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The Triad Design of Order 19

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ABSTRACT: In this paper we present a new method to construct the triad design of order 19,*TD*(19). First we construct the compatible factorization of order 19, *CF*(19).Then we build the starter of triad design of order 19, *STD*(19) using interval techniques of the number of triples in the design. Finally by using addition modular 19 to *STD*(19) we develop *TD*(19).

Keywords: Compatible factorization, Triad design, Starter.

1 INTRODUCTION

A *triad design* on v objects, denoted by $TD(v)$, is a way of arranging the distinct triples of $\binom{v}{3}$into v rows such that:

(i) Row$m$contains $\frac{v-1}{2}$ triples, among which object m meets every other object precisely once, and contains also other distinct triples;
(ii) Each triple occurs exactly once in the design;
(iii) No two elements (entries) occurs together in two or more triples in any row.

$TD(v)$ is a set of distinct triples (3-element) of a v-set of points that deals with counting and listing triples from a graph of v points. It is associated with counting triangles in a graph which is called triangle problemsthat gained recently much practical importance since they are central in so-called complex network analysis [6, 7, 8].

The construction of $TD(v)$ is based on the compatible factorization of a graph of order v, denoted by*CF(v)*, which is a$v \times \frac{v-1}{2}$ array that satisfies the following conditions: [5, 9]

(i) The entries in row m form a near-one-factor with focus m.
(ii) The triples associated with the rows contain no repetitions.

Example 1.1 If $v = 7$, that is a set of 7 points labeled 1, 2, 3, 4, 5, 6, 7, then the compatible factorization*CF(7)* is illustrated in Table 1. Append $C_1$with C_2,$C_1$with C_3 and $C_1$with C_4, we obtain 21 distinct triples in*CF(7)*.Moreover, $TD(7) = CF(7) \cup \overline{CF(7)}$, where $\overline{CF(7)}$ is the completion of *CF(7)*. Clearly, *TD(7)*consists of $\binom{7}{3} = 35$ distinct triples. These triples are associated with the triangles that can be formed from K_7, the complete graph of order 7, see figure 1.

CF(7)				$\overline{CF(7)}$	
C_1	C_2	C_3	C_4		
1	2 7	3 6	4 5	2 3 5	7 6 4
2	3 1	4 7	5 6	3 4 6	1 7 5
3	4 2	5 1	6 7	4 5 7	2 1 6
4	5 3	6 2	7 1	5 6 1	3 2 7
5	6 4	7 3	1 2	6 7 2	4 3 1
6	7 5	1 4	2 3	7 1 3	5 4 2

7	1 6	2 3	3 4	1 2 4	6 5 3

Table 1. $TD(7)$

Table 1 shows that addition modular 7 to the first row, called starter, produces all the distinct triples. Once we construct the starter, listing all distinct triples can be done by addition modular *v*. Our aim in this paper is to present a new method to construct the starter of triad design of order 19, *STD*(19). This method depends on analysing the triples using interval techniques of the triples in the starter.

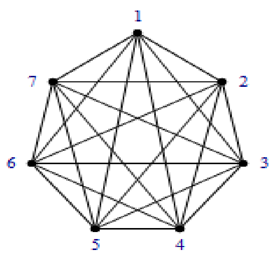

Figure 1. Plot of the complete graph K_7

It is known that *TD(v)* exists if $v \equiv 1 \text{or} 5 (\text{mod} 6)$[2, 3]. The construction of *STD(7)* above is done by the brute force method (trial and error). Moreover, in [1],*STD*(13*)* was constructed by using the brute force method and a new method that depends on the interval techniques of the triples in the starter.

In section 2, we study some properties of thestarter of triad design on v objects, *STD(v*). In section 3, we again use the brute force method to construct *STD*(19*)* and then develop *TD*(19). In section 4, we extend the use of the interval techniques of the triples in the starter to build *STD*(19).

2 PROPERTIES OF THE STARTER

Definition 2.1. The*starter* of triad design on v objects, *STD(v),* is the set of triples on $v$that generates all the triples in *TD(v)* by addition modular v.

The following lemma provides the number of triples in *STD(v)* as well as in $\overline{SCF(v)}$, denoted by |*STD(v)*| and |$\overline{SCF(v)}$|respectively.

Lemma 2.1 If*v* =6*n*+1,then

|*STD*(*v*)| = *n* (6*n*-1*)* and|$\overline{SCF(v)}$| =2*n* (3*n* - 2*).*

Proof: We prove the first and the proof of the second is the same. The number of triples of *TD(v)* is equal to $|TD(v)| = \binom{v}{3} = \binom{6n+1}{3} = n(6n+1)(6n-1)$. By the definition of $CF(v)$, the number of rows of*TD*(*v*) is equal to$v$ = 6*n* + 1. Hence |*STD*(*v*)| = *n* (6*n*-1).

□

Each triple in *STD(v)* consists of three elements (numbers, objects). The first, second and the third element.

Definition 2.2 The *r-th* elements of $STD(v)$are the *r-th* numbers in each triple, denoted by $S_rTD(v)$, for 1≤ *r* ≤ 3.

$SCF(7)$			$\overline{SCF(7)}$	
1 2 7	1 3 6	1 4 5	2 3 5	7 6 4
$\pm 1, \pm 2, \pm 1$	$\pm 2, \pm 3, \pm 2$	$\pm 3, \pm 1, \pm 3$	$\pm 1, \pm 2, \pm 3$	$\pm 1, \pm 2, \pm 3$

Example 2.1 From Example 2.1,

STD(7) = {1 2 7; 1 3 6; 1 4 5; 2 3 5; 7 6 4}. Therefore

S_1TD *(7)*: 1, 1, 1, 2, 7. S_2TD *(7)*: 2, 3, 4, 3, 6. S_3TD *(7)*: 7, 6, 5, 5, 4.

Table 2. Difference sets for $STD(7) = SCF(7) \cup \overline{SCF(7)}$

Note that the difference sets for each triple in *SDT(7)* are listed in table 2 using the facts that ± 6 = ±1, ± 5 = ± 2 and ± 4 = ± 3.

From Table 2, each difference occurs 3 times in $SCF(7)$ and 2 times in$\overline{SCF(7)}$, [4].

In the following section, the triples in *STD*(19) are analyzed in order to construct formulas to generate them and to develop *TD*(19).

3 ALGORITHM FOR*TD*(19)

In this section, addition modular 19 is used to produce the algorithm for*STD*(19). Obviously, *STD*(19)= *SCF*(19) ∪ $\overline{SCF(19)}$. By Lemma 2.1, with *n* = 3, | *STD*(19)| = 51 and$|\overline{SCF(19)}| = 42$. The algorithm for *TD*(19) is as follows:

Step 1. Generate$SCF(19)$ as shown below.
1 2 19 3 18 4 17 5 16
6 15 7 14 8 13 9 12
10 11.Therefore,
$SCF(19) = \{1\ 2\ 19,\ 1\ 3\ 18,\ 1\ 4\ 17,\ 1\ 516,\ 1\ 6\ 15,\ 1\ 7\ 14,\ 1\ \ 8\ \ 13,\ 1\ \ 9\ \ 12,\ 1\ 10\ \ 11\}$.

Note that the difference sets for the triples in$SCF(19)$ are listed in table 4 using the facts that$\pm 18 = \pm 1,\ \pm 17 = \pm 2,\ \pm 16 = \pm 3,\ \pm 15 = \pm 4,\ \pm 14 = \pm 5,\ \pm 13 = \pm 6,\ \pm 12 = \pm 7,\ \pm 11 = \pm 8$ and $\pm 10 = \pm 9$.

1	2	19	± 1	± 2	± 1
	3	18	± 2	± 4	± 2
	4	17	± 3	± 6	± 3
	5	16	± 4	± 8	± 4
	6	15	± 5	± 9	± 5
	7	14	± 6	± 7	± 6
	8	13	± 7	± 5	± 7
	9	12	± 8	± 3	± 8
	10	11	± 9	± 1	± 9

Table 3. Constructing $SCF(19)$

Step 2. Generate$\overline{SCF(19)}$ by using difference set method and addition modular 13.
Note that from Table 3 and 4, each difference occurs 3 times in $SCF(19)$ and 14 times in$\overline{SCF(19)}$.

Table 4. Constructing$SCF(19)$

$\overline{SCF(19)}$Differences						$\overline{SCF(19)}$Differences					
2	3	17	±1	±5	±4	19	18	4	±5	±1	±4
2	4	16	±2	±7	±5	19	17	5	±7	±2	±5
2	5	15	±3	±9	±6	19	16	6	±9	±3	±6
2	6	14	±4	±8	±7	19	15	7	±8	±4	±7
2	7	13	±5	±6	±8	19	14	8	±6	±5	±8
2	8	12	±6	±4	±9	19	13	9	±4	±6	±9
2	9	11	±7	±2	±9	19	12	10	±2	±7	±9
3	4	15	±1	±8	±7	18	17	6	±8	±1	±7
3	5	14	±2	±9	±8	18	16	7	±9	±2	±8
3	6	13	±3	±7	±9	18	15	8	±7	±3	±9
3	7	12	±4	±5	±9	18	14	9	±5	±4	±9
3	8	11	±5	±3	±8	18	13	10	±3	±5	±8
4	5	13	±1	±8	±9	17	16	8	±8	±1	±9
4	6	12	±2	±6	±8	17	15	9	±6	±2	±8
4	7	11	±3	±4	±7	17	14	10	±4	±3	±7
5	6	12	±1	±6	±7	16	15	9	±6	±1	±7
5	7	11	±2	±4	±6	16	14	10	±4	±2	±6
6	7	12	±1	±5	±6	15	14	9	±5	±1	±6
6	8	11	±2	±3	±5	15	13	10	±3	±2	±5
7	8	11	±1	±3	±4	14	13	10	±3	±1	±4
8	9	11	±1	±2	±3	13	12	10	±2	±1	±3

Therefore,$\overline{SCF(19)} = \{2\ \ 3\ \ 17;\ 19\ 18\ \ 4;2416;\ 19\ 17\ 5;\ \ldots;\ 8\ 9\ 11,\ \ 13\ \ 12\ 10\}$.

Step 3: $STD(19) = SCF(19) \cup \overline{SCF(19)}$
$=\{1\ 2\ 19,\ 13\ 18,\ 1\ 4\ 17,\ 15\ 16,\ 16\ 15,\ 1\ 7\ 14,\ 1\ \ 8\ 13,\ 1\ 9\ 12,\ 110\ \ 11\} \cup \{2\ \ 3\ \ 17,\ 19\ \ 18\ \ 4,\ 2\ 4\ 16,\ 19\ 17\ 5,\ldots,\ 8\ 9\ \ 11,$
$13\ 12\ \ 10\}$.

Step 4: Using the starter and addition modular 19 to enumerate $TD(19)$ as shown in table 5.

1	2	19	1	3	18	…	8	9	11	13	12	10
2	3	1	2	4	19	…	9	10	12	14	13	11
3	4	2	3	5	1	…	10	11	13	15	14	12
4	5	3	4	6	2	…	11	12	14	16	15	13
5	6	4	5	7	3	…	12	13	15	17	16	14
6	7	5	6	8	4	…	13	1	16	18	17	15
7	8	6	7	9	5	…	14	2	17	19	18	16
8	9	7	8	10	6	…	15	3	18	1	19	17
9	10	8	9	11	7	…	16	4	19	2	1	18
10	11	9	10	12	8	…	17	5	1	3	2	19
11	12	10	11	13	9	…	18	6	2	4	3	1
12	13	11	12	14	10	…	1	7	3	5	4	2
13	14	12	13	15	11	…	2	8	4	6	5	3
14	15	13	14	16	12	…	3	9	5	7	6	4
15	16	14	15	17	13	…	4	10	6	8	7	5
16	17	15	16	18	14	…	5	11	7	9	8	6
17	18	16	17	19	15	…	6	12	8	10	9	7
18	19	17	18	1	16	…	7	13	9	11	10	8
19	1	18	19	2	17	…	8	14	10	12	11	9

Table 5.$TD(19)$..

Our aim is to construct a new method for developing $TD(19)$. This method depends on building the starter of $TD(19)$ using interval techniques of the number of triples in the design.

4INTERVALS CONSTRUCTIONS $TD(19)$

In this section, we use analyse and divide$STD(19)$into intervals to construct formulas for$S_rTD(19)$,where$1 \le r \le 3$. From Step 3 of the previous section, weare able to summarize$STD(19)$ in terms of $S_rTD(19)$and the triple number k as shown in Table 6.

k	**1**	2	…	**9**	10	11	…	22	**23**	24	25	…	32	**33**	34	35
$S_1TD(19)$	1	1	…	1	2	19	…	2	19	3	18	…	3	18	4	17
$S_2TD(19)$	2	3	…	10	3	18	…	9	12	4	17	…	8	13	5	16
$S_3TD(19)$	19	18	…	11	17	4	…	11	10	15	6	…	11	10	13	8

k	…	38	**39**	40	41	42	**43**	44	45	46	**47**	48	49	50	**51**
$S_1TD(19)$	…	4	17	5	16	5	16	6	15	6	15	7	14	8	13
$S_2TD(19)$	…	7	14	6	15	7	14	7	14	8	13	8	13	9	12
$S_3TD(19)$		11	10	12	9	11	10	12	9	11	10	11	10	11	10

Table 6.*$S_rTD(19)$,* where$1 \leq r \leq 3$

By Lemma 2.1, $1 \leq k \leq 51$.Let $[S_rTD(19)]$ be the k-th element in*$S_rTD(19)$,* for $1 \leq r \leq 3$.. From Table 6, $k$can be divided into 7 intervals (periods). These intervals and the corresponding elements in *$S_1TD(19)$* are illustrated in Table 7.

No. of Intervals	Intervals of k	Corresponding elements in *$S_1TD(19)$*
1	$1 \leq k \leq 9$	1, 1, 1, 1, 1, 1, 1, 1, 1
2	$10 \leq k \leq 23$	2, 19, 2, 19, 2, 19, 2, 19, 2, 19, 2, 19, 2, 19
3	$24 \leq k \leq 33$	3, 18, 3, 18, 3, 18, 3, 18, 3, 18
4	$34 \leq k \leq 39$	4, 17, 4, 17, 4, 17
5	$40 \leq k \leq 43$	5, 16, 5, 16
6	$44 \leq k \leq 47$	6, 15, 6, 15
7	$48 \leq k \leq 51$	7, 14, 8, 13

Table 7. Intervals of k and the corresponding elements in*$S_1TD(19)$*

It could be observed from Table 7 that the corresponding elements in *$S_1TD(19)$*in the last interval of k need a special formula to produce them. Let f denotes the first number of the interval. The formula

$$y = \frac{1}{2}\left[k - f + \operatorname{mod}\left(\frac{k+3}{2}\right) + 1\right] \text{---------} \ (1)$$

can be used to produce the corresponding elements in $S_1TD(19)$ in the indicated interval. From Tables 7, the construction of $S_1TD(19)$is as follows.

$$[S_1TD(19)]_k = \begin{cases} 1 & & 1 \leq k \leq 9 \\ 2 & & 10 \leq k \leq 23, \ k \text{ iseven} \\ 19 & & 10 \leq k \leq 23, \ k \text{ isodd} \\ 3 & & 24 \leq k \leq 33, \ k \text{ iseven} \\ 18 & & 24 \leq k \leq 33, \ k \text{ isodd} \\ 4 & & 34 \leq k \leq 39, \ k \text{ iseven} \\ 17 & if & 34 \leq k \leq 39, \ k \text{ isodd} \\ 5 & & 40 \leq k \leq 43, \ k \text{ iseven} \\ 16 & & 40 \leq k \leq 43, \ k \text{ isodd} \\ 6 & & 44 \leq k \leq 47, \ k \text{ iseven} \\ 15 & & 44 \leq k \leq 47, \ k \text{ isodd} \\ 6+y & & 48 \leq k \leq 51, \ k \text{ iseven} \\ 15-y & & 48 \leq k \leq 51, \ k \text{ isodd} \end{cases}$$

Similarly, intervals of $k$and the corresponding elements in$S_2TD(13)$are shown in Table 8.

No. of Intervals	Intervals of k	Corresponding elements in*$S_2TD(13)$*
1	$1 \leq k \leq 9$	2, 3, 4, 5, 6, 7, 8, 9, 10
2	$10 \leq k \leq 23$	3, 18, 4, 17, 5, 16, 6, 15, 7, 14, 8, 13, 9, 12
3	$24 \leq k \leq 33$	4, 17, 5, 16, 6, 15, 7, 14, 8, 13
4	$34 \leq k \leq 39$	5, 16, 6, 15, 7, 14
5	$40 \leq k \leq 43$	6, 15, 7, 14
6	$44 \leq k \leq 47$	7, 14, 8, 13
7	$48 \leq k \leq 51$	8, 13, 9, 12

Table 8. Intervals of$k$ and the corresponding elements in$S_2TD(19)$

Using formula (1) above to produce the corresponding elements in$S_2TD(19)$in all intervals of k except the first one. Therefore, the construction of $S_2TD(19)$is the following.

$$[S_2TD(19)]_k = \begin{cases} 1+k & & 1 \leq k \leq 9 \\ 2+y & & 10 \leq k \leq 23, \ k \text{ iseven} \\ 19-y & & 10 \leq k \leq 23, \ k \text{ isodd} \\ 3+y & & 24 \leq k \leq 33, \ k \text{ iseven} \\ 18-y & & 24 \leq k \leq 33, \ k \text{ isodd} \\ 4+y & & 34 \leq k \leq 39, \ k \text{ iseven} \\ 17-y & if & 34 \leq k \leq 39, \ k \text{ isodd} \\ 5+y & & 40 \leq k \leq 43, \ k \text{ iseven} \\ 16-y & & 40 \leq k \leq 43, \ k \text{ isodd} \\ 6+y & & 44 \leq k \leq 47, \ k \text{ iseven} \\ 15-y & & 44 \leq k \leq 47, \ k \text{ isodd} \\ 7+y & & 48 \leq k \leq 51, \ k \text{ iseven} \\ 14-y & & 48 \leq k \leq 51, \ k \text{ isodd} \end{cases}$$

Finally, similar to the above discussion,intervals of k and the corresponding elements in$S_3TD(19)$are shown in tables 9.

No. of Intervals	Intervals of k	Corresponding elements in$S_3TD(19)$
1	$1 \leq k \leq 9$	19, 18, 17, 16, 15, 14, 13, 12, 11
2	$10 \leq k \leq 23$	17, 4, 16, 5, 15, 6, 14, 7, 13, 8, 12, 9, 11, 10
3	$24 \leq k \leq 33$	15, 6, 14, 7, 13, 8, 12, 9, 11, 10
4	$34 \leq k \leq 39$	13, 8, 12, 9, 11, 10

5	$40 \le k \le 43$	12, 9, 11, 10
6	$44 \le k \le 47$	12, 9, 11, 10
7	$48 \le k \le 51$	11, 10, 11, 10

Table 9. Intervals of k and the corresponding elements in $S_3TD(19)$

Using the same formula (1), the construction of $S_2TD(19)$ is the following.

$$[S_3TD(19)]_k = \begin{cases} 20-k & 1 \le k \le 9 \\ 18-y & 10 \le k \le 23,\ k \text{ is even} \\ 3+y & 10 \le k \le 23,\ k \text{ is odd} \\ 16-y & 24 \le k \le 33,\ k \text{ is even} \\ 5+y & 24 \le k \le 33,\ k \text{ is odd} \\ 14-y & 34 \le k \le 39,\ k \text{ is even} \\ 7+y \quad if & 34 \le k \le 39,\ k \text{ is odd} \\ 13-y & 40 \le k \le 43,\ k \text{ is even} \\ 8+y & 40 \le k \le 43,\ k \text{ is odd} \\ 13-y & 44 \le k \le 47,\ k \text{ is even} \\ 8+y & 44 \le k \le 47,\ k \text{ is odd} \\ 11 & 48 \le k \le 51,\ k \text{ is even} \\ 10 & 48 \le k \le 51,\ k \text{ is odd} \end{cases}$$

5 CONCLUSION

We have constructed a new method for developing the triad design oforder 19, $TD(19)$. This method analyses the pattern of triples using interval techniques of the number of triples in the design. One can use this method to construct the general cases of $TD(v)$, where $v = 6n + 1$ or $v = 6n + 5$.

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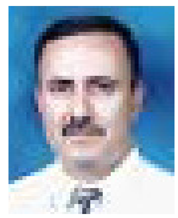

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