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Adaptive Boundary Effect Processing For Empirical Mode Decomposition Using Template Matching

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Abstract: This paper is contributed to the boundary effect problem of the empirical mode decomposition algorithm, which results in a serious distortion in the EMD sifting process. An adaptive method for processing boundary effect in the empirical mode decomposition sifting process is presented, which has exploited the local time- or spatial- scales and the waveform or texture characteristics near boundary of the signal or image to extend the signal or image so that additional subsignal or subimage are obtained. The extended section is taken as the most suited subsignal or subimage to the inner signal or image by template matching operation. The multiple components of the original signal or image are available by applying EMD algorithm to the extended signal or image and then leaving out the extended parts of the decomposed components. Simulation results have proved that the proposed template matching based decomposition method outperforms the neural network extending method, the mirror extrema extending method and the AR model extending method for 1D signals, and perform texture extraction effectively for 2D natural images such as defect-free and defect fabrics.

Keywords: Empirical mode decomposition, boundary effect processing, template matching

1 Introduction

Empirical mode decomposition (EMD) [1] is a newly proposed method for signal processing, which uses intrinsic time scales to decompose the signal instead of predefined basis functions. It is very suitable for the analysis of nonlinear and nonstationary signals. However, in Huang's EMD algorithm, the cubic spline based envelope-mean interpolation creates an effect called boundary swings, which results in distortion near two endpoints of the signal. Aiming at the solution of boundary effect of EMD, certain techniques including the neural network (NN) extending method [2], the mirror extrema (ME) extending method [3] and the autoregressive (AR) model extending method [4] are presented.

Although these previous works have been proved effective by simulation results, they either contaminate the intrinsic nature of a signal such as nonlinearity and nonstationarity or lack use of the waveform characteristics of the signal. It is supposed to exploit the local time scales and the waveform characteristics [5] near endpoints to extend the signal so that additional extrema are obtained. But in [5] the choice of length of moving time window, which is used to segment a signal into a series of vector type data is not adaptive. Based on time scales near the two endpoints, an adaptive method using template matching (TM) is proposed to overcome the boundary effect. The critical step of end extending is to determine the tendencies near the two end points, which most likely emerge in the inner signal especially for the regular signals. If an inner sequence has the most similarity of tendency with the front endpoint or back endpoint, the inner sequence nearest to the searched sequence is most suited as the extended sequence.

When the generalization of classical EMD algorithm to BEMD is performed, boundary effect is still one of the essential issues in the sifting process. Local textures of an image are commonly similar to each other to some extent, especially in the case of fabric image. In the 2D case, the TM method uses the subimage near the boundary as a template to search the most suited subimage in the inner image by correlation operation. If an inner subimage has the most similarity of local texture to the boundary, the

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inner subimages nearest to the searched subimage are the desirable textures for boundary extension.

The proposed method fully makes use of the waveform characteristic of the signal or the similarity of local texture and can thus solve the boundary issue effectively. Applying EMD to the extended signal or image, and then leaving out the extended parts of the decomposed components, the intrinsic mode functions of the original signal or image are formed.

2 Adaptive boundary effect processing

2.1 One-dimensional case

Assume the underlying signal $S(t)$ is a data vector $\mathbf{S} = [s(0),\ s(1), \cdots,\ s(N-1)]^{\mathrm{T}}$. The procedure for processing boundary effect can be formulated in detail as follows.

Step 1. Identify one maximum and one minimum nearest to the front endpoint of $S(t)$. Denote their locations by $t_{\max,\mathrm{front}}$, $t_{\min,\mathrm{front}}$ respectively. Similarly identify one maximum and one minimum nearest to the back endpoint of $S(t)$ and denote by $t_{\max,\mathrm{back}}$, $t_{\min,\mathrm{back}}$ their locations respectively. If $t_{\max,\mathrm{front}} > t_{\min,\mathrm{front}}$, let $t_m = t_{\max,\mathrm{front}}$; otherwise, let $t_m = t_{\min,\mathrm{front}}$. If $t_{\max,back} < t_{\min,back}$, let $\tilde{t}_m = t_{\max,back}$; otherwise, let $\tilde{t}_m = t_{\min,back}$.

Step 2. For the front endpoint and back endpoint extension, construct the matching templates $\mathbf{G}$ and $\tilde{\mathbf{G}}$ respectively as follows:

$$\mathbf{G} = [g(0),\ g(1), \cdots, g(t_m)]^{\mathrm{T}}, \tag{1}$$

where $g(0) = s(0), g(1) = s(1), \ldots, g(t_m) = s(t_m)$, and

$$\tilde{\mathbf{G}} = [\tilde{g}(0),\ \tilde{g}(1), \cdots, \tilde{g}(N-\tilde{t}_m-1)]^{\mathrm{T}}, \tag{2}$$

where $\tilde{g}(0) = s(\tilde{t}_m)$, $\tilde{g}(1) = s(\tilde{t}_m+1)$, $\cdots$, $\tilde{g}(N-\tilde{t}_m-1) = s(N-1)$.

Step 3. Compute the correlation between $\mathbf{G}$ (resp.$\tilde{\mathbf{G}}$) and a series of sub-signal of $s(t)$ to measure the similarity of $\mathbf{G}$ (resp.$\tilde{\mathbf{G}}$) to a certain sub-signal:

$$Corr(\mathbf{G},\mathbf{S}_i) = \frac{\sum\limits_{t=0}^{t_m} g(t)s(t+t_m+i)}{\left[\sum\limits_{t=0}^{t_m} g^2(t) \sum\limits_{t=0}^{t_m} s^2(t+t_m+i)\right]^{1/2}}, \tag{3}$$

where $i = 1,\ 2, \cdots,\ N-2t_m-1$.

$$Corr\left(\tilde{\mathbf{G}},\tilde{\mathbf{S}}_j\right) = \frac{\sum\limits_{t=0}^{N-\tilde{t}_m-1} \tilde{g}(t)\ s(t+2\,\tilde{t}_m+1-N-j)}{\left[\sum\limits_{t=0}^{N-\tilde{t}_m-1} \tilde{g}^2(t) \sum\limits_{t=0}^{N-\tilde{t}_m-1} s^2(t+2\,\tilde{t}_m+1-N-j)\right]^{1/2}}, \tag{4}$$

where $j = 1,\ 2, \cdots,\ 2\tilde{t}_m+1-N$. In Eqs. (3) and (4), $\mathbf{S}_i = [s(t_m+i),\ s(t_m+1+i), \cdots,\ s(2t_m+i)]^{\mathrm{T}}$ and $\tilde{\mathbf{S}}_j = [s(2\tilde{t}_m+1-N-j),\ s(2\tilde{t}_m+2-N-j), \cdots, s(\tilde{t}_m-j)]^{\mathrm{T}}$ are the vector type sub-signals of $s(t)$.

Step 4. For front-endpoint extension, search the maximum of $Corr(\mathbf{G},\mathbf{S}_i)$ and the corresponding value of $\mathbf{S}_i$, which is represented by $\mathbf{S}_{i_0} = \arg\max\limits_i Corr(\mathbf{G},\mathbf{S}_i)$. For back-endpoint extension, search the maximum of $Corr\left(\tilde{\mathbf{G}},\tilde{\mathbf{S}}_j\right)$ and the corresponding value of $\tilde{\mathbf{S}}_j$, which is represented by $\tilde{\mathbf{S}}_{j_0} = \arg\max\limits_j Corr\left(\mathbf{G},\tilde{\mathbf{S}}_j\right)$.

Step 5. For front-endpoint extension, add the sub-signal with one maximum and one minimum ,which is just before $\mathbf{S}_{i_0}$ to the front of $s(t)$. For back-endpoint extension, add the sub-signal with one maximum and one minimum , which is just behind $\tilde{\mathbf{S}}_{j_0}$ to the back of $s(t)$.

Step 6. Perform EMD sifting process (Ref. section 3.1) on the extended signal.

2.2 Bidimensional Case

Step 1. Use the moving spatial windows of size $Q \times Q$ to segment the underlying image $\mathcal{I}(x,y)$ of size $P \times P$ into a series of subimages $\mathcal{B}_i$, where $\mathcal{B}_i = [\mathbf{b}_{i1}, \mathbf{b}_{i2}, \cdots, \mathbf{b}_{iQ}]$. Let $\mathcal{W} = [\mathbf{w}_{i1}, \mathbf{w}_{i2}, \cdots, \mathbf{w}_{iQ}]$ be the template, which at least overlaps one edge of $\mathcal{I}(x,y)$.

Step 2. Compute the similarity measure between $\mathcal{B}_i$ and $\mathcal{W}$ in terms of Eq. (5).

$$Corr(\mathcal{W},\mathcal{B}_i) = \frac{\sum\limits_{q=1}^{Q} \mathbf{w}_{iq}^T \mathbf{b}_{iq}}{\left(\sum\limits_{q=1}^{Q} \mathbf{w}_{iq}^T \mathbf{w}_{iq} \cdot \sum\limits_{q=1}^{Q} \mathbf{b}_{iq}^T \mathbf{b}_{iq}\right)^{1/2}}, \tag{5}$$

Step 3. Search the most similar subimage $\mathcal{B}_{opt}$ to the template $\mathcal{W}$, which satisfies the following expression:

$$\mathcal{B}_{opt} = \arg\max_i Corr(\mathcal{W},\mathcal{B}_i) \tag{6}$$

Step 4. To the left boundary, a subimage of $\mathcal{I}(x,y)$, which is just on the left side of $\mathcal{B}_{opt}$, is attached to the left side of the original image. To the right boundary, a subimage of $\mathcal{I}(x,y)$, which is just on the right side of $\mathcal{B}_{opt}$, is attached to the right side of the original image. To the top boundary, a subimage of $\mathcal{I}(x,y)$, which is just on the top of $\mathcal{B}_{opt}$, is attached to the top of the original image. To the bottom boundary, a subimage of $\mathcal{I}(x,y)$, which is just on the bottom side of $\mathcal{B}_{opt}$, is attached to bottom of the original image. To the diagonal extension, subimages nearest to $\mathcal{B}_{opt}$ along the diagonal direction are attached to outsides along the diagonal direction respectively.

Step 5. Perform EMD sifting process (Ref. section 3.2) on the extended image.

3 Empirical Mode Decomposition

3.1 One Dimensional EMD Algorithm

The objective of EMD algorithm is to extract a number of intrinsic mode functions (IMFs) from a multicomponent signal $s(t)$. For an extracted IMF, the number of extrema and zero-crossings of the signal must be the same or differ by no more than one, and the mean of the envelope defined by the local maxima and the envelope defined by the local minima is always zero. The EMD algorithm can be formulated as follows.

Step 1. Set the residue $r_0(t) = s(t)$ and the IMF number $\ell = 1$.

Step 2. Sift the ℓth IMF component:

i) Set $h_0(t) = r_{\ell-1}(t), k = 1$.

ii) Search all the local maxima and local minima from $h_{k-1}(t)$.

iii) Interpolate all the local maxima and local minima respectively by cubic splines to obtain upper and lower envelopes.

iv) Compute the mean envelope $m_{k-1}(t)$ of upper and lower envelopes.

v) Update $h_k(t) = h_{k-1}(t) - m_{k-1}(t)$.

vi) Repeat steps ii)-v) until $h_k(t)$ being an IMF. If so, the ℓth IMF is $c_\ell(t) = h_k(t)$ and update residue $r_\ell(t) = r_{\ell-1}(t) - c_\ell(t)$.

Step 3. Proceed the sifting process for all the subsequent $r_\ell(t)$'s and finally reconstruct $s(t)$ as

$$s(t) = \sum_{\ell=1}^{L} c_\ell(t) + r_L(t), \tag{7}$$

where L is the number of IMF components and $r_L(t)$ is the final residue.

3.2 Bidimensional EMD Algorithm

The bidimensional EMD is contributed to extract the 2D IMFs from a bidimensional signal $\mathcal{I}(x,y)$ during the sifting process. A 2D IMF is characterized by zero-mean and AM-FM. The bidimensional sifting process can be detailed as:

Step 1. Initialize the residue $\mathcal{R}_0(x,y) = \mathcal{I}(x,y)$ and the IMF number $\ell = 1$.

Step 2. Sift the ℓth IMF component:

i) Set $\mathcal{H}_0(x,y) = \mathcal{R}_{\ell-1}(x,y), k = 1$.

ii) Identify all the local maxima and local minima from $\mathcal{H}_{k-1}(x,y)$.

iii) Interpolate all the local maxima and local minima respectively with the radial basis function (RBF) [6] or the multilevel B-splines [7] to obtain upper and lower envelopes, i.e. $\mathcal{H}_u(x,y)$ and $\mathcal{H}_l(x,y)$.

iv) Construct the mean envelope $\mathcal{M}_{k-1}(x,y)$ of upper and lower envelopes as:

$$\mathcal{M}_{k-1}(x,y) = \frac{\mathcal{H}_u(x,y) + \mathcal{H}_l(x,y)}{2}$$

v) Set $\mathcal{H}_k(x,y) = \mathcal{H}_{k-1}(x,y) - \mathcal{M}_{k-1}(x,y)$.

vi) Repeat steps ii)-v) until the following criterion is satisfied.

$$SD = \sum_{x,y} \left[\frac{|\mathcal{H}_k(x,y) - \mathcal{H}_{k-1}(x,y)|^2}{\mathcal{H}_{k-1}^2(x,y)} \right] < \varepsilon$$

where ε is the predefined value.

Step 3. Represent the ℓth IMF as $\mathcal{C}_\ell(x,y) = \mathcal{H}_k(x,y)$ and update residue $\mathcal{R}_\ell(x,y) = \mathcal{R}_{\ell-1}(x,y) - \mathcal{C}_\ell(x,y)$.

Step 4. Repeat steps 2-3 until there only exists monotonic 2D component $\mathcal{I}_{res}(x,y)$ for horizontal or vertical direction. Finally the bidimensional signal $\mathcal{I}(x,y)$ is reconstructed by

$$\mathcal{I}(x,y) = \sum_{\ell=1}^{L} \mathcal{C}_\ell(x,y) + \mathcal{I}_{res}(x,y). \tag{8}$$

4 Results and Discussion

4.1 Simulation for 1D Case

Consider a multicomponent signal given by

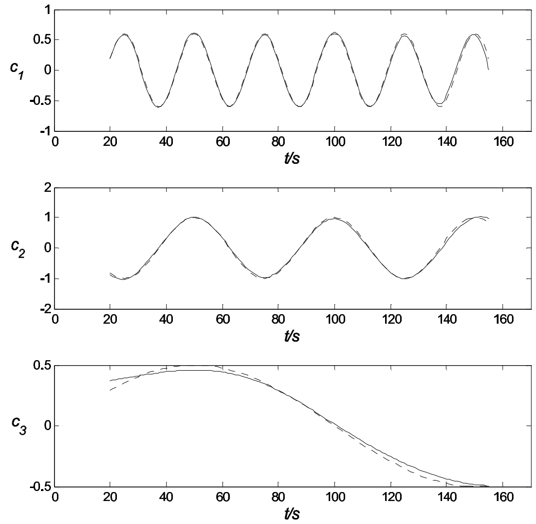
$$s(t) = \cos\left(\frac{\pi}{25}t\right) + 0.6\cos\left(\frac{2\pi}{25}t\right) + 0.5\sin\left(\frac{\pi}{100}t\right), \tag{9}$$

where $t \in [20, 155]$, and $s_1(t) = 0.6\cos\left(\frac{2\pi}{25}t\right)$, $s_2(t) = \cos\left(\frac{\pi}{25}t\right)$, $s_3(t) = 0.5\sin\left(\frac{\pi}{100}t\right)$ are the three components for $s(t)$. As compared to the proposed method, the neural network extending method, the mirror extrema extending method and the AR model extending method are used to suppress boundary effect during the EMD sifting process. To evaluate the processing performance for boundary effect, two criteria called mean absolute error (MAE) and mean squared error (MSE) between real components $s_\ell(t)$ and decomposed components $c_\ell(t)$, $\ell = 1, 2, 3$ are introduced as:

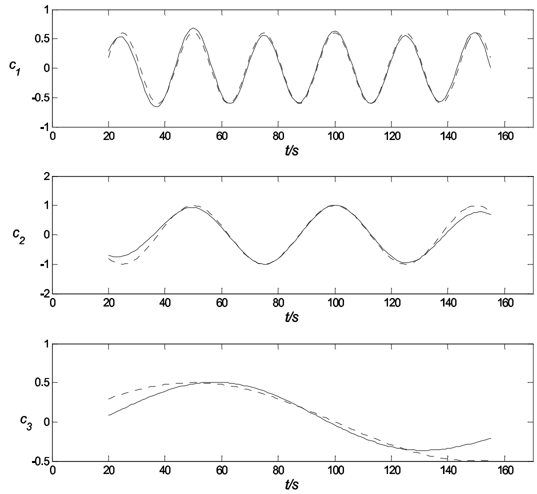
$$\mathrm{MAE} = \frac{1}{N}\sum_{t=1}^{N} |s_\ell(t) - c_\ell(t)|, \ell = 1, 2, 3. \tag{10}$$

$$\mathrm{MSE} = \frac{1}{N}\sum_{t=1}^{N} |s_\ell(t) - c_\ell(t)|^2, \ell = 1, 2, 3. \tag{11}$$

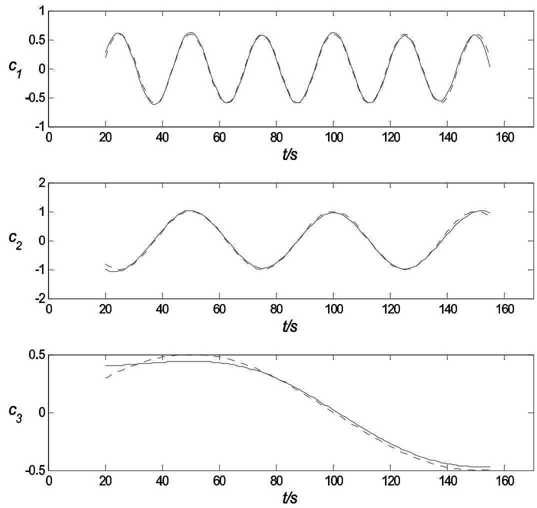
Fig.4.1 illustrates the EMD decomposition results of the simulated signal using four different boundary effect processing methods. It is clearly shown that the decomposed components $c_\ell(t)$, $\ell = 1, 2, 3$ using template matching based boundary effect processing method furthest approximate to the real components,$s_\ell(t)$, $\ell = 1, 2, 3$.



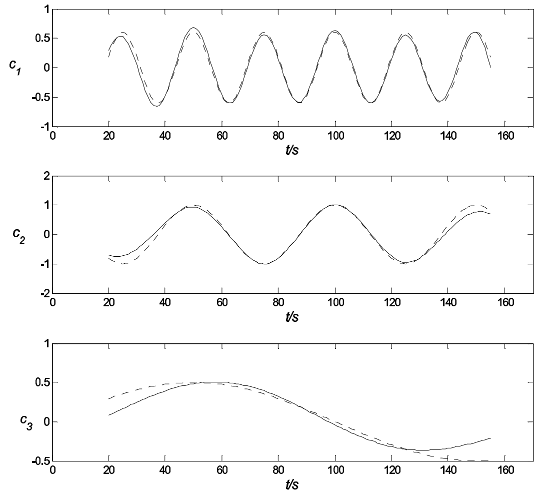
(a)



(b)



(c)



(d)

Figure 4.1 EMD results of the simulated signal with (a) template matching extending method, (b) neural network extending method, (c) mirror extrema extending method and (d) AR model extending method

Tables 4.1-4.3 provide the MAE and MSE of the simulated signal with four different methods.

Table 4.1 MAE and MSE comparison of the first component with four methods.

Criterion	TM	NN	ME	AR
MAE	0.031	0.0667	0.0325	0.0513
MSE	0.0015	0.0066	0.0015	0.0038

Table 4.2 MAE and MSE comparison of the second component with four methods.

Criterion	TM	NN	ME	AR
MAE	0.0358	0.1242	0.0424	0.0873
MSE	0.0022	0.0332	0.0028	0.0165

Table 4.3 MAE and MSE comparison of the third component with four methods.

Criterion	TM	NN	ME	AR
MAE	0.0256	0.1127	0.0301	0.0752
MSE	0.0009	0.0233	0.0012	0.0109

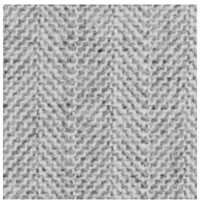
From tables 4.1-4.3, it is shown that the boundary effect processing method based on template matching results in less distortion of decomposed components than any other three boundary effect processing methods. For three components of $s(t)$, the MAEs and MSEs of template matching based method are less than any other boundary effect processing method.

4.2 Simulation for 2D Case

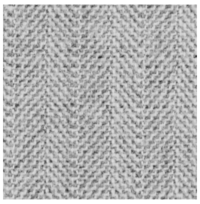
Two real fabric texture images have been used in this section to test and validate the texture extraction performance of the template matching BEMD algorithm.

The decomposition approach is first applied to a defect-free fabric image with boundary extending (see

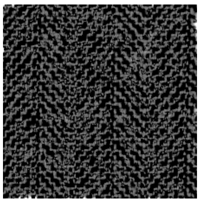
Fig.4.2). This fabric is mainly established by diagonal and vertical structures. Decomposition results are comprised of three components. The first mode corresponds to the woven structure, the second to the pattern and the residue to the vertical stripes.



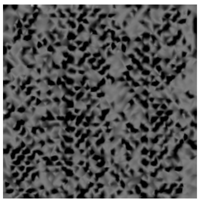
(a)original image



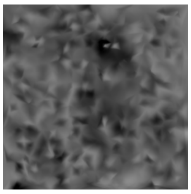
(b)extended image



(c)first mode



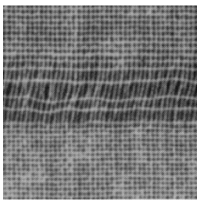
(d)second mode



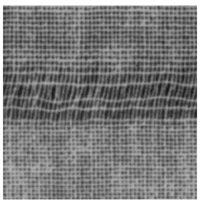
(e)residue

Fig.4.2 Decomposition results of the defect-free fabric using boundary extending algorithm

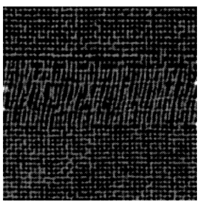
The decomposition approach is then applied to a boundary- extending defect fabric image (see Fig.4.3)[8]. The sifting process as such results in three modes. The first mode contain defect woven information besides horizontal and vertical structures. The second mode is recognized clearly by separate horizontal and vertical pattern and defect pattern.



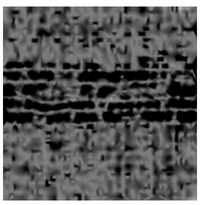
(a)original image



(b)extended image



(c)first mode



(d)second mode

(e)residue

Fig.4.3 Decomposition results of the defect fabric using boundary extending algorithm

5 Conclusion

The proposed template matching based method of processing boundary effect for empirical mode decomposition in this paper uses the sequence near the end (i.e. 1D case) or the local texture (i.e. 2D case) as a template to search the most suited sequence or local texture in the inner signal or image by correlation operation. The searched sequence is taken as the extended signal before or after the original signal, while the the searched texture blocks are added to the original image along horizontal, vertical, and diagonal directions respectively. The simulation results or 1D signal have proved that template matching based method outperforms the classical boundary effect processing method , i.e. neural network extending method, the mirror extrema extending method and the AR model extending method. The simulation results for 2D fabric image have also proved that intrinsic structures with different spatial scales, such as the woven structure, the pattern and the strip, can be recognized effectively in a fully unsupervised way.

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References

[1] Huang N. E., The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis, J. Proc. R. Soc. Lond. A. **454**, 903-995 (1998).

[2] Deng Y. J., Wang W, Qian C. C., et al., Boundary processing technique in EMD method and Hilbert transform, Chinese Science bulletin. **46**, 954-961 (2001).

[3] Rilling, G., P. Flandrin et al., On empirical mode decomposition and its algorithms, IEEE-EURASIP Workshop on Nonlinear Signal and Image Processing. **3**, 8-11 (2003).

[4] Zhang Yushan-Liang Jianwen-Hu Yuxian, Dealing with the boundary problem in EMD method by using autoregressive model, Progress in Natural Science, **13**, 1054-1059 (2003).
[5] Wang J, Peng Y, Peng X, Similarity searching based boundary effect processing method for empirical mode decomposition, Electronics Letters, **43**, 58-59 (2007).
[6] J. C. Nunes, Y. Bouaoune, E. Delechelle, et al., Image analysis by bidimensional empirical mode decomposition, Image and Vision Computing, **21**, 1019-1026 (2003).
[7] Nunes J. C., Guyot, et al., Texture analysis based on local analysis of the Bidimensional Empirical Mode Decomposition, Machine Vision and Applications, **16**, 177-188 (2005).
[8] Textile Texture Database, Version 1.0, Volume 1 of 2(1996).

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