Information Sciences Letters
An International Journal

http://dx.doi.org/10.18576/isl/120230

Extensions of Weak PS-Rings

Sabah A. Nasef[1], *Refaat M. Salem* [2] *and Sarah K. El-Din*[1,3,*]

[1]Mathematics Department, Faculty of Science, Al-Azhar University Girls Branch, Nasr City, Cairo, Egypt
[2]Mathematics Department, Faculty of Science, Al-Azhar University Boys Branch, Nasr City (11884), Cairo, Egypt
[3]Mathematics Department,, College of Science and Arts, Qassim University, Al-Badaya 51951, Saudi Arabia

Received: 2 Jun. 2022, Revised: 12 Aug. 2022, Accepted: 23 Sep. 2022
Published online: 1 Feb. 2023

Abstract: In this paper, we prove that, under mild conditions, if R is a right weak PS-ring, then so does the skew inverse power series ring $R[[x^{-1};\alpha,\delta]]$, the skew generalized power series ring $R[[S,\omega]]$, and the differential inverse power series ring $R[[x^{-1};\delta]]$.

Keywords: weak PS-ring, weak annihilator, skew generalized power series, skew inverse power series, differential inverse power series.

1 Introduction

Throughout this article, all rings R are associative with unity and all modules are unital R-modules unless explicitly indicated otherwise. The study of PS-modules was initiated by Gordon in [1] and Nicholson and Watters in [2]. M_R is called a PS-module if every simple submodule is projective. A ring R is called a left (right) PS-ring if $_RR$ is a left (right) PS-module. It is shown that the class of PS-modules is closed under direct sums, see([2], Theorem 3.3). The notion of PS-rings is not left-right symmetric as in [2]. Many authors investigated the behavior of PS-rings with respect to some extensions. Salem et. al., in [3], characterized PS-modules over Ore extensions and skew generalized power series extensions. Also, Farahat and Al-Harthy, in [4], investigated PS-modules over generalized Mal'cev-Neumann series rings. In [5], Paykan proved that, under given conditions, if R is a right PS-ring, then so the skew inverse power series rings. Farahat and Al-Bogamy, in [6] extend the notation of PS-rings to weak PS-rings. Unlike the class of PS-rings Farahat and Al-Bogamy in [[6], Example 13] shows that the class of weak PS-modules is not closed under direct sums. Farahat and Al-Bogamy study the relation between the ring R and ore extensions, and also for Skew Hurwitz Series [6]. Inspired by the above results, we will prove that for a given ring R, an α-compatible endomorphism α and an α-derivation δ if R is a right weak PS-ring, then the skew generalizd power series ring $R[[S,\omega]]$ is a right weak PS-ring, see section [2]. Also in section [3] we study the skew inverse power series ring $R[[x^{-1};\alpha,\delta]]$. Furthermore, we study the property of weak PS-ring for differential inverse power series ring $R[[x^{-1};\delta]]$. For a nonempty subset X of R, $r_R(X)$ ($l_R(X)$), $Id(R)$, and $nil(R)$ denotes for the right (left) annihilator of X over R, the set of all idempotent elements, and the set of nilpotent elements of R. A ring R is called an NI-ring if $nil(R)$ is a two sided ideal in R. As a generalization of the annihilator concept, Ouyang, in [7], introduced the weak annihilator of a subset X of R,

$$N_R(X) = \{a \in R : xa \in nil(R) \text{ for all } x \in X\}.$$

It can be easily shown that $ab \in nil(R)$ if and only if $ba \in nil(R)$ for all $a, b \in R$. Therefore there is no way to distinguish between the right and the left weak annihilators. Obviously, $r_R(X) \subseteq N_R(X)$ and $l_R(X) \subseteq N_R(X)$. If R is reduced, then $r_R(X) = N_R(X) = l_R(X)$ for any subset X of R. It is easy to see that for any subset $X \subseteq R$, $N_R(X)$ is an ideal of R in NI ring.

Definition 1.[1] *A ring R satisfies the right PS-ring if for every maximal right ideal L of R; either $l_R(L) = 0$ or $L = Re$ (the principal left ideal generated by e), where $e \in Id(R)$.*

* Corresponding author e-mail: sarahgad2612.el@azhar.edu.eg

Theorem 1.*[2]The following are equivalent for a ring R and $e, f \in Id(R)$:*

(1) *R is a PS-ring.*

(2) *R has a faithful PS-module.*

(3) *If L is a maximal left ideal of R then either $r_R(L) = 0$ or $L = Re$.*

(4) *If L is a maximal left ideal of R then $r_R(L) = fR$.*

Now, we recall the definition of weak *PS*-ring as follows in [6]:

Definition 2.*[6] A ring R satisfies the right weak PS-condition if for every maximal right ideal L of R; either $N_R(L) \subset nil(R)$ or $N_R(L) = Re$; (principal left ideal generated by e), where $e \in Id(R)$. Similarly, we can define the left weak PS-condition. A ring R satisfies the weak PS-condition if it satisfies both right and left weak PS-conditions.*

In the definition of weak *PS*-ring, it should be noted that (or) to mean that one of the two options only holds but not both unless $N_R(L) = (0)$, since $nil(R) \cap Id(R) = (0)$. It is clear that in reduced rings weak *PS*-condition and *PS*-condition are equivalent.

The following example shows that the class of weak *PS*-ring is proper class:

Example 1.[6]If $R = Z_4$, then R is a weak *PS*-ring but not a *PS*-ring.

In this paper, we study the transfer of weak *PS*-condition between a base ring *R* and some ring extensions. Namely, the ring of skew generalized power series ring $R[[S,\omega]]$, the skew inverse power series ring $R[[x^{-1};\alpha,\delta]]$, and the diffrential power series ring $R[[x^{-1};\delta]]$.

2 Skew Generalized Power Series Rings Over Weak *PS* -Rings

For a ring *R*, the monoid of endomorphisms of *R* (with composition of endomorphisms as the operation) is denoted by $End(R)$. Let $(S,\leq)$ be an ordered set. Then $(S,\leq)$ is called artinian if every strictly decreasing sequence of elements of *S* is finite and $(S,\leq)$ is called narrow if every subset of pairwise order-incomparable elements of *S* is finite. Thus, $(S,\leq)$ is artinian and narrow if and only if every nonempty subset of *S* has at least one but only a finite number of minimal elements. A monoid *S* (written multiplicatively) equipped with an order $\leq$ is called an ordered monoid if for any $s_1, s_2, t \in S, s_1 \leq s_2$ implies $s_1t \leq s_2t$ and $ts_1 \leq ts_2$. Moreover, if $s_1 < s_2$ implies $s_1t < s_2t$ and $ts_1 < ts_2$, then $(S,\leq)$ is said to be strictly ordered. Now we recall the construction of the skew generalized power series ring introduced in [8]. Let $(S,\leq)$ be a strictly orderd monoid, *R* a ring, $\omega : S \to End(R)$ a monoid homomorphism and let $\omega_s = \omega(s)$ denote the image of $s \in S$ under ω for any $s \in S$. Consider the set *A* of all maps $f : S \to R$ such that

$$supp(f) = \{s \in S : f(s) \neq 0\}$$

is artinian and narrow subset of *S*, with pointwise addition and product operation called convolution defined by

$$fg(s) = \Sigma_{(u,v)\in X_s(f,g)} f(u)w_u(g(v))$$

for each $f,\ g \in A$, where

$$X_s(f,g) = \{(u,v) \in S \times S : uv = s,\ f(u) \text{ and } g(v) \neq 0\}$$

is finite. Hence $A = R[[S,\omega]]$ becomes a ring called skew generalized power series with coefficients in *R* and exponents in *S*. For each $r \in R$ and $s \in S$, we associate elements $c_r, e_s \in R[[S,\omega]]$ defined by

$$c_r(x) = r \text{ if } x = 1 \text{and } c_r(x) = 0 \text{ if } x \in S\backslash\{1\}$$

with the identity map $e : S \to R$ defined by $e_s(x) = 1$ if $x = s$ and $e_s(x) = 0$ for each $x \in S\backslash\{s\}$. It is clear that $r \to c_r$ is a ring embedding of *R* into $R[[S,\omega]]$ and $s \to e_s$ is a monoid embedding of *S* into the multiplicative monoid of the ring $R[[S,\omega]]$ and $e_sc_r = c_{w_s(r)}e_s$. So, the construction of the skew generalized power series rings generalize many classical ring constructions such as polynomial (skew polynomial) rings, Laurent (skew Laurent) polynomial rings, formal power (skew power)series rings, group (skew group) rings, monoid (skew monoid) rings and generalized power series rings.

Definition 3.*[9]Given a ring R, an endomorphism $\alpha : R \to R$ and an α-derivation $\delta : R \to R$. A ring R is said to be α-compatible if $ab = 0 \Leftrightarrow a\alpha(b) = 0$, where $a, b \in R$. One says that R is δ-compatible if $ab = 0 \to a\delta(b) = 0$. If R is both α-compatible and δ-compatible, then R is (α,δ)-compatible.*

Lemma 1.[9] *Let R be a σ-compatible ring. Then*
$ab = 0 \Leftrightarrow a\sigma^i(b) = 0 \Leftrightarrow \sigma^i(a)b = 0$ *for all* $i \geq 0$ *and* $a, b \in R$.

Lemma 2.[10] *Let R be a ring with an endomorphism α. If R is α-compatible and $k_1, k_2, ..., k_n$ are non-negative integers, then:*

$$a_1a_2...a_n \in nil(R) \text{ if and only if}$$

$$\alpha^{k_1}(a_1)\alpha^{k_2}(a_2)...\alpha^{k_n}(a_n) \in nil(R)$$

Definition 4.[10] *Let R be a ring, $(S, \leq)$ a strictly ordered monoid, and $\omega : S \to End(R)$ a monoid homomorphism. We say that R is (S, ω)-nil-Armendariz if $f, g \in R[[S, \omega]]$ satisfy $fg \in nil(R)[[S, \omega]]$, then $f(s)\omega_s(g(t)) \in nil(R)$ for all $s, t \in S$, where $nil(R)$ is the set of nilpotent elements of R.*

Definition 5.[10] *Let R be a ring, $(S, \leq)$ a strictly ordered monoid and $\omega : S \to End(R)$ a monoid homomorphism. The ring R is said to be S-compatible if ω_s is compatible for every $s \in S$.*

Proposition 1.[10] *Let R be a ring, $(S, \leq)$ be a strictly ordered monoid and $\omega : S \to End(R)$ a monoid homomorphism. Assume that R is (S, ω)-nil-Armendariz and S-compatible. If $f_1, f_2, ..., f_n \in R[[S, \omega]]$ such that $f_1f_2...f_n \in nil(R)[[S, \omega]]$, then $f_1(u_1)f_2(u_2)...f_n(u_n) \in nil(R)$, for all $u_1, u_2, ..., u_n \in S$.*

Lemma 3.[11] *Let S be a strictly totally ordered monoid and $\omega : S \to End(R)$ a monoid homomorphism. Let R be an ω-compatible NI ring with $nil(R)$ nilpotent. Then we have*
(1) *$f \in nil(R[[S, \omega]])$ if and only if $f(s) \in nil(R)$ for any $s \in supp(f)$.*
(2) *$fg \in nil(R[[S, \omega]])$ if and only if $f(u)g(v) \in nil(R)$ for any $u \in supp(f)$ and $v \in supp(g)$.*

Theorem 2.*Let $(S, \leq)$ be a strictly totally ordered monoid which satisfied the condition $\forall s \geq 0 \in S$ and let R be an ω-compatible NI ring with $nil(R)$ nilpotent. If R is a right weak PS-ring, then $A = R[[S, \omega]]$ is a right weak PS-ring.*

*Proof.*Let L be a right maximal ideal of A, we will show that either
$N_A(L) \subset nil(A)$ or $N_A(L) = Aq$, where $q = q^2 \in A$, since $(S, \leq)$ is a strictly totally ordered monoid, $supp(f)$ is a nonempty well ordered subset of S, for every $0 \neq f \in A$. We denote $\pi(f)$ the smallest element of $supp(f)$. For any $s \in S$, set

$$I_s = \{f(s) : f \in L, \pi(f) = s\} \subset R$$

$I = \cup_{s \in S} I_s$. Let J be a right ideal of R generated by I.
If $J = R$, then $\exists\ s_1, s_2, ..., s_n \in S$, $f_1, f_2, ..., f_n \in L$ and $r_1, r_2, ..., r_n \in R$ such that

$$1 = f_1(s_1)r_1 + f_2(s_2)r_2 + ... + f_n(s_n)r_n$$

where $f_i(s_i) \in I_{s_i}$, $\pi(f_i) = s_i$ for every $1 \leq i \leq n$. We will show that
$N_A(L) \subset nil(A)$. Let $g \in N_A(L)$, then $\forall f_i \in L$, we have $fg \in nil(A)$, then from Lemma [3] $f(u)g(v) \in nil(R)$ for any $u \in supp(f)$ and $v \in supp(g)$. Therefore, $g(v) \in N_R(J) = N_R(R) = nil(R)$. From Lemma [3] $g \in nil(A)$. Therefore, $N_A(L) \subset nil(A)$.
If $J \neq R$. We will show that J is a maximal right ideal of R. Let $r \in R - J$. If $c_r \in L$, then

$$r = c_r(0) \in I_0 \subset I \subset J$$

and so $r \in J$, contradiction. Therefore $c_r \notin L$. Since L is a maximal right ideal of A, we have $A = L + c_rA$. It follows that $\exists\ f \in L$ and $g \in A$ such that $c_1 = f + c_rg$ such that $c_1(0) = f(0) + (c_rg)(0)$, then $1 = f(0) + rw_0g(0)$, since R is ω-compatible, $1 = f(0) + rg(0)$. If $f(0) = 0$, then $1 = rg(0) \in rR \Rightarrow R = J + rR$. If $f(0) \neq 0$, then $0 \in supp(f)$. Since $0 \leq s$ for every $s \in S$, $\pi(f) = 0$. Thus $f(0) \in I_0 \subset I \subset J$, which implies that $R = J + rR$. Hence J is a maximal right ideal of R. Since R is a right weak PS-ring it follows that either $N_R(J) \subset nil(R)$ or $N_R(J) = Re$. According to that we have the following two cases:-
case 1) Suppose that $N_R(J) \subset nil(R)$, we want to prove $N_A(L) \subset nil(A)$.
Let $g \in N_A(L)$ then $\exists\ f \in A$, $fg \in nil(A)$ then $f(u)g(v) \in nil(R)$ for any $u \in supp(f)$ and $v \in supp(g)$. Therefore, $g(v) \in N_R(L)$. Since R is a right weak PS-ring $N_R(L) \subset nil(R)$, $g(v) \in nil(R)$, from Lemma [3], hence $g \in nil(A)$. Therefore, $N_A(L) \subset nil(A)$.
case 2) Assume that $N_R(J) = Re$, we want to show that $N_A(L) = Aq$, where $q = q^2 \in Id(A)$, i.e
$N_A(L) \subset Aq$ and $Aq \subset N_A(L)$. First, let $g \in N_A(L)$ then $fg \in nil(A)$ for any $f \in L$. Hence $f(u)g(v) \in nil(R)$ for any $u \in supp(f)$ and any $v \in supp(g)$. Since for any $r \in J$, there exist $s_1, ..., s_n \in S$, $f_1, ..., f_n \in L$, and $r_1, ..., r_n \in R$, such that

$$r = f_1(s_1)r_1 + f_2(s_2)r_2 + ... + f_n(s_n)r_n$$

Then

$$g(u)r = g(u)f_1(s_1)r_1 + g(u)f_2(s_2)r_2 + ... + g(u)f_n(s_n)r_n$$

Since $nil(R)$ is nilpotent $g(u)r \in nil(R)$, this leads to $g(u) \in N_R(J) = Re$ for any $u \in supp(g)$, then $g(u) = re$. Also $g = c_r\omega_s(e)$ or $g = c_rq$, where q is an idempotent in A, therefore, $N_A(L) \subset Aq$

Second: suppose that $N_R(J) = Re$, we want to prove $Aq \subset N_A(L)$. Suppose that $f \in L$ then $fq \in L$, then $(fq)(t) = f(t)q$. Since

$$(f(t)q)q = (f(t)q) \in N_R(J) = Re$$

Therefore, $fq \in N_A(L)$, so $N_A(L) \subset Aq$.

3 Skew Inverse Power Series Rings Over Weak *PS* -Rings

Consider R is a ring with identity, α an automorphism of R and δ an α-derivation of the ring R with the properties

$$\delta(a+b) = \delta(a) + \delta(b) \text{ and}$$

$$\delta(ab) = \delta(a)b + \alpha(a)\delta(b) \ \ \forall\ a,\ b \in R$$

The skew inverse power series ring $R[[x^{-1};\alpha,\delta]]$ formed by the formal series $f(x) = \Sigma_{i=0}^{\infty} a_i x^{-i}$ where x is a variable and the coefficients a_i of the series $f(x)$ are elements of the ring R. In the ring $R[[x^{-1};\alpha,\delta]]$ addition is defined as usual and multipilcation is defined with respect to the relation

$$x^{-1}a = \Sigma_{i=1}^{n} \alpha^{-1}(-\delta\alpha^{-1})^{i-1}(a)x^{-i} \qquad\qquad \forall a \in R$$

Proposition 2.*[12] Let R be an NI ring with an automorphism α and an α-derivation δ. Assume that R is (α,δ) compatible. If $f(x) = \Sigma_{i=0}^{\infty} a_i x^{-i}$ and $g(x) = \Sigma_{j=0}^{\infty} b_j x^{-j} \in R[[x^{-1};\alpha,\delta]]$. Then*

$$f(x)g(x) \in nil(R[[x^{-1};\alpha,\delta]]) \text{ then } a_ib_j \in nil(R)$$

$$\text{for each } n \le i,\ m \le j.$$

Theorem 3.*[12]Let R be a ring with an automorphism α and an α-derivation δ. Assume that R is (α,δ) compatible and $nil(R)$ is a nilpotent ideal of R. Then,*

$$nil(R[[x^{-1};\alpha,\delta]] = nil(R)[[x^{-1};\alpha,\delta]]$$

Theorem 4.*Let R be an NI ring, $nil(R)$ nilpotent with an automorphism α and an α-derivation δ. If R is (α,δ) compatible right weak PS-ring, such that $\alpha(e) = e$ and $\delta(e) = 0$ for every $e \in Id(R)$. Then $R[[x^{-1};\alpha,\delta]]$ is right weak PS-ring.*

*Proof.*Let L be a maximal right ideal of $S = R[[x^{-1};\alpha,\delta]]$. We want to show that $N_S(L) \subseteq nil(S)$ or $N_S(L) = Sq$, where $q = q^2 \in S$. Let I be the set of all trailing coefficients of the elements of L. Let J be a right ideal of R generated by

$$J = \langle I \rangle_r = IR$$

If $J = R$.
Let $\varphi(x) = \Sigma_{j=0}^{\infty} b_j x^{-j} \in N_S(L)$. Then for every $f(x) = \Sigma_{i=0}^{\infty} a_i x^{-i} \in L$, we have that

$$f(x)\varphi(x) = (\Sigma_{i=0}^{\infty} a_i x^{-i})(\Sigma_{j=0}^{\infty} b_j x^{-j}) \in nil(S)$$

Since R is an (α,δ)compatible *NI* ring with $nil(R)$ nilpotent, from Proposition 2 $a_ib_j \in nil(R)$ for all integers $i,\ j \ge 0$. For every $a \in I$. This leads to
$Ib_j \in nil(R)$, hence $b_j \in N_R(J) = N_R(R) = nil(R)$. Therefore, we have that $\forall j \ge 0$ $\varphi(x) \in nil(S)$. Hence $N_S(L) \subseteq nil(S)$.
If $I \ne R$.

We want to show that J is a maximal right ideal of R. Let $c \in R - J$, then $c \notin L$. So by maximality of L, $S = L + cS$. It follows that $\exists\ f(x) = \Sigma_{i=0}^{\infty} a_i x^{-i} \in L$ and $g(x) = \Sigma_{j=0}^{\infty} b_j x^{-j} \in S$ such that

$$1 = f(x) + cg(x)$$

thus

$$1 = a_0 + cb_0$$

If $a_0 = 0$, then $1 \in cR$, and so J is a maximal right ideal of R. If $a_0 \neq 0$, then $a_0 \in I \in J$ and then $R = J + cR$. Hence J is a maximal right ideal of R. Since R is a right weak PS-ring $N_R(J) \subseteq nil(R)$ or $N_R(J) = Re$, we have two cases:

Case 1) If $N_R(J) \subseteq nil(R)$ we want to prove $N_S(L) \subseteq nil(S)$.

Let $f(x) \in N_S(L)$, then $Lf(x) \subset nil(S)$, for every $g(x) \in L$, we have that $g(x)f(x) \in nil(S)$. From Theorem [2] $b_j a_i \in nil(R)$ for every $i,\ j$. Then $\forall i$ we have $ba_i \in nil(R)$, $a_i \in N_R(J) \subseteq nil(R)$, $a_i \in nil(R)$. For every i, $\ f(x) \in nil(S)$.

Case 2) If $N_R(J) = Re$, we want to prove that $N_S(L) = Sq$. At first we show that $N_S(L) \subset Sq$, where $q = q^2 \in Id(S)$. Let

$$\varphi(x) = \Sigma_{i=0}^{\infty} b_i x^{-i} \in N_S(L), \varphi(x) \notin nil(A)$$

Then for every $f(x) = \Sigma_{i=0}^{\infty} a_j x^{-j} \in L$, we have that $f(x)\varphi(x) \in nil(S)$. Since R is (α, δ) compatible NI ring with $nil(R)$ nilpotent, we have that $a_j b_i \in nil(R)\ \forall i,\ j \geq 0$. Then for every $a \in I \to ab_i \in nil(R)$ for every $i \geq 0$. For any $m \in J, \exists$ $a_1, a_2, ..., a_n \in I$, $r_1, r_2, ..., r_n \in R$

$$m = a_1 r_1 + a_2 r_2 + ... + a_n r_n$$

$$b_i m = (b_i a_1) r_1 + (b_i a_2) r_2 + ... + (b_i a_n) r_n$$

Hence, $b_i m \in nil(R)\ \forall i \geq 0$.Then $b_i \in N_R(J) = Re$, hence, $b_i = re \to b_i = b_i e\ \forall i \geq 0$.

$$\varphi(x) = \Sigma_{i=0}^{\infty} b_i x^{-i} = \Sigma_{i=0}^{\infty} (b_i e) x^{-i} = \Sigma_{i=0}^{\infty} b_i x^{-i} e$$

since $\alpha(e) = e$ and $\delta(e) = 0$. Therefore,
$\varphi(x) \in Aq$ where $q = q^2 = e = e^2$. Finally we want to show $Sq \subset N_S(L)$, suppose $g(x) \in L$, then $g(x)q = \Sigma_{i=0}^{\infty} a_j x^{-j} q = \Sigma_{i=0}^{\infty} a_j q x^{-j} \in L$. Since $Re \subset N_R(J)$, we have that $a_j q \in N_R(J)$. Since $N_R(J)$ is an ideal of R. Then $g(x)q \in N_S(L)$.

Theorem 5. *Let R be an (α, δ)-compatible NI ring with $nil(R)$ nilpotent. If R is a left weak PS-ring, then $S = R[[x^{-1}; \alpha, \delta]]$ is a left weak PS-ring.*

Proof. The proof is similar to the previous proof of Theorem 4. The only thing we need to note here is that, If L is a maximal left ideal of $S = R[[x^{-1}; \alpha, \delta]]$, then, by analogue manner as above, we get in case (2) that $b_i \in N_R(J) = Re$; for all integers $0 \leq i \leq k$. Therefore there exist $t_i \in R$ such that $b_i = et_i$, for all integers $0 \leq i \leq k$. So

$$\varphi(x) = \Sigma_{i=0}^{\infty} b_i x^{-i} = \Sigma_{i=0}^{\infty} (et_i) x^{-i} = \Sigma_{i=0}^{\infty} eb_i x^{-i}$$

Therefore $N_S(L) = qS$, where $h \in Id(S)$ and the result is proved.

We study the differential inverse power series ring $A = R[[x^{-1}; \delta]]$, where R is a ring equipped with a derivation δ, formed by formal series $\varphi(x) = \Sigma_{i=0}^{\infty} b_i x^{-i}$, where x is a variable and the coefficients a_i of the series $\varphi(x)$ are elements of the ring R. In the ring $A = R[[x^{-1}; \delta]]$, addition is defined as usual and multiplication is defined with respect to the relation

$$x^{-1} a = \sum_{i=0}^{\infty} (-1)^i \delta^i(a) x^{-i-1} \qquad \text{for each } a \in R$$

Corollary 1. *Let R be a δ-compatible NI ring with $nil(R)$ nilpotent, such that $\delta(e) = 0$ for every $e^2 \in Id(R)$. If R is a right weak PS-ring, then $A = R[[x^{-1}; \delta]]$ is a right weak PS-ring.*

Corollary 2. *Let R be a δ-compatible NI ring with $nil(R)$ nilpotent. If R is a left weak PS-ring, then $A = R[[x^{-1}; \delta]]$ is a left weak PS-ring.*

References

[1] R. Gordon, Rings in which minimal left ideals are projective, Pac. J. Math. 31(1969), 679-692.
[2] W. K. Nicholson and J. F. Watters, Rings with projective socle, Proc. Amer. Math. Soc. 102 (1988), 443-450.
[3] R. Salem, M. Farahat, and H. Abd-ELmalk, PS-modules over Ore extensions and skew generalized power series rings, Int. J. Math. Sci., 2015 (2015), 1-6.
[4] M. Farahat and N. Al-Harthy, PS-modules of generalized Mal'cev-Neumann series rings, Hacet. J. Math. Stat., 46(2017), 829-834.
[5] K. Paykan, Skew inverse power series rings over a ring with projective socle, Czechoslovak Math. J., 67(2017), 389-398.
[6] M. Farahat and S. Al-Bogamy, PS-rings over skew Hurwitz series, Eur. J. Pure Appl. Math., 11(2018), 244-259.
[7] L. Ouyang, Ore extensions of weak zip rings, Glasg, Math. J., 51(2009), 525-537.
[8] R. Mazurek and M. Ziembowski, On von Neumann regular rings of skew generalized power series, Comm. Algebra, 36(2008),1855-1868.
[9] S. Annin, Associated primes over ore extension rings, J. Algebr. Appl., 3(2004),193-205.
[10] K. Paykan and A. Moussavi, Nilpotent elements and nil-Armendariz property of skew generalized power series rings, Asian-Eur. J. Math., 10(2017), 1750034-1750062.
[11] L. Ouyang and L. Jinwang, Nilpotent property of skew generalized power series rings, Adv. in Math. (China), 42(2013), 782-794.
[12] K. Paykan and A. Moussavi, Study of skew inverse Laurent series rings, J. Algebra Appl. 16(2017), 175021-175053.