, \nu = 0.01, \tau = 0.3, \omega = -0.01, \Omega = 0.01, \beta(t) = 2t$. Finally, in Fig. 4 we show the graph of (79) at $a_1 = 0.01, a_2 = 0.01, a_3 = 0.001, a_4 = 0.001, \Upsilon = 0.4, \nu = 0.01, \tau = 0.3, \omega = 0.01, \Omega = 0.01, \beta(t) = 2t$.

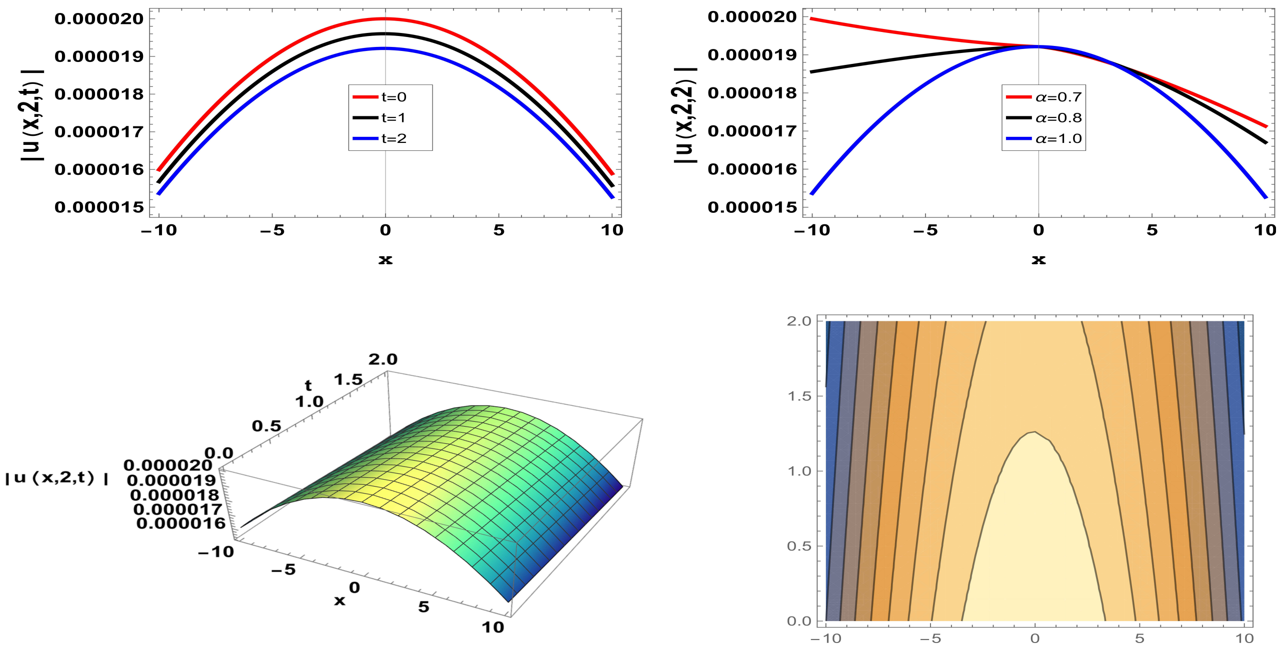

Fig. 1: The graph of (30) at $a_1 = 0.01, a_2 = 0.01, a_3 = 0.001, a_4 = 0.001, \Upsilon = 0.4, \nu = 0.01, \tau = 0.3, \omega = -0.01, \Omega = 0.01, \beta(t) = 2t$.

6 Conclusion

In this paper, we successfully obtained new analytical wave solutions to the (2+1)-dimensional truncated M-fractional stochastic Nizhnik-Novikov-Veselov system by utilizing the modified simplest equation technique. The derived solutions are verified and demonstrate by different plots with the use of Mathematica software. Some of the obtained solutions were also described graphically by 2-dimensional, 3-dimensional and contour plots. The gained solutions can be helpful for future studies of the underlying model. Finally, this technique is simple, reliable and extendable to tackle other nonlinear PDEs. A mathematical system that involves the noise term is called a stochastic system, and the PDEs are called stochastic partial differential equations (SPDEs). This research can be helpful in various fields, including optics and crystal networks.

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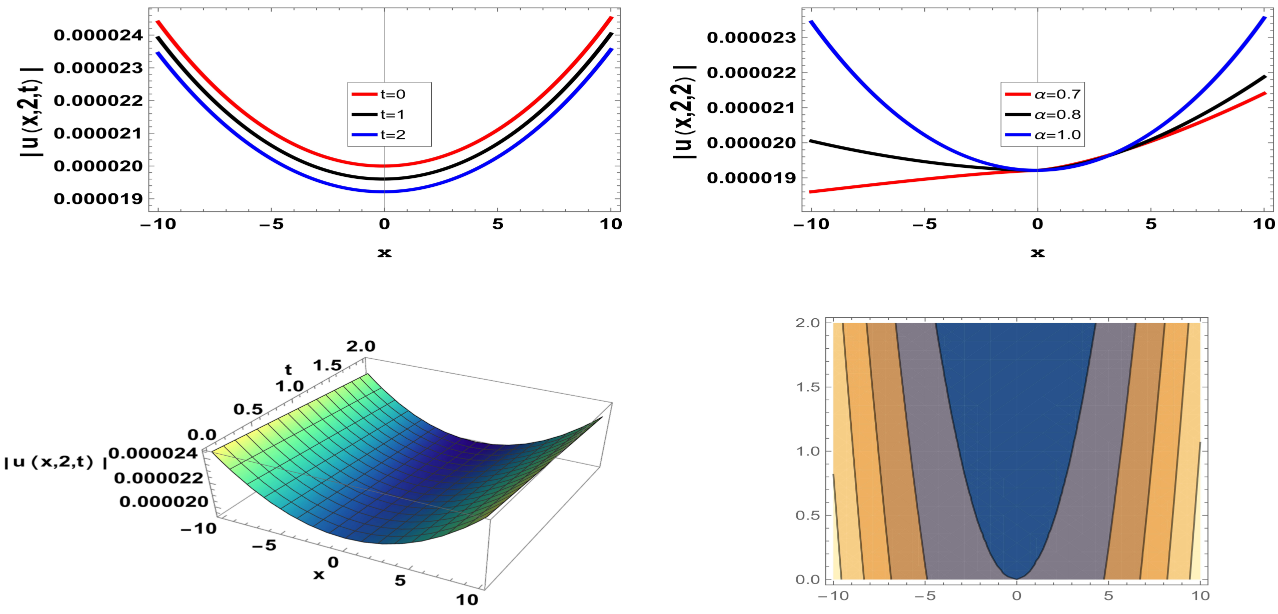

Fig. 2: The graph of (45) at $a_1 = 0.01, a_2 = 0.01, a_3 = 0.001, a_4 = 0.001, \Upsilon = 0.4, \nu = 0.01, \tau = 0.3, \omega = 0.01, \Omega = 0.01, \beta(t) = 2t$.

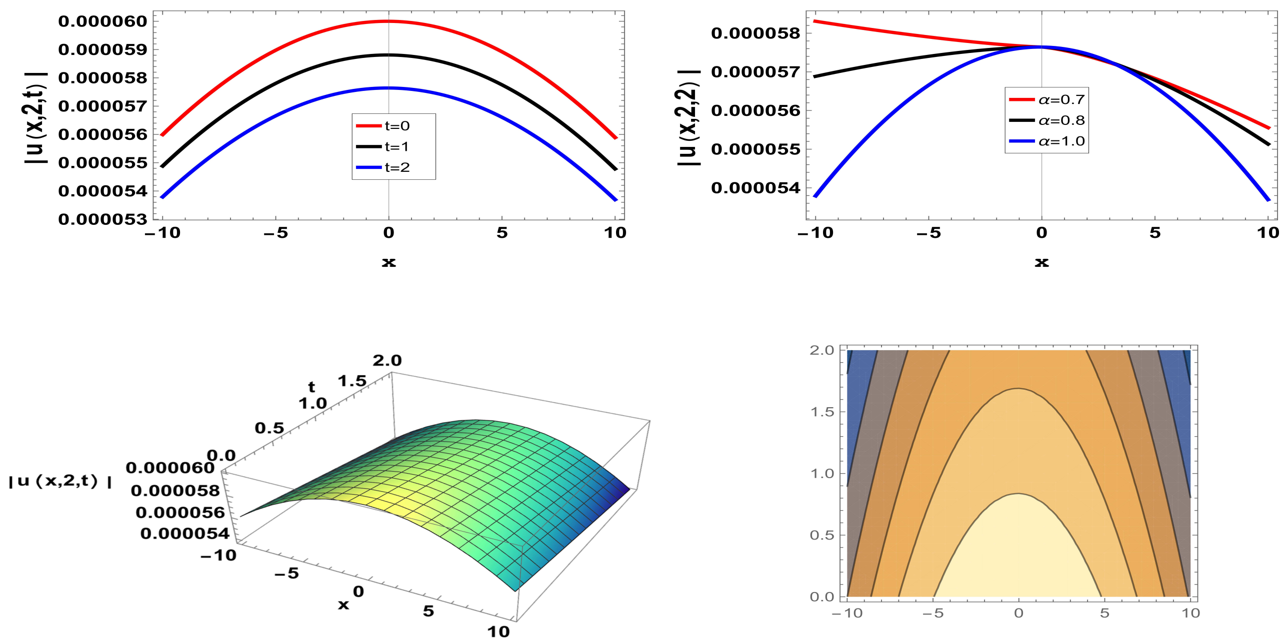

Fig. 3: The graph of (64) at $a_1 = 0.01, a_2 = 0.01, a_3 = 0.001, a_4 = 0.001, \Upsilon = 0.4, \nu = 0.01, \tau = 0.3, \omega = -0.01, \Omega = 0.01, \beta(t) = 2t$.

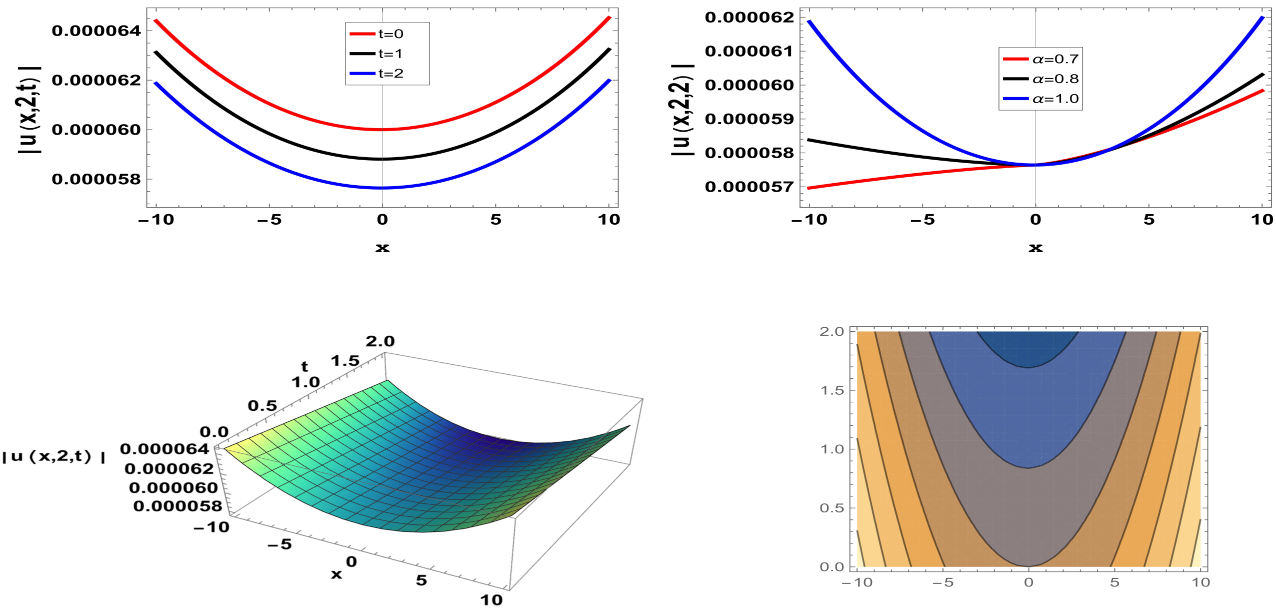

Fig. 4: The graph of (79) at $a_1 = 0.01, a_2 = 0.01, a_3 = 0.001, a_4 = 0.001, \Upsilon = 0.4, \nu = 0.01, \tau = 0.3, \omega = -0.01, \Omega = 0.01, \beta(t) = 2t$.

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